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ENDOGENOUS GROWTH AND HOUSEHOLD LEVERAGE

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We add households with heterogeneous discount factors and constrained credit to a research and development (R&D)-based endogenous growth model. Borrowers' access to credit has profound implications for growth. The direction and magnitude of this effect depend on preferences over labor supply. If labor supply is highly elastic and households do not smooth their labor supply between labor that produces output and R&D, annual growth decreases from 11.6% to approximately zero as the debt-to-capital ratio rises from 0 to 1.38. If households instead have a strong preference for smoothing their labor supply, then growth increases from 2.91% to 3.83% as the debt-to-capital ratio rises from 0 to 1.55. In both cases, less elastic labor supply weakens these effects. The results are similar if existing ideas do not affect the creation of new ideas. Now, when households do not smooth their labor supply, less debt results in faster growth, and productivity and output converge to much higher values.

Keywords: Credit Constraints, Heterogeneous Agents, Financial Markets, Endogenous Growth

1. INTRODUCTION

Over the past decade, a considerable literature has examined the impact of assuming that households can only borrow up to a fraction of their asset holdings. Much of this work has shown that credit constraints allow demand shocks to be amplified and propagated through a financial accelerator effect.[1] In this paper, we add credit constraints with heterogeneous households (differentiated by the discount factor) into a research and development (R&D)-based endogenous growth model similar to Jones (1995).[2] Households must choose how much labor to supply to the productive sector and how much labor to supply to R&D, the latter of which determines the economy's steady-state growth rate of total factor productivity (TFP), output, and consumption. We show that varying the maximum household leverage ratio often has enormous effects on the steady-state growth rate.

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The relationship between access to credit and growth depends primarily on two parameters.[3] The first determines the degree to which households wish to smooth their labor supply between productive and R&D labor. The second is the Frisch elasticity of labor supply. Because the appropriate values of both parameters are unclear, we report several combinations.[4] In one example, we assume both that households have little desire to smooth their labor supply between R&D and productive labor and that aggregate labor supply is highly elastic. Here, the creditors (patient households) are wealthier than the borrowers (impatient households). When households do not have access to credit, the steady-state growth rate is 11.6% per year. After increasing the amount impatient households are allowed to borrow to the point where their debt-to-capital ratio equals 1, growth falls to 3.03%. By the time the leverage ratio reaches 1.4, growth is almost zero. This result occurs because more debt increases the wealth of creditors. The creditors respond by substituting toward leisure and away from labor supply, including R&D labor, which causes a reduction in growth. When we allow labor supply to be less elastic, the effect is smaller, but still meaningful (growth falls from 3.45% to 3.00% as access as debt-to-capital rises from 0 to 1) for even very inelastic labor supply.

We also find cases where more debt increases growth. When we assume that households prefer to smooth their labor between production and R&D and that labor supply is relatively elastic, growth is now driven more by the R&D choices of borrowers. As the borrowers become more indebted, they supply more of all types of labor, including R&D. The steady-state growth rate rises from 2.91% without debt to 3.00% when the leverage ratio equals 1. Furthermore, the steady-state growth rate increases to 3.83% when the leverage ratio equals 1.55. Again, the magnitude of these effects diminishes as labor supply becomes less elastic.

In much of the credit constraint literature, the leverage ratio is treated as an exogenous constant representing the ability of lenders to recover collateral from borrowers in the case of default.[5] We argue that it is important to examine how a variable leverage ratio affects growth, because there is ample evidence that the leverage ratio is neither constant nor immune to policy. Figure 1 plots household debt as a percentage of physical capital for the United States between 1980 and 2011. Figure 1 illustrates that access to credit has both been trending upward and has been volatile around its trend. There is also evidence that global household debt to GDP has been increasing over the period 1960–2012 by over 3 percentage points per year, on average [see Mian et al. (2015) for details].

Because these data are aggregate, however, they do not directly compare to the leverage ratio in our model, which applies only to a specific type of household (impatient). Further decomposing the data by household net worth, Figure 2 shows the leverage ratio (total debt to assets) for households in the lowest two quartiles of net worth from 1989 to 2010. Figure 2 provides additional evidence that the leverage ratio is variable. In addition, Figure 2 shows that, for the poorest quartile of households, the leverage ratio often exceeds 1.[6] In our model, borrowers are almost always poorer than creditors, and we identify cases where allowing borrowers to

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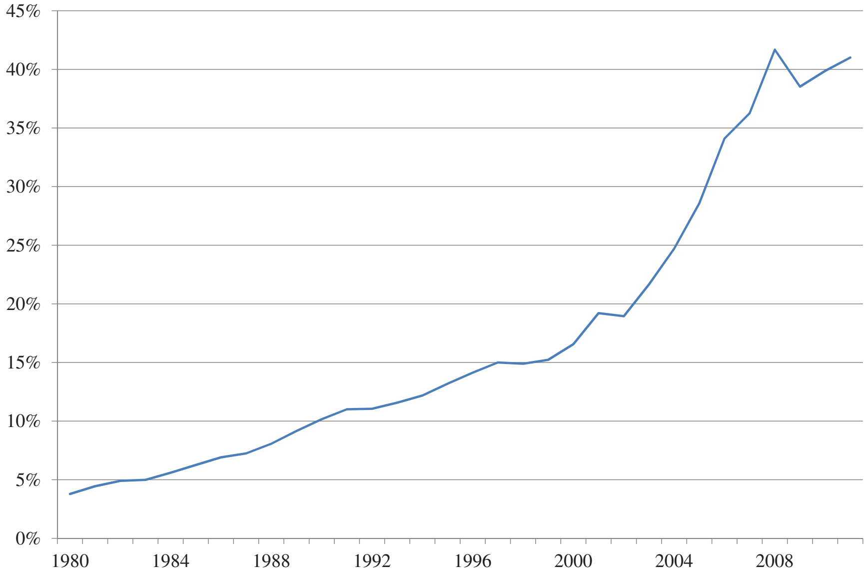

FIGURE 1. The US household debt to capital. Source: Federal reserve economic data, St. Louis Federal Reserve Bank.

go "underwater" (a debt-to-capital ratio greater than 1) has important implications for growth and welfare. When households have little desire to smooth their labor supply across productive and R&D labor, allowing underwater borrowing yields especially low growth rates, often near zero. When households have a strong desire to smooth labor across types, however, allowing underwater borrowing promotes growth.

There is also ample anecdotal evidence of policy makers attempting to influence leverage ratios. The existence of the mortgage giants Fannie Mae and Freddie Mac, government-sponsored enterprises in the United States, may fairly be viewed as a policy attempt to facilitate greater access to mortgage debt. Provisions in the Dodd–Frank Wall Street Reform and Consumer Protection Act of 2010 attempt to induce lower leverage among American firms, and the Basel Accords may be viewed as an international effort to minimize debt-to-capital ratios in the banking sector. The above examples show policy attempts to change access to credit; consequently, we consider it important to understand how changing the leverage ratio affects growth.

The endogenous growth literature splits on whether the creation of new ideas should depend on the existing stock of ideas. Most of our analysis, including the results previously discussed, follows Romer (1990) where old ideas make R&D more productive. We also, however, consider a version of the model closer to Jones (1995), where no such scale effects exist. Our main results are similar. When households do not prefer to smooth their labor supply between R&D and

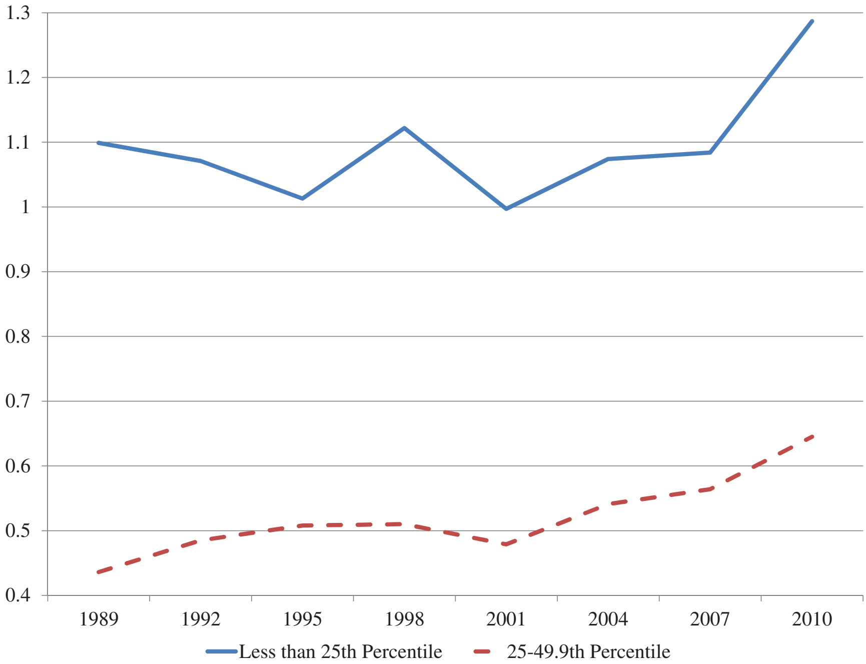

FIGURE 2. Leverage ratio by percentile of net worth. Source: Federal reserve board of governors survey of consumer finances Table 12.

productive labor, the growth rate is always higher for a given value of TFP when debt is lower. Furthermore, TFP converges to a much higher level for low debt. As before, when households have a strong preference for smoothing their labor supply, however, the results are reversed. Lower values of debt now result in lower growth rates and TFP converges to a lower level. Because we believe that credit access is not exogenous to policy, the result from Jones (1995) that policy cannot have very important long-run effects does not apply to our model. Changing the households' maximum debt level now has potentially large effects on the long-run values of TFP, output, and consumption.

