Probability in the Engineering and Informational Sciences, **33**, 2019, 121–135.
doi:10.1017/S0269964818000049

OPTIMAL DESIGN OF SIMPLE STEP-STRESS ACCELERATED LIFE TESTS FOR ONE-SHOT DEVICES UNDER EXPONENTIAL DISTRIBUTIONS

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This paper considers simple step-stress accelerated life tests (SSALTs) for one-shot devices. The one-shot device is an item that cannot be used again after the test, for instance, munitions, rockets, and automobile air-bags. Either left-or right-censored data are collected instead of actual lifetimes of the devices under test. An expectation-maximization algorithm is developed here to find the maximum likelihood estimates of the model parameters based on one-shot device testing data collected from simple SSALTs. Furthermore, the asymptotic variance of the mean lifetime under normal operating conditions is determined under the expectation-maximization framework. On the other hand, the optimal design that minimizes the asymptotic variance of the estimate of the mean lifetime under normal operating conditions in terms of three decision variables, including stress levels, inspection times, and sample allocation is discussed. A procedure then is presented to determine the decision variables when a range of stress levels and the termination time of the test as well as normal operating conditions of the devices are given. The properties of the optimal design and the effects of errors in pre-specified planning values of the model parameters are also investigated. Comprehensive simulation studies show that the procedure is quite reliable for the design of simple SSALTs.

Keywords: cumulative exposure model, EM algorithm, exponential distribution, one-shot devices, optimal design, step-stress accelerated life-tests

1. INTRODUCTION

Due to technology advances coupled with customer expectations on product quality, a significant number of failures occurred in a short period of time is rare. It results in an inevitable challenge to efficiently collect sufficient failure time data under normal operating conditions within a limited time. Therefore, accelerated life tests have become common and popular in reliability engineering. In accelerated life tests, devices are exposed to higher-than-normal stress levels to induce quick failures. An accelerated failure time model coupled with an acceleration model that describes life–stress relationships are then used to extrapolate the collected data outside the elevated stress levels, so as to estimate the mean lifetime

 0269-9648/18 $25.00

under the normal operating conditions. Escobar and Meeker [17] provided a comprehensive literature review on accelerated life tests, which outlines some basic concepts including the most frequently used accelerated failure time models, acceleration models, and sensitivity analyses.

There are many types of accelerated life tests. Constant-stress accelerated life tests wherein each device is subject to only one pre-specific stress level are frequently used. On the other hand, step-stress accelerated life tests (SSALTs) apply stress to devices in the way that stress levels will be changed at pre-specified times. Compared with constant-stress accelerated life tests, SSALTs have advantages that require less samples and are more efficient and less costly to collect lifetime data. Thus, SSALTs have attracted great attention in the literature, and there are three fundamental models for the effect of increased stress levels on the lifetime distribution of a device. These models are tampered random variable model [16], tampered failure-rate model [14], and cumulative exposure model [27,30,31]. Nelson [30] firstly introduced the cumulative exposure model for the SSALTs. The cumulative exposure model assumes that the remaining lifetime of a device depends on the current cumulative fraction failed and current stress, regardless how the fraction is accumulated. Moreover, surviving devices will fail according to the cumulative distribution for the current stress but starting at the previously accumulated fraction failed. Also, only the level of stress has an effect on life but the change in stress does not. The cumulative exposure model is widely used in the reliability study. Miller and Nelson [27], and Alhadeed and Yang [2] studied the optimal simple SSALT plans with the cumulative exposure model under exponential and log-normal distributions, respectively. Bai, Kim, and Lee et al. [3] extended the results of Miller and Nelson [27] to censoring schemes. Ling et al. [23] considered a load-sharing model with the cumulative exposure model to analyze series systems with active redundancy. Thus, it is of great interest to consider the cumulative exposure model in the design of simple SSALTs in this paper.

Many authors have studied SSALTs. Gouno [19] analyzed data collected from SSALTs and subsequently [20] presented optimal design of SSALTs. Zhao and Elsayed [39] analyzed data of light intensity of light emitting diodes collected from SSALTs with four stress levels under Weibull and log-normal distributions. Xiong Zhu, and Ji [38] considered simple step-stress life tests subject to type II censoring under exponential distributions, wherein the stress change time from a low-level stress to a high-level stress is random, and presented exact confidence intervals for the model parameters. However, the literature on SSALTs for one-shot devices is scarce.

The one-shot device that performs its function only once, cannot be used for testing again. For each device, only the condition at an inspection time can be observed in the test. Binary data are collected and the exact failure time cannot be obtained from the test. As a result, the lifetime of the device is either right-censored or left-censored. For instance, Fan, Balakrishnan, and Chang [18] considered electro-explosive devices that are detonated by inducing a current to excite inner powder. Those devices cannot be used any further after detonation, regardless of whether the detonation is successful or not. Moreover, Morris [28] analyzed battery data from destructive life-tests. Shaked and Singpurwalla [35] assessed the effect of various stress levels on the probability of damage to the hull in a submarine based on binary data. Sohn [36] studied one-shot device testing data under destructive inspection. Newby [32] discussed the maintenance and monitoring of one-shot devices such as fire extinguishers and munitions. The lifetimes of those tested items cannot be obtained from the tests. Balakrishnan, Ling, and So [10] provided some popular reliability models for analyzing one-shot device testing data collected from constant-stress accelerated life tests. Analysis of one-shot device testing data has been recently received a great attention in reliability engineering [4–7,11–13,24]. However, the previously published papers considered constant-stress accelerated life tests for one-shot devices. Many reliability studies show that SSALTs are more efficient and less costly than constant-stress accelerated life tests to collect lifetime information of the devices. It is therefore of great interest to study SSALTs for one-shot devices.

Due to the presence of heavily censored data in this study, an expectation-maximization (EM) algorithm is presented to find the maximum likelihood estimates (MLEs) of the model parameters in this paper. The EM algorithm is a suitable and powerful technique to effectively obtain the MLEs in the presence of censored data, and thus many works have been done on the EM algorithm. McLachlan and Krishnan [26] provided an overview on all pertinent details. Ng, Chan, and Balakrishnan [33], Scheike and Sun [34], Kundu and Dey [21], Nandi and Dewan [29], Chen and Lio [15], and Balakrishnan and Mitra [9] developed EM algorithms for various types of censored data. In this paper, an EM algorithm is developed here to find the MLEs of the model parameters based on one-shot device testing data collected from simple SSALTs. Furthermore, the asymptotic variance of the mean lifetime under normal operating conditions is determined under the EM framework.

