

# Polarization effect of a chirped Gaussian laser pulse on the electron bunch acceleration

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**Abstract.** In this paper, we investigated the output characteristics of the electron bunch accelerated by a chirped femtosecond laser pulse in vacuum using linear, elliptical and circular polarization states. To examine the features of the energy and angular spectra, emittances and position distribution of the electron bunch numerically,  $10^5$  electrons are used. It is found that the initial emittances of the electron bunch together with a proper choice of the polarization state of the laser pulse could lead to the efficient electron bunch acceleration.

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## 1. Introduction

Ultra-intense laser pulse can offer promising alternatives to the acceleration of charged particles. For most practical applications, high-quality particle beams with high spatial quality and monoenergetic energy distribution are required. In order to produce high-quality beams from laser-based accelerators, numerous investigations have treated the subject experimentally and theoretically [1–5]. Among the well-known laser-driven acceleration schemes, much work has been carried out studying the vacuum laser acceleration by an ultra-intense short laser pulse [6–11]. In most these materials, a specific polarization has been used to investigate the electron acceleration [12–14]. In order to compare the effective polarization states on electron acceleration, both a single and a bunch of electrons should be studied. Concerning vacuum laser acceleration, there has been much interest in studying the acceleration of electrons by a chirped Gaussian laser pulse [15–22]. In these materials the electron can get considerable energy in tailored parameters such as chirp parameter and polarization state of the laser wave. The polarization effect of the laser wave on vacuum laser acceleration by a non-chirped laser pulse has been considered by Xu et al. [14] and they addressed the advantage and disadvantages of employing circular polarization (CP) field compared to the linear polarization (LP) field for single electron acceleration. In our previous study [16], we found that the linear polarization could be more effective for a single-electron acceleration using a chirped laser pulse. On the other hand, circular and elliptical polarizations seem to be more effective for electron bunch acceleration. The main purpose of this investigation is to examine the detailed output properties of the outgoing accelerated electron bunch for linear, elliptical and circular polarization states. Two different initial electron bunches were employed for considering the output quality of the accelerated electrons. In this work, we study energy and angular spectra, emittances and space distribution

**Table 1.** Conventions for the three polarization states.

Polarization States	$\Delta$	Optimum $b'$	Optimum $\phi_0$	Inclination angle
CP	$\pi/2, 3\pi/2$	0.022	–	–
LP	$0, \pi, 2\pi$	0.012	$0.7\pi$	$0-2\pi$
EP	Otherwise	0.012	$0.7\pi$	$0-2\pi$

of the electron bunch as the main features of the outgoing electrons. It is found that the average number of the accelerated electrons with circular polarization state is greater than that of the linear and elliptical polarizations. It is also shown that the energetic electrons go ahead of the bunch and comprises a fast group. The time characteristics of this group will be examined.

This paper is organized as follows: in Sec. 2, the single particle acceleration will be reviewed in chirped electron acceleration scheme. In Sec. 3, The electron bunch acceleration is discussed. Finally, summary and conclusions will be give in Sec. 4.

## 2. Single-particle acceleration

In this investigation a linear and negative chirp is applied in which the instantaneous frequency assumed to be  $\omega(\xi) = \omega_0 + b\xi$ , where  $\xi = z/c - t$  is the retarded time,  $b$  is the chirp parameter with dimension of  $T^{-2}$  and  $\omega_0$  is the frequency at  $\xi = 0$ . To study the effect of laser pulse polarization states on the electron acceleration, the following assumptions are made to express the electromagnetic fields of the chirped laser pulse with an optional polarization state

$$E_x = E_0 \frac{w_0}{w(z)} \exp\left(-\frac{r^2}{w^2(z)}\right) \exp\left(-\frac{\xi^2}{\tau_p^2}\right) \exp\left\{i\left[\frac{kr^2}{2R(z)} - \varphi(z) + \omega(\xi)\xi + \phi_0\right]\right\}, \quad (1)$$

$$E_y = E_x \exp(i\Delta), \quad (2)$$

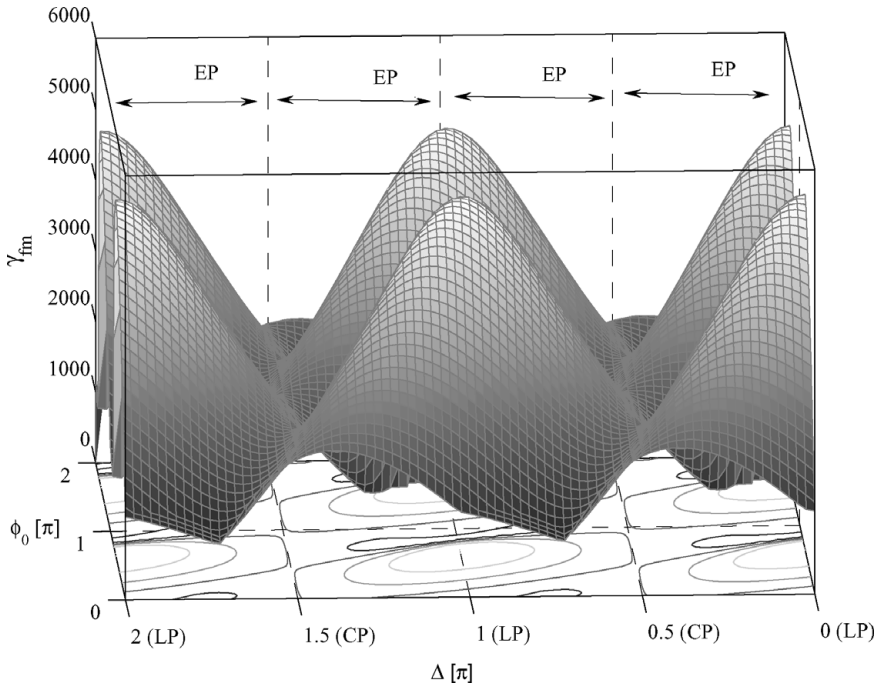
$$E_z = \left[-\frac{x}{R(z)} - \frac{2ix}{kw^2(z)}\right] E_x + \left[-\frac{y}{R(z)} - \frac{2iy}{kw^2(z)}\right] E_y, \quad (3)$$