1.1. Related Literature

Our paper contributes to a small literature that includes financial market imperfections in the endogenous growth framework. Several papers use liquidity constraints in an endogenous growth model wherein growth is determined by capital accumulation instead of technological advancement through R&D. Through this mechanism, Jappelli and Pagano (1994) find that credit constraints increase the savings rate of households, which leads to a higher growth rate. Amable et al. (2004) use a model in which net worth determines borrowing capacity and examine the impact of a change in the interest rate on growth. They conclude that when the interest rate increases, growth decreases due to the negative impact on retained earnings and collateral to borrow against. Bencivenga and Smith (1993) consider a model in which all investment activities are externally financed and find that an increase in capital production technology magnifies the negative effects of credit rationing and reduces growth. These papers, however, either do not examine the effects of changes in leverage on growth, or do not find that the leverage ratio has a significant impact on growth rates. In contrast, we find that the effects of altering the leverage ratio are often dramatic.

Comin and Gertler (2006) use an endogenous growth model with endogenous technological adoption to match features of the U.S. data that exhibits medium-term (decades) shifts between high and low trend growth. Guerron-Quintana and Jinnai (2015) use an endogenous growth model with financial frictions to argue that a financial shock reduced the economy's trend growth rate in the aftermath of the Great Recession. Ates and Saffie (2016) combine an endogenous growth model with a small open economy Real Business Cycle model to show that sudden stops to incoming capital can cause a large permanent income loss.

This paper is also related to a literature that examines the effects of credit on growth through a very different mechanism [see Acemoglu and Zilibotti (1997) and Bencivenga and Smith (1991)]. Here, more efficient financial markets allow capital to be better directed toward promising projects and may therefore be growth enhancing.

In addition, our paper contributes to the mostly empirical literature on growth and financial development. There is considerable debate over whether growth causes financial development or vice-versa. Levine (2005) provides an overview and argues that the bulk of the evidence suggests that greater financial development does promote growth. Although it is not obvious how to equate leverage and general financial development, our paper presents a novel channel through which changes in the financial sector affect growth.

This paper is organized as follows. Section 2 outlines the theoretical model. Section 3 shows the steady-state results and the impact of varying access to credit. Section 4 shows that our main conclusions hold even if we eliminate the scale effect where existing ideas facilitate the creation of new ideas. Section 5 concludes.

2. THE MODEL

Following the related literature on credit constraints, we assume that a set of patient households, with relatively high discount factors, lend to a set of impatient households. Also consistent with the previous literature, we impose a collateral constraint on the impatient households who borrow from the patient households. Our contribution is to include a R&D-based endogenous growth sector similar to Romer (1990). Both types of households are infinitely lived and exist on a continuum on the unit interval. Patient and impatient households produce, consume, and supply labor to the R&D and the production sector. Productivity, common to both lenders and borrowers, evolves according to a recursive structure:

$$A_{t+1} - A_t = \mu A_t (L_{a,t}^{\lambda} + L_{a,t}^{'\lambda}). \quad \textbf{(1)}$$

The growth rate is then defined using

$$g_{t+1} = \frac{A_{t+1} - A_t}{A_t} = \mu (L_{a,t}^{\lambda} + L_{a,t}^{'\lambda}), \quad \textbf{(2)}$$

where A_t represents the productivity in time t, and $L_{a,t}$ and $L_{a,t}^{'}$ are the labor supplied to the technology sector by the lenders and borrowers, respectively. The returns-to-scale parameter on R&D labor for both lenders and borrowers is represented by λ.[7] For each simulation, we select a value of μ that yields a growth rate of approximately 3% when the debt-to-capital ratio for households equals 1.

Patient households are savers and lend to impatient households. They maximize expected lifetime utility, which is a function of consumption (c_t), hours worked for R&D ($L_{a,t}$), and hours worked for production ($L_{y,t}$):

$$\text{Max}_{c_t, L_{a,t}, L_{y,t}, k_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\ln c_t}{U_t} - \frac{\chi (L_{a,t}^{\epsilon} + L_{y,t}^{\epsilon})^{\frac{\eta}{\epsilon}}}{\eta} \right],$$

where the expectation operator is E_0, the discount factor is β (calibrated at a value of 0.99), χ is the weight placed on the disutility associated with supplying labor, ϵ is a parameter that dictates the substitutability between labor supplied to R&D and production, and η is the inverse Frisch elasticity of labor supply. $\log(U_t)$ is a mean zero, white noise preference shock. Patient households are subject to a budget constraint:

$$c_t + k_{t+1} + b_t \leq R_{t-1} b_{t-1} + Y_t + (1 - \delta) k_t,$$

where c_t is the patient household consumption, k_t is the patient household capital stock, δ is the depreciation rate, and Y_t is the patient household output. The borrower and lender relationship between the two types of households is defined by their relative discount factors. The discount factor of the impatient households is less than that of patient households. As a result, the impatient households are the borrowers and patient households are the lenders in the model. The loan structure is such that in time t, a loan of amount b_t is made from the patient households to the impatient households. In the subsequent period, the impatient households pay off the debt at an interest rate of r_t, where $R_t = 1 + r_t$. Therefore, in time t, the value of the loan plus interest from time $t - 1$ ($R_{t-1} b_{t-1}$) is an additional source of income for the patient households.

In addition, patient households produce according to the following constant returns to scale (in labor and capital, given productivity) production function:

$$Y_t = (A_t Z_t L_{y,t})^{\alpha} k_t^{(1-\alpha)}.$$

There are two components of TFP: a permanent component (A_t) and a transitory component (Z_t) that is subject to random productivity shocks.

The first-order conditions for the patient households are

$$\frac{1+g_{t+1}}{\tilde{c}_t U_t} = E_t\left\{\frac{\beta}{\tilde{c}_{t+1}U_{t+1}}\left[1-\delta+Z_{t+1}^{\alpha}(1-\alpha)L_{y,t+1}^{\alpha}\tilde{k}_{t+1}^{-\alpha}\right]\right\} \quad (3)$$

$$\frac{1+g_{t+1}}{c_t\tilde{U}_t} = E_t\left[\frac{\beta R_t}{\tilde{c}_{t+1}U_{t+1}}\right] \quad (4)$$

$$\frac{\alpha Z_t^{\alpha}L_{y,t}^{\alpha-1}\tilde{k}_t^{1-\alpha}}{\tilde{c}_t U_t} = \chi(L_{a,t}^{\epsilon}+L_{y,t}^{\epsilon})^{\frac{\eta}{\epsilon}-1}L_{y,t}^{\epsilon-1} \quad (5)$$

$$\chi(L_{a,t}^{\epsilon}+L_{y,t}^{\epsilon})^{\frac{\eta}{\epsilon}-1}L_{a,t}^{\epsilon-1} = E_t\left[\frac{\beta\lambda\mu\alpha Z_{t+1}^{\alpha}L_{a,t}^{\lambda-1}L_{y,t+1}^{\alpha}\tilde{k}_{t+1}^{1-\alpha}}{\tilde{c}_{t+1}(1+g_{t+1})U_{t+1}}\right.$$
$$\left.+\beta\chi L_{a,t+1}^{\epsilon-1}(L_{a,t+1}^{\epsilon}+L_{y,t+1}^{\epsilon})^{\frac{\eta}{\epsilon}-1}\left(\frac{L_{a,t}}{L_{a,t+1}}\right)^{\lambda-1}\left(\frac{1+g_{t+2}}{1+g_{t+1}}\right)\right], \quad (6)$$

where equation (3) is the patient household demand for capital, equation (4) is the Euler equation, equation (5) is the patient household labor supply to the production sector, and equation (6) is the patient household labor supply to the R&D industry. Consumption, capital, and debt have been detrended such that $\tilde{c}_t = \frac{c_t}{A_t}$, $\tilde{k}_t = \frac{k_t}{A_t}$, and $\tilde{b}_t = \frac{b_t}{A_t}$.

To see the intuition behind equation (6), the supply of R&D labor, consider a case where $L_{a,t}$ changes by one unit such that $dL_{a,t} = 1$. Solving for $dL_{a,t+1}$ yields

$$dL_{a,t+1} = -\left(\frac{L_{a,t}}{L_{a,t+1}}\right)^{\lambda-1}\left(\frac{1+g_{t+2}}{1+g_{t+1}}\right). \quad (7)$$

In order for A_{t+2} to remain unchanged, if labor is increased by one unit in time t, labor in time $t+1$ changes by $dL_{a,t+1}$. Equation (6) equates the benefits and costs of changing $L_{a,t}$ and $L_{a,t+1}$ by these amounts.