Optimal design of accelerated life tests has a long history. Miller and Nelson [27] discussed optimal test plans that minimize the asymptotic variance of the MLE of the mean lifetime under normal operating conditions. Later, Bai et al. [3] studied similar optimal simple SSALT plans. Alhadeed and Yang [1] discussed optimal simple step-stress test plans for a specific model. Balakrishnan and Ling [8] presented constant-stress accelerated life test plans for one-shot devices. Thus, it is of great interest to obtain simple SSALT plans for one-shot devices, which minimize the asymptotic variance of the MLE of the mean lifetime under normal operating conditions in terms of three decision variables, namely, stress levels, inspection times, and sample allocation. A procedure then is presented to determine the decision variables when a range of stress levels and the termination time of the test as well as normal operating conditions of the devices are given. The properties of the optimal design and the effects of errors in pre-specified planning values of the model parameters are also investigated.

The rest of this paper is organized as follows. Section 2 formulates the problem of simple SSALTs for one-shot devices under exponential distributions. The corresponding mean lifetime under normal operating conditions is also derived. Section 3 presents the EM algorithm for finding the MLEs of the model parameters as well as the mean lifetime. Also, the information matrix and the asymptotic variance of the MLEs are presented. Section 4 describes a procedure for the determination of the optimal design of simple SSALTs for one-shot devices. Section 5 presents several numerical examples to illustrate the proposed procedure and also the results of a sensitivity analysis to examine the robustness of the optimal design to misspecification of planning values of the model parameters. Finally, some concluding remarks are made in Section 6.

2. MODEL DESCRIPTION

Consider simple SSALTs wherein the stress level is changed only once from the test. Suppose that $0 < IT_1 < IT_2, 0 < K_1 < K$, and $x_1 < x_2$, and that all K devices are exposed to the same initial stress level x_1. K_1 devices are selected to be tested at a pre-specified inspection time IT_1, the number of failures n_1 are recorded. Then, the stress level is increased to x_2. All the remaining $K_2 = K - K_1$ devices are to be tested at another pre-specified inspection time IT_2, and the number of failures n_2 are recorded. The one-shot device testing data thus observed can be summarized as in Table 1. Given one-shot device testing data $\boldsymbol{z} = \{IT_i, K_i, n_i, x_i, i = 1, 2\}$.

Table 1. One-shot device testing data under simple step-stress accelerated life-tests.

Stage	Inspection time	# of tested devices	# of failures	Stress level
1	IT_1	K_1	n_1	x_1
2	IT_2	K_2	n_2	x_2

Let T denote the lifetime of the device that follows exponential distributions with corresponding cumulative hazard function, reliability function, and probability density function as

$$H(t) = \begin{cases} \alpha_1 t, & 0 < t \leq IT_1 \\ \alpha_1 IT_1 + \alpha_2(t - IT_1), & t > IT_1 \end{cases}, \tag{1}$$

$$R(t) = \exp(-H(t)) = \begin{cases} \exp(-\alpha_1 t), & 0 < t \leq IT_1 \\ \exp(-(\alpha_1 IT_1 + \alpha_2(t - IT_1))), & t > IT_1 \end{cases}, \tag{2}$$

and

$$f(t) = -R'(t) = \begin{cases} \alpha_1 \exp(-\alpha_1 t), & 0 < t \leq IT_1 \\ \alpha_2 \exp(-(\alpha_1 IT_1 + \alpha_2(t - IT_1))), & t > IT_1 \end{cases}, \tag{3}$$

where $\alpha_1 > 0, \alpha_2 > 0$ are the rate parameters at stages 1 and 2, respectively. We further assume that the rate parameters are related to the stress level in a log-linear form [37] as

$$\alpha_i = \exp\left(a_0 + a_1 x_i\right). \tag{4}$$

The log-linear form includes many popular acceleration models, such as Arrhenius, inverse power law, and Eyring models (possibly transformed stress levels), and is frequently used in accelerated life tests.

For notational convenience, we denote $\boldsymbol{\theta} = \{a_0, a_1\}$ as the model parameters to be estimated. Furthermore, the mean lifetime under the normal operating condition x_0 is given by

$$\mu(x_0) = \alpha_0^{-1} = \exp\left(-a_0 - a_1 x_0\right). \tag{5}$$

3. EM ALGORITHM

The EM algorithm is a powerful technique for finding the MLEs of the model parameters in the presence of censored data. Interested readers may refer to [9,11,26,33] for its other applications. The EM algorithm simply proceeds by alternating between the expectation (E-step) and the maximization step (M-step). At the E-step,

$$Q(\boldsymbol{\theta}, \boldsymbol{\theta}^{(m)}) = E_{\boldsymbol{\theta}^{(m)}}[\ell_c(\boldsymbol{\theta})|\boldsymbol{z}], \tag{6}$$

the expected log-likelihood of the complete data conditional on the observed data, $\boldsymbol{z}$, and the current estimates of the parameters, $\boldsymbol{\theta}^{(m)}$, is computed. At the M-step, updated estimates of the parameters, $\boldsymbol{\theta}^{(m+1)}$, are computed by maximizing the expected log-likelihood function, $Q(\boldsymbol{\theta}, \boldsymbol{\theta}^{(m)})$. The updated estimates of the parameters, $\boldsymbol{\theta}^{(m+1)}$, are then used to compute the expected log-likelihood of the complete data, $Q(\boldsymbol{\theta}, \boldsymbol{\theta}^{(m+1)})$ at the E-step. This process is repeated until convergence occurs to a desired level of accuracy. It can be seen that the

https://doi.org/10.1017/S0269964818000049 Published online by Cambridge University Press

EM algorithm involves approximating the censored data at the E-step and maximizing the corresponding likelihood function at the M-step in each iteration.

In the EM algorithm, we first consider the log-likelihood function based on complete data given by

$$\begin{aligned}\ell_c(\boldsymbol{\theta}) &= \sum_{i=1}^{2}\sum_{j=1}^{K_i}\log(f(t_{ij};\boldsymbol{\theta})) + c\\ &= \sum_{i=1}^{2}\sum_{j=1}^{K_i}(\log\alpha_1 - \alpha_1 t_{ij})\,\mathbb{I}_{0<t_{ij}\le IT_1} + (\log\alpha_2 - \alpha_1 IT_1 - \alpha_2(t_{ij}-IT_1))\,\mathbb{I}_{t_{ij}>IT_1} + c,\end{aligned} \tag{7}$$

where $\mathbb{I}$ is an indicator function and c is a constant.

In our case, the expected log-likelihood of the complete data is given by

$$Q(\boldsymbol{\theta},\boldsymbol{\theta}^{(m)}) = \sum_{i=1}^{2} n_{i1}^*\,(\log\alpha_1 - \alpha_1 t_1^*) + n_{i2}^*\,(\log\alpha_2 - \alpha_1 IT_1 - \alpha_2(t_2^* - IT_1)) + c, \tag{8}$$

where n_{i1}^* and n_{i2}^* are the expected numbers of failures in the ith stage at the first and second inspection times, IT_1 and IT_2, respectively. It is noted that $n_{11}^* = n_1, n_{12}^* = K_1 - n_1, n_{21}^* = n_2(1-R(IT_1))/(1-R(IT_2))$, and $n_{22}^* = K_2 - n_2(1-R(IT_1))/(1-R(IT_2))$. Moreover, t_1^* and t_2^* are unobserved and required computation at the E-step.