where the parameter  $\Delta$  determines the type of polarization state. The magnetic field components are also obtained using the Maxwell equation

$$\vec{B} = \frac{-ic}{\omega} \vec{\nabla} \times \vec{E}, \quad (4)$$

where  $\tau_p$  is the laser pulse duration,  $w(z) = w_0 \sqrt{1 + (z/z_R)^2}$  the laser beam waist,  $R(z) = z[1 + (z_R/z)^2]$  the radius of wave front curvature,  $z_R = k_0 w_0^2 (1 + b\xi/\omega_0)/2$  the Rayleigh length,  $\varphi(z) = \tan^{-1}(z/z_R)$  the Gouy phase.  $\phi_0$  is the initial phase and  $E_0$  the amplitude of the electric field. We modified the laser beam characteristics by correcting the Rayleigh length via well-known Gaussian laser beam parameters assuming  $\omega = \omega_0 + b\xi$ . Table 1 clarifies the linear, elliptical and circular polarization conventions for different polarization states. For linear and elliptical polarization states the inclination angles ranging from 0 to 360 degrees are employed. To investigate the electron dynamics, three-dimensional test particle simulations are employed to solve the relativistic Newton–Lorentz equation of motion

$$\frac{d\beta_x}{d\tau} = \frac{1}{\gamma} [E_x(\beta_x^2 - 1) + E_z(\beta_x\beta_z) + B_y\beta_z - B_z\beta_y], \quad (5)$$



**Figure 1.** Single electron final energy as a function of  $\Delta$  and  $\phi_0$ . The parameter  $\Delta = 3\pi/2$  and  $\pi/2$  correspond to CP field,  $\Delta = 0, \pi$  and  $2\pi$  correspond to LP field. The other values represent EP field. Both the laser and the electron initial parameters were addressed in the text.

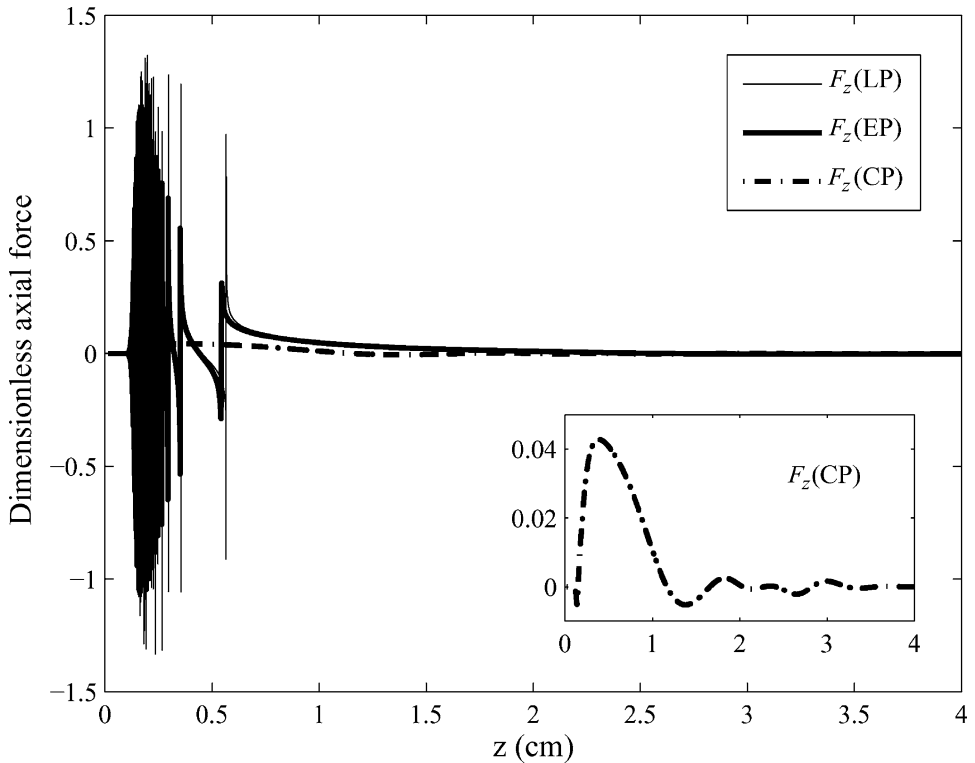
$$\frac{d\beta_y}{d\tau} = \frac{1}{\gamma} [E_x(\beta_y\beta_x) + E_z(\beta_y\beta_z) - B_x\beta_z + B_z\beta_x], \tag{6}$$

$$\frac{d\beta_z}{d\tau} = \frac{1}{\gamma} [E_x(\beta_x\beta_z) + E_z(\beta_z^2 - 1) - B_y\beta_x + B_x\beta_y], \tag{7}$$

