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TREND INFLATION, RIGIDITIES, AND HUMAN CAPITAL GROWTH

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A wage setting process defined in terms of wage per hour is the key factor for obtaining negative optimal trend inflation in a closed economy. However, this inflation will be zero if the process is established on the wage per unit of human capital. The origin of both results is a dynamic mechanism that, with some differences, makes possible the attainment of a situation equivalent to wage flexibility. Finally, while the effect of trend inflation on the long-run growth rate is tiny in the first case, it is much more important in the second, highlighting the relevance of this approach.

Keywords: Nominal Rigidity, Trend Inflation, Long-Term Growth, Staggered Price Setting

1. INTRODUCTION

There exists a vast literature on the optimal inflation rate that has been revitalized recently as a consequence of the constraints on monetary policy caused by the zero lower bound problems. At the same time, it is great the concern about the simultaneous occurrence of low growth rates among advanced economies, which has reopened the debate on a "secular stagnation." The relationship between trend inflation and long-run growth appears therefore of broad interest.

In this context we analyze whether the maximization of the long-run growth rate for a negative long-run inflation rate of around 2–3%, obtained in Amano et al. (2009) for an exogenous growth model and Amano et al. (2012) for an endogenous growth model as in Romer (1990), can be generalized for any other engine of growth with sticky prices and wages. In order to answer this question, we have alternatively considered Schumpeterian technological change, as in Aghion and Howitt (1992), and human capital, as in Lucas (1988).

After analyzing the impact of price and wage rigidities on the long-run growth rate in both models, we can conclude that the trend inflation rate that makes long-run growth maximum is not always negative. Consequently, the main result of Amano et al. (2009) and Amano et al. (2012) cannot be generalized. First because, with only price rigidity, the long-run relationship between inflation and growth

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is not relevant, at least for admissible values of quarterly inflation or deflation rates, so the neutrality of trend inflation is the conclusion in this case regardless of growth engines.[1]

Second, this result cannot be generalized either when we consider sticky wages since the model based on human capital accumulation reaches the maximum growth rate for a null inflation. While the annual inflation rate that maximizes the growth is within the interval $[-2\%, -3\%]$ when we consider stickiness in nominal wage per hour (Schumpeterian model[2]), we find that the model of human capital reaches its maximum growth for a null inflation rate as a consequence of its stickiness in nominal wage per unit of human capital.

What is the cause of this difference? It lies in the fact that, in the first model, the wage-setting process must adjust the nominal value in order to compensate inflation and growth. Consequently, a negative trend inflation rate with an absolute value equal to the long-run growth rate makes unnecessary a nominal revision, so the situation is the same as if there were wage and price flexibility. As a consequence, the maximum growth is the same as in the case of flexibility. Any deviation of the trend inflation rate from that negative value implies a lower growth rate (lower the greater the deviation) because it has the effect of a negative productivity shock. In this way, the decreasing prices implied by the negative optimal trend inflation rate avoid the distortion on growth introduced by wage stickiness. The long-run real wage that individuals receive will grow at the same rate as with flexibility thanks to the falling trend of prices.

However, the wage setting process in the human capital model has not to compensate the effect of growth because wages respond without lag to the human capital accumulation process carried out by individuals. As nominal wages are set per unit of human capital and they grow in the long run at the rate of trend inflation, the compensation for inflation is sufficient to recover the equilibrium real value of the wage per unit of human capital. Then the maximum growth rate is reached with null trend inflation in a situation that is also equivalent to flexibility. The long-run real wage that individuals receive will grow thanks to long-run human capital accumulation.

On the basis of the above analysis, we identify that the ultimate reason behind negative or null optimal trend inflation is the attainment of a situation that is equivalent to wage flexibility, with the result depending on the type of wage unit considered in the wage settlements. This finding is a clear contribution to show the mechanism that clarifies the meaning and the costs of nominal wage rigidities in the long-run when trend inflation is not the optimal one.

Although four have been the models analyzed in order to obtain our results, this paper is focused on the two alternative growth engines mentioned previously. The steady-state systems of equations for both models have been obtained, calibrated, and simulated in order to replicate the type of results required to answer the question posed. The calibration and simulation of these models have been carried out by means of Dynare. The results of the other two models [Physical capital externality as in Romer (1986); and technological change as in Romer (1990)] are briefly commented as a robustness argument, being available also their complete derivation from the authors upon request.

An additional and outstanding aspect of the models with wages per hour is that trend inflation has a very small effect on long-run growth. However, the results of the human capital model show a much more important non-neutrality phenomenon, given that the distortion directly affects not only the labor demand, but also the effort devoted to human capital accumulation and, hence, the growth rate. This result suggests the convenience of taking into account the role played by human capital (or job skill) when studying the influence of wage rigidity on growth in the long run. All the previous studies have considered the influence of nominal rigidities on wages per hour, ignoring the important role played by human capital in the wage setting process.

Effectively, the practical counterpart of the wage per unit of human capital is reflected in the relevance of job categories or occupations in remuneration settlements. Homogeneity in job categories of firms is nowadays not very common. Relative wages between categories can be considered as constant and the periodic nominal wage revision are generally applied considering this relative wage as given. This basic scheme can have complements, as bonus payments, whose achievement is not warranted by the general wage settlements because they depend on the individual productivity performance.

Unfortunately, DSGE models used for the analysis of monetary policy have, until recently, avoided the introduction of trend inflation and long-run growth and, consequently, the implications of their interactions are not still well known. The long-run consequences of trend inflation have been studied by Ascari (2004) and Amano et al. (2007), among others. After these contributions, several attempts appeared trying to show the relationship between inflation and growth from different perspectives.

The study of the interactions of long-run growth and monetary policy were initiated by Amano et al. (2009), followed by Amano et al. (2012). Amano et al. (2009) show clearly that the steady optimal inflation rate is negative with nominal wage rigidity. They assume an exogenous growth rate and conclude the optimal inflation rate from simulations with price and wage rigidities. The contribution of Amano et al. (2012) extended the analysis to an endogenous growth context and confirmed the result in a model of technological change as in Romer (1990).

Annicchiarico et al. (2011) introduce an endogenous growth in an standard NK model with staggered prices and wages concluding that (i) monetary volatility negatively affects long-run growth; (ii) the relation between nominal volatility and growth depends on the persistence of the nominal shocks and on the Taylor rule considered; and (iii) a Taylor rule with smoothing increases the negative effect of nominal volatility on mean growth.

Vaona (2012) explores the influence of inflation on economic growth merging an endogenous growth model of knowledge externalities [Romer (1986)] with a New Keynesian one with sticky wages, showing that, taking into account the influence of the quantity of money, the intertemporal elasticity of substitution of working time is a key parameter for the inflation–growth nexus. When it is zero, the inflation–growth nexus is weak and hump-shaped. When it is positive, inflation has a sizeable and negative effect on growth. In a cross-country/time-series data set, he found that increasing inflation reduces real economic growth, consistent with the theoretical model with a positive intertemporal elasticity of substitution of working time.