At the M-step, we find the next iterate of the estimate $\boldsymbol{\theta}^{(m+1)}$ by maximizing the conditional expectation $Q(\boldsymbol{\theta},\boldsymbol{\theta}^{(m)})$, for which the first-order derivatives of $Q(\boldsymbol{\theta},\boldsymbol{\theta}^{(m)})$ with respect to the model parameters a_0 and a_1 are set to zero. The solution of the system of equations is the estimates of the model parameters. The required first-order derivatives are, respectively,

$$\begin{aligned}\frac{\partial Q(\boldsymbol{\theta},\boldsymbol{\theta}^{(m)})}{\partial a_0} &= \sum_{i=1}^{2} n_{i1}^*\,(1-\alpha_1 t_1^*) + n_{i2}^*\,(1-\alpha_1 IT_1 - \alpha_2(t_2^*-IT_1))\\ &= (n_{11}^* + n_{21}^* + n_{12}^* + n_{22}^*) - \alpha_1 M_1 - \alpha_2 M_2,\end{aligned} \tag{9}$$

$$\begin{aligned}\frac{\partial Q(\boldsymbol{\theta},\boldsymbol{\theta}^{(m)})}{\partial a_1} &= \sum_{i=1}^{2} n_{i1}^*\,(x_1 - x_1\alpha_1 t_1^*) + n_{i2}^*\,(x_2 - x_1\alpha_1 IT_1 - x_2\alpha_2(t_2^*-IT_1))\\ &= ((n_{11}^* + n_{21}^*)x_1 + (n_{12}^* + n_{22}^*)x_2) - \alpha_1 M_1 x_1 - \alpha_2 M_2 x_2,\end{aligned} \tag{10}$$

where

$$M_1 = n_{11}^* t_1^* + n_{21}^* t_1^* + n_{12}^* IT_1 + n_{22}^* IT_1, \tag{11}$$

$$M_2 = (n_{12}^* + n_{22}^*)(t_2^* - IT_1). \tag{12}$$

Then, we obtain the estimates of the model parameters, a_0 and a_1, respectively, as follows.

$$\hat{a}_0 = \frac{x_1\log\hat{\alpha}_2 - x_2\log\hat{\alpha}_1}{x_1 - x_2}, \tag{13}$$

$$\hat{a}_1 = \frac{\log\hat{\alpha}_1 - \log\hat{\alpha}_2}{x_1 - x_2}, \tag{14}$$

where

$$\hat{\alpha}_1 = \left(t_1^* + IT_1\left(\frac{n_{12}^* + n_{22}^*}{n_{11}^* + n_{21}^*}\right)\right)^{-1}, \tag{15}$$

$$\hat{\alpha}_2 = (t_2^* - IT_1)^{-1}. \tag{16}$$

At the E-step, the required conditional expectations are

$$\begin{aligned} t_1^* &= E[T_{ij}|0 < T_{ij} \leq IT_1] = \frac{\int_0^{IT_1} t\alpha_1 \exp(-\alpha_1 t)\, dt}{1 - R(IT_1)} \\ &= \frac{1 - \exp(-\alpha_1 IT_1) - \alpha_1 IT_1 \exp(-\alpha_1 IT_1)}{\alpha_1(1 - \exp(-\alpha_1 IT_1))} \\ &= \frac{1}{\alpha_1} - IT_1\left(\frac{R(IT_1)}{1 - R(IT_1)}\right), \end{aligned} \tag{17}$$

$$\begin{aligned} t_2^* &= E[T_{ij}|T_{ij} > IT_1] = \frac{\int_{IT_1}^{\infty} t\alpha_2 \exp(-(\alpha_1 IT_1 + \alpha_2(t - IT_1)))\, dt}{R(IT_1)} \\ &= \frac{\exp(-\alpha_1 IT_1) + \alpha_2 IT_1 \exp(-\alpha_1 IT_1)}{\alpha_2 \exp(-\alpha_1 IT_1)} \\ &= \frac{1}{\alpha_2} + IT_1. \end{aligned} \tag{18}$$

Moreover, the choice of initial values of the model parameters is often important issue in the EM algorithm. When K_1 and K_2 are sufficiently large,

$$p_1 = \frac{n_1}{K_1} \approx 1 - \exp(-\alpha_1 IT_1), \tag{19}$$

$$p_2 = \frac{n_2}{K_2} \approx 1 - \exp(-\alpha_1 IT_1 - \alpha_2(IT_2 - IT_1)), \tag{20}$$

respectively. Then, let

$$X_1 = \begin{bmatrix} IT_1 & 0 \\ IT_1 & IT_2 - IT_1 \end{bmatrix}, \quad X_2 = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \end{bmatrix}, \quad A_1 = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}, \quad A_2 = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix},$$
$$Y = \begin{bmatrix} -\log(1 - p_1) \\ -\log(1 - p_2) \end{bmatrix}.$$

It is observed that

$$X_1 A_1 = Y \tag{21}$$

$$X_2 A_2 = -\log(A_1). \tag{22}$$

Thus, the EM algorithm proceeds as follows:

Step 1: (Initial Step) compute $A_1 = (X_1' X_1)^{-1} X_1' Y$;
Step 2: (Initial Step) compute $A_2 = -(X_2' X_2)^{-1} X_2' \log(A_1)$;
Step 3: (E Step) compute t_1^* and t_2^* by using Eqs. (17) and (18), respectively;
Step 4: (M Step) compute $\hat{\alpha}_1$ and $\hat{\alpha}_2$ by using Eqs. (15) and (16), respectively;

Step 5: (M Step) compute $\hat{a}_0$ and $\hat{a}_1$ by using Eqs. (13) and (14), respectively;

Step 6: repeat Steps 3–5 until convergence occurs to a desired level of accuracy.

