CORRESPONDENCE

To the Editor of the Journal of the Royal Aeronautical Society.

The Solution of Simultaneous Linear Equations

Dear Sir,—

I was considerably interested in the method given by Captain Morris in the April number of the Journal for solving certain forms of simultaneous equations. The example worked out by the method shows a set of equations to which it is very applicable, namely, a set in which in the first equation the x term has a pronounced preponderance over the x terms in the other equations; in the second equation the same holds for the y term, and so on. In fact, the first equation may be looked upon as an equation which would give the value of x directly if it were not for the fact that this value has to have slight linear corrections for the values y, z and w. This, of course, is a type of equation which often occurs in physical calculations. I also understand that equations of this kind occur in the calculation of stresses in framed structures. In his approximations as tabulated it will be noted that after three cycles only of approximation, the values x, y, z are within 0.01 per cent. of their true values. Although so very useful in the above class of work the method is not nearly so useful for simultaneous equations in general, even though the condition for "convergency" may be satisfied, nor does Captain Morris claim this. For example, I tried a set of three equations in which the major terms were not quite so preponderant as in the example used by Captain Morris. These were:—

$$8x + y + 5z = 88 \qquad (1)$$
$$x + 15y + 3z = 146 \qquad (2)$$
$$5x + 6y + 10z = 182 \qquad (3)$$

The actual values of x, y and z are 2, 7 and 13 respectively. The progress of approximation in this case is as follows:—Not until after the seventh cycle of approximation do the values of y and z approach to within 0.1 per cent. of their true values; while the value of x after the seventh approximation is only within 1 per cent. of its true value, and would require many more cycles for it to approach to within 0.1 per cent. Each cycle of approximations occupies three lines. As a further example, I took the same three equations except that in equation (1) the coefficient of z was increased from 5 to 20 (and the constant term from 88 to 283). In this case, the first equation contains the predominant x term as well as the predominant z term, and it was found that the approximations became divergent, showing that the method is not applicable. For simultaneous equations in general containing three variables, the chance that the practical condition of applicability, as above stated, is seen to be 7 to 2 against. It will be seen that the practical condition as stated above for applicability ensures that each of the "ratios of distribution" shall be less than one.

The ordinary determinantal method of solving simultaneous equations, which is of course applicable in every case, is not however so formidable as might be imagined if, when evaluating the determinant, we use these two common properties of a determinant, viz.: (1) That the interchange of any two columns (or rows) simply changes the sign of the determinant; (2) that the subtraction of any multiple of the terms in any column (or row) from the corresponding terms in any other column (or row) does not alter the value of the determinant. By the use of these properties any determinant can be quickly altered so that every figure in the first column except the top one is reduced to o and the determinant can then be reduced to one of smaller degree. This method has, I believe, been recommended by Mr. Chio, but I have not got the reference to his work.

As an example, take the set of three equations last considered, viz., equations (2) and (3) and the modified form of equation (1), *i.e.*,

$$8x + y + 20z = 283 \qquad . \quad . \quad . \quad . \quad . \qquad (1a)$$

The solution is

$$x/\Delta_x = -y/\Delta_y = z/\Delta_z = -1\Delta,$$

where Δ_x, Δ_y, Δ_z and Δ are the determinants obtained by striking out from the whole array of coefficients and final constants those columns containing the three final constants respectively, thus:—

$$\Delta_x = \begin{vmatrix} 1 & 20 & 283 \\ 15 & 3 & 146 \\ 6 & 10 & 182 \end{vmatrix} = \begin{vmatrix} 1 & 20 & 283 \\ 0 & 297 & 4099 \\ 0 & 110 & 1516 \end{vmatrix} \begin{matrix} \\ \ldots * \\ \ldots \dagger \end{matrix}$$

$$= \begin{vmatrix} 297 & 4099 \\ 110 & 1516 \end{vmatrix}$$

$$= -450252 + 450890$$
$$= 638$$

$$\Delta_y = \begin{vmatrix} 20 & 283 & 8 \\ 3 & 146 & 1 \\ 10 & 182 & 5 \end{vmatrix} = -1 \begin{vmatrix} 1 & 3 & 146 \\ 8 & 20 & 283 \\ 5 & 10 & 182 \end{vmatrix} = -1 \begin{vmatrix} 4 & 885 \\ 5 & 548 \end{vmatrix}$$

$$= 2192 - 4425$$
$$= -2233$$

$$\Delta_z = \begin{vmatrix} 283 & 8 & 1 \\ 146 & 1 & 15 \\ 182 & 5 & 6 \end{vmatrix} = \begin{vmatrix} 1 & 283 & 8 \\ 15 & 146 & 1 \\ 6 & 182 & 5 \end{vmatrix} = \begin{vmatrix} 4099 & 119 \\ 1516 & 43 \end{vmatrix}$$

$$= -176257 + 180404$$
$$= 4147$$

$$\Delta = \begin{vmatrix} 8 & 1 & 20 \\ 1 & 15 & 3 \\ 5 & 6 & 10 \end{vmatrix} = -1 \begin{vmatrix} 1 & 8 & 20 \\ 15 & 1 & 3 \\ 6 & 5 & 10 \end{vmatrix} = -1 \begin{vmatrix} 119 & 297 \\ 43 & 110 \end{vmatrix}$$

$$= -13090 + 12771$$
$$= -319$$
$$\therefore x/638 = y/2233 = z/4147 = 1/319$$
$$\therefore x = 2, \; y = 7, \; z = 13.$$

Yours, etc.,

T. W. K. Clarke.

* Formed by multiplying line 1 of the previous determinant by 15 and subtracting from line 2.

† Formed by multiplying line 1 of the previous determinant by 6 and subtracting from line 3.