where  $\beta_x = v_x/c$ ,  $\beta_y = v_y/c$  and  $\beta_z = v_z/c$  are x, y and z components of dimensionless velocity of the electron.  $\tau = t/\tau_p$  is the the dimensionless time. The rate of change of the electron energy can be rewritten in the form of

$$\frac{d\gamma}{dt} = \frac{1}{m_0c^2} \vec{F} \cdot \vec{v}, \tag{8}$$

where  $\vec{F}$  is the relativistic Newton–Lorentz force. Equations (5)–(8) have been solved numerically using the forth-order Runge–Kutta method. It is assumed that the laser time duration  $\tau_p = 100$  fs,  $\lambda_0 = 0.8$   $\mu\text{m}$ ,  $w_0 = 100$   $\mu\text{m}$ ,  $\gamma_0 = 1.15$  MeV and the initial dimensionless electron position  $(x, y, z = 0, 0, 1200)$ . The dimensionless parameter  $a = 0.61 \times 10^{-9} \lambda_0 (\mu\text{m}) \sqrt{I(\text{W}/\text{cm}^2)}$  is assumed to be 4.3 which specifies the magnitude of the laser field intensity for the polarization states. The optimum dimensionless chirp parameter  $b' = b/\omega_0^2$  is found to be  $b' = 0.022$  for CP field and  $b' = 0.012$  for elliptic polarization (EP) and LP fields (Table 1). The results show that the right and left handed circularly polarized fields transfer the same energy to the electron. It is also verified that the electron acceleration does not depend on the inclination angles of the linear polarization states. Figure 1 demonstrates the dependence of

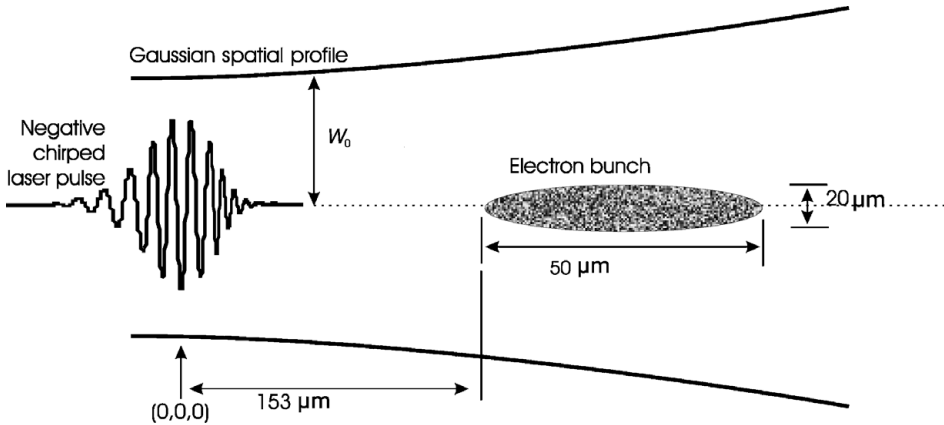


**Figure 2.** Comparison of longitudinal magnetic forces  $F_{\parallel} = -e/c |\vec{V} \times \vec{B}|_{\parallel}$  exerted on a single electron for three polarization states with the same parameters as indicated in the text. For EP field  $\Delta = 0.9\pi$ .

electron final energy as a function of initial phase  $\phi_0$  and parameter  $\Delta$ .  $\Delta$  specifies the polarization states. For example,  $\Delta = 0, \pi, 2\pi$  denote LP field,  $\Delta = \pi/2, 3\pi/2$  denote CP field, otherwise it denotes EP field. It is seen that the electron final energy is independent of the initial phase  $\phi_0$  for CP field. In comparison to the elliptical and circular polarizations, the linear polarization seems to be more effective for a single-particle acceleration using a chirped laser pulse. One can attribute the difference in efficiency of electron acceleration for three polarizations to magnitude of the longitudinal Lorentz force. As can be seen in Fig. 2, the magnitude of the longitudinal Lorentz force for CP field is one order of magnitude less than that of both EP and LP fields. As indicated in [15], the longitudinal magnetic force is found to be responsible for axial electron acceleration and the efficiency in turns states the phase synchronization scheme in this subject. Therefore, the greater longitudinal force, the higher acceleration efficiency and gradient. A discussion about the axial acceleration of electrons using non-chirped CP and LP fields based on capture and acceleration scenario model has been given in [12, 13].

### 3. Electron bunch acceleration

Using the three polarization states, the efficiency of the electron bunch acceleration differs from that of a single-electron acceleration. It was pointed out in [16] that