Annicchiarico and Rossi (2013) consider a NK model with an endogenous growth "a la Romer (1986)" and nominal rigidities due to staggered prices "a la Calvo (1983)." They consider the interplay between growth and business cycle and concentrate on the relationship between volatility and growth concluding that it leads to disregard the implied optimal monetary policy prescriptions to restore the efficiency.

Ascari and Sbordonne (2014) show that the conduct of monetary policy should be analyzed by appropriately accounting for the positive trend inflation targeted by policy makers. They construct a Generalized New Keynesian model that accounts for a positive trend inflation, where an increase in trend inflation is associated with a more volatile and unstable economy and tends to destabilize inflation expectations. Their analysis offers a note of caution regarding proposals to address the existing zero lower bound problem by raising the long-run inflation target.

From all these papers, the more concrete result is the main conclusion of Amano et al. (2009) and Amano et al. (2012) stating that the value of the trend inflation rate that maximizes welfare and the long-run growth rate is clearly negative (-1.8% and -3%, respectively). From the endogenous growth point of view, this result requires confirmation because it has been obtained through the introduction of a particular type of growth engine. Other engines, alternative to technological change as in Romer (1990), could provide different results when they are integrated into the same context of a DSGE model with trend inflation and wage and price rigidities with Taylor contracts [Taylor, (1980)].

Before proceeding with the rest of the paper, we must point out that we are only going to talk about growth to identify the optimal trend inflation, even though the ultimate goal of individuals is welfare maximization. This is because, given our interest in the steady state, talking in terms of growth is equivalent to talk in terms of welfare, as in Gomme (1993) and Amano et al. (2012), since the maximum growth also involves both maximum labor and consumption.

Section 2 contains the details of the two benchmark models of both kinds of wage setting processes (Schumpeterian and human capital models) and concludes with the steady-state systems of equations that are systematically collected in Appendix B. Section 3 analyses the long-run effects of nominal rigidities on the growth rate. Section 4 summarizes and compares the main impacts of the two kinds of models. The transmission mechanisms of each model and some outstanding results are commented in Section 5. Finally, Section 6 summarizes the main findings.

2. DSGE MODELS WITH ENDOGENOUS GROWTH AND STAGGERED WAGE AND PRICE SETTING

We present in this section the two benchmark models, which we have advanced in the previous section, one for each type of wage unit. First, we consider the Schumpeterian technological change model, which introduce nominal stickiness in wage per hour, as Amano et al. (2012). Second, we will analyze the human capital model that represents the case of nominal stickiness in wage per unit of human capital. We develop both models in the detail required to analyze thoroughly the impact of nominal rigidities on growth.

Special mention deserves the human capital model, where human capital accumulation raises the productivity of both labor and physical capital. Its basic idea is that people divide their time between work and training, from where a trade-off arises since people do not receive income when taking part in training but they increases their future productivity and, therefore, their future wages. It is a question of postponing income today (and hence consumption) for a greater income tomorrow.

We present the elements of both models following the same structure. First, we describe the behavior of the main agents in the economy. Second, we will obtain the mechanisms of price and wage setting with rigidities, where we can observe the main differences between both wage-setting processes. Hereafter, the source of growth will be explained in detail. To end the presentation of each model, we will conclude with the equilibrium conditions. A reference to the steady-state systems of equations for the two models closes the section.

We maintain in both models the assumption that there is no money, following the "cashless economy" hypothesis [Woodford (2003), Galí (2008)] typically adopted in New Keynesian macroeconomic models.

2.1. Schumpeterian Model

Households. Household members offer labor to final good producers, consume the final good, and hold bonds. Households are composed of infinite horizon individuals and are uniformly distributed in a continuum [0, 1].

Their expected utility takes the form

$$E_o \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \frac{1}{1+\nu} \int_0^1 L_{st}^{1+\nu}\, ds \right), \tag{1}$$

where $\beta \in (0,1)$ is the utility discount factor, ν (>0) the disutility of labor parameter, C is consumption, and L_s represents the supply of labor service s.

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Furthermore, households must satisfy their budget constraint in time t.

$$C_t + \frac{B_t}{P_t} + R\&D_t = \frac{B_{t-1}}{P_t} R_t + D_t + \int_0^1 \frac{W_{st}}{P_t} L_{st}\, ds, \tag{2}$$

where C_t is consumption, B_t nominal value of the stock of one-period life bonds that households hold in their portfolios; P_t the price of the final good, $R\&D_t$ investment in "research and development," R_t real gross interest rate, D_t firms' dividends, and W_{st} nominal wage for labor service s.

Moreover, we must consider the following restriction to avoid Ponzi schemes [Galí (2008)]:

$$\lim_{T \to \infty} E_t\,(B_t) \geq 0. \tag{3}$$

Final goods producers. According to Aghion and Howit (1992), intermediate good is a continuum of indexed goods [0,1] and the final goods production function is the following:

$$Y_t = L_t^{1-\alpha} \int_0^1 A_{it}^{1-\alpha} x_{it}^{\alpha}\, di, \tag{4}$$

where x_{it} is the intermediate good i used at t, $0 < \alpha < 1$, L_t is the composite demand of labor services $L_t = (\int_0^1 L_{st}^{\frac{(\sigma-1)}{\sigma}}\, ds)^{\sigma/_{\sigma-1}}$ and A_{it} is its productivity of the intermediate good i (quality level). The productivity evolves according to an innovation process, which will be explained later.

The final good producing sector is perfectly competitive, with firms choosing their inputs to maximize their profits. Consequently, the final good producers´ profits can be represented as follows:

$$F_{Yt} = P_t \int_0^1 (A_{\text{it}} L_t)^{1-\alpha} x_{\text{it}}^{\alpha} di - \int_0^1 W_{st} L_{st}\, ds - \int_0^1 P_{\text{it}} x_{\text{it}}\, di, \tag{5}$$

where P_{it} is the price of the intermediate good i.

Once obtained the demand function for labor service s, the demand function for L_t is

$$L_t = \frac{(1-\alpha)\, Y_t}{\Delta_{wt}}, \tag{6}$$

where $\Delta_{wt} = [\int_{s=0}^1 (\frac{W_{st}}{P_t})^{1-\sigma}\, ds]^{\frac{1}{1-\sigma}}$ represents the average real wage.

Intermediate good firms. Monopolistically competitive firms obtain intermediate goods. This sector operates a simple technology that generates one unit of a given intermediate good from one unit of final output. The profit for the firm producing i will be

$$F_{it} = P_{it} x_{it} - P_t x_{it}. \tag{7}$$

They sell their goods to final goods firms and set the prices according to Taylor contracts for I periods.