Impatient households, denoted by the prime symbol, work, consume, produce, and borrow from the patient households. They maximize expected lifetime utility:

$$\text{Max}_{c_t',L_{a,t}',L_{y,t}',k_{t+1}'} E_0\sum_{t=0}^{\infty}\beta'^t\left[\frac{\ln c_t'}{U_t}-\frac{\chi(L_{a,t}'^{\epsilon}+L_{y,t}'^{\epsilon})^{\frac{\eta}{\epsilon}}}{\eta}\right],$$

subject to a budget constraint:

$$c_t' + k_{t+1}' + R_{t-1}b_{t-1} = b_t + Y_t' + (1-\delta)k_t',$$

and a credit constraint:

$$R_t b_t \le m_t k_{t+1}',$$

where m_t evolves according to some stationary, exogenous process. The detrended credit constraint is given by

$$\tilde{b}_t \leq \frac{(1+g_t)m_t\tilde{k}'_{t+1}}{R_t}. \qquad \textbf{(8)}$$

The collateral constraint on impatient households requires that the value of their debt plus interest cannot exceed the amount of recoverable future capital. In this setup, the threat of default matters, but borrowers are not allowed to actually be insolvent.[8] The variable m_t represents access to credit and may be interpreted as a leverage ratio. It is analogous to capital requirements for firms or loan-to-value (LTV) ratios for households. In the credit constraints literature, m is both treated as a constant and often interpreted to be 1 minus the cost of recovering collateral in the event of default. This latter interpretation constrains m to be less than 1. However, as discussed in Section 1, the data show that m is both volatile and, for subsets of households, possibly greater than 1. Thus, we simply interpret m as the exogenous level of access to credit and only constrain it to be nonnegative.

It is not obvious whether we should allow underwater borrowing. In all of our simulations, the steady-state utility of impatient households significantly declines when $m > 1$ and is greater when $m = 0$ than when households can go underwater. If we assume that lenders can only respond to default by first seizing borrowers' capital and, second, denying access to credit in the future, then impatient households would rationally choose to default even if it meant their inability to borrow in the future. In this case, patient households would constrain lending to $m \leq 1$. Under this interpretation, the reader should consider any result where welfare is maximized for $m > 1$ as instead being a corner solution wherein optimality occurs at $m = 1$.

It is reasonable, however, to assume that m can be greater than 1 if, in the event of default, the patient households can recover future wealth or income in order to satisfy their debt obligations.[9] We leave this interpretation to the reader and provide results for $m > 1$ with the caveats discussed above.

Impatient households also produce using their own labor and capital:

$$Y'_t = (A_t Z_t L'_{y,t})^{\alpha} k_t^{'(1-\alpha)}.$$

The first-order conditions for the impatient households are

$$\frac{(1+g_{t+1})}{\tilde{c}'_t} = E_t\left\{\frac{\beta'[1-\delta+Z^{\alpha}_{t+1}(1-\alpha)L^{'\alpha}_{y,t+1}\tilde{k}^{'-\alpha}_{t+1}]}{\tilde{c}'_{t+1}} + m\tilde{\gamma}_t(1+g_{t+1})\right\} \qquad \textbf{(9)}$$

$$\tilde{\gamma}_t R_t + E_t\left[\frac{\beta' R_t}{\tilde{c}'_{t+1}(1+g_{t+1})U_{t+1}}\right] = \frac{1}{\tilde{c}'_t U_t} \qquad \textbf{(10)}$$

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$$\frac{\alpha Z_t^{\alpha} L_{y,t}^{'(\alpha-1)} \tilde{k}_t^{'(1-\alpha)}}{\tilde{c}_t{}' U_t} = \chi (L_{a,t}^{'\epsilon} + L_{y,t}^{'\epsilon})^{\frac{\eta}{\epsilon}-1} L_{y,t}^{'(\epsilon-1)} \tag{11}$$

$$\chi (L_{a,t}^{'\epsilon} + L_{y,t}^{'\epsilon})^{\frac{\eta}{\epsilon}-1} L_{a,t}^{'(\epsilon-1)} = E_t \Bigg[\frac{\beta' \lambda \mu \alpha Z_{t+1}^{\alpha} L_{a,t}^{'(\lambda-1)} L_{y,t+1}^{'\alpha} \tilde{k}_{t+1}^{'(1-\alpha)}}{\tilde{c}_{t+1}^{'}(1+g_{t+1})U_{t+1}}$$
$$+ \beta' \chi L_{a,t+1}^{'(\epsilon-1)} (L_{a,t+1}^{'\epsilon} + L_{y,t+1}^{'\epsilon})^{\frac{\eta}{\epsilon}-1} \left(\frac{L_{a,t}^{'}}{L_{a,t+1}^{'}} \right)^{\lambda-1} \left(\frac{1+g_{t+2}}{1+g_{t+1}} \right) \Bigg], \tag{12}$$

where equation (9) is the impatient household demand for capital, equation (10) is the Euler equation, equation (11) is the impatient household labor supply to the production sector, and equation (12) is the impatient household labor supply to the R&D industry. The Lagrange multiplier on the credit constraint is detrended by $\tilde{\gamma}_t = \gamma_t A_t$.

Other equations in the system include

$$\tilde{Y}_t = L_t^{\alpha} \tilde{k}_t^{1-\alpha} \tag{13}$$

$$\tilde{Y}_t^{'} = L_t^{'\alpha} \tilde{k}_t^{'(1-\alpha)} \tag{14}$$

$$\tilde{c}_t + \tilde{k}_{t+1}(1+g_{t+1}) + \tilde{b}_t \leq \frac{R_{t-1}\tilde{b}_{t-1}}{(1+g_t)} + \tilde{Y}_t + (1-\delta)\tilde{k}_t \tag{15}$$

$$\tilde{c}_t^{'} + \tilde{k}_{t+1}^{'}(1+g_{t+1}) + \frac{R_{t-1}\tilde{b}_{t-1}}{(1+g_t)} \leq \tilde{b}_t + \tilde{Y}_t{}' + (1-\delta)\tilde{k}_t{}', \tag{16}$$

where equations (13) and (14) are the detrended production function for the patient and impatient households, respectively, and equations (15) and (16) are the detrended budget constraints for the patient and impatient households, respectively. Finally, we assume the transitory part of TFP (Z_t) and the shock to preferences (U_t) follow stationary, exogenous preferences.

3. THE BALANCED GROWTH PATH AND LEVERAGE

This section examines the model's steady state and shows that the effects of varying the leverage ratio on the growth rate may be in either direction, depending on the calibration, and are potentially large. Two parameters are critical: ϵ and η. When ϵ is close to 1, suggesting that households have little desire to smooth their labor supply, higher leverage reduces growth. High values of ϵ, which implies a desire to smooth labor supply, however, cause higher leverage ratios to increase growth. The magnitude of the effects is large when η is low (suggesting elastic labor supply) and smaller when it is high. Because ϵ is novel to our paper and the correct calibration of η is controversial, we consider several alternate calibrations. Table 1 reports the values for each simulation.[10]

TABLE 1. Calibrations for different simulations

	ϵ	η	μ	$g(m=0)$	$g(m=1)$	$m^*(U)$	$m^*(R)$
Simulation # 1	1.1	1.1	0.01222	11.60%	3.03%	1.38	0
Simulation # 2	10	1.1	0.0042	2.91%	3.00%	1.55	0.88
Simulation # 3	1.1	3	0.0148	3.45%	3.00%	1.50	0.86
Simulation # 4	10	3	0.0044	2.94%	3.00%	1.61	0.87

TABLE 2. Calibration

α	Labor's share in production function	0.67
β	Patient households' discount factor	0.99
β'	Impatient households' discount factor	0.95
χ	Weight on labor supply in utility function	1
λ	Returns to scale on R&D	1
δ	Capital depreciation rate	0.025

We also consider the optimal leverage ratio, calibrated using a social welfare function evaluated at the steady state. These results are sensitive to both our calibration and our choice of social welfare function, specifically, whether we use a Benthamian (hereafter, Utilitarian) social welfare function that sums the utility of both agents that generally tracks the utility of patient households or a Rawlsian function that generally tracks the utility of impatient households. We find cases where optimality occurs at $m = 0$, suggesting that allowing any debt is welfare reducing, and cases where the optimal leverage ratio is greater than 1, which implies that households should be allowed to go underwater on their debt. If we prohibit underwater borrowing, utilitarian social welfare is always a corner solution where $m = 1$.