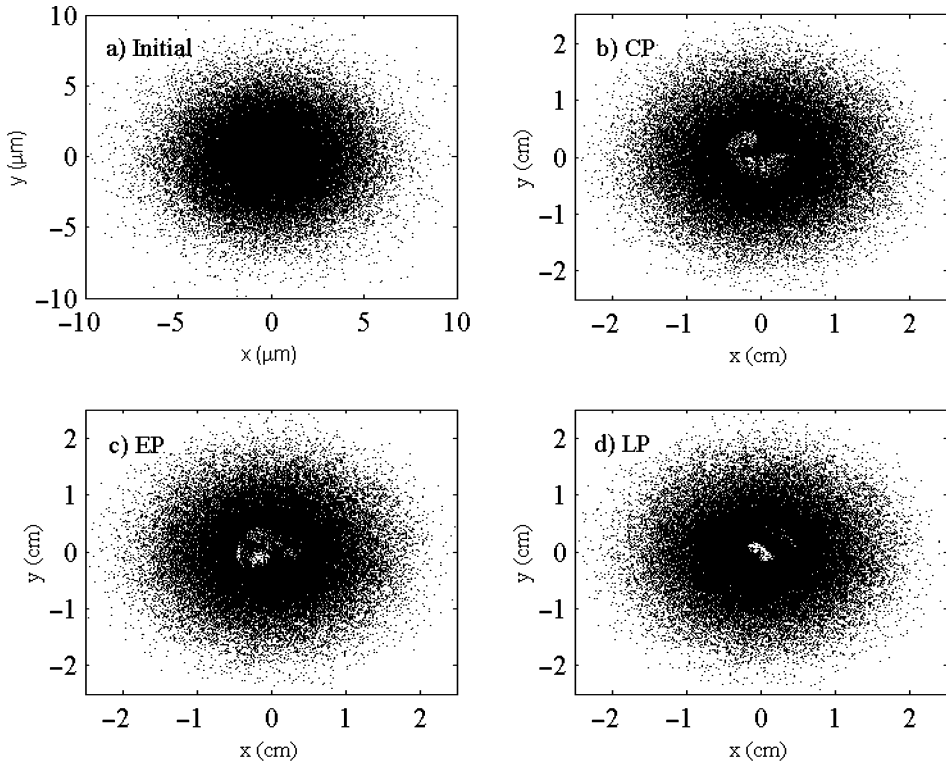
**Figure 3.** Schematic view of the electron bunch interaction with a chirped laser pulse.

the three polarization states have different efficiencies for an ensemble of electron. However, the output properties of an ensemble of electron should be examined in detailed. To study the subject, we performed three-dimensional particle modelling using electron bunches containing  $10^5$  particles. In this modelling, the electron-electron interaction is neglected. It is assumed that the initial electron bunch is an ellipsoid which has the traverse and longitudinal bunch sizes of  $25\lambda$  and  $62.5\lambda$ , respectively. The electrons in the ellipsoidal bunch are assumed to be distributed Gaussian in both space and velocity. Figure 3 illustrates the schematic view of the electron bunch acceleration by a chirped Gaussian laser pulse in vacuum. The root mean square (RMS) emittance is used to evaluate the quality of the output bunch. The transverse RMS emittances can be obtained from the following formula [22]:

$$\epsilon_x = \{ \overline{(x - \bar{x})^2} \overline{(x' - \bar{x}')^2} - [\overline{(x - \bar{x})(x' - \bar{x}')}]^2 \}^{1/2},$$

$$\epsilon_y = \{ \overline{(y - \bar{y})^2} \overline{(y' - \bar{y}')^2} - [\overline{(y - \bar{y})(y' - \bar{y}')}]^2 \}^{1/2}.$$

Here  $x' = dx/dz$  and  $y' = dy/dz$  are the inclination of the electron trajectory relative to the z-axis. Two different initial emittances  $\epsilon_0 = 1 \text{ mm.mrad}$  and  $\epsilon_0 = 0.1 \text{ mm.mrad}$  are used to investigate the output electron bunch characteristics. In the case of  $\epsilon_0 = 1 \text{ mm.mrad}$ , the initial mean velocity of the electron bunch is assumed to be  $0.9c$  and the spread in vertical and parallel electron velocity are  $0.1c$  and  $0.82c$ , respectively. In order to evaluate the quality of the accelerated electrons, we show the cross-section of the electron bunch distribution in  $y$ - $x$  plane for three polarization states in Fig. 4. It is seen that the initial bunch with transverse sizes of  $20 \mu\text{m}$  spreads to a circle with radius of about  $2 \text{ cm}$  for three polarization states after interaction with the laser pulse. As indicated in Figs. 5 and 6, the cross-section distribution of the electron bunch in  $x$ - $z$  and  $y$ - $z$  planes show that the electron bunch is separated into two main groups. The first group which escapes ahead of the bunch consists of fast and energetic electrons with energies of the order of hundreds of mega-electronvolts. This group corresponds to about 45% of the electron bunch for CP field and 39% for both EP and LP fields. A snapshot of the accelerated electron number distributions on the  $z$ -axis are shown in Fig. 7 for three polarization states



**Figure 4.** Initial  $y$ - $x$  cross-section distribution of  $10^5$  electrons with the initial emittance of  $\epsilon_0 = 1$  mm.mrad (a), final  $y$ - $x$  cross-section distribution of the electrons after acceleration for CP field (b), EP field (c) and LP field (d). For EP field  $\Delta = 0.9\pi$ .

with  $\epsilon_0 = 1$  mm.mrad. It is seen that the electrons scatter and separate into two main groups. The leading group consists of the energetic electrons and the second group which follows the first one comprises slow electrons. As can be seen from Fig. 7, the number of energetic electrons with CP field are greater than that of both EP and LP fields. In Fig. 8, the accelerated electron number distribution on the  $z$ -axis are shown for  $\epsilon_0 = 0.1$  mm.mrad. Unlike the case  $\epsilon_0 = 1$  mm.mrad, the accelerated electrons in this case are mainly comprise one group at the leading edge of the bunch. However, we took the fast electrons in the case of  $\epsilon_0 = 0.1$  mm.mrad as  $\gamma > 150$  and illustrated the characteristics in Fig. 9(a). In Fig. 9, the distribution of energetic electrons (first group) on the  $z$ -axis are shown for two different initial emittances of  $\epsilon_0 = 1$  and 0.1 mm.mrad. If one overview to this group as a pulse of electron beam, the full width at half maximum(FWHM) of the electron number can be estimated to be about 10 fs for both emittances with CP field. It can be seen from this figure that the higher energetic electrons will be produced with lower emittances. The ejection angle of the accelerated electrons is shown in Fig. 10 with  $\epsilon_0 = 1$  mm.mrad. It is seen that the mean ejection angle of the electron bunch for the CP field is less than that of both EP and LP fields. The electron energy distribution on the  $z$ -axis is shown in Fig. 11. Although in the case of  $\epsilon_0 = 1$  mm.mrad, the maximum acquired energy by the electrons belong to the LP field, but the maximum