Wage and price setting. We assume the existence of price and wage rigidities that leads to a Taylor-type process of staggered price and wage setting. In both cases, this process takes into account the preferences of the agents involved. The preferences of the workers are relevant in the case of wage rigidity, given the assumption of equality between supply and demand for labor services, while the profit maximization strategies of the firms play the corresponding role in the case of price rigidities.

Wage setting. Final goods firms are who set wages for J periods that must satisfy households' preferences, given that we assume the equality between labor supply and demand. Therefore, the optimal wage for any type of labor service will be obtained from the maximization of the total discounted utility for every interval of J periods. The solution leads to the expression

$$W^* = \left[\frac{\sigma}{\sigma-1}\,\frac{\sum_{\tau=0}^{J-1}\beta^\tau L_{st+\tau}^{1+v}}{\sum_{\tau=0}^{J-1}\lambda_{t+\tau}L_{st+\tau}P_{t+\tau}^{-1}}\right]^{\frac{1}{1+\sigma v}}.$$

Substituting the expression of L_{st}, we can obtain the steady-state real wage normalized by the final good output.

$$\frac{W^*}{PY} = \left(\frac{\sigma(1-\alpha)^v}{\sigma-1}\,\frac{C}{Y}\,\frac{\sum_{\tau=0}^{J-1}\beta^\tau\left(g^\tau\Delta_w^Y\right)^{(\sigma-1)(1+v)}\Pi^{\sigma(1+v)\tau}g^{(1+v)\tau}}{\sum_{\tau=0}^{J-1}\beta^\tau\left(g^\tau\Delta_w^Y\right)^{(\sigma-1)}\Pi^{(\sigma-1)\tau}}\right)^{\frac{1}{1+\sigma v}}, \tag{8}$$

where $\Delta_w^Y = \frac{\Delta_W}{Y}$ and the variables have not subscript t because indicating that is a relation for the long run. We can see that the normalized steady-state real wage depends on the growth rate g, the average propensity to consume C/Y, the trend inflation Π, and the normalized average wage. Consequently, monetary policy could affect the real wage through Π. In addition, we can observe that nominal wage grows at the rate g $+\pi$, where $\pi = \Pi - 1$.

Price setting. Intermediate goods producers, *i*, are who set for the every I periods the price that maximizes their expected profits:

$$P_{it}^* = \frac{1}{\alpha}\,\frac{\sum_{\tau=0}^{I-1}\frac{\lambda_{t+\tau}}{\lambda_t}x_{it+\tau}\left(P_{it}^*\right)}{\sum_{\tau=0}^{I-1}\frac{\lambda_{t+\tau}}{\lambda_t}\frac{x_{it+\tau}\left(P_{it}^*\right)}{P_{t+\tau}}}, \tag{9}$$

where $x_{it+\tau}(P_{it}^*)$ is the demand of the intermediate good i in $t+\tau$ with the price fixed in the value P_{it}^*, which will be the same in steady state for all I with the expression

$$\frac{P^*}{P} = \frac{1}{\alpha}\,\frac{\sum_{\tau=0}^{I-1}\left(\beta(\Pi)^{1/1-\alpha}\right)^\tau}{\sum_{\tau=0}^{I-1}\left(\beta(\Pi)^{\alpha/1-\alpha}\right)^\tau}. \tag{10}$$

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Growth and innovation. This model displays Schumpeterian growth because it occurs by increasing the quality of intermediate goods [Aghion and Howitt (1992)]. By *quality*, we must understand technological (or productivity) level of the capital goods.

According to the intermediate goods demand function, the profit of the intermediate good producer i in t will be

$$F_{it} = \alpha^{\frac{1}{1-\alpha}} \left(\frac{P_{it}}{P_t} - 1 \right) \left(\frac{P_{it}}{P_t} \right)^{-\frac{1}{1-\alpha}} A_{it} L_t. \tag{11}$$

So that, taking into account price rigidity during I periods, the average expected profit VF_{it} in a period t for the intermediate producers, after having had success in innovation, is equal to

$$VF_{it} = \alpha^{\frac{1}{1-\alpha}} A_{it} L_t \frac{1}{I} \sum_{s=0}^{I-1} \left(\frac{P^*_{t-s}}{P_t} \right)^{-\frac{1}{1-\alpha}} \left(\frac{P^*_{t-s}}{P_t} - 1 \right). \tag{12}$$

We assume the following diminishing returns probability function for the success of the innovation:

$$\phi\left(n_{it}\right) = n_{it}^{\chi} \; 0 < \chi < 1, \tag{13}$$

with $\phi'(n_{it}) = \chi n_{it}^{\chi-1} > 0$ and $\phi''(n_{\mathrm{it}}) = \chi(\chi - 1) n_{\mathrm{it}}^{\chi-2} < 0$.

If innovation is successful, expected profits will be

$$\phi\left(n_{it}\right) VF^*_{it}, \tag{14}$$

where $n_{it} = {}^{R_{it}}/_{A^*_{it}}$, R_{it} being the quantity of final goods devoted to innovation and A^*_{it} the intermediate goods productivity achieved if innovation is successful. Consequently, the expected profit of the R&D activity that can provide an innovation is

$$\phi\left({}^{R_{it}}/_{A^*_{it}} \right) VF^*_{it} - R_{it}. \tag{15}$$

The optimal value of n_{it} will be common for all entrepreneurs due to the fact that n only depends on market elements

$$n_{it} = n = \left[\chi \alpha^{\frac{1}{1-\alpha}} L_t \frac{1}{I} \sum_{s=0}^{I-1} \left(\frac{P^*_{t-s}}{P_t} \right)^{-\frac{1}{1-\alpha}} \left(\frac{P^*_{t-s}}{P_t} - 1 \right) \right]^{\frac{1}{1-\chi}}. \tag{16}$$

According to the law of large numbers, the proportion of successful innovators will be $\mu = \emptyset(n)$. Consequently, the technological level of the economy will be

$$A_t = \mu \gamma A_{t-1} + (1 - \mu) A_{t-1}$$
$$A_t = \int_0^1 A_{it} di. \tag{17}$$

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Then, the gross growth rate will be as follows:

$$g_t = \frac{A_t}{A_{t-1}} = \frac{Y_t}{Y_{t-1}}, \tag{18}$$

$$g_t = \mu\,(\gamma - 1) + 1. \tag{19}$$

Considering $\mu = \emptyset(n) = n^{\chi}$, the gross growth rate in the steady state has the following expression:

$$g = \left[\chi\alpha^{\frac{1}{1-\alpha}} L \frac{1}{I} \sum_{s=0}^{I-1} \left(\frac{P^*_{-s}}{P}\right)^{-\frac{1}{1-\alpha}} \left(\frac{P^*_{-s}}{P} - 1\right)\right]^{\frac{\chi}{1-\chi}} (\gamma - 1) + 1. \tag{20}$$