In reliability engineering, it is also of interest to examine the variability in the estimates. However, there is no closed-form expression for the MLEs of the model parameters and that we cannot develop an exact inference. We describe here the information matrix to estimate the standard errors. When the EM algorithm is employed for finding the MLEs based on censored data, the Missing Information Principle developed by Louis [25] is commonly used to extract the information matrix. This method requires complete information and missing information matrices. These matrices are given by

$$I_{\text{complete}} = -E\left[\frac{\partial^2(\ell_c(\boldsymbol{\theta}))}{\partial\boldsymbol{\theta}^2}\right] \quad \text{and} \quad I_{\text{missing}} = -E\left[\frac{\partial^2(\log(f(t_{ij}|\boldsymbol{z},\boldsymbol{\theta})))}{\partial\boldsymbol{\theta}^2}\right], \tag{23}$$

respectively. Using these matrices, we will then obtain the information matrix as

$$I(\boldsymbol{\theta}) = I_{\text{complete}} - I_{\text{missing}}. \tag{24}$$

Subsequently, Balakrishnan and Ling [6] found that, when failure times are all censored, the information matrix by using the missing information principle is equivalent to the expectation of the second-derivatives of the observed log-likelihood function. In the one-shot device testing data, the observed log-likelihood function is given by

$$\ell(\boldsymbol{\theta}) = \sum_{i=1}^{2} n_i \log(1 - R(IT_i;\boldsymbol{\theta})) + (K_i - n_i)\log(R(IT_i;\boldsymbol{\theta})). \tag{25}$$

The second-derivative of the observed log-likelihood function is derived as follows:

$$\begin{aligned}\frac{\partial^2\ell(\boldsymbol{\theta})}{\partial a_p \partial a_q} =& \sum_{i=1}^{2}\left(\frac{\partial^2 R(IT_i;\boldsymbol{\theta})}{\partial a_p \partial a_q}\right)\left(-\frac{n_i}{1-R(IT_i;\boldsymbol{\theta})} + \frac{K_i - n_i}{R(IT_i;\boldsymbol{\theta})}\right)\\ &- \sum_{i=1}^{2}\left(\frac{\partial R(IT_i;\boldsymbol{\theta})}{\partial a_p}\right)\left(\frac{\partial R(IT_i;\boldsymbol{\theta})}{\partial a_q}\right)\left(\frac{n_i}{(1-R(IT_i;\boldsymbol{\theta}))^2} + \frac{K_i - n_i}{(R(IT_i;\boldsymbol{\theta}))^2}\right),\end{aligned} \tag{26}$$

where

$$\frac{\partial R(IT_i;\boldsymbol{\theta})}{\partial a_0} = -d_{i0}R(IT_i), \tag{27}$$

$$\frac{\partial R(IT_i;\boldsymbol{\theta})}{\partial a_1} = -d_{i1}R(IT_i), \tag{28}$$

$$\frac{\partial^2 R(IT_i;\boldsymbol{\theta})}{\partial a_0 \partial a_0} = -d_{i0}R(IT_i) + d_{i0}^2 R(IT_i), \tag{29}$$

$$\frac{\partial^2 R(IT_i;\boldsymbol{\theta})}{\partial a_0 \partial a_1} = -d_{i1}R(IT_i) + d_{i1}d_{i0}R(IT_i), \tag{30}$$

$$\frac{\partial^2 R(IT_i;\boldsymbol{\theta})}{\partial a_1 \partial a_1} = -d_{i2}R(IT_i) + d_{i1}^2 R(IT_i), \tag{31}$$

$$d_{1m} = \alpha_1 IT_1 x_1^m, \tag{32}$$

$$d_{2m} = \alpha_1 IT_1 x_1^m + \alpha_2(IT_2 - IT_1)x_2^m. \tag{33}$$

Furthermore, the information matrix is

$$I(\boldsymbol{\theta}) = -E\left[\frac{\partial^2 \ell(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}^2}\right]. \tag{34}$$

The asymptotic covariance matrix of the MLEs of the model parameters can then be obtained by inverting the above information matrix. The variance of the MLEs of the mean lifetime under normal operating conditions can also be computed by using the delta method [6,7] that requires the asymptotic covariance matrix of the MLEs of the model parameters and the first-order derivatives of the mean lifetime with respect to the model parameters. These derivatives are as follows.

$$\begin{aligned}\frac{\partial \mu(x_0)}{\partial a_0} &= -\alpha_0^{-1}\\ \frac{\partial \mu(x_0)}{\partial a_1} &= -\alpha_0^{-1} x_0.\end{aligned} \tag{35}$$

The variance of the MLEs of the mean lifetime under the normal operating condition is

$$V_\mu = P' V_{\boldsymbol{\theta}} P, \tag{36}$$

where $V_{\boldsymbol{\theta}} = I^{-1}(\boldsymbol{\theta})$ and $P = [-\alpha_0^{-1}, -\alpha_0^{-1} x_0]'$ is a 2×1 column vector.

Since $\hat{a}_0$ and $\hat{a}_1$ are the MLEs of the model parameters of a_0 and a_1, it follows that $\hat{\boldsymbol{\theta}} \sim N_2(\boldsymbol{\theta}, V_{\boldsymbol{\theta}})$, where

$$\boldsymbol{\theta} = \begin{pmatrix} a_0 \\ a_1 \end{pmatrix} \quad \text{and} \quad V_{\boldsymbol{\theta}} = \begin{pmatrix} \sigma_0^2 & \sigma_0\sigma_1\rho \\ \sigma_0\sigma_1\rho & \sigma_1^2 \end{pmatrix}. \tag{37}$$

Then, it can be easily seen that the logarithm of the estimated mean lifetime under the normal operating condition, $\log(\hat{\mu}(x_0)) = \hat{a}_0 + \hat{a}_1 x_0$, is asymptotically normal distributed.

4. OPTIMAL DESIGN OF SSALT

Furthermore, the information matrix is useful for design of SSALTs for one-shot devices. The issue about how to choose the optimal settings of decision variables, such as (a) stress levels, (b) inspection times, and (c) sample allocation, will be discussed in this section.

For $p = 0, 1$ and $q = 0, 1$,

$$-E\left[\frac{\partial^2 \ell(\boldsymbol{\theta})}{\partial a_p \partial a_q}\right] = \sum_{i=1}^{2}\left(\frac{K_i}{R(IT_i)} + \frac{K_i}{1 - R(IT_i)}\right)\left(\frac{\partial R(IT_i)}{\partial a_p}\right)\left(\frac{\partial R(IT_i)}{\partial a_q}\right), \tag{38}$$

Let $K_i = K p_i$ with $p_2 = 1 - p_1$, $A_i = R(IT_i)^{-1} + (1 - R(IT_i))^{-1}$ and $X_{ik} = \partial R(IT_i)/\partial a_k$.

