ANZIAM J. **62** (2020), 318–333
doi:10.1017/S1446181120000206

MODELLING HUMAN CARRYING CAPACITY AS A FUNCTION OF FOOD AVAILABILITY

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(Received 16 January, 2020; accepted 11 September, 2020; first published online 18 December, 2020)

Abstract

We assume that human carrying capacity is determined by food availability. We propose three classes of human population dynamical models of logistic type, where the carrying capacity is a function of the food production index. We also employ an integration-based parameter estimation technique to derive explicit formulas for the model parameters. Using actual population and food production index data, numerical simulations of our models suggest that an increase in food availability implies an increase in carrying capacity, but the carrying capacity is "self-limiting" and does not increase indefinitely.

2020 *Mathematics subject classification*: primary 34A12; secondary 65R32, 92B05.

Keywords and phrases: logistic model, variable carrying capacity, food production index, mathematical modelling.

1. Introduction

According to the Population Reference Bureau, the world population is projected to reach 9.8 billion in 2050, representing a 31% increase from the estimated 7.5 billion population in 2017 [11, 12]. Urbanization is expected to increase, with approximately 70% of the world's population living in cities or urban areas. Rapid human population growth is one of the most detrimental environmental issues that we face, and thus deserves our serious attention [9].

The logistic model for population growth has been utilized extensively to study the cause-and-effect relationship between the so-called "carrying capacity" (that is, the population size that available resources can support) and the population size [2, 6, 19, 26]. It is typically assumed that the carrying capacity is constant in time. Consequently, the logistic model exhibits a sigmoidal shape when the population is

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plotted as a function of time. It has also been studied in the case where the population oscillates due to periodic seasonality [13].

However, human population growth exhibits more complex behaviour, unlike population species grown in laboratory cultures which have a fixed amount of space and resources. It is, therefore, more realistic to assume a time-varying carrying capacity when using the logistic model for describing human population dynamics.

Meyer [16], as well as Meyer and Ausubel [17], considered a bi-logistic model, which is essentially a logistic model but with a sigmoidal time-varying carrying capacity. The bi-logistic model was applied to the English and Japanese populations, where a second growth occurred due to a shift from an agriculture-based society to an industrialized one [17]. Cohen [3] proposed a human population growth model with a variable carrying capacity, which in turn changes as a function of the population itself. A conclusion from the above models is that the inclusion of a variable carrying capacity is more reflective of the human condition.

Safuan et al. [22] considered a coupled system that describes the interaction between the population and carrying capacity. Their model eliminates the need for prior knowledge of the carrying capacity, or constraints to be placed upon the initial conditions. The same authors found an exact solution of a nonautonomous logistic equation with a special form for the carrying capacity and expressed the solution as a series [23]. Shepherd and Stojkov [24] studied the logistic equation with a slowly varying carrying capacity, and used multiple scale analysis to obtain an approximate closed-form solution. Other assumptions for the carrying capacity that have been considered are either periodic [14, 21] or stochastic [1].

Cohen [3] and Meyer and Ausubel [17] have attempted to illustrate human carrying capacity for the purpose of presenting robust models to accurately estimate the human population size that can be supported. However, there is no consensus with regard to appropriate models for human carrying capacity [3]. It is more or less accepted that human carrying capacity is influenced by several factors such as changes in technology, culture and economics. Specific examples of new technologies and resources are those that have permitted the increase in crop yields, as well as other innovations that have brought about the increase in human food availability [8]. Hence food availability is deemed an important factor that affects human population growth and defines its carrying capacity [8, 9].

To quantify food production data, a measure of global food availability must be established. The Food and Agriculture Organization (FAO) obtains data from official and semi-official reports of crop yields, area under production and livestock numbers. The food production index covers food crops that are considered edible and that contain nutrients (see [8] for more details on how the FAO determines this food production index, as well as a related livestock production index).

As a first step in modelling human population growth with a variable carrying capacity, we follow Hopfenberg [8] and postulate that food production data is the sole variable that influences human carrying capacity. In fact, Hopfenberg [8] assumed a simple linear relationship between human carrying capacity and food production index. Using the available FAO food production data and despite a crude fitting procedure, model parameters were estimated that yielded population estimates that closely approximate actual population numbers. However, population forecasting was not discussed in [8].

In this paper we propose more sophisticated human population growth models that relate human carrying capacity to food production data. The outline of this paper is as follows. In Section 2 we give three general classes of population models with variable carrying capacities that depend on food availability. In Section 3 we derive an integration-based method for model fitting and obtain explicit formulas for the parameters. Results of numerical simulations are presented and discussed in Section 4. We conclude with brief remarks in Section 5.

2. Logistic model with variable carrying capacity that depends on food supply

Let us consider the classical logistic equation and assume a variable carrying capacity, namely,

$$\frac{dN}{dt} = rN\left[1 - \frac{N}{K(t)}\right], \tag{2.1}$$

where $N(t)$ is the human population size at time t, r is the (constant and positive) intrinsic growth rate and $K(t)$ is the carrying capacity at time t. As stated in the previous section, we suppose for simplicity that food production data is the only variable that influences human carrying capacity. More precisely, we assume that $K(t) = f(I(t))$ (where $I(t)$ is the food production index at time t) for some suitable smooth function f of I such that $f(0) = 0$ and $f(I) > 0$ for $I > 0$. The former says that there is zero carrying capacity if no food is available (this is a mathematical idealization, since $I(t) > 0$ in practice; hence $K(t) > 0$), while the latter is due to the fact that carrying capacity is a positive quantity.