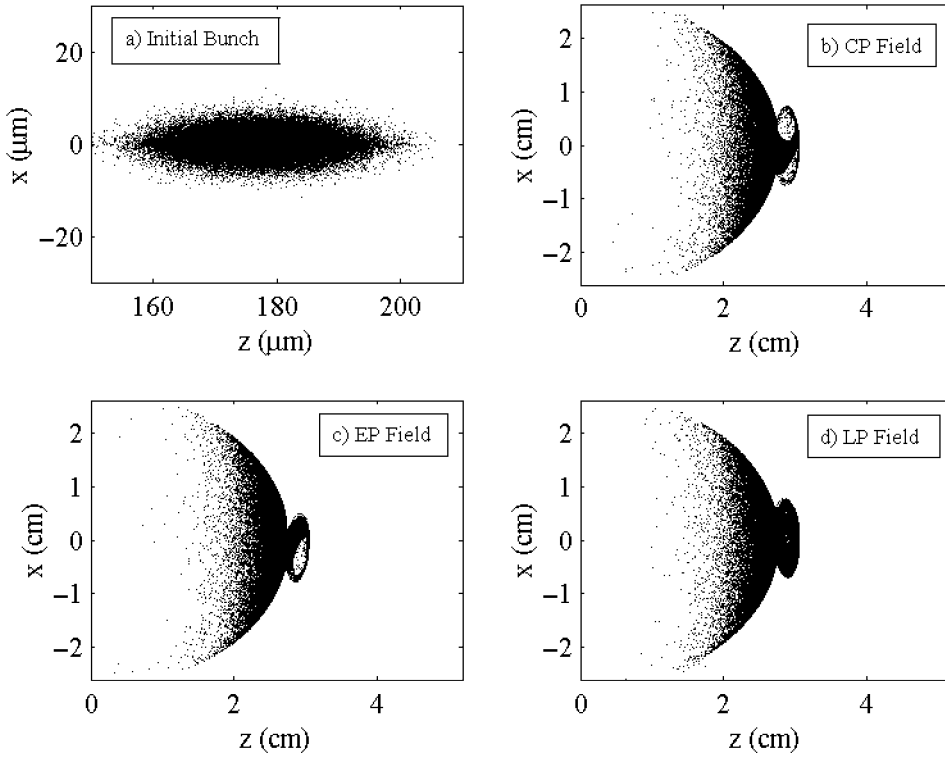


Figure 5. The same as Fig. 4 for x-z cross-section distribution.

