Laser and Particle Beams (2011), *29*, 407–414.

doi:10.1017/S0263034611000504

Self-focusing of laser beam in collisional plasma and its effect on Second Harmonic generation

ARVINDER SINGH AND KESHAV WALIA
Department of Physics, National Institute of Technology Jalandhar, India
(RECEIVED 2 June 2011; ACCEPTED 6 August 2011)

Abstract

This paper presents an investigation of self-focusing of Gaussian laser beam in collisional plasma and its effect on second harmonic generation. Due to non-uniform heating, collisional non-linearity arises, which leads to redistribution of carriers and hence affects the plasma wave, which in turn affects the second harmonic generation. Effect of the intensity of the laser beam/plasma density on the harmonic yield is studied in detail. We have set up the non-linear differential equations for the beam width parameters of the main beam, plasma wave, second harmonic generation and second harmonic yield by taking full non-linear part of the dielectric constant of collisional plasma with the help of moment theory approach. It is predicted from the analysis that harmonic yield increases/decreases due to increase in the plasma density/intensity of the laser beam respectively.

Keywords: Collisional Plasma; Plasma Wave; Second harmonic generation; Self-focusing

1. INTRODUCTION

There has been considerable interest in the non-linear propagation of intense laser beams in plasmas because of its relevance to laser induced fusion and charged particle acceleration. In laser induced fusion, the most important problem is the efficient coupling of laser energy to plasmas to heat the latter. In coupling process, many non-linear phenomena such as self-focusing, stimulated raman scattering and stimulated brillouin scattering, and several others (Deutsch *et al.*, 1996; Esarey *et al.*, 1996; Tajima & Dawson, 1979) play a crucial role. However, self-focusing continues to be a subject of great fascination due to its relevance to ionospheric radio propagation, optical harmonic generation, X-ray lasers, and other important applications (Burnett & Corkum, 1989; Amendt *et al.*, 1991; Hora, 1981; Jones *et al.*, 1988; Liu & Kaw, 1976; Ginzburg, 1970; Shi, 2007; Merdji *et al.*, 2000). The self-focusing of laser beams, having non-uniform distribution of irradiance in a plane, normal to direction of propagation leads to non-uniform distribution of carriers along the wavefront, which further leads to a change in dielectric constant of plasma. The collisional non-linearity occurs because of electrons acquiring temperature higher than other species on account of net effect of ohmic heating and energy lost by electrons due to collisions (Sodha *et al.*, 1974; Umstadter, 2001) with heavy particles (atoms/molecules and ions) and by thermal conduction (Sodha *et al.*, 1973, 1975, 1976). These analyses consider only one type of energy loss viz. collisions or thermal conduction. Generation of harmonics of electromagnetic waves in plasmas engaged the attention of a number of researchers due to its practical value for many applications. Harmonic generation in intense laser plasma interaction has been studied extensively both experimentally and theoretically (Hafizi *et al.*, 2000; Young *et al.*, 1989; Engers *et al.*, 1991; Parashar & Pandey, 1992; Esarey *et al.*, 1993). The early work on harmonic generation in collisional plasmas has been reviewed (Sodha & Kaw, 1969). This phenomenon arises on account of the second harmonic component in the isotropic part of the distribution function of electron velocities in a plasma caused by a high irradiance electromagnetic wave. The presence of second harmonic term in the current density, which is proportional to the square of the electric vector of the fundamental wave gives rise to second harmonic. A number of investigations on optimization of conditions for maximum magnitude of generated harmonics were later published. All these investigations on generations of harmonics and combination frequencies in collisional plasma are applicable, when the fundamental wave is a plane wave, with uniform irradiance along the wavefront.

Address correspondence and reprint requests to: Arvinder Singh, Department of Physics, National Institute of Technology Jalandhar, India. Email: arvinder6@lycos.com

Since most of the electromagnetic beams have non-uniform distribution of irradiance along the wavefront, there was a need to take in account this non-uniformity in the theory of harmonic generation. It is well known that such beams exhibit the phenomenon of self-focusing/self-defocusing. Since for a given power of beam, the average of square of electric vector in the wavefront is much higher for non-uniform irradiance distribution than that for uniform irradiance distribution; the magnitude of the generated harmonics is higher in the case of non-uniform irradiance. This provides a strong motivation for the study of the second harmonic yield by taking self-focusing in to account.

The interest in the interaction of laser beam with plasma is not limited only to harmonic generation. The propagation of intense electromagnetic waves in underdense plasmas can also excite plasma wave. This instability is of interest in laser inertial confinement fusion because it can generate energetic electrons that can preheat the fuel and reduce the implosion efficiency. Another case where plasma wave is of increasing interest is in laser particle acceleration. This plasma wave interacts with the plasma particles and transfers its energy to particles by wave particle interaction (Fibich, 1996) and hence acceleration of particles takes place. Harmonic generation has been studied by number of workers (Malka *et al.*, 1997; Baton *et al.*, 1993; Brandi *et al.*, 2006; Gupta *et al.*, 2007; Ganeev *et al.*, 2007; Schifano *et al.*, 1994; Ozaki *et al.*, 2006, 2007, 2008; Nuzzo *et al.*, 2000). In most of the above mentioned works, investigations have been carried out in the paraxial approximation due to small divergence angles of the laser beams involved. In some experiments, where solid state lasers are used, wide angle beams are generated for which the paraxial approximation is not applicable. Also, if the beam width of laser beam used is comparable to the wavelength of the laser beam, paraxial approximation is not valid. Paraxial theory approach (Akhmanov *et al.*, 1968; Sodha *et al.*, 1974, 1976) takes in to account only paraxial region of the beam, which in turn leads to large error in the analysis. In this theory, non-linear part of the dielectric constant is taylor expanded up to second order term and higher order terms are neglected. However, moment theory (Vlasov *et al.*, 1971; Lam *et al.*, 1977) is based on the calculation of moments and does not suffer from this defect. In moment theory approach, non-linear part of the dielectric constant is not taylor expanded, rather taken as a whole in calculations. (Sodha *et al.*, 1979; Sinha & Sodha, 1980; Singh & Walia, 2010, 2011; Singh & Singh, 2010, 2011*a*, 2011*b*, 2011*c*; Walia & Singh, 2011). Moment theory is difficult to apply wherever the propagation of more than one wave is involved and therefore one always prefer to apply paraxial theory, in which the mathematical calculations become simpler as compared to moment theory approach. To the best of our knowledge, so far no one has used moment theory approach to study the second harmonic generation. So, the novelty of the present work is that we have considered the full non-linear part of the dielectric constant in the present investigation.