As close as possible, we follow the related literature in calibrating the remainder of the model. We set $\alpha = 0.67$, a standard value for the Cobb–Douglas production function. We set $\beta = 0.99$, implying a real interest rate of 4% for quarterly data. We set $\beta' = 0.95$, and, as in Jones (1995), we set $\lambda = 1$.[11] We set $\delta = 0.025$, another standard value. For each simulation, we fix μ so that the steady-state growth rate for $m = 1$ is near 3%. We then examine the effects of changing m. Table 2 below summarizes the baseline calibration.

3.1. Simulation 3.1: $\epsilon = 1.1$, $\eta = 1.1$, $\mu = 0.01222$

In this simulation, ϵ is low, suggesting that households have little taste for smoothing their labor supply across types. In addition, labor supply is highly elastic. The key implication of having a low ϵ is that impatient households, having a lower

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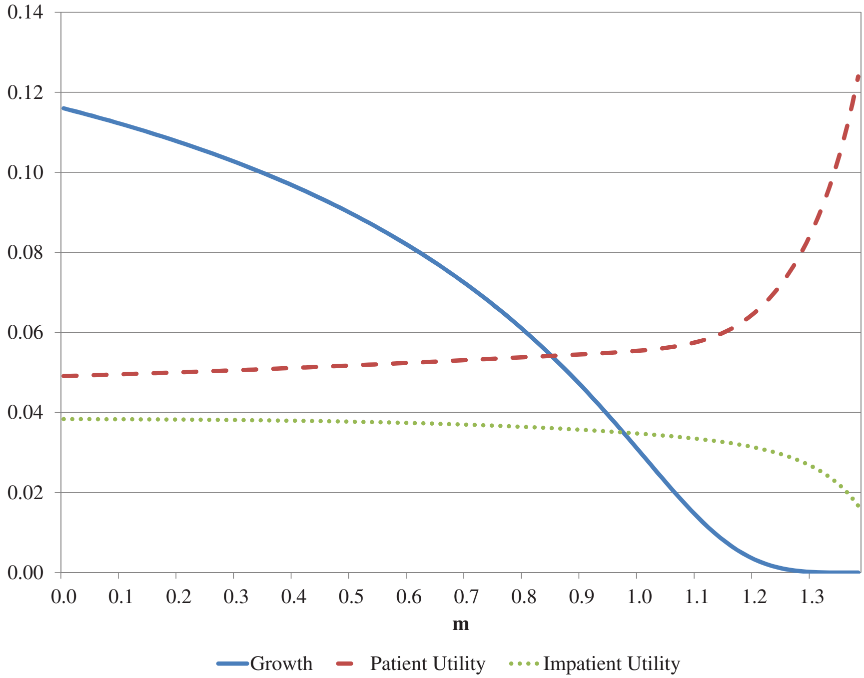

FIGURE 3. Effects of varying m on growth and utility.

discount factor, supply very little R&D labor. As a result, the R&D decisions of patient households almost entirely drive growth.

In this simulation, growth equals 11.60% when $m = 0$. Here, there is no interaction between the two types of households, except for an uncorrected positive externality, where each household's R&D benefits all others. As shown in Figure 3, the growth rate continues to fall as m increases, reaching 3.03% at $m = 1$, and it is near zero by $m = 1.38$.

The effect of a change in access to credit on growth in this simulation is dramatic. Growth becomes virtually nonexistent when impatient households are allowed to be highly leveraged. At the steady state, higher leverage transfers wealth from impatient households to patient households. Impatient households respond by increasing their productive labor. But, because they supply almost no R&D labor, this has virtually no impact on growth. Figure 4 summarizes the impact of a change in access to credit on impatient households.

As m increases, patient households become wealthier. They respond by substituting away from both types of labor toward leisure. Since the patient households drive growth, the reduction in their R&D labor causes the aggregate growth rate to collapse. For very high leverage ratios, patient households are content to live almost entirely on debt payments while supplying little labor, resulting in very low

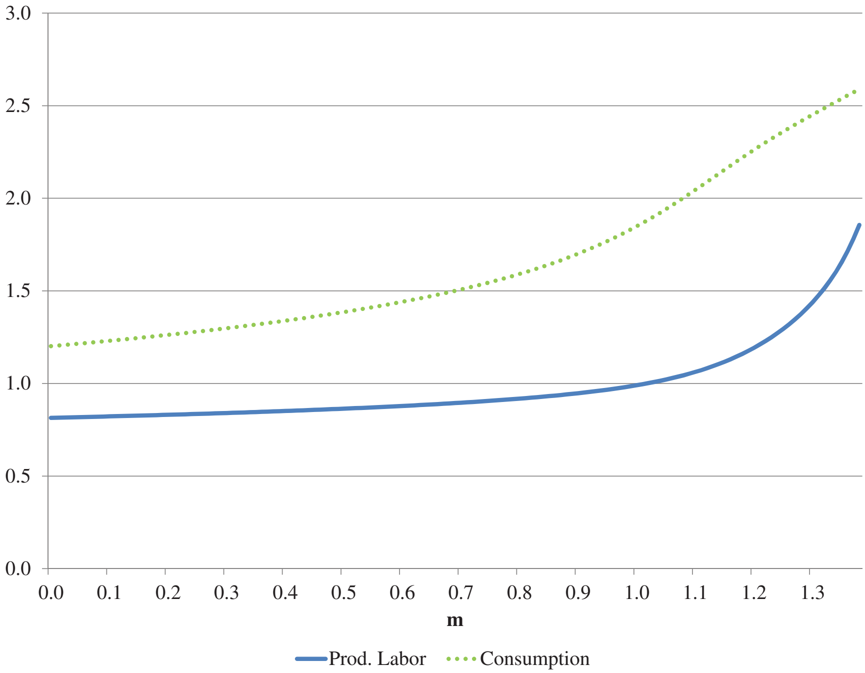

FIGURE 4. Effects of varying m on impatient households.

growth. Figure 5 shows the effects of a change in m for the patient households. The values for all variable are normalized so that they equal 100 when $m = 0$.

We now evaluate the welfare implications for different values of m using the steady-state levels of utility for each type of household. Iterating the utility functions forward at the steady state and taking infinite geometric series yields

$$U = \frac{1}{1-\beta}\left[\ln(c) - \chi\frac{(L_a^\epsilon + L_y^\epsilon)^{\frac{\eta}{\epsilon}}}{\eta} + \frac{\ln(g)}{1-\beta}\right] \quad (17)$$

$$U' = \frac{1}{1-\beta}\left\{\ln(c') - \chi\frac{\left[(L_a')^\epsilon + (L_y')^\epsilon\right]^{\frac{\eta}{\epsilon}}}{\eta} + \frac{\ln(g)}{1-\beta}\right\}. \quad (18)$$

These utility levels are included in Figure 3. Steady-state utility is maximized for impatient households at $m = 0$, because their wealth and growth are highest, and for patient households at $m = 1.38$ (the largest value of m in this calibration). A utilitarian social welfare function based on steady-state utilities is maximized at $m = 1.38$, while a Rawlsian is welfare function is maximized at $m = 0$, because it perfectly tracks impatient utility.

This simulation illustrates how higher leverage can cause a dramatic decline in growth. Increasing access to credit causes both lower growth and much higher

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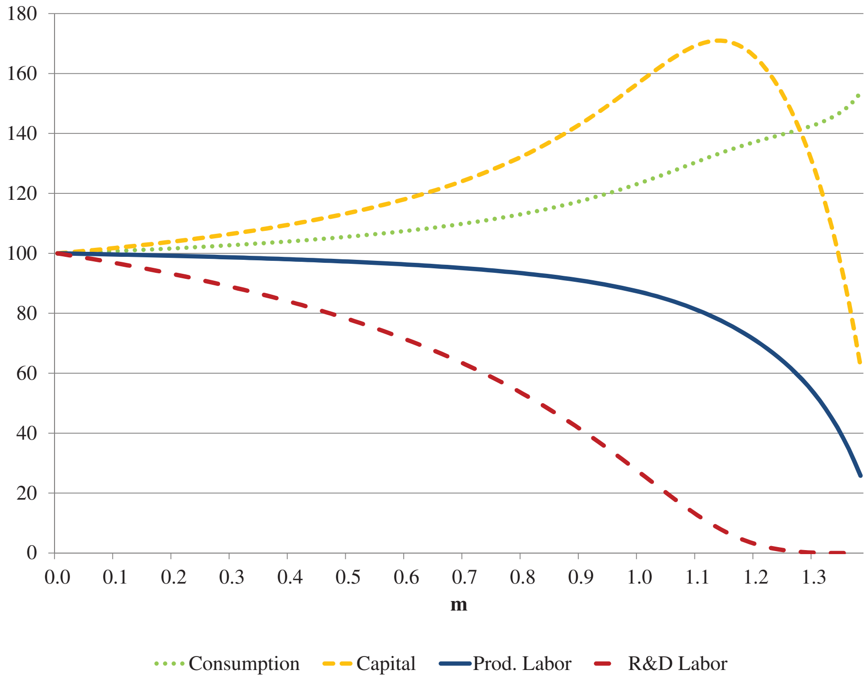

FIGURE 5. Effects of varying m on patient households.

levels of income inequality. At the steady state, the effects of higher leverage are thus particularly detrimental to the poorer (impatient) households. Determining whether an increase in m is actually detrimental to them, however, would require that we consider the transition between steady states where they exhibit higher consumption. This is left for future research.