Moreover, we will consider three models depending on the properties of the function f.

(a) $f'(I) > 0$ for $I > 0$ and $f(\infty) = \infty$.
This model assumes that human carrying capacity increases indefinitely with increasing food production. A family of examples is $f(I) = \alpha I^p$, where $\alpha > 0$ and $p \geq 0$. When $p = 1$, we recover Hopfenberg's model [8], while $p = 0$ reduces to the classical logistic equation with a constant carrying capacity.

(b) $f'(I) > 0$ for $I > 0$ and $0 < f(\infty) < \infty$.
This is similar to the previous model in that human carrying capacity increases with increasing food production, but it does not do so indefinitely and tends to some finite positive limiting value. We say that the human carrying capacity is "self-limiting". Some examples are $f(I) = \alpha I/(1 + I)$ or $f(I) = \alpha(1 - e^{-I})$, where $\alpha > 0$. It is easy to see that $0 < f(\infty) = \alpha < \infty$.

(c) There exists $I^* > 0$ such that $f'(I) > 0$ for $0 < I < I^*$ and $f'(I) < 0$ for $I > I^*$, that is, f has a unique global maximum at I^*. Furthermore, $0 < f(\infty) < \infty$.

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Here we assume that there is a critical threshold value for the food production index. If the food production index is below the threshold, then the carrying capacity increases with the food supply, as in models (a) and (b). However, too much food production (and hence a food production index greater than the threshold) leads to a lowering of the carrying capacity. Some examples are $f(I) = \alpha I(1 + I)/(1 + I^2)$ and $f(I) = (I - 1)e^{-I} + 1$, where $\alpha > 0$.

The above examples for f can be expressed in the form $f(I) = \alpha g(I)$, where $\alpha > 0$ and $g(I) > 0$ for $I > 0$. The parameter α is to be estimated by fitting the model to the population data, while the functional form for g is specified according to the behaviour desired for the human carrying capacity. Of course, in principle, g may also depend on one or more parameters that will also have to be estimated. However, as an initial attempt at modelling and to keep the parameter estimation tractable, we assume that g does not depend on any unknown parameters.

3. Integration-based parameter estimation method

If we set $K(t) = \alpha g(I(t))$, then equation (2.1) becomes

$$N'(t) = rN(t) - \frac{r}{\alpha}\frac{N(t)^2}{g(I(t))}. \tag{3.1}$$

Suppose for the moment that $N(t)$ and $I(t)$ are known for all $0 \le t \le T$ for some positive T. Our goal here is to find explicit formulas for α and/or r using the integration-based method of Holder and Rodrigo [10].

Let $w = w(t; s)$ be a suitable positive weight function parametrized by $s \ge 0$. Examples of possible weight functions are $w(t; s) = e^{-st}$ and $w(t; s) = 1/(1 + t)^s$. If we choose the exponential function, then $\int_0^T w(t; s)N(t)\,dt$ can be viewed as a finite Laplace transform. Multiplying both sides of (3.1) by $w(t; s)$ and integrating by parts, we obtain

$$\begin{aligned} w(T; s)N(T) - w(0; s)N(0) - \int_0^T w'(t; s)N(t)\,dt \\ = r\int_0^T w(t; s)N(t)\,dt - \frac{r}{\alpha}\int_0^T \frac{w(t; s)N(t)^2}{g(I(t))}\,dt. \end{aligned} \tag{3.2}$$

To simplify the notation, define

$$\begin{aligned} a(s) &= \int_0^T w(t; s)N(t)\,dt, \\ b(s) &= -\int_0^T \frac{w(t; s)N(t)^2}{g(I(t))}\,dt, \\ c(s) &= w(T; s)N(T) - w(0; s)N(0) - \int_0^T w'(t; s)N(t)\,dt, \end{aligned} \tag{3.3}$$

so that (3.2) becomes

$$ra(s) + \frac{r}{\alpha} b(s) = c(s). \tag{3.4}$$

Note that in equation (3.4), $a(s)$, $b(s)$ and $c(s)$ are known quantities for a fixed s. We can think of (3.4) as a "generating equation" that is used to generate algebraic equations for α and/or r by assigning specific values to s. In principle, the same values for α and/or r should be obtained for any value of $s \geq 0$, provided the logistic equation (2.1) were an exact model of human population growth. In practice, of course, this may not be the case. However, if we believe in the validity of the logistic model, the parameter values thus obtained should be robust with respect to the choice of s, although only a heuristic justification of this was given by Holder and Rodrigo [10].

3.1. Rate *r* is known If the intrinsic growth rate r is assumed to be known as in [8], then choosing $s = s_0 \geq 0$ in (3.4) yields the explicit formula

$$\alpha = \frac{rb(s_0)}{c(s_0) - ra(s_0)}, \tag{3.5}$$

provided α is positive.

