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BOOK REVIEW

Holomorphic Dynamics

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Holomorphic dynamics includes the iteration of rational maps of the sphere (Julia and Fatou circa 1918), transcendental entire functions (Fatou 1926) and holomorphic automorphisms of $\mathbb{C}^* = \mathbb{C}\backslash\{0\}$ (Rådström 1953). In these cases the Fatou set $F(f)$ of the map f appears as the maximal open set where the iterates f^n, $n \in \mathbb{N}$, form a normal family. $F(f)$ is divided into components by its complement, the Julia of chaotic set $J(f)$ and these components are permuted by the map. There is an obvious analogy with the theory of Kleinian groups, in which the limit set of the group corresponds to $J(f)$. This analogy is now formalized in Sullivan's 'dictionary'. There are extensions of the above theories to holomorphic maps of $\mathbb{C}^n$. One may also iterate a general meromorphic function g in $\mathbb{C}$, (despite the fact that the iterates are no longer in general meromorphic in $\mathbb{C}$) by defining the Fatou set of g as the largest open set in which the iterates g^n are both defined and form a normal family; the results remain broadly similar to the other cases.

The theory has been very active since about 1980, partly because of the availability of computer graphics to study the elaborate patters which arise (Julia sets are usually fractal), and partly because of the use of powerful new tools such as quasiconformal surgery. An enormous literature deals with the special case of quadratic polynomials and the associated Mandelbrot set. In the last decade four admirable introductory books have appeared, all on rational dynamics [**1, 3, 6, 7**], each with a different character, and for entire and meromorphic functions there is a book [**5**] and a useful survey [**2**].

The book under review provides a survey of rational dynamics, some treatment of transcendental entire functions, the Sullivan dictionary and some dynamics in $\mathbb{C}^n$ usually $\mathbb{C}^2$. For this last, one can also see [**4**]. The authors cover a lot of ground, some of it quite advanced. Thus Chapter 1 on polynomials starts simply with the filled Julia set introduced as the complement of the set of 'escaping' points, whose orbits tend to infinity. However by p. 22 we have reached a sketch of the proof of Douady and Hubbard, that the Mandelbrot set is connected, and we proceed on through the Hausdorff dimension, polynomial-like mappings, the measurable Riemann mapping theorem and infinitely renormalizable polynomials.

In Chapter 2, on the Fatou set and the Julia set, the authors give the main properties of these sets for rational and entire functions, including the density of repelling cycles in J and the significance of singular values. Chapter 3 deals with the dynamics of entire functions and includes a study of escaping points, special properties of the exponential family and the existence of new types of Fatou components, such as wandering domains which do not occur in the rational case. Chapter 4 gives more on rational functions; Newton's method; Sullivan's proof that wandering Fatou components do not occur; and Shishikura's results on the possible number of non-repelling cycles. Chapter 5 is Sullivan's dictionary.

The last four chapters are devoted to topics in dynamics in $\mathbb{C}^n$, which will perhaps be the least familiar part to most readers. One of the earliest results of the theory was the existence of Fatou–Bieberbach domains, that is proper subdomains of $\mathbb{C}^2$ which are biholomorphically equivalent to $\mathbb{C}^2$. These arise naturally out of the study of polynomial automorphisms of $\mathbb{C}^2$. These latter include the generalized Hénon maps $(x, y) \to (y, P(y) - \delta x)$, $\delta \neq 0$, P denotes polynomial. The Hénon maps are studied here in terms of their escaping sets. The final chapters survey the theory of pluripotentials in order to expound results of Bedford and Smillie on polynomial automorphisms.

The translation from the Japanese occasionally uses non-idiomatic or non-standard expressions, but this should not cause problems. There is frequent use of results from later in the book when discussing examples or even in proving theorems, which can be irritating. All in all this is a survey of a large body of material, much of it at a rather advanced level. In particular, it is one of the few presentations of dynamics of several complex variables. It should be useful as an introduction to this area and as giving a broad coverage of rational dynamics.

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