3.2. Simulation 3.2: $\epsilon = 10$, $\eta = 1.1$, $\mu = 0.0042$

In this case, we continue to assume that labor supply is highly elastic. But now, we set $\epsilon = 10$ so that households try to smooth their labor supply between the two types. We also lower μ in order to keep growth rates close to 3% at $m = 1$. Figure 6 shows the effects of varying m on the growth rate and utility. When $m = 0$, the annualized growth rate is 2.91%. As m rises to 1, the growth rate exhibits a small but important increase to 3.00%. As m increases to 1.55, however, the increase in the growth rate accelerates, rising to 3.83%.

By assuming that households wish to smooth their labor across types, we now induce impatient households to supply significant levels of R&D labor. As m increases and wealth is transferred from impatient households to patient households, the former now respond by increasing their supply of both types of labor, while

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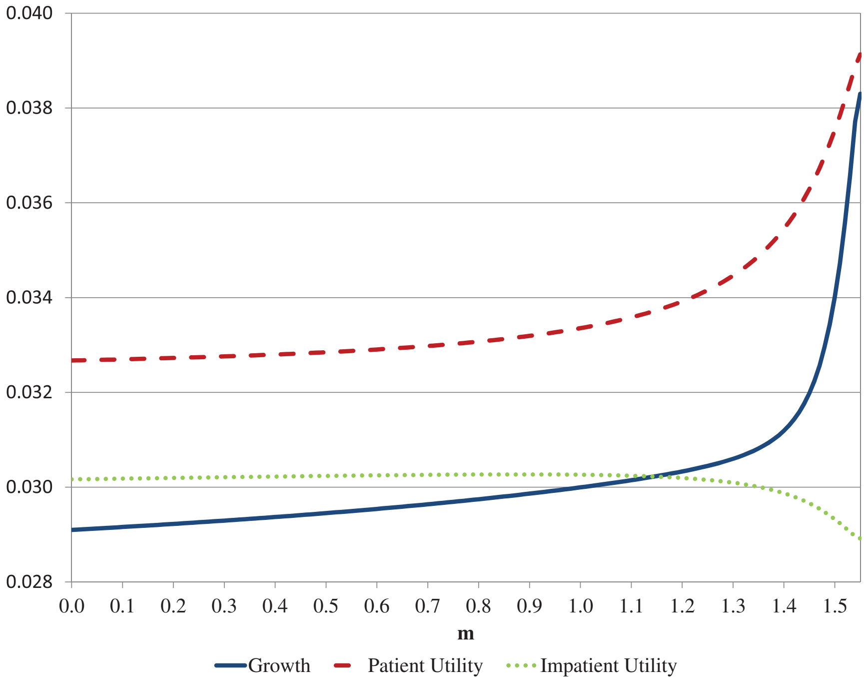

FIGURE 6. Effects of varying m on growth and utility.

the latter respond by decreasing both of their labor supplies. In this simulation, the effect on impatient households is dominant and growth increases along with m.

Figure 7 shows how impatient households change their steady-state levels of labor and consumption for different values of m, while Figure 8 shows the effects on patient households.[12]

A striking result is how the effect on the growth rate accelerates when households are allowed to be significantly underwater on their debt. To understand this result, consider the model when m is below 1. When impatient households are allowed to borrow more, they do so (at the steady state), and their steady-state consumption falls accordingly. They respond to this both by supplying more productive labor and acquiring more capital. With an LTV ratio of less than 1, any additional capital increases debt less than one-to-one and increased capital thus has a positive effect on the impatient households' wealth. This secondary effect dampens the overall reduction in wealth and, as a result, all variables exhibit relatively small changes, including the growth rate.

Now suppose that m is above 1. Once again, increased access to credit reduces the impatient households' wealth and increases the impatient households' capital. Now, however, an extra unit of capital results in a more than one-to-one increase in debt. If m is far enough above 1, the effect of extra debt payments will overcome that of the positive marginal product of capital, so that more capital actually reduces

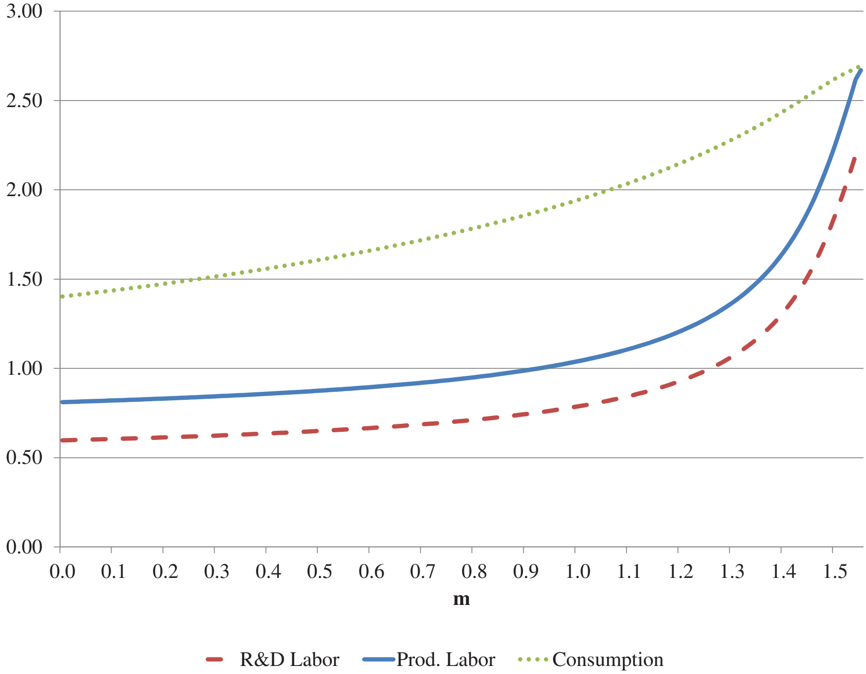

FIGURE 7. Effects of varying m on impatient households.

the impatient households' wealth. The initial effect on wealth is now amplified instead of dampened so that all variables, including the growth rate, exhibit much larger changes in response to varying m.

Impatient household welfare and a Rawlsian social welfare function are maximized when $m = 0.88$. Because growth is now increasing in m, impatient households no longer are better off without any debt, even though no debt eliminates the negative wealth effect. Patient household welfare and a Utilitarian social welfare function are highest when $m = 1.55$ (the largest value in this calibration).

3.3. Simulation 3.3: $\epsilon = 1.1$, $\eta = 3$, $\mu = 0.0148$

Our first two simulations assume that labor supply is highly elastic. Now, we consider a variation of Simulation 3.1 wherein we continue to assume that $\epsilon = 1.1$, but labor supply is now inelastic and calibrated at $\eta = 3$. Recall, a low ϵ implies that households do not have a strong preference for smoothing their labor supply across types. Figure 9 reports the results. The growth rate is 3.45% at $m = 0$ and is 3.00% at $m = 1$. Growth decreases to around 0.75% as access to credit increases up to the point where $m = 1.50$.

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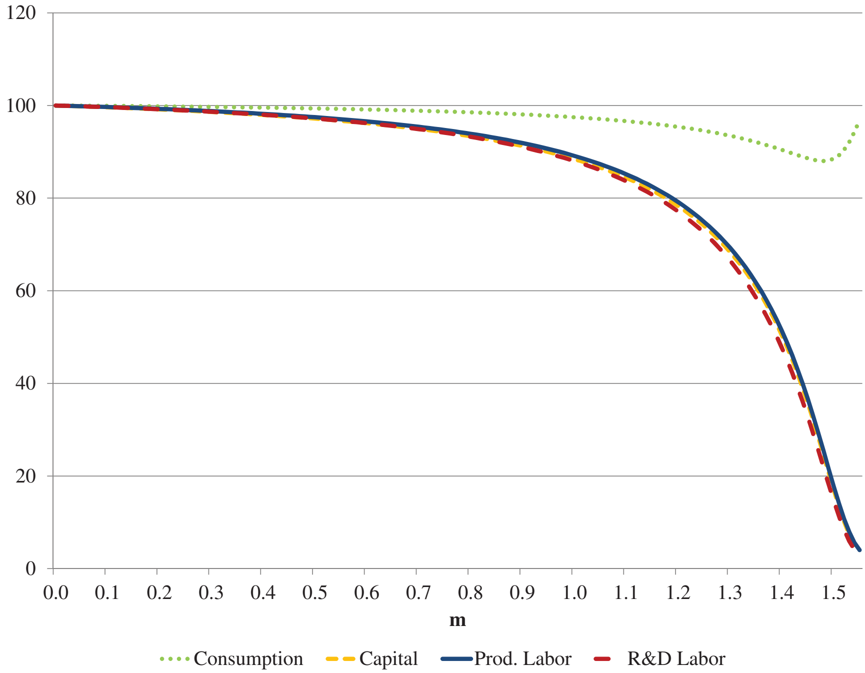

FIGURE 8. Effects of varying m on patient households.