REMARK 3.1. From (3.3) we see that $b(s_0) < 0$. Suppose that there exists $M > 0$ such that $0 < N(t) < M$ for all $0 \leq t \leq T$. For definiteness, assume that $w'(t; s_0) < 0$ for all $0 \leq t \leq T$. This is the case, for example, when $w(t; s) = e^{-st}$ or $w(t; s) = 1/(1 + t)^s$. The case when $w'(t; s_0) > 0$ for all $0 \leq t \leq T$ can be treated similarly. Then

$$\begin{aligned} w(T; s_0)N(T) - w(0; s_0)N(0) - \int_0^T w'(t; s_0)N(t)\,dt \\ \geq w(T; s_0)N(T) - w(0; s_0)N(0) - M[w(T; s_0) - w(0; s_0)], \end{aligned}$$

which gives

$$c(s_0) - ra(s_0) \geq -[M - N(T)]w(T; s_0) + [M - N(0)]w(0; s_0) - rM \int_0^T w(t; s_0)\,dt.$$

If s_0 is such that

$$[M - N(T)]w(T; s_0) - [M - N(0)]w(0; s_0) + rM \int_0^T w(t; s_0)\,dt < 0, \tag{3.6}$$

then $c(s_0) - ra(s_0) > 0$ and, therefore, $\alpha < 0$, a contradiction. For example, if $w(t; s) = e^{-st}$, then inequality (3.6) simplifies to

$$[M - N(T)]e^{-s_0 T} - [M - N(0)] + \frac{rM}{s_0}(1 - e^{-s_0 T}) < 0. \tag{3.7}$$

Since the limit of the left-hand side of (3.7) as $s_0 \to \infty$ is $-[M - N(0)] < 0$, we deduce that s_0 cannot be taken to be too large. For a more general weight function $w(\cdot; s_0)$, s_0 should not be chosen so that (3.6) holds.

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REMARK 3.2. Here we investigate the robustness of α in (3.5) with respect to s_0. One way is to sketch α against s_0, and determine subintervals of s_0 where α is "almost constant" and positive. We then choose any s_0 in such subintervals. Another way is to consider $d\alpha/ds_0$. Differentiating (3.5) with respect to s_0 yields

$$\frac{d\alpha}{ds_0} = r\frac{b'(s_0)c(s_0) - b(s_0)c'(s_0) - ra(s_0)b'(s_0) + ra'(s_0)b(s_0)}{[c(s_0) - ra(s_0)]^2}. \tag{3.8}$$

In particular, if $w(t; s) = e^{-st}$, then (3.3) gives

$$\begin{aligned}
a'(s_0) &= -\int_0^T te^{-s_0 t} N(t)\, dt,\\
b'(s_0) &= \int_0^T \frac{te^{-s_0 t} N(t)^2}{g(I(t))}\, dt,\\
c'(s_0) &= -Te^{-s_0 T} N(T) + \int_0^T e^{-s_0 t} N(t)\, dt - s_0 \int_0^T te^{-s_0 t} N(t)\, dt.
\end{aligned}$$

If $N = N(t)$ is an exact solution of (3.1), then of course $d\alpha/ds_0 = 0$ for any s_0. Otherwise, sketching $d\alpha/ds_0$ against s_0 would indicate subintervals of s_0 where the graph is close to the s_0-axis. We then choose s_0 in one of these subintervals.

3.2. Rate r is unknown If the intrinsic growth rate r is not assumed to be known, then we need to determine α and r simultaneously. For this we choose two convenient nonnegative values of s, for example, s_1 and s_2 with $s_1 \neq s_2$, in (3.4) to produce the linear algebraic system

$$\begin{bmatrix} a(s_1) & b(s_1) \\ a(s_2) & b(s_2) \end{bmatrix} \begin{bmatrix} r \\ r/\alpha \end{bmatrix} = \begin{bmatrix} c(s_1) \\ c(s_2) \end{bmatrix}$$

for r and r/α. More specifically,

$$\alpha = \frac{c(s_1)b(s_2) - c(s_2)b(s_1)}{a(s_1)c(s_2) - a(s_2)c(s_1)}, \qquad r = \frac{c(s_1)b(s_2) - c(s_2)b(s_1)}{a(s_1)b(s_2) - a(s_2)b(s_1)}, \tag{3.9}$$

assuming that both quantities are positive. Conditions analogous to (3.6) to ensure the positivity of α and r in (3.9) can also be derived, which give restrictions on s_1 and s_2. Similarly, the robustness of α and r with respect to s_1 and s_2 can be investigated by looking at regions in the $s_1 s_2$-plane, where either (i) the surfaces α and r given in (3.9) are "almost constant", or (ii) the surfaces $\partial\alpha/\partial s_1$, $\partial\alpha/\partial s_2$, $\partial r/\partial s_1$ and $\partial r/\partial s_2$ are "close" to the $s_1 s_2$-plane.

REMARK 3.3. In practice, $N(t)$ and $I(t)$ are not known for all $0 \le t \le T$. Rather, discrete values N_j and I_j, where $j = 0, 1, \ldots, n$, are given at corresponding times t_j such that $t_0 = 0$ and $t_n = T$. Thus, the integrals appearing in (3.3) will be evaluated using numerical quadrature.