$$\begin{aligned}-E\left[\frac{\partial^2 \ell(\boldsymbol{\theta})}{\partial a_p \partial a_q}\right] &= K\begin{bmatrix} p_1 A_1 X_{10}^2 + (1-p_1) A_2 X_{20}^2 & p_1 A_1 X_{11} X_{10} + (1-p_1) A_2 X_{21} X_{20} \\ p_1 A_1 X_{11} X_{10} + (1-p_1) A_2 X_{21} X_{20} & p_1 A_1 X_{11}^2 + (1-p_1) A_2 X_{21}^2 \end{bmatrix}\\ &= K\begin{bmatrix} r_{00} + s_{00} p_1 & r_{10} + s_{10} p_1 \\ r_{10} + s_{10} p_1 & r_{11} + s_{11} p_1 \end{bmatrix},\end{aligned}$$

where $r_{kk} = A_2 X_{2k}^2, r_{10} = A_2 X_{21} X_{20}, s_{kk} = A_1 X_{1k}^2 - A_2 X_{2k}^2, s_{10} = A_1 X_{11} X_{10} - A_2 X_{21} X_{20}$. The asymptotic covariance matrix of the MLEs of the model parameters becomes

$$V_{\boldsymbol{\theta}} = I^{-1}(\boldsymbol{\theta}) = \frac{1}{D}\begin{bmatrix} r_{11} + s_{11} p_1 & -(r_{10} + s_{10} p_1) \\ -(r_{10} + s_{10} p_1) & r_{00} + s_{00} p_1 \end{bmatrix}, \tag{39}$$

https://doi.org/10.1017/S0269964818000049 Published online by Cambridge University Press

where $D = K((r_{00}+s_{00}p_1)(r_{11}+s_{11}p_1) - (r_{10}+s_{10}p_1)^2)$. Then,

$$V_\mu = P'V_{\boldsymbol{\theta}}P = \frac{(r_{11}-2r_{10}x_0+r_{00}x_0^2)+(s_{11}-2s_{10}x_0+s_{00}x_0^2)p_1}{K\alpha_0^2(r_{00}r_{11}-r_{10}^2)+(r_{00}s_{11}+r_{11}s_{00}-2r_{10}s_{10})p_1+(s_{00}s_{11}-s_{10}^2)p_1^2} \quad (40)$$

$$= \frac{(r_{11}-2r_{10}x_0+r_{00}x_0^2)+(s_{11}-2s_{10}x_0+s_{00}x_0^2)p_1}{K\alpha_0^2(r_{00}s_{11}+r_{11}s_{00}-2r_{10}s_{10})p_1(1-p_1)}. \quad (41)$$

The objective of the optimal design is to minimize the standard error of the mean lifetime subject to sample allocation as follows:

$$p_1 = \arg\min_{0<p_1<1} V_\mu = \arg\min_{0<p_1<1} \frac{c_1+c_2p_1}{p_1(1-p_1)} = \arg\min_{0<p_1<1} \frac{c_1}{p_1} + \frac{c_1+c_2}{1-p_1}. \quad (42)$$

Setting $\partial V_\mu/\partial p_1 = 0$, the solution to the equation is

$$p_1 = \left(1+\sqrt{\frac{c_1+c_2}{c_1}}\right)^{-1} = \left(1+\sqrt{\frac{A_1(X_{11}-X_{10}x_0)^2}{A_2(X_{21}-X_{20}x_0)^2}}\right)^{-1}. \quad (43)$$

Given K, IT_1, IT_2, x_0, x_1, and x_2, the numbers of devices to be tested at IT_1 and IT_2 can be determined, that is, $K_1 = Kp_1$ and $K_2 = K(1-p_1)$ (rounded to nearest integers). The standard error of the mean lifetime under the design is then given by

$$se(\hat{\mu}) = \sqrt{V_\mu} = \frac{\sqrt{A_1(X_{11}-X_{10}x_0)^2}+\sqrt{A_2(X_{21}-X_{20}x_0)^2}}{\sqrt{KA_1A_2}\,\alpha_0(X_{11}X_{20}-X_{10}X_{21})} = \frac{C}{\sqrt{K}}. \quad (44)$$

It can be easily seen that standard error of the mean lifetime under the design is in inverse proportional to the square root of the sample size K. Also, it is important to point out that C is a non-linear function of x_1, x_2, IT_1, and IT_2. The non-linear function can be minimized by using existing optimization tools, namely, **fminsearch** in Matlab, or **optim** in R. Here, we present the following procedure to decide the stress level and the inspection time for each of the two stages.

1. Set the range of stress level $(x_{\mathrm{L}}, x_{\mathrm{H}})$, the normal operating condition x_0, and the termination time T;
2. Define $x_i = x_{\mathrm{L}} + (x_{\mathrm{H}} - x_{\mathrm{L}})(1-\exp(-\sum_{k=1}^{i}\exp(u_k)))$ and $IT_i = T(1-\exp(-\sum_{k=1}^{i}\exp(v_k)))$, for $i = 1, 2$;
3. Find (u_1, u_2, v_1, v_2) that minimize C by using an optimization tool;
4. Compute (x_1, x_2, IT_1, IT_2) with (u_1, u_2, v_1, v_2).

Furthermore, $(A_1, A_2, X_{11}, X_{10}, X_{21}, X_{20})$ can be obtained from (x_1, x_2, IT_1, IT_2). Consequently, given the standard error of the mean lifetime, the minimum required sample size can be determined as follows:

$$K \geq \left(\frac{C}{se(\hat{\mu})}\right)^2. \quad (45)$$

Finally, (K_1, K_2) can be determined from $(K, A_1, A_2, X_{11}, X_{10}, X_{21}, X_{20})$. It is noting that Step 2 guarantees that $0 \leq x_{\mathrm{L}} \leq x_1 < x_2 \leq x_{\mathrm{H}}$ and $0 < IT_1 < IT_2 \leq T$. Also, C is a non-linear function of x_1, x_2, IT_1, and IT_2 in Eq. (44), u_1, u_2, v_1, and v_2 in Step 2 are all identical as long as $x_{\mathrm{L}}, x_{\mathrm{H}}$, and T are the same, regardless of $se(\hat{\mu})$. It leads to the fact that x_1, x_2, IT_1, and IT_2 are also all identical.

5. NUMERICAL EXAMPLES

Fan et al. [18] and Balakrishnan and Ling [4] considered exponential distributions to analyze data of electro-explosive devices collected from constant-stress accelerated life tests. Here we present the optimal design of simple SSALTs for the electro-explosive devices. In this problem, the simple SSALTs are run to estimate the mean lifetime of the devices under the normal operating condition, $x_0 = 25°$C. The planning values (a_0^*, a_1^*) are set to be $(-5.3185, 0.0473)$ with $\mu(x_0) = 62.552$.

In this section, several examples are presented in Tables 2 and 3 to demonstrate the optimal designs of simple SSALTs for one-shot devices with different settings. Moreover, a simulation study under each of the optimal designs is carried out based on 10,000 experiments to determine the corresponding mean $\bar{x}_{\hat{\mu}}$ and standard deviation $s_{\hat{\mu}}$ of the MLE of the mean lifetime under the normal operating condition, so as to compare the experimental and theoretical results.