In the present paper, second harmonic generation is studied in detail by taking the plasma wave as a source for generating a second harmonic in collisional plasma ($t > \tau_E$, where τ_E is the energy relaxation time). The non-linearity arising through non-uniform heating leads to redistribution of carriers, which modifies the background plasma density profile in a direction transverse to pump beam axis and hence generates the plasma wave at pump frequency. This plasma wave in turn interacts with incident laser beam and a second harmonic is generated. In Section 2, we have set up and solved wave equation for the laser beam with the help of moment theory approach. In Section 3, we have derived an expression for the density perturbation associated with the electron plasma wave. In Section 4, second harmonic yield is estimated. Last, a brief discussion of the results is presented in Section 5.

2. SOLUTION OF THE WAVE EQUATION

Consider the propagation of a laser beam of angular frequency ω_0 in a homogeneous plasma along z-axis. The initial intensity distribution of beam along the wavefront at z = 0 is given by

$$E_0.E_0^{\star}|_{z=0} = E_{00}^2 \exp\left[-r^2/r_0^2\right], \tag{1}$$

where $r^2 = x^2 + y^2$ and r_0 is initial width of the main beam, and r is radial co-ordinate of the cylindrical coordinate system. For collisional plasma i.e., for the case of non-uniform heating type non-linearity, the modified electron concentration may be written as Sodha *et al.*, (1976)

$$N_{0e} = N_0 \cdot \left[1 + \alpha/2EE^{\star}\right]^{\frac{S}{2}-1}. \tag{2}$$

Where S is a parameter characterizing the nature of collisions. In plasmas, various types of collisions take place; e.g., $S = -3$ corresponds to collisions between electrons and ions, $S = 2$ for collisions between electrons and diatomic molecules, and $S = 0$ corresponds to collisions, which are velocity dependent and α is the non-linearity constant given by

$$\alpha = \frac{e^2 M}{6m^2\omega_0^2 K_B T_0}. \tag{3}$$

Here, K_B is the Boltzmann constant, ω_0 is the angular frequency of laser beam, T_0 is equilibrium temperature of plasma, e and m are charge and mass of electron, respectively, and M is mass of ion. Slowly varying electric field E_0 of the laser beam satisfies the following wave equation.

$$\nabla^2 E_0 - \nabla(\nabla.E_0) + \frac{\omega_0^2}{c^2}\varepsilon E_0 = 0. \tag{4}$$

In the Wentzel-Kramers-Brillouin approximation, the second term $\nabla\ (\nabla.E_0)$ of Eq. (4) can be neglected, which is justified when $\frac{c^2}{\omega_0^2}|\frac{1}{\varepsilon}\nabla^2 \ln \varepsilon| \ll 1$,

$$\nabla^2 E_0 + \frac{\omega_0^2}{c^2}\varepsilon E_0 = 0, \tag{5}$$

and

$$\varepsilon = \varepsilon_0 + \Phi(AA^\star). \tag{6}$$

Where ε_0 and Φ (AA^*) are linear and non-linear parts of the dielectric constant, respectively.

$$\varepsilon_0 = 1 - \frac{\omega_p^2}{\omega_0^2}, \tag{7}$$

and

$$\Phi(AA^\star) = \frac{\omega_p^2}{\omega_0^2}\left[1 - \frac{N_{0e}}{N_0}\right]. \tag{8}$$

Where $\omega_p = \sqrt{4\pi N_0 e^2/m}$ is the electron plasma frequency.
Further, taking E_0 as

$$E_0 = A(r, z)\exp[\iota\{\omega_0 t - k_0 z\}], \tag{9}$$

where $A(r, z)$ is a complex function of its argument. The behavior of the complex amplitude $A(r, z)$ is governed by the following parabolic equation obtained from the wave Eq. (5) by assuming variations in the z direction being slower than those in the radial direction,

$$-2\iota k_0\frac{\partial A}{\partial z} + \nabla_\perp^2 A + \frac{\omega_0^2\Phi(AA^\star)A}{c^2} = 0. \tag{10}$$