Equilibrium conditions. The aggregate equilibrium of the economy is the equality between final output and the sum of consumption and gross investment. We assume, for the sake of simplicity, that there are neither public expenditures nor an external sector. Specifically, demand for good output is composed of consumption, investment in R&D, and intermediate goods production. As a result, the ratio consumption/output satisfices the following expression in steady state:

$$\frac{C}{Y} = 1 - \alpha^{\frac{1}{1-\alpha}} L \frac{1}{I} \sum_{s=0}^{I-1} \left(\frac{P^*_{-s}}{P}\right)^{-\frac{1}{1-\alpha}} \frac{A}{Y}$$
$$- \left[\chi\,\alpha^{\frac{1}{1-\alpha}}\, L \frac{1}{I} \sum_{s=0}^{I-1} \left(\frac{P^*_{-s}}{P}\right)^{-\frac{1}{1-\alpha}} \left(\frac{P^*_{-s}}{P} - 1\right)\right]^{\frac{1}{1-\chi}} \frac{A}{Y}, \tag{21}$$

where additionally

$$\frac{A}{Y} = \frac{1}{\left(\alpha^{\frac{1}{1-\alpha}} \frac{1}{I} \sum_{s=0}^{I-1} \left(\frac{P^*_{-s}}{P}\right)^{-\frac{1}{1-\alpha}}\right)^{\alpha} L}. \tag{22}$$

2.2. Human Capital Model

Households. In this model, household members offer labor to intermediate good producers, consume the final good, accumulate human capital, and hold bonds. Households again are composed of infinite horizon individuals and are uniformly distributed in a continuum [0, 1]. Their expected utility is the same as in the Schumpeterian model, but budget constraint includes as a noteworthy difference the dynamics associated to physical capital K:

$$C_t + \frac{B_t}{P_t} + K_{t+\tau+1} = \frac{B_{t-1}}{P_t} R_t + D_t + \int_0^1 \frac{W_{st}}{P_t} L_{st}\, ds + (1 + R_{t+\tau} - \delta)\, K_{t+\tau}. \tag{23}$$

Again, we must consider the previous restriction to avoid Ponzi schemes (Galí, 2008).

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As in Christiano et al. (2005), the representative household holds a stock of physical capital, rents it to the intermediate goods producers, and decides how much physical capital to accumulate. For simplicity, we assume in this model that there are no adjustment costs of investment. Then, the law of motion of physical capital is given as follows:

$$K_{t+1} = (1 - \delta) K_t + I_t, \tag{24}$$

where δ represents a depreciation rate of physical capital and I_t gross investment.

In addition, human capital requires a special mention related to the effective supply of every labor service s. We suppose individuals make two decisions. First, each individual chooses the total time devoted to nonleisure activities, that is, production activity plus accumulation of human capital, N_{st} . Second, each household member also chooses the fraction of every time unit devoted to the production activity, u_{st} ($u_{st} \in [0, 1]$), and the fraction devoted to human capital accumulation, $1 - u_{st}$ Therefore, we define the effective labor supply as follows:

$$L_{st} = u_{st} N_{st} h_{st}. \tag{25}$$

As in the previous model, L_{st} represents the supply of labor service s with $s \in [0, 1]$ and $L_t = (\int_0^1 L_{st}^{\frac{(\sigma-1)}{\sigma}} ds)^{\sigma/\sigma-1}$ the composite supply of labor services, being σ the elasticity of substitution.

It is assumed that human capital accumulation has the following technology:

$$h_{st+1} = [1 + \xi (1 - u_{st}) N_{st}] h_{st}, \tag{26}$$

where ξ is the productivity parameter of the accumulation process. The law of motion for the economy's total human capital is the following:

$$h_{t+1} = \int_0^1 h_{st} di = \left\{ \int_0^1 [1 + \xi (1 - u_{st}) N_{st}] \frac{h_{st}}{h_t} ds \right\} h_t. \tag{27}$$

Intermediate goods firms. Intermediate good producers are indexed by $j \in [0, 1]$ and has a Cobb–Douglas production function of the type

$$Y^i_{jt} = A K^{\alpha}_{jt} L^{1-\alpha}_{jt}, \tag{28}$$

where Y^i_{jt} is the output of a homogeneous intermediate good, A is total factor productivity, K_{jt} the stock of physical capital, and L_{jt} the composite index of differentiated labor services.

With regard to the labor demand, from profit maximization, we obtain the demand for labor service s of the firm j

$$L_{sjt} = \left[(1 - \alpha) A K_{jt}{}^{\alpha}\right]^{\sigma} \left(\frac{W_{st}}{P^i_t}\right)^{-\sigma} L^{1-\sigma\alpha}_{jt}, \tag{29}$$

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where L_{sjt} is the demand of the differentiated labor service s. The aggregated demand for labor is as follows:

$$L_t = \left[\frac{(1-\alpha)\,A}{\Delta^i_{wt}}\right]^{\frac{1}{\alpha}} K_t\,, \quad L_t = \int_0^1 L_{jt} dj\,, \quad K_t = \int_0^1 K_{jt} dj, \tag{30}$$

where Δ^i_{wt} again represents average real wage in the intermediate goods industry. We can rewrite the intermediate goods producer's optimal conditions as follows:

$$L_t = \left[\left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{(1-\alpha)\,A}{\Delta_{wt}}\right]^{\frac{1}{\alpha}} K_t, \tag{31}$$

$$R_t = \alpha\left[A\left(\frac{\varepsilon-1}{\varepsilon}\right)\right]^{\frac{1}{\alpha}}\left[\frac{1-\alpha}{\Delta_{wt}}\right]^{\frac{1-\alpha}{\alpha}}, \tag{32}$$

$$\Delta_{wt} = \left[\int_0^1 \left(\frac{W_{st}}{P_t}\right)^{1-\sigma} ds\right]^{1/_{1-\sigma}}. \tag{33}$$

Retail firms or final goods producers. There are an infinite number of retail firms over the continuum [0,1], which repackage the homogeneous intermediate goods and sell them to households. We assume that they have the same simplified production technology that converts one unit of homogeneous intermediate good into one unit of differentiated final good. Consequently, the final output Y_t is composed of a continuum of retail final goods

$$Y_t = \left(\int_0^1 Y_{rt}^{(\varepsilon-1)/_\varepsilon} dr\right)^{\varepsilon/_{\varepsilon-1}}, \tag{34}$$

where Y_{rt} is the output of retailer r. If users of the final output minimize costs, the demand for each differentiated final good r is

$$Y_{rt} = \left(\frac{P_{rt}}{P_t}\right)^{-\varepsilon} Y_t, \tag{35}$$

$$P_t = \left(\int_0^1 P_{rt}^{1-\varepsilon} dj\right)^{1/_{1-\varepsilon}}, \tag{36}$$

where P_{rt} is the price of Y_{rt} and P_t is the price index of the final output. They sell their goods to households and set the price according to Taylor contracts each interval of I periods.