REMARK 3.4. Equation (2.1) is a Bernoulli equation whose exact solution is

$$N(t; r, \alpha) = \frac{1}{e^{-rt}/N(0) + (r/\alpha)\int_0^t e^{-r(t-u)}/g(I(u))\,du}. \tag{3.10}$$

A nonlinear least squares approach to estimating r and α involves the minimization of the squared error

$$E(r, \alpha) = \sum_{j=0}^{n} [N(t_j; r, \alpha) - N_j]^2.$$

In the case of a constant carrying capacity, the integral appearing in (3.10) can be evaluated explicitly, and partial derivatives of E with respect to r and α can be calculated, in principle. Here, however, this is not straightforward since one of the unknown parameters r appears inside the integral, which cannot be evaluated explicitly since it depends on $g(I(u))$. The integration-based method we use in this paper is easy to implement, as we have explicit formulas for α and/or r involving integrals that can be evaluated numerically.

4. Numerical simulations and discussion

We now present the results of the model fitting and population forecasting. The world population data [25] and world food production index data [5] can be downloaded from the World Bank website. Both data sets are visualized in Figure 1.

The food production index is a measure of the net food production of a country's agricultural sector per person. This covers all edible agricultural products that contain nutrients. The FAO determines these numbers relative to the average food production for three years, and sets the average for these three years equal to 100. Hopfenberg [8]

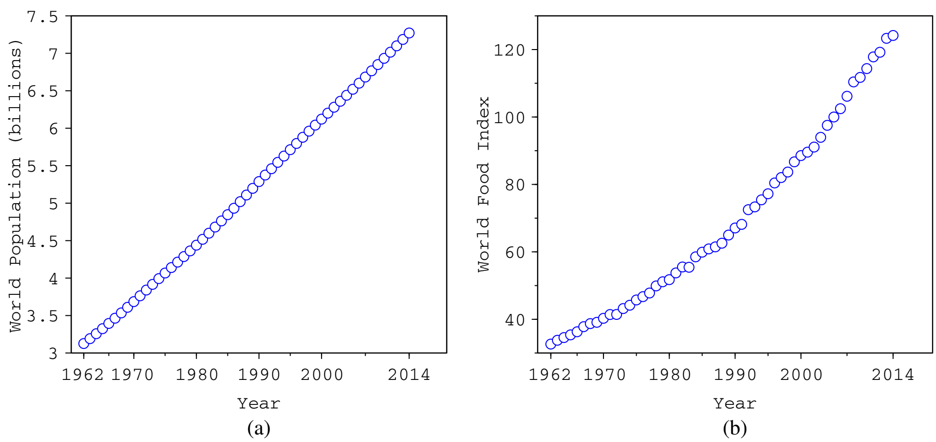

FIGURE 1. Data sets for (a) world population (billions) and (b) World Food Index, the net food production of the agricultural sector in the world per person, from years 1962 to 2014.

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used the three-year period from 1989 to 1991, while we use the current three-year period from 2004 to 2006. In Figure 1(b), an index value greater than 100 means food production is increasing with respect to the base years 2004–2006; otherwise it is decreasing.

There are three steps to be implemented in the procedure. The "parameter estimation" step applies the integration-based technique from the previous section, and makes use of the data from 1962 ($t = 0$) to 1991 ($t = 29$); thus $T = 30$. We use three different values of s_0 or (s_1, s_2) to find out which one gives the best estimates for α and/or r, respectively. The choice of three values is guided by the heuristic arguments given in Remark 3.2. The "error estimation" step uses the data from 1992 ($t = 30$) to 2014 ($t = 53$). Here we use the estimated parameters from the parameter estimation step, and solve the logistic equation (3.1) numerically to approximate the population from 1962 ($t = 0$) to 2014 ($t = 53$). Note that the last available population data are for 2014. Then we calculate the root mean square (RMS) error between the numerically obtained population size and the actual population data from 1992 ($t = 30$) to 2014 ($t = 53$). The magnitude of the errors will give an indication of which of the models (a), (b) or (c) with corresponding s_0 or (s_1, s_2) gives the best fit to the given data. Finally, the "population forecasting" step is to solve (3.1) numerically from 2015 ($t = 54$) to 2120 ($t = 158$), thus predicting the population trend after 2014.

As we can observe in Figure 1(b), the food production index exhibits an exponential trend. Therefore it is reasonable to implement a linear least squares technique to obtain the approximate curve $I(t) \simeq 32.86e^{0.025t}$ using the data from 1962 ($t = 0$) to 1991 ($t = 29$).

For the weight function we take $w(t; s) = e^{-st}$. Note that numerical simulations were also performed with the weight function $w(t; s) = 1/(1 + t)^s$ and similar results were obtained.

4.1. Rate *r* is known Here we estimate the parameter α only and fix $r = 0.03$, similarly to Hopfenberg's work [8]. By trying out three different values for s_0, the model fitting step using (3.5) and then the error estimation step are implemented. Table 1(a) gives the results for the three carrying capacity models. It shows that for model (a) with $f(I) = \alpha I$, the value $\alpha = 0.23$ gives the best estimate since the RMS error has the lowest value. Meanwhile, for models (b) with $f(I) = \alpha I/(1 + I)$ and (c) with $f(I) = \alpha I(1 + I)/(1 + I^2)$, the values $\alpha = 11.38$ and $\alpha = 10.97$ are the respective best approximations.