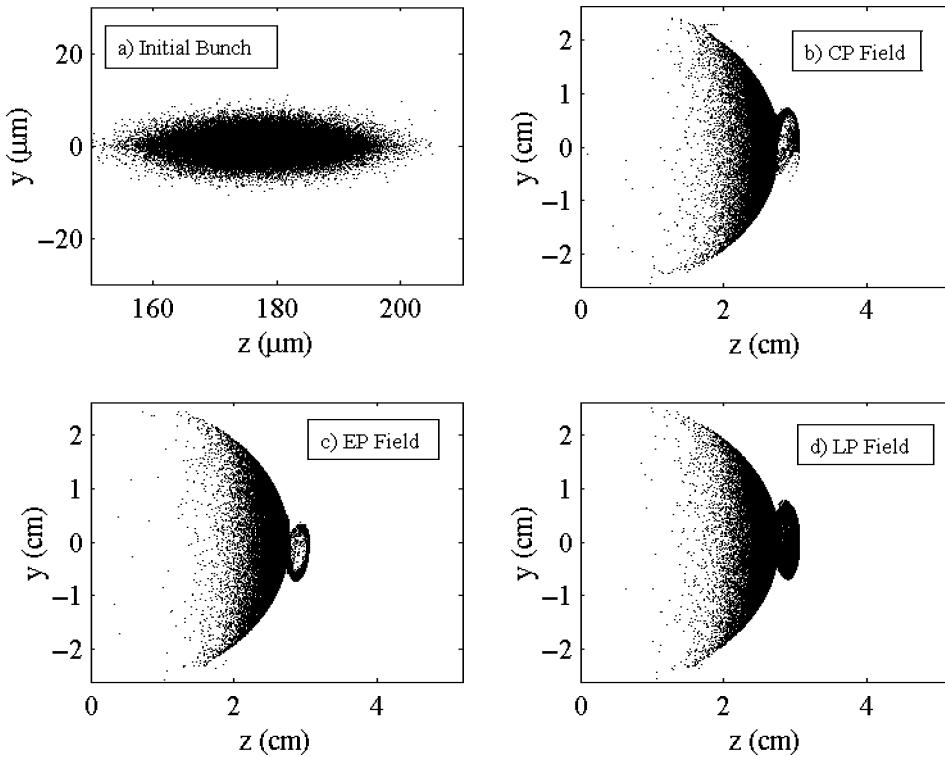
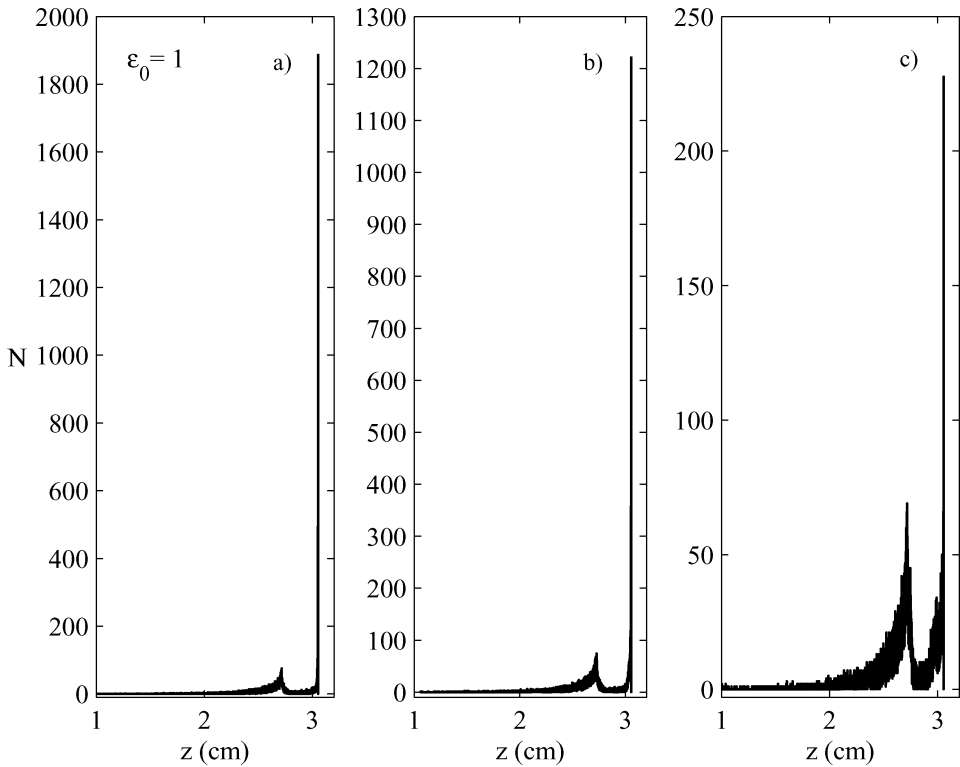


Figure 6. The same as Fig. 4 for y-z cross-section distribution.



**Figure 7.** A snapshot of  $10^5$  electron distribution on the  $z$ -axis with initial emittance of  $\epsilon_0 = 1$  mm.mrad for CP field (a), EP field (b) and LP field (c). For EP field  $\Delta = 0.9\pi$ .

**Table 2.** Characteristics of the outgoing fast electrons with initial  $\epsilon_0 = 1$  mm.mrad.

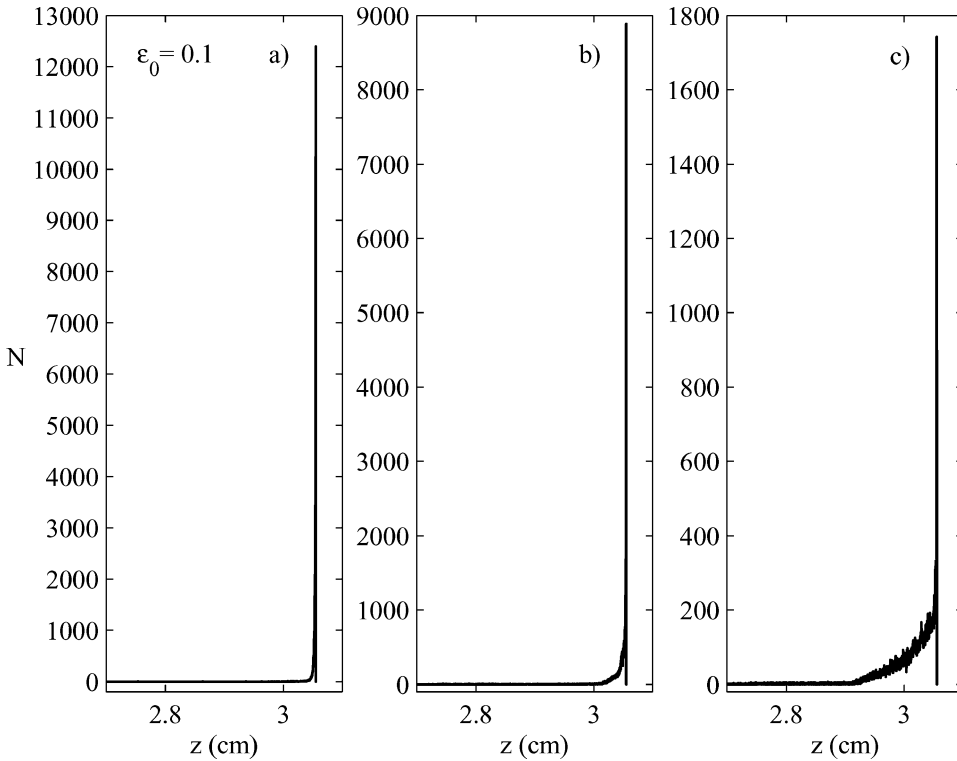
	Initial bunch	CP field	EP field	LP field
Emittance [mm.mrad]	$\epsilon_0=1$	$\epsilon_x=2511$	$\epsilon_x=1962$	$\epsilon_x=708$
Bunch length	50 $\mu\text{m}$	0.34 cm	0.33 cm	0.32 cm
Transverse size	20 $\mu\text{m}$	1.5 cm	1.3 cm	1.48 cm
Electron number	$10^5$	45128	38906	39229
Mean energy [MeV]	1.15	175	128	30.5

mean energy belong to the CP field. On the other hand, the mean energy of the fast electrons is considerable for the case of  $\epsilon_0 = 0.1$  mm.mrad. It is greater than that of the former case by two order of magnitudes. The outgoing characteristics of the electron bunch are summarized in Tables 2 and 3 for three polarization states and two different initial emittances.