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Growth and innovation. We have derived the growth process in human capital model from the solution of a dynamic optimization problem recorded in Appendix A for price and wage flexibility and for staggered wage and price setting. As a consequence, final output Y, intermediate goods production Y^i, physical capital stock K, and effective labor L grow at the same rate in steady state, that is the growth rate of average human capital h. Letting $g(.)$ be the growth rate of a variable at steady state, this situation implies the following relationships:

$$g\,(Y) = g\left(Y^i\right) = g\,(K) = g\,(L) = g\,(h)\,,$$

$$g\,(h) = \begin{cases} \left[1 + \xi\,(1 - u_{ss})\,N_{ss}\right] & \text{Wage Flexibility} \\ \left[1 + \xi\left(1 - u^1\right)N^1\right]\left(1 + g\left(h^1\right)\right)\left(\frac{J-2}{J}\right) & \\ \quad + \left[1 + \xi\left(1 - u^{01}\right)N^1\right]\left(1 + g\left(h^{01}\right)\right)\left(\frac{1}{J}\right) + & \\ + \left[1 + \xi\left(1 - u^0\right)N^0\right]\left(1 + g\left(h^0\right)\right)\left({}^1\!/_J\right) & \text{Wage rigidity.} \end{cases} \tag{37}$$

where u_{ss} y N_{ss} are steady-state values with wage flexibility, while u^1, h^1, and N^1 are the decisions for labor services with constant nominal wage for $s \in [0, \mathrm{J} - 3)$, u^{01}, h^{01}, and N^1 for $s \in [\mathrm{J} - 3, \mathrm{J} - 2)$, and u^0, h^0, and N^0 for s$\in [J - 2, J - 1]$ the corresponding for labor services that will reset nominal wage in the following period.

Wage and price setting.

Wage setting. Unlike previous model, intermediate goods producers are who set wages for J periods in the models with human capital. They set the wage W^* at t for J periods according to households' preferences, given the equality between labor supply and demand. Therefore, again, we obtain the optimal wage W_t^* for any type of labor service from the maximization of total discounted utility for an interval of J periods from t

$$W_t^* = \left(\frac{\sigma}{\sigma - 1}\right)\frac{E_t \sum_{\tau=0}^{J-1} \beta^\tau P_{t+\tau}^{\sigma} K_{t+\tau}^{\sigma\alpha} L_{t+\tau}^{1-\sigma\alpha} N_{\tau t+\tau}^{\upsilon} (u_{\tau t+\tau} h_{\tau t+\tau})^{-1}}{E_t \sum_{\tau=0}^{J-1} \beta^\tau C_{t+\tau}^{-1} P_{t+\tau}^{\sigma-1} K_{t+\tau}^{\sigma\alpha} L_{t+\tau}^{1-\sigma\alpha}}. \tag{38}$$

$$\frac{W^*}{P} = \left[\left(\frac{\sigma}{\sigma - 1}\right)\left(\left(\frac{\varepsilon}{\varepsilon - 1}\right)\frac{\Delta_w^{1-\alpha\sigma}}{(1-\alpha)\,A}\right)^{\frac{1}{\alpha}}\frac{C}{K}\frac{\sum_{\tau=0}^{J-1}\beta^\tau N_\tau^{1+\upsilon}}{\sum_{\tau=0}^{J-1}\beta^\tau \Pi^{(\sigma-1)\tau}}\right]^{\frac{1}{1-\sigma}}. \tag{39}$$

$N_\tau = N^1$ for $\tau = 0,\ 1,\ 2,\ \ldots,\ J - 2 \qquad N_\tau = N^0$ for $\tau = J - 1$.

Unlike previous model, nominal wages per unit of human capital grow at rate π in the steady state.

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Price setting. Retail firms are who set for I periods from t the price P_t^* that maximizes their expected profits in that time interval:

$$P_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t \sum_{\tau=0}^{I-1} \beta^\tau (P_{t+\tau})^\varepsilon \frac{Y_{t+\tau}}{C_{t+\tau}} P_{t+\tau}^i}{E_t \sum_{\tau=0}^{I-1} \beta^\tau (P_{t+\tau})^{\varepsilon-1} \frac{Y_{t+\tau}}{C_{t+\tau}}}. \quad \mathbf{(40)}$$

From this expression, the optimal relative price in steady-state will be

$$\frac{P^*}{P} = \frac{\sum_{\tau=0}^{I-1} (\beta \Pi^\varepsilon)^\tau}{\sum_{\tau=0}^{I-1} \left(\beta \Pi^{\varepsilon-1}\right)^\tau}. \quad \mathbf{(41)}$$

Equilibrium conditions. We again assume, for the sake of simplicity, that there are neither public expenditures nor an external sector. Final good output is composed of consumption and investment, and the steady-state consumption to physical capital ratio in steady state, C/K, will be as follows:

$$\frac{C}{K} = \frac{Y}{K} - g(K) - \delta. \quad \mathbf{(42)}$$

Since the right-hand side is constant over time in steady state, consumption and capital grow at the same rate, and therefore

$$\frac{C}{K} = A^{\frac{1}{\alpha}} \left[\left(\frac{\varepsilon - 1}{\varepsilon} \right) \frac{1-\alpha}{\Delta_t^W} \right]^{\frac{1-\alpha}{\alpha}} - g(C) - \delta, \quad \mathbf{(43)}$$

$$g(Y) = g\left(Y^i\right) = g(K) = g(L) = g(C). \quad \mathbf{(44)}$$

2.3. Steady State

Considering that our objective is to analyze the long-term performance of the economy, we must define the steady state and the system of equations that determine the values of the endogenous variables in this situation. Since our models incorporate economic growth and some variables grow in steady state, these variables must be normalized.

On one hand, in Schumpeterian model, we carry out the normalization of all the growing variables dividing them by the production level of the final good Y. The system of equations is presented in Section B.1. The endogenous variables are $\frac{P^*}{P}$, $\frac{P^*_{-s}}{P}$, g, L, Δ_W^Y, $\frac{W^*}{PY}$, $\frac{W^*_{-s}}{P}$, $\frac{C}{Y}$, $\frac{A}{Y}$, and R .