To justify the choices of s_0 in Table 1(a), Figure 2 depicts the RMS error on the interval $[0, 1]$, where the larger s_0 yields the larger value of the error. The value of s_0 that results in the smallest RMS error is then used to approximate the population number as provided in Figure 4(a). On the other hand, Figure 3(a) shows the graph of α against s_0 from (3.5) and Figure 3(b) shows $d\alpha/ds_0$ against s_0 from (3.8). We see that any s_0 in the subinterval $[0.00, 0.5]$ gives an "almost constant" and positive α and an "almost zero" $d\alpha/ds_0$; hence the choices of $s_0 = 0.00, 0.01, 0.10$ in Table 1(a).

TABLE 1. Parameter estimation for three carrying capacity models when (a) r is known to be $r = 0.03$ and (b) r is unknown.

Model (a): $f(I) = \alpha I$			Model (a): $f(I) = \alpha I^{1/4}$			
s_0	α	RMS	(s_1, s_2)	r	α	RMS
0.00	0.23	**0.415**	(0.00, 0.01)	0.032	3.86	0.021
0.01	0.23	0.461	(0.00, 0.10)	0.032	3.78	0.018
0.10	0.26	0.970	(0.05, 0.10)	0.032	3.75	**0.016**
Model (b): $f(I) = \alpha I/(1 + I)$			Model (b): $f(I) = \alpha I/(1 + I)$			
s_0	α	RMS	(s_1, s_2)	r	α	RMS
0.00	11.41	0.0004	(0.00, 0.01)	0.027	13.36	0.011
0.01	11.38	**0.0003**	(0.00, 0.10)	0.028	12.93	0.008
0.10	11.16	0.0017	(0.05, 0.10)	0.028	12.71	**0.006**
Model (c): $f(I) = \alpha I(1 + I)/(1 + I^2)$			Model(c): $f(I) = \alpha I(1 + I)/(1 + I^2)$			
s_0	α	RMS	(s_1, s_2)	r	α	RMS
0.00	10.97	**0.0001**	(0.00, 0.01)	0.027	13.32	0.011
0.01	10.94	0.0002	(0.00, 0.10)	0.027	12.88	0.008
0.10	10.66	0.0065	(0.05, 0.10)	0.028	12.65	**0.006**

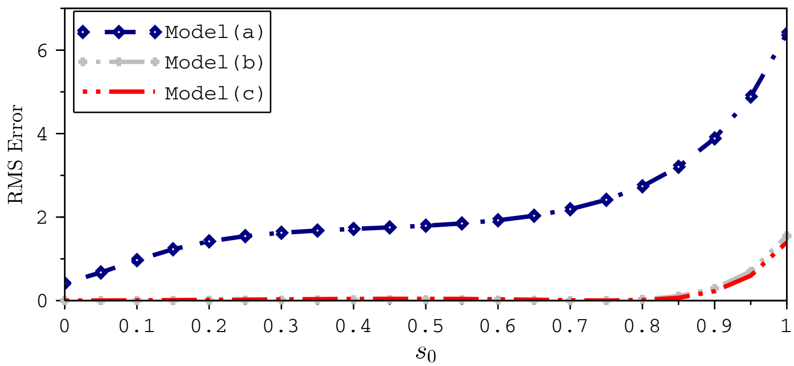

FIGURE 2. Root mean square error between numerical and actual population numbers as a function of s_0 for three carrying capacity models.

From these estimations we can now implement the population forecasting step to find out if the population will grow without bound, which is arguably unrealistic, or if it tends to some limiting population value for large times. Figure 4(b) shows that only models (b) and (c) give a reasonable result as the population number reaches a "limiting carrying capacity" α. This is because when t is large, then $I(t)$ is also large as it is approximated by an exponentially increasing function. Since $K = f(I)$, when I is large, the carrying capacity will tend towards α. On the other hand, when model (a)

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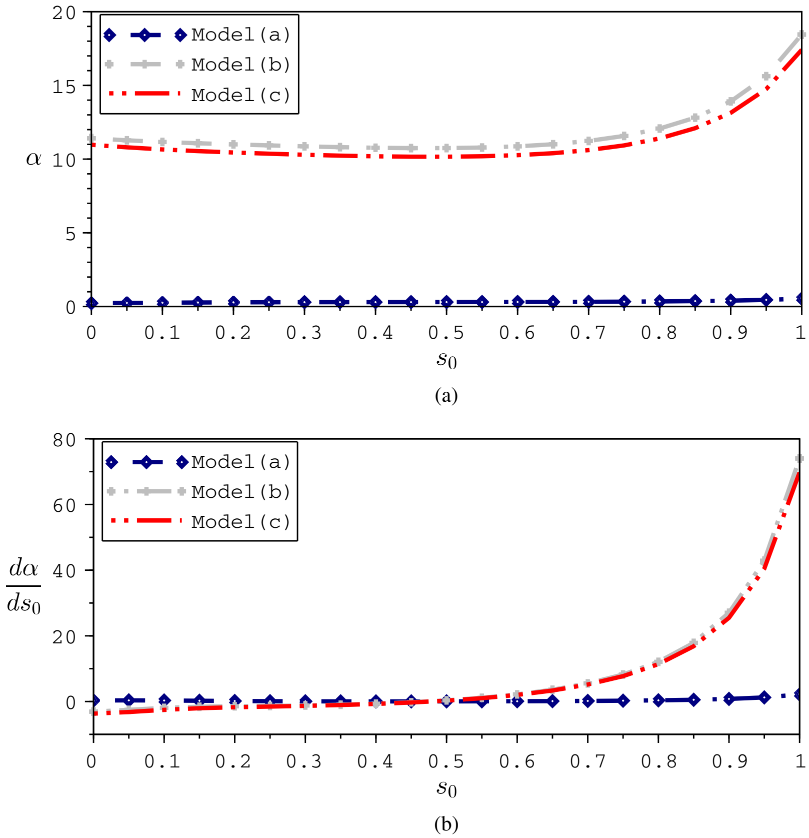