This equation is also known as the quasi-optic equation. Now, Eq. (10) can be written as

$$\iota\frac{\partial A}{\partial z} = \frac{1}{2k_0}\nabla_\perp^2 A + \chi(AA^\star)A, \tag{11}$$

where $\chi(AA^\star) = \frac{k_0}{2\varepsilon_0}(\varepsilon - \varepsilon_0)$ and $\varepsilon = \varepsilon_0 + \Phi(|AA^\star|)$, where $\varepsilon_o = 1 - \frac{\omega_p^2}{\omega_0^2}$ and $\Phi(|AA^\star|)$ are the linear and non-linear parts of the dielectric constant, respectively. Also, $k_0 = \frac{\omega_0}{c}\sqrt{\varepsilon_0}$ and ω_p are propagation constant and plasma frequency, respectively. Now from the definition of the second order moment, the mean square radius of the beam is given by

$$<a^2> = \frac{\iint (x^2 + y^2)AA^\star dxdy}{I_0}. \tag{12}$$

From here one can obtain the following equation:

$$\frac{d^2 <a^2>}{dz^2} = \frac{4I_2}{I_0} - \frac{4}{I_0}\iint Q(|A|^2)dxdy, \tag{13}$$

where I_0 and I_2 are the invariants of Eq. (11) (Vlasov *et al.*, 1971)

$$I_0 = \iint |A|^2 dxdy, \tag{14}$$

$$I_2 = \iint \frac{1}{2k_0^2}(|\nabla_\perp |A|^2 - F)dxdy, \tag{15}$$

with (Lam *et al.*, 1977)

$$F(|A|^2) = \frac{1}{k_0}\int \chi(|A|^2)d(|A|^2), \tag{16}$$

and

$$Q(|A|^2) = \left[\frac{|A|^2\chi(|A|^2)}{k_0} - 2F(|A|^2)\right]. \tag{17}$$

For $z > 0$, we assume an energy conserving gaussian ansatz for the laser intensity (Akhmanov *et al.*, 1968; Sodha *et al.*, 1974, 1976)

$$AA^\star = \frac{E_{00}^2}{f_0^2}\exp\left\{-\frac{r^2}{r_0^2 f_0^2}\right\}. \tag{18}$$

From Eqs. (12), (14), and (18) it can be shown that

$$I_0 = \pi r_0^2 E_{00}^2, \tag{19}$$

$$<a^2> = r_0^2 f_0^2. \tag{20}$$

Where f_0 is dimensionless beam width parameter and r_0 is beam width at $z = 0$. Now, from Eqs. (13)–(20) we get

$$\frac{d^2 f_0}{d\xi^2} + \frac{1}{f_0}\left(\frac{df_0}{d\xi}\right)^2 = \frac{2k_0^2}{\pi E_{00}^2 f_0}[I_2 - \iint Q(|E_0|^2)dxdy]. \tag{21}$$

Where $\xi = (z/k_0 r_0^2)$ is the dimensionless propagation distance. Eq. (21) is a basic equation for studying the self-focusing of a gaussian laser beam in a non-linear, non-absorptive medium. Now, by making use of Eqs. (2), (8), (15)–(18), and (21) we get

$$\frac{d^2 f_0}{d\xi^2} + \frac{1}{f_0}\left(\frac{df}{d\xi}\right)^2 = \frac{1}{f_0^3} - \frac{2f_0}{3\alpha E_{00}^2}\left(\frac{\omega_p r_0}{c}\right)^2 \times\left[\left[1 + \frac{\alpha E_{00}^2}{f_0^2}\right]^{\frac{-3}{2}} - 1 - \log\left(\frac{\left[1 + \frac{\alpha E_{00}^2}{f_0^2}\right]^{\frac{1}{2}} - 1}{\left[1 + \frac{\alpha E_{00}^2}{f_0^2}\right]^{\frac{1}{2}} + 1}\right) + 2\left(\frac{1}{\left[1 + \frac{\alpha E_{00}^2}{f_0^2}\right]^{\frac{1}{2}}} - 1\right)\right]. \tag{22}$$

Initial conditions of plane wavefront are $\frac{df_0}{d\xi} = 0$ and $f_0 = 1$ at $\xi = 0$. Eq. (22) describes the change in the beam width parameter of a gaussian beam on account of the

https://doi.org/10.1017/S0263034611000504 Published online by Cambridge University Press

competition between diffraction divergence and nonlinear focusing terms as the beam propagates in the collisional plasma.

3. PLASMA WAVE GENERATION

We consider the interaction of a weak plasma wave and a Gaussian laser beam in a collisional plasma. Due to non-uniformity in heating, the background electron density gets modified, which further leads to change the amplitude of the plasma wave, which depends on the background electron density. Following the standard procedure, the equation governing the electron plasma wave generation can be written as,

$$\frac{\partial^2 N}{\partial t^2} - v_{th}^2 \nabla^2 N + 2\Gamma_e \frac{\partial N}{\partial t} - \frac{e}{m} \nabla \cdot [NE] = \nabla \cdot \left[\frac{N}{2} \nabla (V \cdot V^*) - V \frac{\partial N}{\partial t}\right]. \tag{23}$$

Where $2\Gamma_e$ is landau damping factor, v_{th} is the electron thermal speed, E is the sum of electric vectors of electromagnetic wave and self-consistent field, V is the sum of drift velocity of electron in electromagnetic field and self-consistent field, m is mass of electron. The density component varying at pump wave frequency (N_1) can be written as

$$-\omega_0^2 N_1 + v_{th}^2 \nabla^2 N_1 + 2\iota\Gamma_e \omega_0 N_1 + \omega_p^2 \left[\frac{N_{0e}}{N_0}\right] N_1 \cong \frac{e}{m}(N_{0e} \nabla \cdot E_0 + E_0 \cdot \nabla N_{0e}). \tag{24}$$

Where N_0 is the equilibrium electron density, $\omega_p^2 = \frac{4\pi N_0 e^2}{m}$ is the electron plasma frequency, V_0 is the oscillation velocity of the electron in the pump wave field, and ω_0 is the pump wave frequency.It is obvious from the source term of Eq. (24) that one component of N_1 varies as E_0 and that the second component is the solution of homogeneous Eq. (24). Therefore, N_1 can be written as

$$N_1 = N_{10}(r, z) \exp(-ikz) + N_{20}(r, z) \exp(-ik_0 z). \tag{25}$$

Where N_{10} (r, z) and N_{20} (r, z) are the complex functions of their arguments and satisfy the following equations.