On the other hand, taking into account the representative household's optimal control problem of human capital model developed in Appendix A, the steady-state system of equations is different depending on the existence or not of wage rigidity. If wages are flexible, the equations system contains seven unknowns: W^*/P, C/K, g, N_{ss}, u_{ss}, P^*/P, and Δ_P . If there is wage rigidity, the equations system contains 13 unknowns according to Appendix A: W^*/P, W^*_{-s}/P, Δ_t^W,

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TABLE 1. Parameter values

Parameter	Description	Schumpeterian model	Human capital model
δ	Capital depreciation rate		0.025
α	Output elasticity with respect to capital	0.332	0.332
β	Utility discount factor	0.995	0.995
ε	Elasticity of substitution among retail goods		5
σ	Elasticity of substitution among labor services	10	10
ν	Relative utility weight of labor	1	1
I	Periods it takes to reset prices	1 or 2	1 or 2
J	Periods it takes to reset wages	1 or 4	1 or 4
γ	Productivity upgrade after each innovation	1.009	
χ	Probability of innovation success elasticity	0.1	
ξ	Productivity of human capital accumulation		0.008
A	Constant total factor productivity		1

C/K, g, N^0, N^1,u^1, u^0,u^{01}, $P^*/P, P^*_{-s}/P$, and Δ_P. We present the system of equations in Section B.2.

3. NOMINAL RIGIDITIES AND THE INFLATION–GROWTH RELATIONSHIP IN THE LONG RUN

This section presents the results obtained from the simulation of the relationship between trend inflation and long-run growth in the two models for different kinds of rigidity. The chosen values of the parameters for each model are presented in Table 1. These values correspond to quarterly data and are the same if they are present in both models in order to consider comparable economies.

The values for α and β are typical in simulations of DSGE models. We have chosen the value of capital depreciation δ in order to obtain plausible values of the annual growth rates (around the interval 2–3%), and the values 5 and 10 for the elasticity of substitution for retail or intermediate goods (ϵ) and differentiated labor services (σ), respectively. The first value is coherent with the evidence of Basu and Fernald (1997), and the second is closed to the used in Amano et al. (2012) quoting Basu (1996). We have assigned a value of 1 for the disutility of labor parameter ν, as in Hornstein and Wolman (2005).

The length of price contracts I will be 2 when there is no price flexibility ($I = 1$), based on results reported in Bils and Klenow (2004). The length of wage contracts J will be 4 when there is no wage flexibility ($J = 1$), as in Erceg et al. (2000) and Huang and Liu (2002). Taylor (1999) provided a review of the empirical literature

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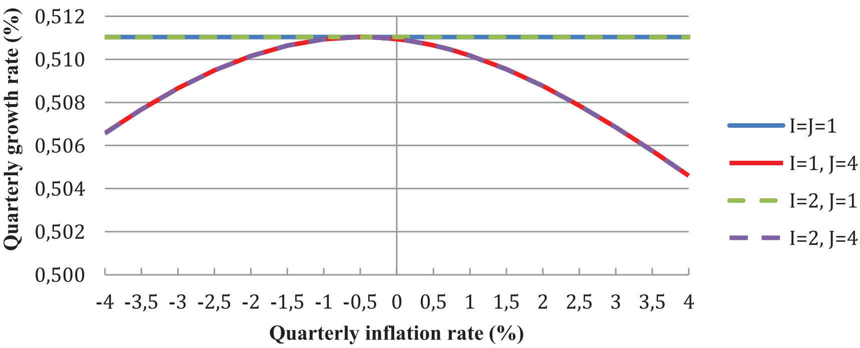

FIGURE 1. Long-term inflation–growth relationship for different types of rigidity Schumpeterian growth model.

and concludes that the average frequency of wage changes is about one year. The other parameters (γ, ξ, A) are present only in one model and the values are very plausible for each of them.

3.1. Model with Nominal Wage Per Hour

The Schumpeterian technical change model has been our election as benchmark for the models with stickiness in wage per hour. Figure 1 contains the results for the different types of wage and price rigidities.

First, we will start with the model of price and wage flexibility ($I = J = 1$). Figure 1 shows with a horizontal continuous blue line how the quarterly growth rate remains constant at 0.511% whatever the inflation rate in this case of total flexibility.

Second, if we calibrate the model with only price rigidity (I=2, J=1) and simulate it for different values of trend inflation, we obtain the same relationship between inflation and growth as for flexibility for admissible values of trend inflation (dotted green line). As a consequence, we conclude that trend inflation has no impact on the growth rate with only price rigidity.

Regarding only wage rigidity ($I = 1$, $J = 4$), we obtain the relationship between inflation and growth displayed in Figure 1 as a continuous red line with a maximum of 0.511% (the same value as when wages are flexible). However, this value is reached for a deflation rate of −0.511% (−2.06% annual). When the inflation or deflation rate is different from this value, the long-run growth rate is lower the greater the difference. Consequently, the long-run relationship between inflation and growth has an inverted-U shape. In other words, there exists a distortion in the allocation of resources for values of the trend inflation rate different from −0.511%, the growth rate corresponding to flexibility with a negative sign. If we calibrate the model simultaneously with price and wage rigidity, the second dominates the first and the outcome is the same as with only wage rigidity (an inverted-U shape with a maximum growth rate 0.511% for a trend inflation rate of −0.511%, dotted purple line in Figure 1).

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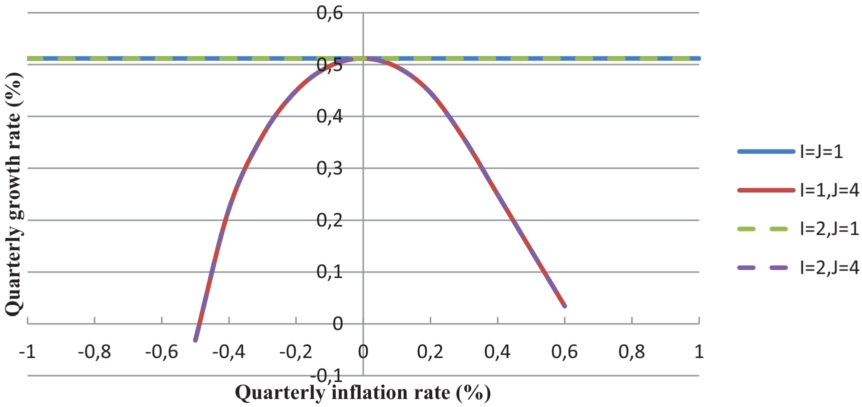

FIGURE 2. Long-term inflation–growth relationship for different types of rigidity human capital model.