FIGURE 3. Graphs showing (a) the parameter α estimated as a function of s_0 (calculated by (3.5)) and (b) the gradient of α with respect to s_0 (obtained from (3.8)).

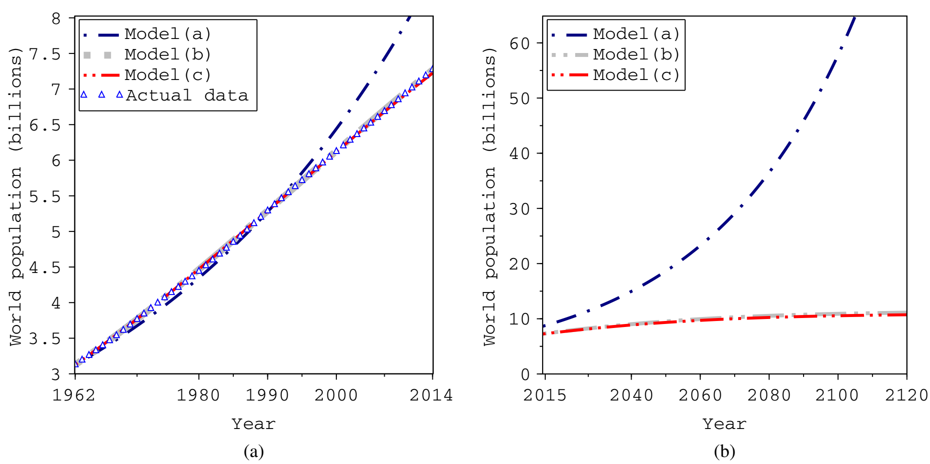

FIGURE 4. Comparison of the best three carrying capacity models (when r is known) with respect to (a) approximation with the actual data for 1962–2014 and (b) forecasting for 2015–2120.

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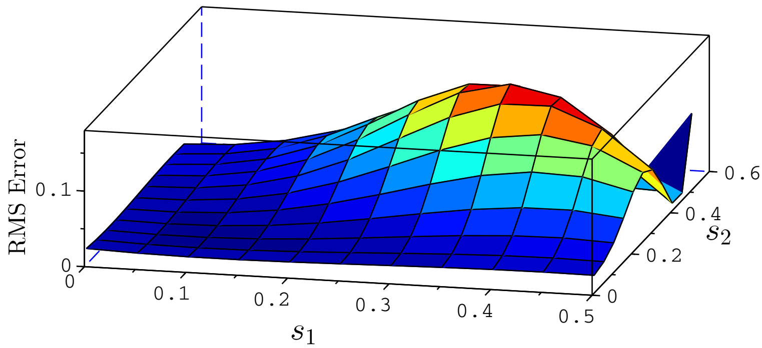
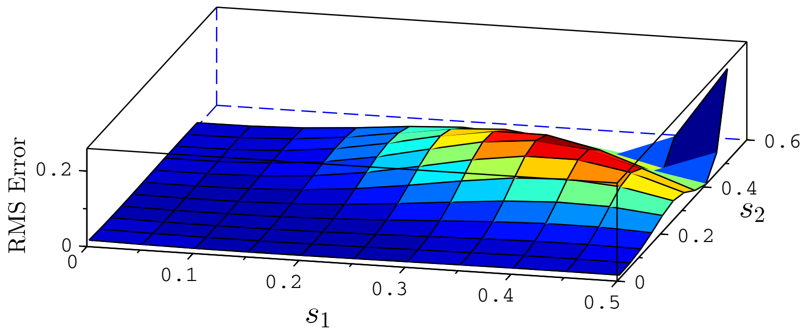
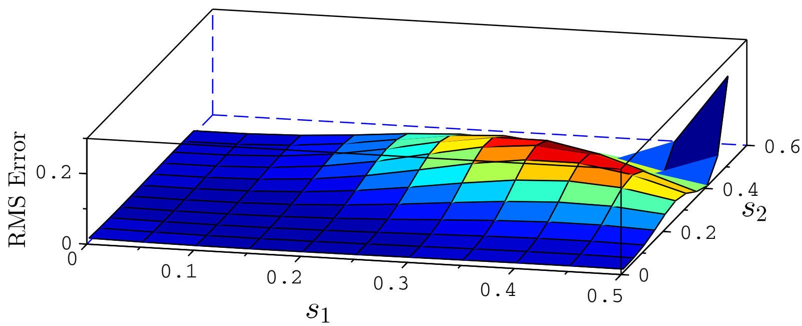


(a)



(b)



(c)

FIGURE 5. Root mean square error between numerical and actual population numbers as a function of s_1 and s_2 for three carrying capacity models.

is applied, then the carrying capacity will approach infinity since I becomes large, therefore the population number increases without bound.