$$-\omega_0^2 N_{10} - v_{th}^2 \nabla_\perp^2 N_{10} + 2ikv_{th}^2 \frac{\partial N_{10}}{\partial z} + k^2 v_{th}^2 N_{10} + 2\iota\Gamma_e \omega_0 N_{10} + \omega_p^2 \left[\frac{N_{0e}}{N_0}\right] N_{10} = 0, \tag{26}$$

and

$$-\omega_0^2 N_{20} - v_{th}^2 \nabla^2 N_{20} + 2\iota\Gamma_e \omega_0 N_{20} + \omega_p^2 \left[\frac{N_{0e}}{N_0}\right] N_{20} \cong -\frac{N_{0e} e E_{00}}{m f_0} \exp\left[\frac{-r^2}{2r_0^2 f_0^2}\right]\left[\frac{y}{r_0^2 f_0^2}\right] I_3, \tag{27}$$

where

$$I_3 = \left[1 - \frac{5\alpha E_{00}^2 \exp\left(-\frac{r^2}{r_0^2 f_0^2}\right)}{2f_0^2\left(1 + \frac{\alpha E_{00}^2 \exp\left(-\frac{r^2}{r_0^2 f_0^2}\right)}{2f_0^2}\right)}\right].$$

Now, Eq. (26) can be written as,

$$\frac{\partial N_{10}}{\partial z} = -\frac{i}{2k} \nabla_\perp^2 N_{10} - iP_1 N_{10} - \frac{\omega_0 \Gamma_e N_{10}}{k v_{th}^2}, \tag{28}$$

Where, $P_1 = \frac{\omega_p^2}{2kv_{th}^2}\left[1 - \frac{N_{0e}}{N_0}\right]$.

Now, from the definition of second order moment

$$<a^2> = \frac{1}{I_0} \iint (x^2 + y^2) N_{10} N_{10}^* dxdy, \tag{29}$$

where I_0 is zeroth order moment and can be written as

$$I_0 = \iint N_{10} N_{10}^* dxdy. \tag{30}$$

Now, solution of Eq. (28) is of the form,

$$N_{10}^2 = \frac{B^2}{f^2} \exp\left(-\frac{r^2}{a_0^2 f^2}\right) \exp(-k_i z). \tag{31}$$

Now, from Eqs. (29), (30), and (31), it can be shown that

$$I_0 = \pi B^2 a_0^2, \tag{32}$$

and

$$<a^2> = a_0^2 f^2 \exp(-2k_i z). \tag{33}$$

Now, with the help of Eqs. (29) and (33), one can get

$$\frac{d^2 f}{d\xi^2} + \frac{1}{f}\left(\frac{df}{d\xi}\right)^2 = \frac{1}{4f^3} - \frac{1}{4}\left(\frac{\omega_p r_0}{v_{th}}\right)^2 \frac{1}{f}\left(1 - \frac{f_0^2}{f^2} I_4 - \frac{f_0^4}{f^4} I_5\right), \tag{34}$$

where

$$I_4 = \int \frac{t^{\beta_1}}{\left[1 + \frac{\alpha E_{00}^2 t}{f_0^2}\right]^{5/2}} dt$$

$$I_5 = \int \log(t) t^{\beta_1} \left[1 - \frac{1}{\left[1 + \frac{\alpha E_{00}^2 t}{f_0^2}\right]^{5/2}}\right] dt$$

$\beta_1 = \alpha_1 - 1$ where $\alpha_1 = \left(\frac{r_0 f_0}{a_0 f}\right)^2$.

https://doi.org/10.1017/S0263034611000504 Published online by Cambridge University Press

Now, Solution of Eq. (27) gives the second harmonic source equation as

$$N_{20} = -\frac{N_{0e}eE_{00}}{mf_0}\exp\left[\frac{-r^2}{2r_0^2f_0^2}\right]\left[\frac{y}{r_0^2f_0^2}\right]I_3\frac{1}{\left[\omega_0^2 - k_0^2v_{th}^2 - \omega_p^2\frac{N_{0e}}{N_0}\right]}. \quad (35)$$

4. SECOND HARMONIC POWER

Generated plasma wave can interact with the incident laser beam to produce second harmonic. The electric vector of the second harmonic (A_2) satisfy the following equation.

$$\nabla^2 A_2 + \frac{\omega_2^2}{c^2}\varepsilon_2(\omega_2)A_2 = \frac{\omega_p^2 N_1}{c^2 N_0}A_0, \quad (36)$$

where $\omega_2 = 2\ \omega_0$ and $\varepsilon_2\ (\omega_2)$ is the effective dielectric constant of plasma at the second harmonic frequency and is given by

$$\varepsilon_2(\omega_2) = \varepsilon_{2f}(\omega_2) + \Phi_2(A\cdot A^*), \quad (37)$$

where $\Phi_2\ (A\cdot A^*)$ is non-linear part of the dielectric constant and is given by

$$\Phi_2(A\cdot A^*) = \frac{\omega_p^2}{\omega_2^2}\left[1 - \frac{N_{0e}}{N_0}\right]. \quad (38)$$