The mechanisms for this simulation are qualitatively the same as for Simulation 3.1. Impatient households again supply almost no R&D labor. As patient households become wealthier, they substitute away from R&D labor (and productive labor) toward leisure and growth declines. But because labor supply is less elastic, the changes in R&D labor and growth are less dramatic compared to Simulation 3.1, but the effects are still large.

Figure 10 shows the effects on impatient households. As in Simulation 3.1, impatient household labor supply to the R&D sector is basically zero. As m increases, impatient households once again acquire more capital and supply more R&D labor. However, the change in the growth rate is driven by patient households, as shown in Figure 11.[13] Patient households again supply less R&D and productive labor, but their consumption increases due to the increase in wealth from debt payments. Now, the decline in patient households' R&D labor causes the decline in growth seen in Figure 9.

Patient household utility is increasing throughout this range of m. Impatient household utility peaks at $m = 0.86$. A Rawlsian social welfare function again tracks the impatient household utility in this simulation. A utilitarian social welfare function more closely tracks the patient household utility. Once again, the optimal value of m depends on the social welfare function specification but lies between 0.86 and 1.50.

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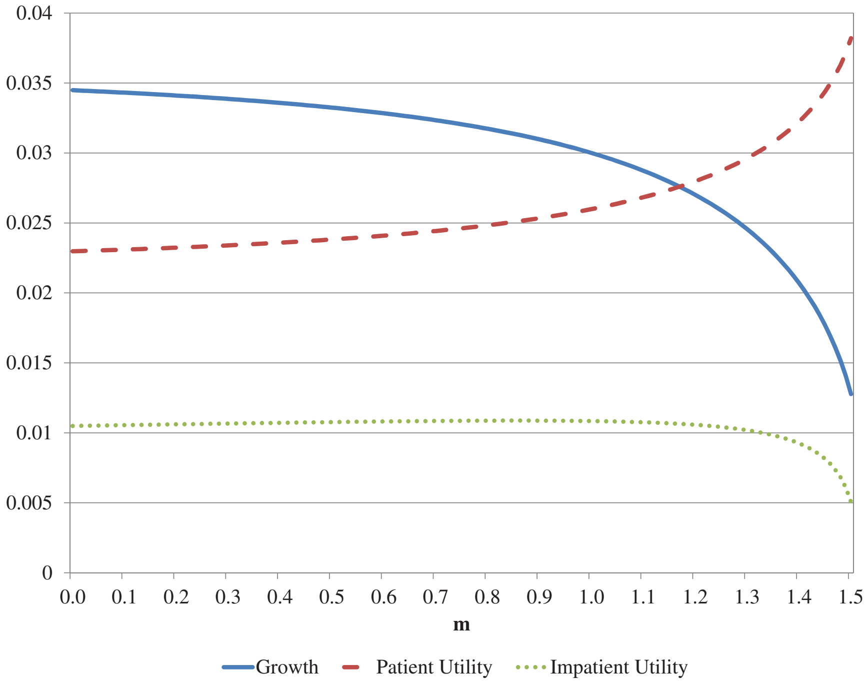

FIGURE 9. Effects of varying m on growth and utility.

3.4. Simulation 3.4: $\epsilon = 10$, $\eta = 3$, $\mu = 0.0044$

In the last simulation, labor supply is less elastic and households prefer to smooth their labor between production and R&D labor, as in Simulation 3.2. The results here are similar to that of Simulation 3.2, except that the effects are dampened due to the reduction in labor supply elasticity. Figure 12 shows the impact of a change in m on growth and utility. Growth increases from 2.94% when $m = 0$ to 4.77% when $m = 1.69$. The growth is driven by an increase in impatient household labor supply to the production sector, and, as a result of the preference for smoothing labor across types, R&D labor supply also increases. Patient household utility is increasing as access to credit increases. Impatient household utility begins to decline significantly once the LTV ratio exceeds 1.

Impatient household utility and Rawlsian social welfare are maximized when $m = 0.87$. The LTV ratio is much higher at $m = 1.69$ (the largest possible value) when maximizing patient household utility. Utilitarian welfare is maximized when $m = 1.61$.

3.5. Stability Around the Steady State

We now analyze the model's local stability of equilibrium around its steady state using the well-known technique of Blanchard and Kahn (1980). Stability is critical

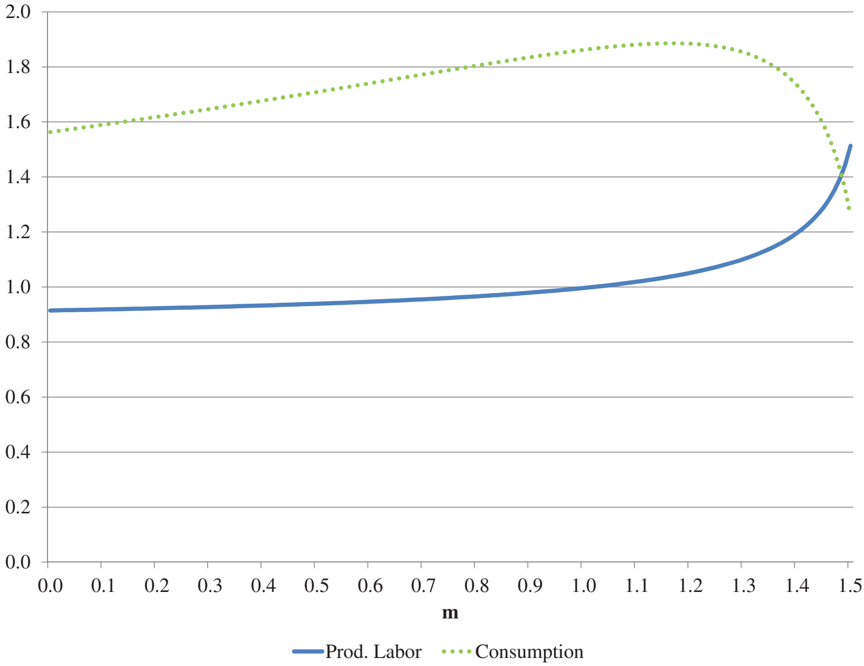

FIGURE 10. Effects of varying m on impatient households.

for the comparative statics results of this section to be valid. We first log-linearize the system (using "hats" to indicate log-linearized variables) around its steady state. We then eliminate the two variables for productive labor ($L_{y,t}$, and $L^{'}_{y,t}$) using (5) and (11), and the two variables for output (Y_t, and $Y^{'}_t$) using (13) and (14). The system may now be written as

$$\hat{X}_t = D\hat{X}_{t+1} + Fe_t, \tag{19}$$

where $\hat{X}_t = [\hat{A}_t, \hat{k}_t, \hat{k}^{'}_t, \hat{c}_t, \hat{c}^{'}_t, \hat{L}_{a,t}, \hat{L}^{'}_{a,t}, \hat{b}_t, \hat{R}_t]'$. The first three elements of $\hat{X}_t$ are the model's three predetermined variables. The latter six elements are the model's endogenous variables. The vector e_t includes the model's three shocks ($\hat{Z}_t$, $\hat{U}_t$, $\hat{m}_t$,) and endogenous expectational errors for each of the model's six endogenous variables in period $t+1$.

The model's general local stability conditions then depend on the number of eigenvalues of D with modulus inside the unit circle. Each such eigenvalue provides a saddle condition that pins down one endogenous variable. The model thus has a unique equilibrium path around the steady state if there are exactly six eigenvalues inside the unit circle. If five or fewer such eigenvalues exist, however, then equilibrium is indeterminate. In this case, multiple equilibria paths exist around the steady state including those which depend on extraneous expectational

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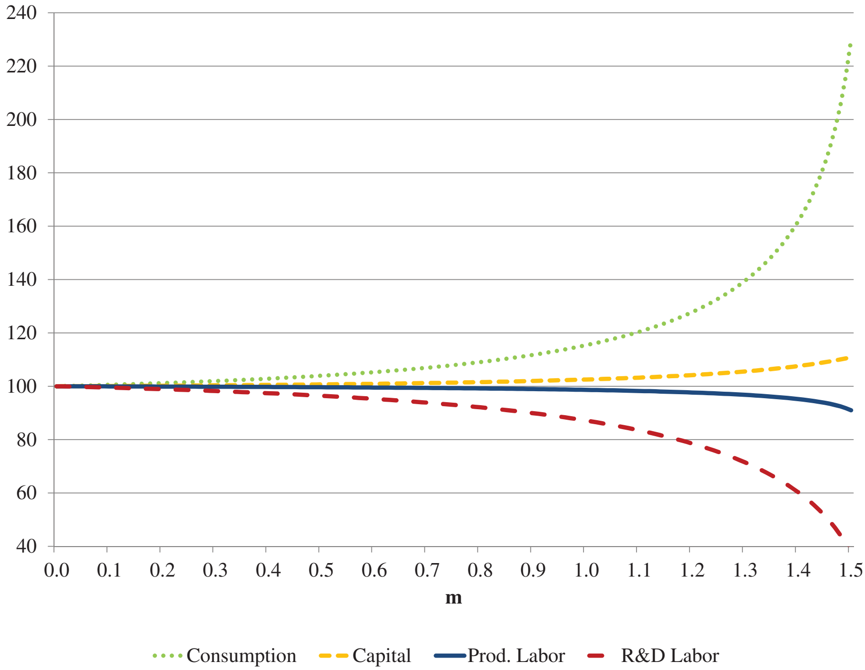

FIGURE 11. Effects of varying m on patient households.

errors known as "sunspots." If seven or more eigenvalues are inside the unit circle, then a stable equilibrium path does not exist around the steady state.