To summarize the simulations of this model, we must consider the shape of the lines as well as the importance of the effects trend inflation has on the growth rate. Three characteristics can be highlighted. The first is the no impact of trend inflation on long-run growth with price rigidity. The second, the symmetry around the inflation rate value -0.511% in the case of wage rigidity (with or without price rigidity). The third, highly remarkable characteristic is the very low effect that trend inflation rate has on the long-run growth rate under wage rigidity. For example, a change of 4 percentage points in the annual inflation rate from -2.06% affects the growth rate by only less than one hundredth of a percentage point, which is a very low effect.

3.2. Models with Nominal Wage Per Unit of Human Capital

We do not find in Figure 2 any difference between human capital model and the previous one regarding flexibility and only price rigidity: the long-run inflation–growth relationship is a horizontal line in the value 0.512% for the growth rate, which shows that long-run growth and trend inflation are independent.

The biggest difference with the previous model is found in the case of wage rigidity: the growth rate is maximized for a null inflation. At that point, the growth rate is the same as in the case of flexible wages (0.512% quarterly, 2.06% annual). As a consequence of the two previous results, if we consider both kinds of rigidity, the growth rate will be also maximum for null inflation at the same value as when only wages rigidity exists.

We can appreciate the existence of a clear asymmetry around null inflation for rigid wages: greater deflation negatively affects more the growth rate than greater inflation. However, this model is not only different from the previous one in the value of trend inflation that maximizes the long-run growth rate and in the asymmetry. We also find a sharp difference when we consider the magnitude of the effect of trend inflation on long-run growth. Now, the units are not hundredths or tenths of a percentage point. A positive change in the annual inflation rate of two percentage points from 0 is the cause of a decline of 1.49 percentage points in the long-run growth rate, while a negative change of two percentage points origins a decline of 2.2 percentage points.

4. ASSESSMENT OF THE MAIN IMPACTS OF NOMINAL RIGIDITIES ON THE RELATIONSHIP BETWEEN LONG-RUN GROWTH AND INFLATION

The independence between long-run growth and trend inflation when only price rigidity exists is a result that appears in both types of models. Under this type of rigidity, the impact of trend inflation on long-run growth is so limited that the growth rate remains constant for admissible values of trend inflation. In fact, the relationship between these two variables is a question of greater or lesser degree. For quarterly, values lower than 12–13%, the relationship is negligible. Using an appropriate scale, it would be possible to appreciate graphically not only the effect but also that the maximum growth is attained when trend inflation is null. Despite that situation we think it is appropriate to conclude the lack of relation for usual values of the inflation rate.

This irrelevance of the pricing frictions is a consequence of the indirect mechanisms trough which they affects growth and the very low direct impact of the changes in the inflation rate. These frictions hardly affect first the consumption/output ratio, then the optimal wage, then the employment, and finally the growth rate. By contrast, wage rigidity directly affects employment and, hence, long-term growth rate.

Table 2 summarizes the optimal trend inflation and maximum growth rates depending on the different types of rigidities and models. It is noteworthy that the behavior when both rigidities exist is the same as with only wage rigidity, but, above all, the standing out result is that the long-run growth rate is maximum for null inflation when wages are rigid in the human capital model.

Wage rigidity reduces the long-run growth rate for any model, except for the value of trend inflation where the long-run growth rate is the same as for flexibility. For values differing from this one, a distortion is introduced at least in the demand for labor, which reduces the long-run growth rate to a greater extent the greater the difference. Moreover, the maximum growth rate with wage rigidity is the same as with price and wage flexibility indicating that, in fact, the growth rate is reached because the two situations are, for this trend inflation rate, equivalent. Effectively, with this inflation rate, the revision of nominal wages is not necessary because deflation adjusts the real wage just to reach the real wage target.

If we observe Schumpeterian model results, the value of the trend inflation rate is equal in absolute value to the maximum growth rate, showing a clear compensation between the two rates. These results indicate that the revision of wages excessively elevates the average real wage when trend inflation is different from a rate equal

TABLE 2. Optimal trend inflation and maximum long-run growth (quarterly rates)

	Schumpeterian model		Human capital model	
Types of rigidity	Π (%)	g (%)	Π (%)	g (%)
Total flexibility*	–	0.511	–	0.512
Price rigidity*	–	0.511	–	0.512
Wage rigidity	−0.511	0.511	0	0.512
Price and wage rigidity	−0.511	0.511	0	0.512

*Long-run growth and trend inflation are independent.

to the growth rate corresponding to price and wage flexibility with a negative sign, which decreases labor demand and the long-run growth rate. In fact, inflation acts as a negative productivity shock. When trend inflation is negative at exactly the same value as the long-run growth rate, nominal wage revision is not necessary. It is exactly a situation equivalent to wage flexibility. In this kind of models, the long-run real wage that individuals receive will grow as with flexibility due to the falling trend of prices with this negative trend inflation.

In the human capital model, the rigidity in the wage per unit of human capital does not involve a growth component as in the case of the wage per hour. The wage setting process revises with flexibility the skill component of the contracts and the real distortion of the previous model is not present. This is the reason why the maximum growth takes place for a null inflation. The long-run real wage that individuals receive without inflation will grow as with flexibility due to the long-run human capital accumulation. This accumulation process is precisely the origin of the greater sensibility of long-run growth to trend inflation compared to the other approach that human capital model shows, because the distortion affects additionally the growth process through the time devoted to the accumulation of human capital.

According to the general equilibrium results, we can conclude that the key variable in the distortion is Δ_{W}, the average wage, given the direct effect that it has on employment and growth. The human capital model shares with the model with wage per hour the direct effect of Δ_W on employment and from employment on growth. The maximum long-run growth rate in both types of models takes place for the value of the trend inflation rate for which Δ_W is minimum.

But in the human capital model there is an additional effect of Δ_W, a direct effect on the growth rate due to the effort to accumulate human capital. The growth rate increases after a reduction in Δ_W as a result of the sum of the two effects as indicates equation (B.15) in Appendix B of the revised version. This reduction implies an increase in employment as equation (23) shows and in the effort devoted to human capital accumulation as a higher growth rate requires, all this jointly with an increase in the return to capital as equation (24) shows.

TABLE 3. Summary of the main results (optimal trend inflation and maximum growth rate)

	Physical capital externality model		Technological change model	
Types of rigidity	Π (%)	g (%)	Π (%)	g(%)
Total flexibility*	–	0.541	–	0.571
Price rigidity*	–	0.541	–	0.571
Wage rigidity	−0.541	0.541	−0.571	0.571
Price and wage rigidity	−0.541	0.541	−0.571	0.571

*Long-run growth and trend inflation are independent.

We have also analyzed two additional growth engines in search of robustness for our results: the physical capital externality model as in Romer (1986) and the technological change model as in Romer (1990). Both models consider nominal wages per hour, the same type of stickiness than our Schumpeterian model. Their growth engines allow us to confirm that this kind of wage setting processes involves adjusting the nominal wages value through compensation of inflation and growth. We have confirmed exactly all the results of the Schumpeterian model in all their details (Table 3). The complete results of these models are available from the authors upon request.