In summary, since models (b) and (c) give smaller errors than model (a) after fitting actual population data, the numerical simulations indicate that around 100 years from

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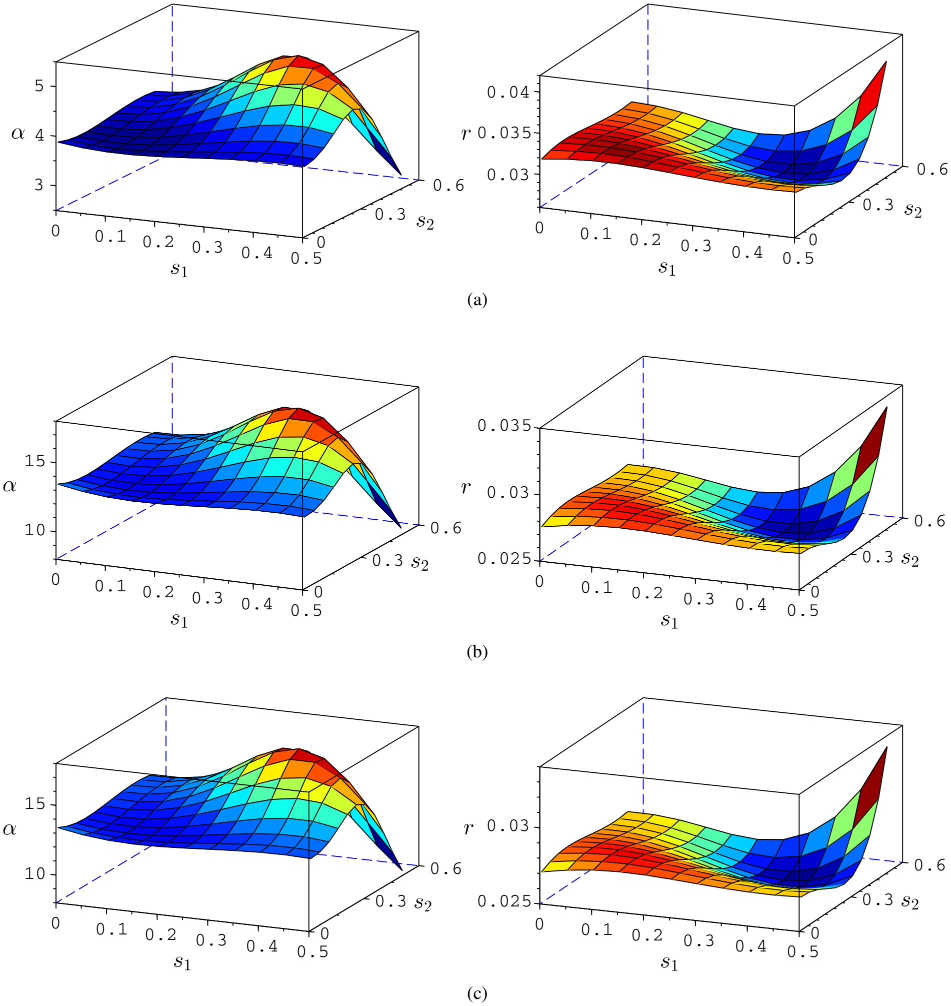

FIGURE 6. The estimated values of α and r which both depend on s_1 and s_2, calculated by (3.9).

now, the projected world population is about 11 billion (the approximate value of α in models (b) and (c)). This is in stark contrast to the projected population from model (a), which is around 92 billion. Note that model (a) with $f(I) = \alpha I$ is identical to the model proposed by Hopfenberg [8].

4.2. Rate r is unknown Next we estimate both parameters α and r using (3.9). This time, we choose three pairs of values for (s_1, s_2). We again choose $f(I) = \alpha I/(1 + I)$ for model (b) and $f(I) = \alpha I(1 + I)/(1 + I^2)$ for model (c) as before when r is assumed to be given. However, if we choose $f(I) = \alpha I$ for model (a), then (3.9) yields negative values for r as (s_1, s_2) is made to vary. Thus we take $f(I) = \alpha I^{1/4}$ instead (other forms

https://doi.org/10.1017/S1446181120000206 Published online by Cambridge University Press

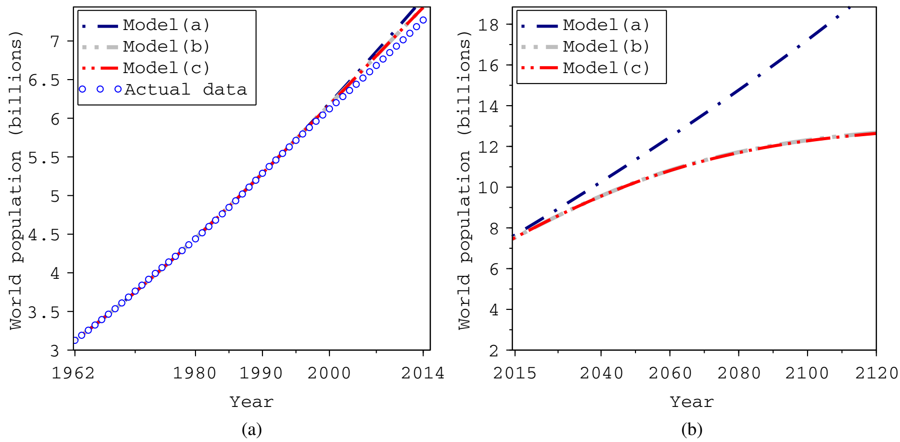

FIGURE 7. Comparison of the best three carrying capacity models (when r is unknown) with respect to (a) approximation with the actual data for 1962–2014 and (b) forecasting for 2015–2120.