#### 4. Summary and conclusions

The effect of polarization states of a chirped Gaussian laser pulse was investigated on electron bunch acceleration, numerically. Using the optimum parameters for a single electron localized initially at the laser axis, it was found that the LP filed could accelerate the charged particle effectively. This effect was attributed to the



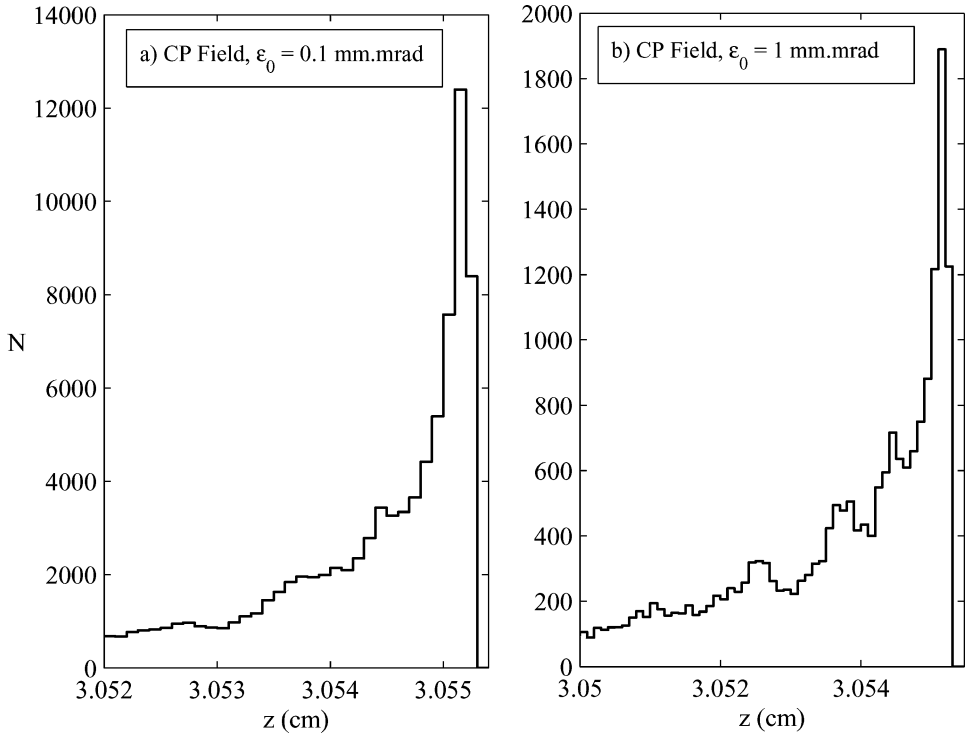


**Figure 8.** A snapshot of  $10^5$  electron distribution on the z-axis with initial emittance of  $\epsilon_0 = 0.1$  mm.mrad for CP field (a), EP field (b) and LP field (c).

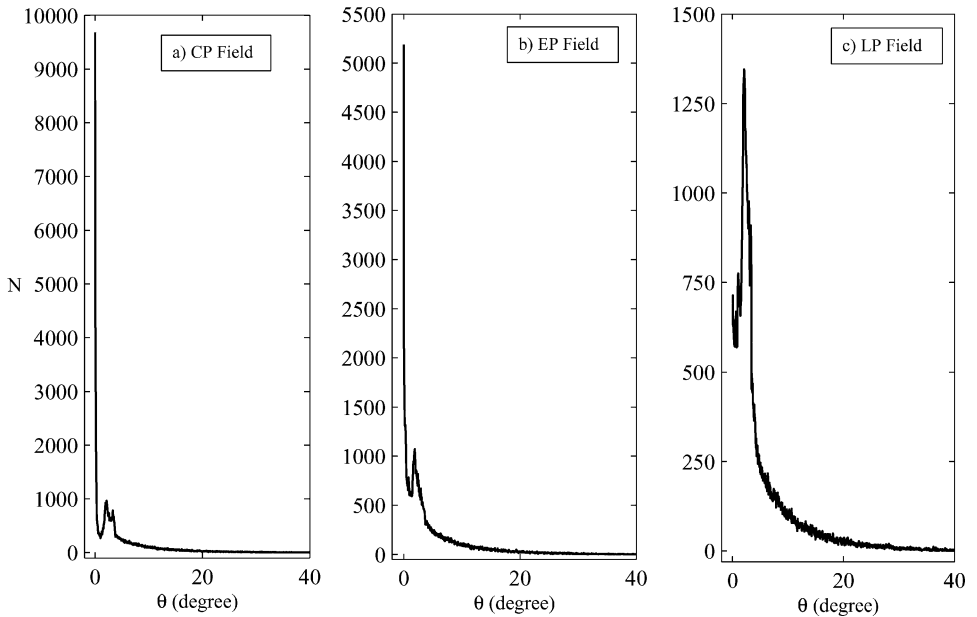
**Table 3.** Characteristics of the outgoing fast electrons with initial  $\epsilon_0 = 0.1$  mm.mrad.