Now, the solution of Eq. (36) can be written as

$$A_2 = A_{20}(r,z)\exp(-ik_2z) + A_{21}(r,z)\exp(-2ik_0z), \quad (39)$$

where A_{20} and A_{21} are the complex functions of their arguements and satisfy following equations

$$2ik_2\frac{\partial A_{20}}{\partial z} = \nabla_\perp^2 A_{20} + \Phi_2 A_{20}, \quad (40)$$

and

$$\nabla_\perp^2 A_{21} - 4ik_0\frac{\partial A_{21}}{\partial z} - 4k_0^2A_{21} + (\varepsilon_2(\omega_2))A_{21} = \frac{\omega_p^2 N_1}{c^2 N_0}A_0. \quad (41)$$

Now, from the definition of second order moment,

$$<a^2> = \frac{1}{I_0}\iint (x^2+y^2)A_{20}A_{20}^*dxdy, \quad (42)$$

where I_0 is zeroth order moment and can be written as

$$I_0 = \iint A_{20}A_{20}^*dxdy. \quad (43)$$

Now, solution of Eq. (40) is of the form,

$$A_{20}^2 = \frac{B'^2}{f_2^2}\exp\left(-\frac{r^2}{b_0^2f_2^2}\right). \quad (44)$$

Now, from Eqs. (42), (43), and (44), it can be shown that

$$I_0 = \pi B'^2 b_0^2, \quad (45)$$

and

$$<a^2> = a_0^2 f_2^2. \quad (46)$$

Now, with the help of Eqs. (42) and (46), one can get

$$\frac{d^2f_2}{d\xi^2} + \frac{1}{f_2}\left(\frac{df_2}{d\xi}\right)^2 = \frac{k_2^2}{k_0^2}\left[\frac{1}{f_2^3} - (\frac{\omega_p r_0}{c})^2\frac{1}{f_2}\left(1 - \frac{f_0^2}{f_2^2}I_6 - \frac{f_0^4}{f^4}I_7\right)\right], \quad (47)$$

where

$$I_6 = \int\frac{t^{\beta_2}}{\left[1+\frac{\alpha E_{00}^2 t}{f_0^2}\right]^{5/2}}dt$$

$$I_7 = \int log(t)t^{\beta_2}\left[1 - \frac{1}{\left[1+\frac{\alpha E_{00}^2 t}{f_0^2}\right]^{5/2}}\right]dt$$

$\beta_2 = \alpha_2 - 1$ where $\alpha_2 = \left(\frac{r_0f_0}{b_0f_2}\right)^2$.

Now, solution of equation (41) can be written as,

$$A_{21} = \frac{\omega_p^2 N_{20}E_{00}}{c^2\ N_0\ f_0}\exp\left[\frac{-r^2}{2r_0^2f_0^2}\right]\frac{1}{\left[k_2^2 - 4k_0^2 + \Phi_2(A\cdot A^*)\right]}. \quad (48)$$

Now, the constants B' and b_0 are obtained from the boundary condition that second harmonic wave is zero at $z = 0$.

$$B' = -\frac{\omega_p^2 N_{20}}{c^2\ N_0}\left[\frac{E_{00}}{k_2^2 - 4k_0^2 + \Phi_2(A\cdot A^*)}\right], \quad (49)$$

and $b_0 = r_0$, respectively.

Now, the second harmonic yield can be written as

$$\frac{P_2}{P_0} = 2\frac{\omega_p^4 N_{0e}^2(z=0)e^2E_{00}^2}{c^4N_0^2\ m^2f_0^2r_0^2}I_3^2I_8I_9I_{10}, \quad (50)$$

where

$$I_8 = \frac{1}{\left(k_2^2 - 4k_0^2 + \Phi_2(A\cdot A^*)\right)^2}, \quad (51)$$

$$I_9 = \frac{1}{\left(\omega_0^2 - k^2v_{th}^2 - \omega_p^2(1 + (\frac{\alpha E_{00}^2 exp(-1.0)}{2f_0^2})^{\frac{-5}{2}})\right)^2}, \quad (52)$$

https://doi.org/10.1017/S0263034611000504 Published online by Cambridge University Press

and

$$I_{10} = \left[\frac{f_2^4}{2(f_0^2+f_2^2)^2} + \frac{1}{8f_0^2} - \frac{cos(k_2 - 2k_0)z \cdot 2f_0f_2}{(3f_2^2+f_0^2)}\right]. \quad (53)$$

5. DISCUSSION

Eq. (22) governs the behavior of dimensionless beam width parameter f_0 of a beam as a function of dimensionless distance of propagation ξ $(=zc/\omega_0 r_0^2)$. This equation has been solved numerically for the following set of parameters; $\omega_0 = 1.778\times10^{14}$ rads^{-1}, $r_0 = 30\ \mu$m, $\alpha E_{00}^2 = 1.27,\ 1.37$, $\omega_p^2/\omega_0^2 = 0.17,\ 0.20$.

The first term on the right-hand side of Eq. (22) represents diffraction phenomenon of the laser beam. The second term which arises due to collisional non-linearity represents the non linear refraction. The relative magnitude of these terms determines the focusing/defocusing behavior of the beam. Figure 1 describes the variation of beam width parameter f_0 of a beam with normalized distance of propagation $\xi = zc/\omega_0 r_0^2$ for different values of intensity parameter $\alpha E_{00}^2 = 1.27, 1.37$ at a fixed value ofplasma density, $\omega_p^2/\omega_0^2 = 0.20$. It is observed from Figure 1 that extent of self-focusing of the beam decreases with increase in intensity. This is due to the fact that the non-linear refractive term is very sensitive to the intensity of the laser beam. So, with the increase in the intensity of the laser beam, diffractive term relatively becomes stronger but not enough to overpower the non-linear refractive term, as a result beam remains in a self-focusing mode. So, one can infer that as the intensity of the laser beam is increased, there is decrease in the non-linear term, which leads to decrease in self-focusing.