such as $f(I) = \alpha I^{1/2}$ may be taken as well). Table 1(b) summarizes the results for the α and r estimates together with the corresponding RMS errors.

We see that all three models give very good estimates when $(s_1, s_2) = (0.05, 0.10)$, and again models (b) and (c) are (marginally) better than model (a).

To justify the choices of (s_1, s_2) in Table 1(b), Figure 5 presents the RMS error of all three models, where the subinterval $[0.0, 0.15] \times [0.0, 0.15]$ gives consistent values. Therefore, by this subinterval, any (s_1, s_2) yields "almost constant" and positive α and r, as shown by Figure 6. This is why we chose $(s_1, s_2) = (0.00, 0.01), (0.00, 0.10), (0.05, 0.10)$ in Table 1(b). Population approximations for the best value (s_1, s_2) for each model are provided in Figure 7(a). Meanwhile, population projections using the best approximation for each of the models are shown in Figure 7(b).

We summarize as follows. Similarly to the case when r was assumed known, from the numerical simulations we infer that around 100 years from now, the projected world population is about 13 billion (the approximate value of α in models (b) and (c)). However, this time model (a) predicts a population of around 19 billion when α and r are simultaneously estimated, compared to around 92 billion when r was fixed and only α was estimated. It should be noted, of course, that the functional forms for f are different, although they both belong to the class of functions in model (a).

5. Concluding remarks

In this paper, we assumed that the human carrying capacity is a function of food availability. Extending the classical logistic equation, we proposed three different classes of models (a), (b) and (c) that describe how the carrying capacity varies with the food production index. Model (a) assumed that the human carrying capacity increases without bound as food production increases, whereas models (b) and (c) assumed that there is a limit to the carrying capacity, even as food production is increased indefinitely.

We also proposed an integration-based method to estimate the parameters and gave explicit formulas for them. The method provides an alternative to a nonlinear least squares approach when an explicit analytical formula for the solution to the differential equation is not available or is not easy to implement. In essence, instead of minimizing the squared error, the integration-based method "averages out the potential errors" by taking the integrals of associated functions. This statement was not proved by Holder and Rodrigo [10], but can be heuristically motivated here as follows. A naive discretization of (3.1) is

$$\frac{N(t_{j+1}) - N(t_j)}{t_{j+1} - t_j} - rN(t_j) + \frac{r}{\alpha}\frac{N(t_j)^2}{g(I(t_j))} = 0.$$

Suppose that r is given and we wish to estimate α. For a fixed j_0, we substitute t_{j_0}, $I(t_{j_0})$ and $N(t_{j_0})$ into the above equation and solve for α (which is basically a collocation method). However, for each $j \neq j_0$, the left-hand side will introduce a residual term which may be positive or negative. By multiplying (3.1) by a weight function and integrating over $[0, T]$, in effect we are "averaging out the potential errors". A more rigorous mathematical analysis of the integration-based method is still work in progress by the second author.

From the integration-based model fitting using actual world population and food production index data, our results suggest that models (b) and (c) give the best fit. This implies that although an increase in food availability implies an increase in carrying capacity, there is an upper limit to the carrying capacity, which is not unreasonable to expect. In fact, looking at Figures 4(b) and 7(b), our models (b) and (c) predict human population in 2050 to be roughly 10 billion, which is comparable to the Population Reference Bureau prediction of 9.8 billion mentioned in the Introduction.

Potential extensions of our work here would be to include other factors that influence human carrying capacity, for example, water supply, living space and environmental conditions [7, 20, 27]. Agriculture requires water for food production and accounts for almost 70% of all water withdrawals, and up to 95% in some developing countries [18]. Thus water supply can be considered as a factor that influences human carrying capacity, since greater food production leads to a decrease in water supply, which in turn could potentially decrease human carrying capacity. Food production may also reduce the population size due to deaths from diseases caused by food plant infection. This is one mechanism that explains model (c), for example. One well-known example of dieback (see [4] as well as [15] for more information) occurred in Ireland after a fungus infection destroyed the potato crop in 1845 (the Irish potato famine). It was reported that as a result of the potato famine, approximately 1 million people died and 3 million more emigrated to other countries. The challenge of mathematical modelling is of course how to quantify such factors.

In this paper, although we elected to model human carrying capacity explicitly as a function of the food production index only, it is not unreasonable to expect that the effects of other factors (for example, water supply) are implicitly reflected in the observed population and food production data, and such effects are encapsulated in the parameters estimated via the model fitting procedure.

Acknowledgement

D. Zulkarnaen would like to thank the Indonesia Endowment Fund for Education (LPDP) for financial support for this research.

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