	Initial bunch	CP field	EP field	LP field
Emittance [mm.mrad]	$\epsilon_0=0.1$	$\epsilon_x=347$	$\epsilon_x=258$	$\epsilon_x=338$
Bunch length [ $\mu\text{m}$ ]	50	7.6	8	6
Transverse size [ $\mu\text{m}$ ]	20	1000	460	500
Electron number	$10^5$	48430	28322	39311
Mean energy [MeV]	1.15	861	1200	1380

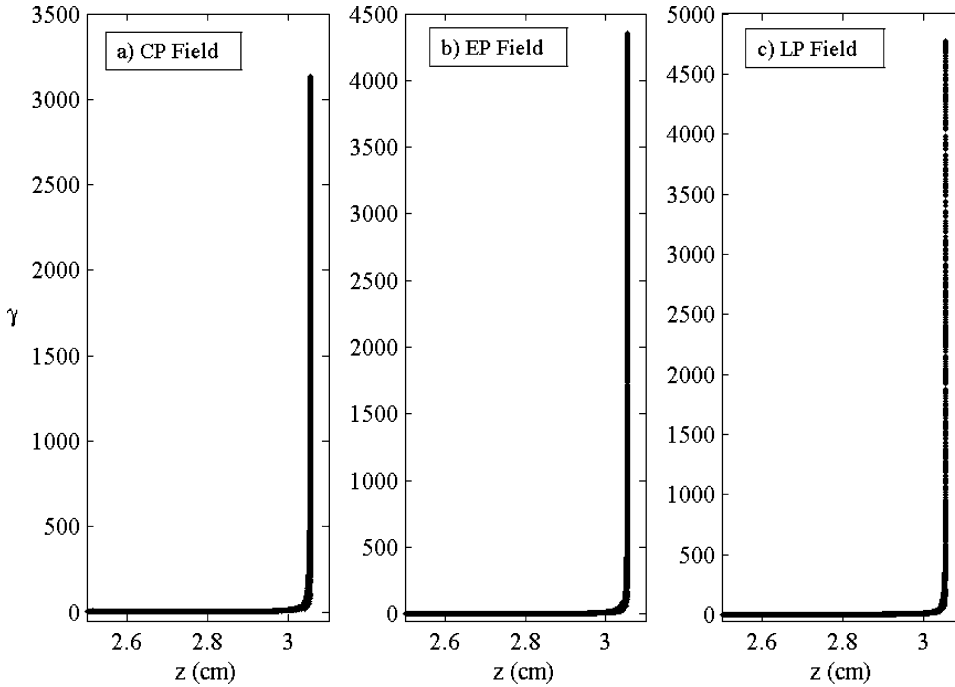
magnitude of the longitudinal magnetic force  $F_{\parallel} = -e/c | \vec{V} \times \vec{B} |_{\parallel}$ . The situation differs when a bunch of electron is used in which the electrons distributed Gaussian on the transverse plane. Since the acceleration efficiency of the three polarization states depend on transverse evolution of the axial magnetic force, electron initial position and energy, then the gained energy by an electron far away from the laser axis reduces, drastically. The reduction rates are not the same for these polarizations. On the other hand, the electron trajectories and instant transverse distance from the laser axis are also different for the three polarization states during the interaction zone. For example helical and zigzag trajectories are obtained for CP and LP fields, respectively. These reasons clarifies that for an ensemble of electrons not only the polarization states but also the initial characteristics of the electrons play important



**Figure 9.** A snapshot of fast electron distribution (group one) on the  $z$ -axis for CP field with initial emittance of  $\epsilon_0 = 0.1$  mm.mrad (a), and  $\epsilon_0 = 1$  mm.mrad (b). For EP field  $\Delta = 0.9\pi$ .



**Figure 10.** The same as Fig. 6 a snapshot of the electron number versus ejection angle for CP field (a), EP field (b) and LP field (c).



**Figure 11.** The energy distribution of the outgoing  $10^5$  electrons with  $\epsilon_0 = 1$  mm.mrad at the final stage for CP field (a), EP Filed (b) and LP field (c).

roles in acceleration efficiency. For example, for an initial beam with low enough emittances, quasi-monoenergy particles and low divergence, the LP field could be more efficient in electron acceleration. On the other hand, for a typical electron bunch with relative poor quality, the CP filed could be efficient in electron acceleration. In this work, we choose two different initial emittances for the electron bunch with  $\epsilon = 1$  mm.mrad and  $\epsilon = 0.1$  mm.mrad. It was found that in the former case two main groups can apparently distinguished in space after interaction while for the latter case it dose not take place. However, we took the fast electrons in the latter case as  $\gamma > 150$  and illustrated the characteristics in Figs. 7 and 8. The characteristics of the accelerated electrons were summarized in Tables 2 and 3. It can be seen that the circular polarization could be more efficient in electron bunch acceleration compared to EP and LP fields. Regarding Tables 2 and 3, one can conclude that both the initial electron ensemble parameters and the field polarization state are very important in electron acceleration with a chirped laser pulse in vacuum. In this scheme, the chirp parameter plays the essential role in phase synchronization between the electron phase velocity and the electromagnetic forces.

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