Figure 2 describes the variation of beam width parameter f_0 of a beam with normalized distance of propagation ξ for different values of plasma density $\omega_p^2/\omega_0^2 = 0.17, 0.20$ and at a fixed value of intensity parameter $\alpha E_{00}^2 = 1.27$. It is observed that with increase in plasma density extent of self-focusing of the beam increases. This is due to the fact that the refractive term dominates the diffractive term as we increase the value of plasma density.

Figure 3 depicts the variation of second harmonic yield P_2/P_0 with the normalized distance of propagation ξ for different values of intensity parameter $\alpha E_{00}^2 = 1.27,\ 1.37$ and at $\omega_p^2/\omega_0^2 = 0.20$. It is observed that second harmonic yield decreases with increase in intensity. This is due to the reason that with increase in intensity, the self-focusing of the laser beam decreases, which results in decrease in non-uniform heating of the carriers in the focal region and hence decreases the amplitude of plasma wave generation and ultimately the second harmonic yield.

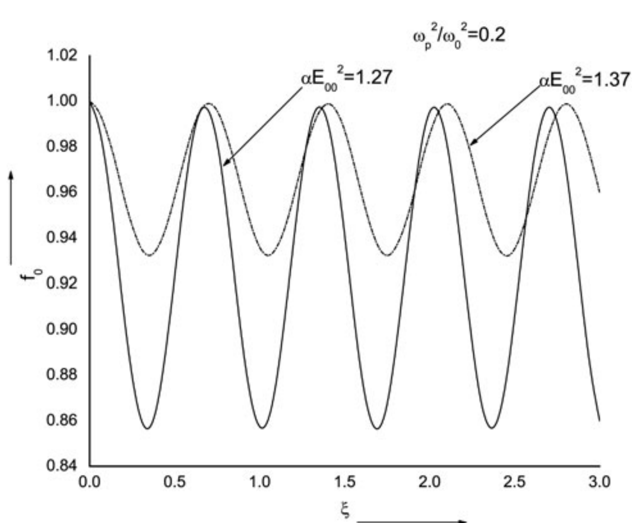

Fig. 1. Variation of beam width parameter f_0 against the normalized distance of propagation $\xi(=zc/\omega_0 r_0^2)$ for plasma density $\omega_p^2/\omega_0^2 = 0.20$ and for intensity $\alpha E_{00}^2 = 1.27, 1.37$.

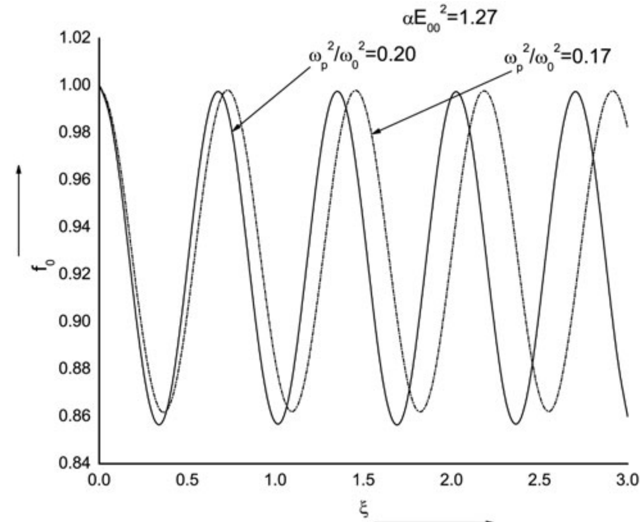

Fig. 2. Variation of beam width parameter f_0 against the normalized distance of propagation ξ for intensity $\alpha E_{00}^2 = 1.27$ and for plasma density $\omega_p^2/\omega_0^2 = 0.17, 0.20$.

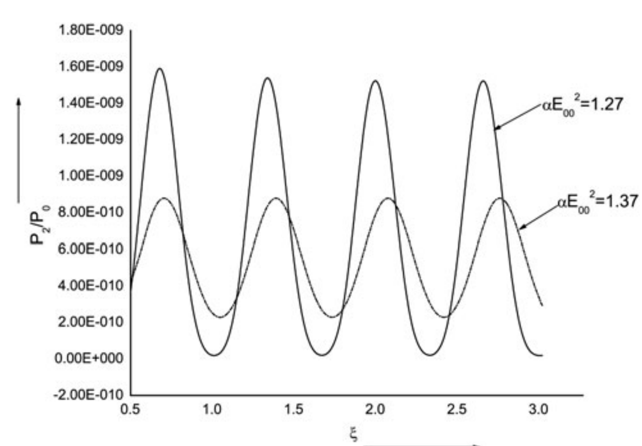

Fig. 3. Variation of second harmonic yield P_2/P_0 against the normalized distance of propagation ξ for plasma density $\omega_p^2/\omega_0^2 = 0.20$ and for intensity $\alpha E_{00}^2 = 1.27, 1.37$.

https://doi.org/10.1017/S0263034611000504 Published online by Cambridge University Press

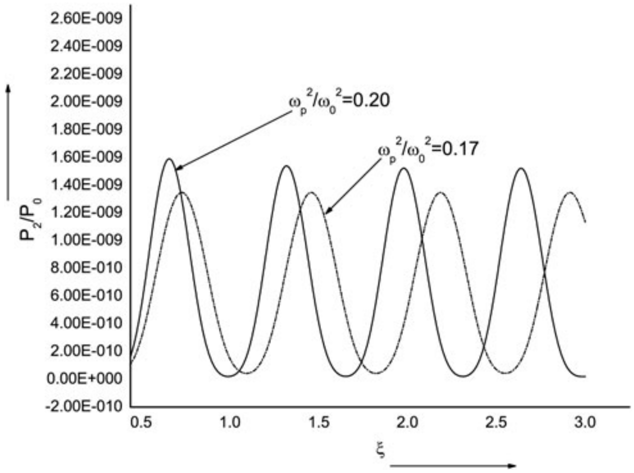

Fig. 4. Variation of second harmonic yield P_2/P_0 against the normalized distance of propagation ξ for intensity $\alpha E_{00}^2 = 1.27$ and for plasma density $\omega_p^2/\omega_0^2 = 0.17, 0.20$.