The size of D prevents us from obtaining general stability conditions in terms of the model's structural parameters. We do analyze stability numerically for each of the four simulations from Section 3. We typically find that a unique equilibrium exists, and we never find cases where no stable equilibrium exists. For Simulations 2 and 4, equilibrium around the steady state is always determinate. Simulations 1 and 3 do exhibit regions of indeterminacy for the very high steady-state values of m. For Simulation 1, this region exists for $m \geq 1.54$. For Simulation 3, this region occurs for $m \geq 1.82$.

4. THE MODEL WITHOUT THE SCALE EFFECT

The results of the preceding section show that differences in leverage can have dramatic effects on the growth rate. Our results must thus confront a common criticism of the endogenous growth literature that notes that the growth rate in the United States and many other developed economies has been quite stable for many decades.[14] It is notable, however, that if TFP flows relatively freely across countries, and then endogenous growth models are best interpreted as models of the global economy and country-by-country comparisons are dubious.

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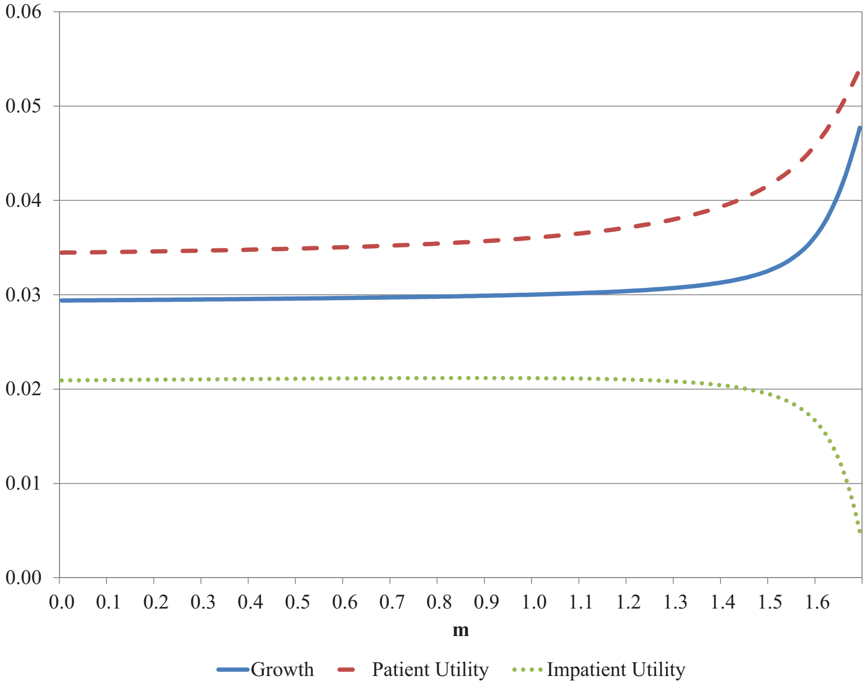

FIGURE 12. Effects of varying m on growth and utility.

It is certainly true that global growth rates have increased over a very long time horizon. Although there are not good data on per-capita debt on a global scale, our results suggest that changes to such a variable could potentially explain the increase in observed global growth rates.

Our baseline model follows Romer (1990) in that TFP formation is increasing in A_t, as captured by its inclusion on the right-hand side of equation (1). This assumption has been criticized, including by Jones (1995), on the grounds that models without this scale effect fit the data much better. In this section, we show that our story extends to a version of the model without the scale effect. Here, TFP formation follows:

$$A_{t+1} - A_t = \mu(L^{\lambda}_{a,t} + L'^{\lambda}_{a,t}). \quad \mathbf{(20)}$$

Because our model includes no population growth, a constant value of ΔA_{t+1} would result in a growth rate that goes to zero as $A \to \infty$. The growth rate now equals

$$g_{t+1} = \frac{A_{t+1} - A_t}{A_t} = \frac{\mu(L^{\lambda}_{a,t} + L'^{\lambda}_{a,t})}{A_t}. \quad \mathbf{(21)}$$

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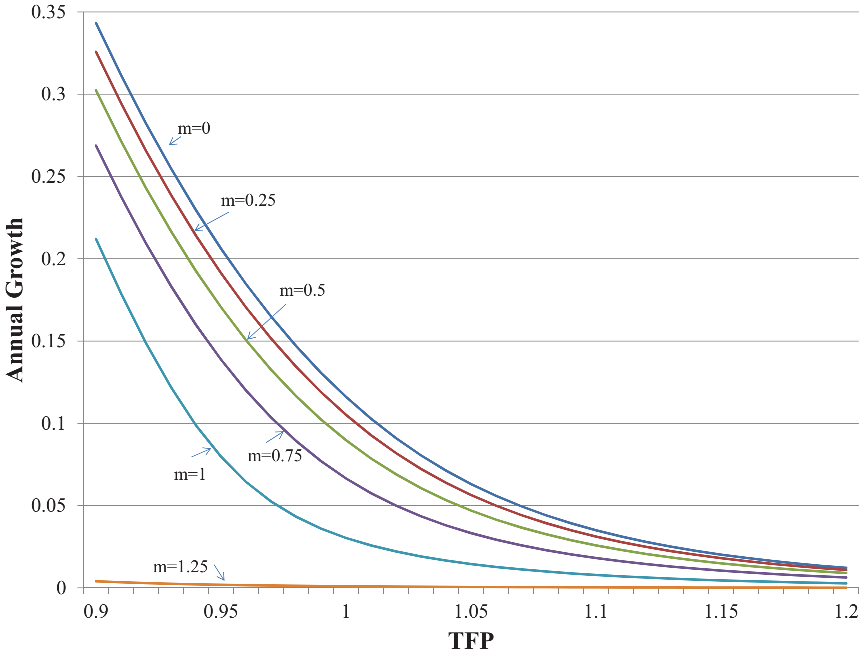

FIGURE 13. Growth rate for different values of A_t (no scale effect).

Because this modification affects the return to R&D, equations (6) and (12) are now replaced with

$$\chi(L_{a,t}^{\epsilon}+L_{y,t}^{\epsilon})^{\frac{\eta}{\epsilon}-1}L_{a,t}^{\epsilon-1}=E_t\left[\frac{\beta\lambda\mu\alpha Z_{t+1}L_{a,t}^{\lambda-1}L_{y,t+1}^{\alpha}\tilde{k}_{t+1}^{1-\alpha}}{\tilde{c}_{t+1}A_t(1+g_{t+1})U_{t+1}}\right.$$
$$\left.+\beta\chi L_{a,t+1}^{\epsilon-1}(L_{a,t+1}^{\epsilon}+L_{y,t+1}^{\epsilon})^{\frac{\eta}{\epsilon}-1}\left(\frac{L_{a,t}}{L_{a,t+1}}\right)^{\lambda-1}\right] \quad (22)$$

$$\chi(L_{a,t}^{'\epsilon}+L_{y,t}^{'\epsilon})^{\frac{\eta}{\epsilon}-1}L_{a,t}^{'(\epsilon-1)}=E_t\left[\frac{\beta'\lambda\mu\alpha Z_{t+1}L_{a,t}^{'(\lambda-1)}L_{y,t+1}^{'\alpha}\tilde{k}_{t+1}^{'(1-\alpha)}}{\tilde{c}'_{t+1}A_t(1+g_{t+1})U_{t+1}}\right.$$
$$\left.+\beta'\chi L_{a,t+1}^{'(\epsilon-1)}(L_{a,t+1}^{'\epsilon}+L_{y,t+1}^{'\epsilon})^{\frac{\eta}{\epsilon}-1}\left(\frac{L'_{a,t}}{L'_{a,t+1}}\right)^{\lambda-1}\right]. \quad (23)$$

There are two differences in the R&D labor supply rules. First, the term $\frac{1+g_{t+2}}{1+g_{t+1}}$ no longer appears in the final term on the right-hand side of each equation. However, our earlier analysis focused on a steady state where g is constant and this change