It is observed that the standard deviations of the MLE of the mean lifetime are close to the theoretical standard errors for the settings considered. In addition, due to the fact that the MLE of the mean lifetime follows log-normal distribution with

$$E[\hat{\mu}(x_0)] = E[\exp(-\hat{a}_0 - \hat{a}_1 x_0)] = \exp\left(-a_0 - a_1 x_0 + \frac{\sigma_0^2 + \sigma_1^2 x_0^2 + 2\rho\sigma_0\sigma_1 x_0}{2}\right), \quad \textbf{(46)}$$

the mean of the MLE of the mean lifetime are slightly overestimated. Moreover, when the standard error is fixed, increasing the highest stress level x_{H} significantly reduces sample sizes. The numerical examples show that when the highest stress level x_{H} is set to be sufficiently high, increasing the number of failures by prolonging experimental time T does not provide more information for the mean lifetime estimation. In other words, prolonging experimental time T may not be the most effective design to collect data. It is also realized that simple SSALTs with equally spaced inspection times generally would not efficiently collect lifetime data. The results also suggest the optimal design wherein more devices are

TABLE 2. Optimal design of step-stress accelerated life tests for electro-explosive devices with planning values $(a_0^*, a_1^*) = (-5.3185, 0.0473)$ under different settings, along with the corresponding mean $\bar{x}_{\hat{\mu}}$ and standard deviation $s_{\hat{\mu}}$ of the MLE of the mean lifetime $\mu(25) = 62.552$.

Setting				SSALT plan				Simulated results			
x_{L}	x_{H}	T	$se(\hat{\mu})$	x_1	x_2	IT_1	IT_2	K_1	K_2	$\bar{x}_{\hat{\mu}}$	$s_{\hat{\mu}}$
35	55	15	10	35	55	7.78	15.00	797	204	63.43	10.267
35	55	30	10	35	55	14.97	30.00	459	132	63.39	10.252
35	55	60	10	35	55	27.99	60.00	311	123	63.85	10.510
35	80	15	10	35	80	11.15	15.00	307	45	63.46	10.359
35	80	30	10	35	80	22.38	30.00	186	36	63.99	10.401
35	80	60	10	35	80	38.30	49.44	151	43	63.62	9.939
35	55	15	15	35	55	7.78	15.00	354	91	64.79	15.927
35	55	30	15	35	55	14.97	30.00	204	59	64.77	16.140
35	55	60	15	35	55	27.99	60.00	138	55	65.96	16.188
35	80	15	15	35	80	11.15	15.00	137	20	65.38	16.736
35	80	30	15	35	80	22.38	30.00	83	16	65.15	16.225
35	80	60	15	35	80	38.30	49.44	67	19	62.62	14.372

TABLE 3. Optimal design of step-stress accelerated life tests for electro-explosive devices with planning values $(a_0^*, a_1^*) = (-5.3185, 0.0473)$ under different settings, along with the corresponding mean $\bar{x}_{\hat{\mu}}$ and standard deviation $s_{\hat{\mu}}$ of the MLE of the mean lifetime $\mu(25) = 62.552$.

Setting				SSALT plan				Simulated results			
x_L	x_H	T	$se(\hat{\mu})$	x_1	x_2	IT_1	IT_2	K_1	K_2	$\bar{x}_{\hat{\mu}}$	$s_{\hat{\mu}}$
45	55	15	10	45	55	5.12	15.00	3922	1681	63.20	10.194
45	55	30	10	45	55	9.46	30.00	2364	1197	63.39	10.129
45	55	60	10	45	55	13.90	48.62	1876	1239	63.74	10.356
45	80	15	10	45	80	9.05	15.00	497	145	63.69	10.433
45	80	30	10	45	80	18.44	29.34	347	158	64.29	10.534
45	80	60	10	45	80	18.44	29.34	347	158	64.04	10.561
45	55	15	15	45	55	5.12	15.00	1743	748	64.56	15.744
45	55	30	15	45	55	9.46	30.00	1051	532	64.61	15.908
45	55	60	15	45	55	13.90	48.62	834	551	64.66	15.983
45	80	15	15	45	80	9.05	15.00	221	65	65.02	16.168
45	80	30	15	45	80	18.44	29.34	155	70	65.91	16.271
45	80	60	15	45	80	18.44	29.34	155	70	65.72	16.242

to be tested at the first inspection time with lower stress level and less devices are to be tested at the second inspection time with higher stress level. Moreover, more devices are required to be tested in the simple SSALTs to maintain $se(\hat{\mu})$ when the lowest stress level x_L is increased from 35 to 45.

The effect of misspecification of the planning values on the design needs to be studied to evaluate its robustness feature. Because the planning values (a_0^*, a_1^*) are likely to depart from the true model parameters (a_0, a_1), we assume here that the planning values have small and moderate errors of the form $(a_0^*, a_1^*) = (a_0(1+\epsilon_1), a_1(1+\epsilon_2))$, where $\epsilon_i = \{-0.05, -0.02, 0.00, 0.02, 0.05\}$, thus allowing for under-specification as well as over-specification from the true values of the model parameters. Suppose that $(x_L, x_H, T, se(\hat{\mu}))$ are set to be $(35, 55, 30, 15)$. Tables 4 and 5 present the sensitivity analysis of the choice of planning values to the design of simple SSALTs, along with the corresponding mean $\bar{x}_{\hat{\mu}}$

TABLE 4. Sensitivity analysis of the choice of planning values (a_0^*, a_1^*) with small errors to the design of simple step-stress accelerated life tests, along with the corresponding mean $\bar{x}_{\hat{\mu}}$ and standard deviation $s_{\hat{\mu}}$ of the MLE of the mean lifetime $\mu(25)$.

ϵ_1	ϵ_2	a_0^*	a_1^*	$\mu(25)$	x_1	x_2	IT_1	IT_2	K_1	K_2	$\bar{x}_{\hat{\mu}}$	$s_{\hat{\mu}}$
0.00	0.00	−5.3185	0.0473	62.55	35	55	14.97	30.00	204	59	64.77	16.140
0.00	+0.02	−5.3185	0.0482	61.09	35	55	15.01	30.00	189	55	64.90	16.907
0.00	−0.02	−5.3185	0.0464	64.04	35	55	14.92	30.00	220	63	64.59	15.492
+0.02	0.00	−5.4249	0.0473	69.57	35	55	15.08	30.00	273	76	64.30	13.604
+0.02	+0.02	−5.4249	0.0482	67.94	35	55	15.13	30.00	252	71	64.21	14.359
+0.02	−0.02	−5.4249	0.0464	71.23	35	55	15.03	30.00	295	82	64.23	12.938
−0.02	0.00	−5.2121	0.0473	56.24	35	55	14.84	30.00	154	45	65.42	18.865
−0.02	+0.02	−5.2121	0.0482	54.92	35	55	14.88	30.00	142	43	65.47	19.450
−0.02	−0.02	−5.2121	0.0464	57.58	35	55	14.79	30.00	166	48	65.35	18.213

https://doi.org/10.1017/S0269964818000049 Published online by Cambridge University Press

Table 5. Sensitivity analysis of the choice of planning values (a_0^*, a_1^*) with moderate errors to the design of simple step-stress accelerated life tests, along with the corresponding mean $\bar{x}_{\hat{\mu}}$ and standard deviation $s_{\hat{\mu}}$ of the MLE of the mean lifetime $\mu(25)$.