Figure 4 depicts the variation of second harmonic yield P_2/P_0 with the normalized distance of propagation ξ for different values of plasma density $\omega_p^2/\omega_0^2 = 0.17, 0.20$ and at fixed value of intensity parameter $\alpha E_{00}^2 = 1.27$. It is observed from Figure 4 that second harmonic yield increases with increase in plasma density. This is due to the reason that with increase in plasma density, the self-focusing of the laser beam increases which results in increase in non-uniform heating of the carriers in the focal region, which in turn leads to increase the amplitude of plasma wave generation and hence the second harmonic yield.

6. CONCLUSION

In the present work, moment theory has been developed to study the second harmonic generation of laser beam, when Collisional non-linearity is operative. Following important observations are made from present analysis.

(1) The effect of increase of laser beam intensity/plasma density is to increase/decrease the self-focusing length.

(2) Self-focusing of the laser beam becomes stronger with increase in the plasma density and becomes weaker with increase in laser beam intensity.

(3) There is an increase in the second harmonic yield with increase in the plasma density and also with decrease in laser beam intensity. Thus laser power and plasma density parameters are crucial to harmonic generation.

Results of the present analysis are useful in understanding the physics of high power laser driven fusion in which second harmonic generation play important role.

ACKNOWLEDGMENTS

The authors are thankful to the Department of Science and Technology(DST), Government of India for providing financial assistance for carrying out this work.

REFERENCES

Akhmanov, S.A., Sukhorukov, A.P. & Khokhlov, R.V. (1968). Self-focusing and diffraction of light in a non-linear medium. *Sov. Phys.Uspekhi*. **10**, 609–636.

Amendt, P., Eder, D.C. & Wilks, S.C. (1991). X-ray lasing by optical-field-induced ionization. *Phys. Rev. Lett*. **66**, 2589–2592.

Baton, S.D., Baldis, H.A., Jalinaud, T. & Labaune, C. (1993). Fine-Scale Spatial and Temporal Structures of Second-Harmonic Emission from an Underdense Plasma. *Europhys. Lett*. **23**, 191.

Brandi, F., Giammanco, F. & Ubachs, W. (2006). Spectral Redshift in Harmonic Generation from Plasma Dynamics in the Laser Focus. *Phys. Rev. Lett*. **96**, 123904.

Burnett, N.H. & Corkum, P.B. (1989). Cold-plasma production for recombination extreme ultraviolet lasers by optical-field-induced ionization. *J. Opt. Soc. Am. B* **6**, 1195–1199.

Deutsch, C., Furukawa, H., Mima, K., Murakami, M. & Nishihara, K. (1996). Interaction Physics of the Fast Ignitor Concept. *Phys Rev Lett*. **77**, 2483–2486.

Engers, T., Fendel, W., Schuler, H., Schulz, H. & von der Linde, D. (1991). Second-harmonic generation in plasmas produced by femtosecond laser pulses. *Phys. Rev. A* **43**, 4564–4567.

Esarey, E., Sprangle, P., Krall, J. & Ting, A. (1996). Overview of plasma-based accelerator concepts. *IEEE Trans Plasma Sci*. **24**, 252–288.

Esarey, E., Ting, A., Sprangle, P., Umstadter, D. & Liu, X. (1993). Nonlinear analysis of relativistic harmonic generation by intense lasers in plasmas. *IEEE Trans. Plasma Sci*. **21**, 95–104.

Fibich, G. (1996). Small Beam Non-paraxiality arrests Self-Focusing of Optical Beams. *Phys. Rev. Lett*. **76**, 4356–4359.

Ganeev, R.A., Suzuki, M., Baba, M. & Kuroda, H. (2007). High-order harmonic generation from laser plasma produced by pulses of different duration. *Phys. Rev. A* **76**, 023805.

Ginzburg, V.L. (1970). *The Propagation of Electromagnetic Waves in Plasmas*. Pergamon: Oxford, NY.

Gupta, M.K., Sharma, R.P. & Mahmoud, S.T. (2007). Generation of plasma wave and third harmonic generation at ultra relativistic laser power. *Laser part. Beams* **25**, 211–218.

Hafizi, B., Ting, A., Sprangle, P. & Hubbard, R.F. (2000). Relativistic focusing and ponderomotive channeling of intense laser beams. *Phys. Rev. E* **62**, 4120–4125.

Hora, H. (1981). *Physics of Laser Driven Plasmas*. Wiley: New York.

Jones, R.D., Mead, W.C., Coggeshall, S.V., Aldrich, C.H., Norton, J.L., Pollak, G.D. & Wallace, J.M. (1988). Self-focusing and filamentation of laser light in high Z plasmas. *Phys. Fluids* **31**, 1249–1272.

Lam, J.F., Lippmann, B. & Tappert, F. (1977). Self-trapped laser beams in plasma. *Phys. Fluids* **20**, 1176–1179.

Liu, C.S. & Kaw, P.K. (1976). *Advances in Plasma Physics*. **6**, 83.