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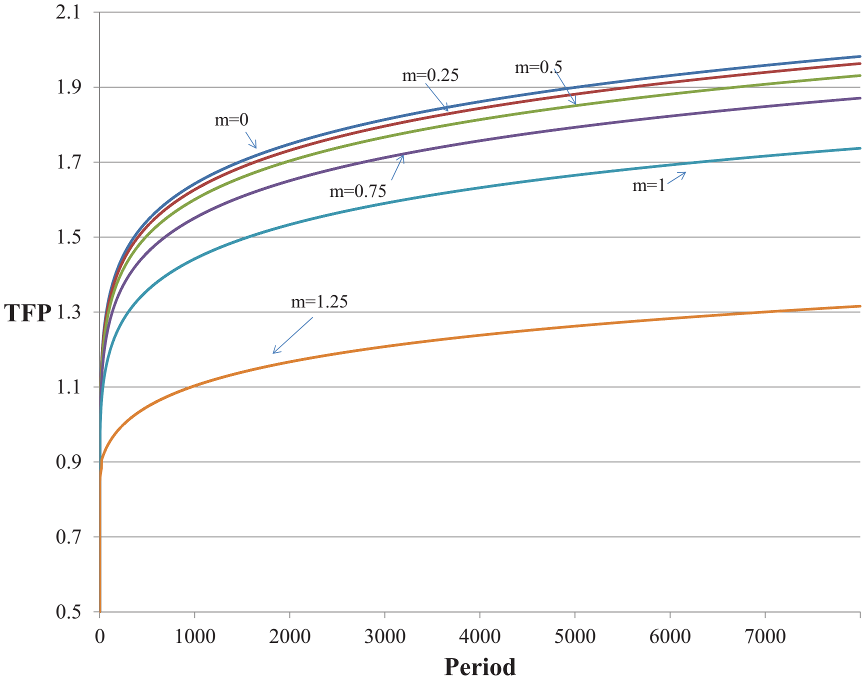

FIGURE 14. TFP over time.

thus has no effect. Second, the first term on the right-hand side now includes A_t in its denominator. This shows that as TFP increases, households value the marginal effect of new TFP less and therefore supply less R&D labor. As a result, neither R&D labor nor ΔA_{t+1} are constant. Instead, as TFP grows, R&D labor falls, accelerating the more obvious decline in the growth rate that comes from having A_t in the denominator of equation (21). TFP thus does not grow without bound, but instead converges to a finite value. The rest of the model is unchanged from Section 2.

We now repeat Simulation 3.1, where labor supply is highly elastic and households have little desire to smooth labor supply across types, without the scale effect. Figure 13 shows the growth rate for different values of m and different values of A_t. The main result is preserved. Higher values of m yield a lower growth rate for all values of A_t. As in Section 3.1, the effects of m are potentially large and last indefinitely. We now impose an initial condition $A_0 = 0.5$ and simulate the model without shocks for 10,000 periods. Figure 14 shows the effects on the level of TFP and Figure 15 shows the effects on TFP growth. The level effect after 10,000 periods of $m = 0$ versus $m = 1.25$ is large, TFP is 51% higher for the former value.

We now repeat Simulation 3.2, where labor supply is still highly elastic but households have a strong taste for smoothing labor supply across types, without the scale effect. Our earlier results once again carry through. When households

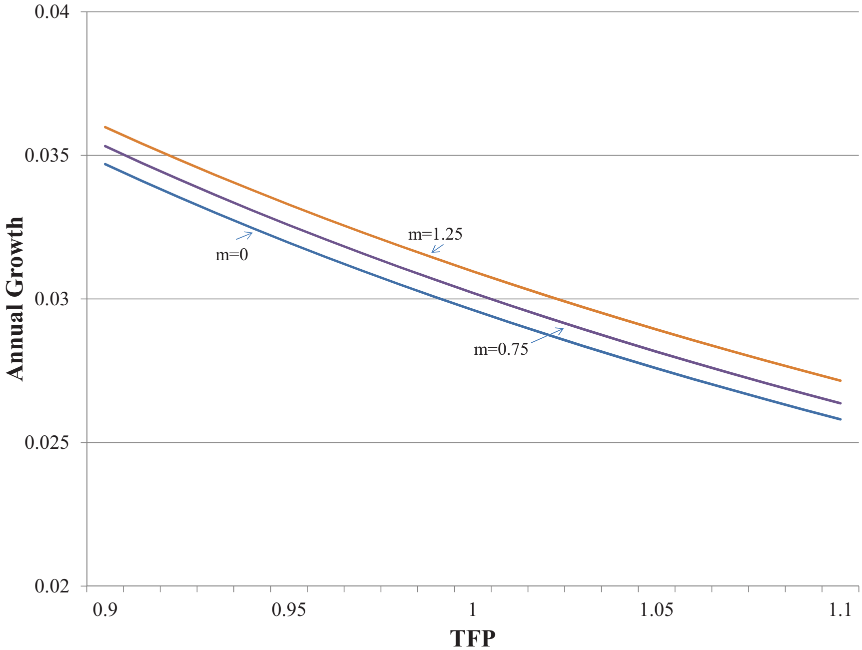

FIGURE 15. Growth rate for different values of A_t (no scale effect).

wish to smooth their labor supply, higher values of m result in more growth for all values of A_t. The magnitude of the effect on growth is again smaller than for Simulation 3.1 that includes the scale effect.[15]

A main point of Jones (1995) is that, without the scale effect, the growth rate depends on parameters such as population growth that are typically thought of as exogenous to policy. As discussed in Section 1, we view access to credit as a parameter that is regularly influenced by policy. By adding credit to the endogenous growth framework, we thus introduce a novel channel through which policy makers might affect growth. As in Section 3, however, how credit affects growth depends on whether or not households prefer to smooth their labor supply across types.[16]

5. CONCLUSION

The events of the Great Recession have helped spur increased attention to the macroeconomic effects of changes in access to credit. So far, most research has focused on how credit constraints may contribute to short-term volatility. This paper, however, provides conditions where varying the leverage ratio of households affects not just the level of output, but its steady-state growth rate. We have shown that if labor supply is elastic, then these effects may be very large and that the direction of the effect depends on whether households prefer to smooth their labor supply between R&D and productive labor.

Throughout the paper, we have not taken a firm position on the appropriate calibration of labor supply elasticity and the desire of households to smooth their labor supply. The former calibration remains exceptionally controversial. We note that if the reader finds the magnitude of the effects from Simulations 3.1 and 3.2 to be implausibly large, then our results may be seen as an additional piece of support for a more inelastic labor supply, which reduces the magnitude of the effects on the growth rate. Because the second calibration is novel to our paper, we are unaware of other work that illuminates its correct value. Empirical work estimating its value might pin down the direction of how access to credit affects growth and is left for future research.

NOTES

1. See Kiyotaki and Moore (1997) and Iacoviello (2005).
2. For a more complete discussion of models with growth driven by endogenous technological change, see Romer (1990), Grossman and Helpman (1991a,b,c), and Aghion and Howitt (1992).
3. We use the terms access to credit, debt-to-capital ratio, leverage ratio, and loan-to-value ratio interchangeably. While they do represent slightly different measures in the data, in our model, they are synonymous.
4. See Chetty et al. (2013) for further discussion of the controversy over the correct calibration of the Frisch elasticity of labor supply. Highly elastic labor supply tends to allow macroeconomic theory models to better fit the data, while microeconometric studies tend to suggest less elasticity. The reader may reasonably conclude, however, that the effect of access to credit on growth is implausibly high when labor supply is very elastic in our model and that less elastic labor supply thus yields better fit. Because our model introduces households' desire to smooth their labor supply across types as a new parameter, we are unaware of any existing work that attempts to calibrate its value.
5. This interpretation dates from the seminal work of Kiyotaki and Moore (1997).
6. Much of the variation in these leverage ratios results from the volatility of housing prices. Figure 2 uses current housing prices. An alternate approach would be to try to estimate long-term housing prices. This is more challenging but might result in smoother leverage ratios.
7. In Table 2, we calibrate λ to be equal to 1. Alternate values of λ did not have any significant effects on our results.
8. Marshall and Shea (2017) relax this assumption, allowing agents to explicitly default and show that default causes a discrete drop in asset prices and output.
9. In the United States, there are examples of both types of debt. Mortgage lenders can usually only seize the borrowers' housing in the case of default. For student loan debt, however, the lender (typically the federal government) is able to pursue claims against the borrowers' future assets and income.
10. Table 1 also reports the growth rates for when $m = 0$ and $m = 1$, and the values of m that maximize social welfare for a Utilitarian welfare function $[m^*(U)]$ and a Rawlsian welfare function $[m^*(R)]$.
11. As $\beta' \to \beta$ debt approaches zero, the effects changing m also go to zero.
12. For Figure 8, the values for all variables are normalized so that they equal 100 when $m = 0$.
13. Again, the values of the variables are normalized to equal 100 when $m = 0$.
14. See Jones (2002) for a discussion of this issue. He argues that U.S. growth has been well above its balanced growth rate because of long-lasting transitional effects.
15. In addition, after 1,000 periods, TFP is 7.1% higher when $m = 1.25$ compared to when $m = 0$.
16. The stability of the model without the scale effect is very similar to that of the model with the scale effect.

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