ϵ_1	ϵ_2	a_0^*	a_1^*	$\mu(25)$	x_1	x_2	IT_1	IT_2	K_1	K_2	$\bar{x}_{\hat{\mu}}$	$s_{\hat{\mu}}$
0.00	0.00	−5.3185	0.0473	62.55	35	55	14.97	30.00	204	59	64.77	16.140
0.00	+0.05	−5.3185	0.0497	58.96	35	55	15.08	30.00	168	50	65.15	17.570
0.00	−0.05	−5.3185	0.0449	66.36	35	55	14.85	30.00	248	70	64.60	14.589
+0.05	0.00	−5.5844	0.0473	81.60	35	55	15.24	30.00	424	115	63.32	10.833
+0.05	+0.05	−5.5844	0.0497	76.92	35	55	15.37	30.00	347	96	64.08	12.028
+0.05	−0.05	−5.5844	0.0449	86.57	35	55	15.11	30.00	518	139	63.47	9.734
−0.05	0.00	−5.0526	0.0473	47.94	35	55	14.62	30.00	101	31	67.30	24.182
−0.05	+0.05	−5.0526	0.0497	45.19	35	55	14.74	30.00	84	28	68.35	27.069
−0.05	−0.05	−5.0526	0.0449	50.86	35	55	14.52	30.00	121	37	66.74	22.105

and standard deviation $s_{\hat{\mu}}$ of the MLE of the mean lifetime under the normal operating condition, $x_0 = 25°\text{C}$. It is realized that, within small (±2%) and moderate (±5%) errors of (a_0, a_1), the designs of simple SSALTs are quite robust, only when the estimated mean lifetime is close to the true mean lifetime. In general, the stress levels and the termination time are constant and the change times to increase the stress level from x_1 to x_2 are similar among the designs. But, the required sample sizes highly depend on the estimated mean lifetime. For example, when ϵ_1 increases by 5%, the estimated mean lifetime is much larger than the actual mean lifetime. As a result, the required sample size increases and thus the standard deviation of the MLE of the mean lifetime becomes smaller. However, the numerical results show that the means of the MLE of the mean lifetime are similar.

6. CONCLUDING REMARKS

In this paper, simple SSALTs for one-shot devices were studied. An EM algorithm was developed to find the MLEs of the model parameters as well as the mean lifetime under normal operating conditions. Furthermore, the information matrix was obtained and used for the design of simple SSALTs. The procedure to choose decision variables including the stress levels, the inspection times, and sample allocation was discussed. The optimal design is effectively collecting one-shot device testing data in the sense that the asymptotic variance of the MLE of the mean lifetime is minimized. In addition, the asymptotic variance can be used to construct the confidence interval. Interested readers may refer to [6,7].

Comprehensive simulation studies show that the procedure is quite reliable for design of simple SSALTs, as the theoretical and simulated standard deviations of the mean lifetime are similar. There are several observations from the simulation studies. (1) When the standard error is fixed, increasing the highest stress level x_H significantly reduces sample sizes. (2) When the highest stress level x_H is set to be sufficiently high, prolonging experimental time T may not be the most effective design to collect data. (3) Simple SSALTs with equally spaced inspection times is generally not the most effective design. (4) To effectively collect data, more devices are to be tested at the first inspection time with lower stress level and less devices are to be tested at the second inspection time with higher stress level.

A sensitivity analysis was also carried out to determine the effect of misspecification of planning values to the design. It is realized that the design is quite robust within small and moderate errors of the true parameters. The required sample size highly depends on the estimated mean lifetime. When the estimated mean lifetime is larger than the actual mean lifetime, the required sample size increases. In practice, it means that the cost to run the tests would slightly increase. From this observation, it is of great interest to design simple SSALTs with consideration of budgets for further investigations.

The mean lifetime under normal operating conditions is considered in this study, because of its advantage that explicit forms of sample allocation (K_1, K_2) exist. The determination of sample allocation enables us to select appropriate change time and stress level as well as the termination time at ease. In practice, especially for high-reliability products, we are also interested in a certain percentile of the lifetime, say, 0.1 or even 0.01, rather than the mean lifetime. Theoretically, it is possible to minimize the standard error of a certain percentile at normal operating conditions. But, explicit forms of sample allocation may not exist, which lead to a less efficient procedure for test planning.

Moreover, the proposed procedure can be modified for design of multi-SSALTs. The key to the optimal design is the determination of sample allocation. In the simple SSALTs with only two stress level, the sample allocation, (K_1, K_2), can be explicitly determined. In the multi-SSALTs with m stress levels, finding sample allocation, $K_i, i = 1, 2, \ldots, m$, with $\sum_{i=1}^{m} K_i = K$, becomes challenging.

The simulation results show that $x_1 = x_{\mathrm{L}}$ and $x_2 = x_{\mathrm{H}}$ in all cases. It is of great interest to justify whether $x_1 = x_{\mathrm{L}}$ and $x_2 = x_{\mathrm{H}}$ are always obtained in the optimal design. The problem can be simplified further by determining only the inspection times, if the asymptotic variance of the MLE of the mean lifetime is minimized when $x_1 = x_{\mathrm{L}}$ and $x_2 = x_{\mathrm{H}}$. However, Eq. (44) is a non-linear function of x_1, x_2, IT_1 and IT_2. The theoretical justification for $x_1 = x_{\mathrm{L}}$ and $x_2 = x_{\mathrm{H}}$ is not obvious, a further study will be carried out for this problem.

Besides, it is of great practical interest for design of SSALTs for one-shot devices under broader and more flexible lifetime distributions, namely gamma and Weibull distributions. These two lifetime distributions contain the exponential distribution as a special case and are more frequently used in the real world to describe the lifetime for device. From our experience, the probability density functions of these two popular lifetime distributions are more complicated than that of the exponential distribution. The formulation of the design becomes more challenging. However, this present work could provide a good insight for the further investigations. On the other hand, model mis-specification error might be a considerably critical issue in both theoretical and practical point of view. Model mis-specification analysis [22] would also be helpful for reliability engineering. Work on these flexible distributions is currently under progress and I hope to report these findings in a future paper.

Acknowledgement

This work was supported by the Early Career Scheme Fund (Ref. 28300114), granted by the Research Grants Council of Hong Kong. Thanks are also due to the Associate Editor and anonymous reviewers for their valuable comments and suggestions on an earlier version of this manuscript which led to this improved version.

References

1. Alhadeed, A.A. & Yang, S.S. (2002). Optimal simple step-stress plan for Khamis–Higgins model. *IEEE Transactions on Reliability* 51: 212–215.
2. Alhadeed, A.A. & Yang, S.S. (2005). Optimal simple step-stress plan for cumulative exposure model using log-normal distribution. *IEEE Transactions on Reliability* 54: 64–68.
3. Bai, D.S., Kim, M.S., & Lee, S.H. (1989). Optimum simple step-stress accelerated life tests with censoring. *IEEE Transactions on Reliability* 38: 528–532.