Malka, V., Modena, A., Najmudin, Z., Dangor, A.E., Clayton, C.E., Marsh, K.A., Joshi, C., Danson, C., Neely, D. & Walsh, F.N. (1997). Second harmonic generation and its interaction with relativistic plasma waves driven by forward Raman instability in underdense plasmas. *Phys. Plasmas*. **4**, 1127–1131.

Merdji, H., Guizard, S., Martin, P., Petite, G., Quéré, F., Carré, B., Hergott, J.F., Dé Roff, L., Salie'Res, P., Gobert, O., Meynadier, P. & Perdrix, M. (2000). Ultrafast electron relaxation measurements on a-SiO2 using high-order harmonics generation. *Laser Part. Beams* **18**, 489–494.

Nuzzo, S., Zarcone, M., Ferrante, G. & Basile, S. (2000). A simple model of high harmonic generation in a plasma. *Laser Part. Beams* **18**, 483–487.

Ozaki, T., Kieffer, J.C., Toth, R., Fourmaux, S. & Bandulet, H. (2006). Experimental prospects at the Canadian advanced laser light source facility. *Laser Part. Beams* **24**, 101–106.

Ozaki, T., Bom, L.E., Ganeev, R., Kieffer, J.C., Suzuki, M. & Kuroda, H. (2007). Intense harmonic generation from silver ablation. *Laser Part. Beams* **25**, 321–325.

Ozaki, T., Bom, L.E. Ganeev, R.A. (2008). Extending the capabilities of ablation harmonics to shorter wavelengths and higher intensity. *Laser Part. Beams* **26**, 235–240.

Parashar, J. & Pandey, H.D. (1992). Second-harmonic generation of laser radiation in a plasma with a density ripple. *IEEE Trans. Plasma Sci*. **20**, 996.

Schifano, E., Baton, S.D., Biancalana, V., Giuletti, A., Giuletti, D., Labaune, C. & Renard, N. (1994). Second harmonic emission from laser-preformed plasmas as a diagnostic for filamentation in various interaction conditions. *Laser Part. Beams* **12**, 435–444.

Shi, Y.J. (2007). Laser electron accelerator in plasma with adiabatically attenuating density. *Laser Part. Beams* **25**, 259–265.

Singh, A., & Singh, N. (2010). Optical guiding of a laser beam in an axially nonuniform plasma channel. *Laser Part. Beams* **28**, 263–268.

Singh, A., & Singh, N. (2011*a*). Relativistic guidance of an intense laser beam through an axially non-uniform plasma channel. *Laser Part. Beams*. DOI: 10.1017/S0263034611000292

Singh, A., & Singh, N. (2011*b*). Guidance of a Laser Beam Through an Axially Non-Uniform Plasma Channel in the Weakly Relativistic Limit. Fusion Energ DOI: 10.1007/s10894-011-9425-0.

Singh, A., & Singh, N. (2011*c*). Guiding of a Laser Beam in Collisionless Magnetoplasma Channel. Published in *J. Opt. Soc. Am. B*

Singh, A., & Walia, K. (2010). Relativistic self-focusing and self-channeling of Gaussian laser beam in plasma. *Applied Physics B: Lasers and Opticss* **101**, 617–622.

Singh, A., & Walia, K. (2011). Self-focusing of Gaussian Laser Beam Through Collisionless Plasmas and Its Effect on Second Harmonic Generation. Fusion Energ DOI 10.1007/s10894-011-9426-z.

Sinha, S.K. & Sodha, M.S. (1980). Transverse self-focusing of a gaussian beam: Moment method. *Phys. Rev. A* **21**, 633–638.

Sodha, M.S., Ghatak, A.K. & Tripathi, V.K. (1976). *Progress in Optics*. Amsterdam: North Holland. 13, p 171.

Sodha, M.S., Ghatak, A.K. & Tripathi, V.K. (1974). *Self Focusing of Laser Beams in Dielectrics, Semiconductors and Plasmas* Tata-McGraw-Hill: Delhi.

Sodha, M.S., Kaushik, S.C. & Kumar, A. (1975). Steady state and transient self-interaction of a laser beam in a strongly ionized moving plasma. *App. Phys*. **7**, 187–193.

Sodha, M.S. & Kaw, P.K. (1969). Harmonics in Plasmas in Advances in Electronics and Electron Physics, edited by L. Marton. Academic: New York. Vol. **27** p. 187–293.

Sodha, M.S., Khanna, R.K. & Tripathi, V.K. (1973). Self-focusing of a laser beam in a strongly ionized plasma. *Opto-electronics* **5**, 533–538.

Sodha, M.S., Sinha, S.K. & Sharma, R.P. (1979). The self-focusing of laser beams in magnetoplasmas: the moment theory approach. *J. Phys. D: Appl. Phys*. **12**, 1079–1091.

Tajima, T. & Dawson, J.M. (1979). Laser Electron Accelerator. *Phys. Rev. Lett*. **43**, 267–270.

Umstadter, D. (2001). Review of physics and applications of relativistic plasmas driven by ultra-intense lasers. *Phys. Plasmas* **8**, 1774–1785.

Vlasov, S.N., Petrishchev, V.A. & Talanov, V.I. (1971). Averaged Description of Wave Beams in Linear and Nonlinear Media(the Method of Moments) *Radiophys. Quantum Electron*. **14**, 1062–1070.

Walia, K. & Singh, A. (2011). Comparison of Two Theories for the Relativistic Self Focusing of Laser Beams in Plasma. *Contrib. Plasma Phys*. **51**, 375–381.

Young, P.E., Baldis, H.A., Johnston, T.W., Kruer, W.L. & Estabrook, K.G. (1989). Filamentation and second-harmonic emission in laser-plasma interactions. *Phys. Rev. Lett*. **63**, 2812–2815.