4. Balakrishnan, N. & Ling, M.H. (2012). EM Algorithm for one-shot device testing under the exponential distribution. *Computational Statistics and Data Analysis* 56: 502–509.
5. Balakrishnan, N. & Ling, M.H. (2012). Multiple-stress model for one-shot device testing data under exponential distribution. *IEEE Transactions on Reliability* 61: 809–821.
6. Balakrishnan, N. & Ling, M.H. (2013). Expectation maximization algorithm for one shot device accelerated life testing with Weibull lifetimes, and variable parameters over stress. *IEEE Transactions on Reliability* 62: 537–551.
7. Balakrishnan, N. & Ling, M.H. (2014). Gamma lifetimes and one-shot device testing analysis. *Reliability Engineering & System Safety* 126: 54–64.
8. Balakrishnan, N. & Ling, M.H. (2014). Best constant-stress accelerated life-test plans with multiple stress factors for one-shot device testing under a Weibull distribution. *IEEE Transactions on Reliability* 63: 944–952.
9. Balakrishnan, N. & Mitra, D. (2012). Left truncated and right censored Weibull data and likelihood inference with an illustration. *Computational Statistics and Data Analysis* 56: 4011–4025.
10. Balakrishnan, N., Ling, M.H., & So, H.Y. (2016). Constant-stress accelerated life-test models and data analysis for one-shot devices. In L. Fiondella & A. Puliafito (eds.), *Principles of performance and reliability modeling and evaluation*, Switzerland: Springer International Publishing, pp. 77–108.
11. Balakrishnan, N., So, H.Y., & Ling, M.H. (2015). EM algorithm for one-shot device testing with competing risks under exponential distribution. *Reliability Engineering & System Safety* 137: 129–140.
12. Balakrishnan, N., So, H.Y., & Ling, M.H. (2016). A Bayesian approach for one-shot device testing with exponential lifetimes under competing risks. *IEEE Transactions on Reliability* 65: 469–485.
13. Balakrishnan, N., So, H.Y., & Ling, M.H. (2016). EM algorithm for one-shot device testing with competing risks under Weibull distribution. *IEEE Transactions on Reliability* 65: 973–991.
14. Bhattacharyya, G.K. & Soejoeti, Z. (1989). A tampered failure rate model for step-stress accelerated life test. *Communications in Statistics: Theory and Methods* 18: 1627–1643.
15. Chen, D.G. & Lio, Y.L. (2010). Parameter estimations for generalized exponential distribution under progressive type-I interval censoring. *Computational Statistics and Data Analysis* 54: 1581–1591.
16. DeGroot, M.H. & Goel, P.K. (1979). Bayesian estimation and optimal designs in partially accelerated life testing. *Naval Research Logistics Quarterly* 26: 223–235.
17. Escobar, L.A. & Meeker, W.Q. (2006). A review of accelerated test models. *Statistical Science* 21: 552–577.
18. Fan, T.H., Balakrishnan, N., & Chang, C.C. (2009). The Bayesian approach for highly reliable electro-explosive devices using one-shot device testing. *Journal of Statistical Computation and Simulation* 79: 1143–1154.
19. Gouno, E. (2001). An inference method for temperature step-stress accelerated life testing. *Quality and Reliability Engineering International* 17: 11–18.
20. Gouno, E. (2007). Optimum stepstress for temperature accelerated life testing. *Quality and Reliability Engineering International* 23: 915–924.
21. Kundu, D. & Dey, A.K. (2009). Estimating the parameters of the Marshall–Olkin bivariate Weibull distribution by EM algorithm. *Computational Statistics and Data Analysis* 53: 956–965.
22. Ling, M.H. & Balakrishnan, N. (2017). Model mis-specification analyses of Weibull and gamma models based on one-shot device test data. *IEEE Transactions on Reliability* 66: 641–650.
23. Ling, M.H., Ng, H.K.T., Chan, P.S., & Balakrishnan, N. (2016). Autopsy data analysis for a series system with active redundancy under a load-sharing model. *IEEE Transactions on Reliability* 65: 957–968.
24. Ling, M.H., So, H.Y., & Balakrishnan, N. (2016). Likelihood inference under proportional hazards model for one-shot device testing. *IEEE Transactions on Reliability* 65: 446–458.
25. Louis, T.A. (1982). Finding the observed information matrix when using the EM algorithm. *Journal of the Royal Statistical Society, Series B* 44: 226–233.
26. McLachlan, G.J. & Krishnan, T. (2008). *The EM Algorithm and Extensions*, 2nd ed. Hoboken, New Jersey: John Wiley & Sons.
27. Miller, R.W. & Nelson, W.B. (1983). Optimum simple step-stress plans for accelerated life testing. *IEEE Transactions on Reliability* 32: 59–65.
28. Morris, M.D. (1987). A sequential experimental design for estimating a scale parameter from quantal life testing data. *Technometrics* 29: 173–181.
29. Nandi, S. & Dewan, I. (2010). An EM algorithm for estimating the parameters of bivariate Weibull distribution under random censoring. *Computational Statistics and Data Analysis* 54: 1559–1569.
30. Nelson, W.B. (1980). Accelerated life testing—step-stress models and data analysis. *IEEE Transactions on Reliability* 29: 103–108.

31. Nelson, W.B. (1990). *Accelerated testing—statistical models, test plans, and data analyses*. New York: Wiley.
32. Newby, M. (2008). Monitoring and maintenance of spares and one shot devices. *Reliability Engineering & System Safety* 93: 588–594.
33. Ng, H.K.T., Chan, P.S., & Balakrishnan, N. (2002). Estimation of parameters from progressively censored data using EM algorithm. *Computational Statistics and Data Analysis* 39: 371–386.
34. Scheike, T.H. & Sun, Y.Q. (2007). Maximum likelihood estimation for tied survival data under cox regression model via EM-algorithm. *Lifetime Data Analysis* 13: 399–420.
35. Shaked, M. & Singpurwalla, N.D. (1990). A Bayesian approach for quantile and response probability estimation with applications to reliability. *Annals of the Institute of Statistical Mathematics* 42: 1–19.
36. Sohn, S.Y. (1997). Accelerated life-tests for intermittent destructive inspection, with logistic failure-distribution. *IEEE Transactions on Reliability* 46: 122–129.
37. Wang, W.D. & Kececioglu, D.B. (2000). Fitting the Weibull log-linear model to accelerated life-test data. *IEEE Transactions on Reliability* 49: 217–223.
38. Xiong, C., Zhu, K., & Ji, M. (2006). Analysis of a simple step-stress life test with a random stress-change time. *IEEE Transactions on Reliability* 55: 67–74.
39. Zhao, W. & Elsayed, E.A. (2005). A general accelerated life model for step-stress testing. *IIE Transactions* 37: 1059–1069.