5. TRANSMISSION MECHANISMS

Having evaluated and compared through simulations the consequences of the nominal rigidities on the long-run relationship between inflation and growth, it is necessary to identify from the steady-state equations (Appendix B) the main mechanisms that make this relationship similar in some cases and different in others depending on the kind of model and the sort of rigidity. To do so, in this section, we emphasize the distortion in the labor market introduced by wage rigidity as the key factor in the dependence or independence between trend inflation and long-run growth and the reason why the human capital model shows highly differentiated results: maximum long-run growth rate for null trend inflation, asymmetry and a significant impact of trend inflation on long-run growth.

5.1. Model with Nominal Wage Per Hour

A direct inspection of the equations in Section B.1 of Appendix B with $I = J = 1$ leads immediately to the conclusion that g is independent of Π in the case of flexibility. With only price rigidity ($I = 2$, $J = 1$) trend inflation barely has an influence on the long-run growth rate. Although the terms $\frac{P^*_{-s}}{P}$ change with Π, the effects on $\frac{1}{I}\sum_{s=0}^{I-1}\left(\frac{P^*_{-s}}{P}\right)^{-\frac{1}{1-\alpha}}\left(\frac{P^*_{-s}}{P}-1\right)$, $\frac{1}{I}\sum_{s=0}^{I-1}\frac{P^*_{-s}}{P}$ and $\left(\frac{1}{I}\sum_{s=0}^{I-1}\frac{P^*_{-s}}{P}\right)^{-\frac{1}{1-\alpha}}$

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are scant and on Δ_W^Y, C/Y and A/Y negligible in (B.5), (B.8) and (B.9). The effect on L is also insignificant due to the fact that it only receives an indirect effect through consumption. As a result, long-run growth rate in (B.3) remains constant.

The situation is different with wage rigidity for any value of Π because the distortion introduced by the inflation rate in the mark-up of the wage affects directly Δ_w^Y in (B.5), W^*/PY in (B.6), L in (B.4) and, finally, the growth rate in (B.3). The main difference between this effect and the previous one is that wage rigidity affects directly the employment, the main variable that drives the growth according to (B.3). In fact, L has a maximum when Δ_w^Y is minimum and coincides for the value of Π that maximize the growth. Then, according to expression (B.3), the maximum growth rate occurs for a quarterly deflation rate of -0.511% with L maximum. When rigidity takes place in wages and prices, the result is the same as in the situation with only wage rigidity due to the independence between trend inflation and growth with only price rigidity. Consequently, we find that the key role is played by the distortion in the labor market introduced in the variable Δ_w^Y as a consequence of wage rigidity. Hence, the maximum growth is reached when this variable is minimum (because the distortion is null) and the employment maximum.

5.2. Model with Nominal Wage Per Unit of Human Capital

The results in this kind of model have many aspects in common with the previous one but they are different and much more significant in some key aspects. From the equations in Section B.2 of Appendix B with $I = J = 1$, we can conclude immediately that g is independent of Π in the case of flexibility. Regarding price rigidity, the effects of Π on Δ_p, P^*/P, C, and Δ_W are negligible and so limited on L that they are not noticeable until quarterly inflation/deflation rate values are greater than 12–13%. Consequently, in (B.15) g and Π are independent for admissible values of trend inflation, showing the same behaviur as with flexibility in prices and wages.

However, if we consider wage rigidity the effect of Π on g is a consequence of the variation in both the wage per unit of human capital (B.11) and in Δ_W (B.13). The distortion in the labor market is only present due to the inflation rate, as is reflected in (B.13). As (B.15) establishes a univocal (and inverse) relationship between Δ_W and g, the maximum growth is reached when Δ_W is minimum for a null inflation.

Regarding wage and price rigidity, the situation is similar to the previous model. As with price rigidity, the relationship between Π and g is null for admissible inflation rates, the behavior under both rigidities is similar to that with only wage rigidity.

The features that increase the effect of Π on g in this model are those related to the process of human capital accumulation. A greater average wage, Δ_W, affects not only L (negatively), but also directly g (negatively). The effects on the last two variables imply a lower effort in the accumulation of human capital (expressions

B.16–B.20). This lower effort means a direct reduction in the long-run growth rate, which is the difference in the magnitude of the effects of Π on g compared to the other models in which only the effect on L is present in growth path.

According to the general equilibrium results, we can conclude that the key variable is Δ_{W}, the average wage, given the direct effect that it has on employment and growth. The human capital model shares with the other models with wages per hour the direct effect of Δ_{W} on employment and of employment on growth. The long-run growth rate is maximized in all the models for the trend inflation rate value for which Δ_{W} has the minimum.

But in the human capital model there is first an additional effect of Δ_W, a direct effect on the growth rate due to the response of the effort to accumulate human capital. A higher Δ_W than the minimum shifts hours from human capital to market work, not only as a direct substitution effect but also as an indirect wealth effect caused by a fall in the return to physical capital. The sum of the two effects is showed by equation (B.15) in Appendix B. The rise in Δ_W implies a reduction in employment, as equation (23) shows, and in the effort devoted to human capital accumulation as the lower growth rate requires, all this jointly with a decrease in the return to physical capital as can be seen in equation (24).

Moreover, there is another feature derived from the definition of L in both types of models that also plays an important role in the greater magnitude of the effects in the model with stickiness in the wage per unit of human capital. In this model, the employment L has three dimensions or factors: N, u, and h. The models with wage per hour have only one, N. When the effects on N, u, and h go in the same direction a clear amplification effect appears.

6. CONCLUSIONS

We have carried out an analysis of two DSGE models with different growth engines for understanding how nominal price and wage rigidities affect the relationship between trend inflation and long-run growth. The results confirm the non-neutrality of the trend inflation when wages are sticky in a context of endogenous growth, and the key role played by the wage units to find the optimal trend inflation.

Our main objective was to verify whether the conclusion of Amano et al. (2009) and Amano et al. (2012) stating the maximum long-run growth rate for a negative trend inflation when price and wage rigidity exists, can be generalized whatever the growth engine. Our results lead us to reject the general validity of this conclusion. First because, when only price rigidity exists, the long-run growth rate is independent of trend inflation for usual values of inflation or deflation rates regardless the growth engine. Second, although we confirm the result in the Schumpeterian model, the maximum long-run growth rate is reached when trend inflation is zero in the human capital model when wages are sticky.

Considering the Schumpeterian model, where steady nominal wages grow at a rate that is the sum of the rates of trend inflation and long-run growth, the long-run growth rate is maximum when trend inflation rate is negative at exactly the same absolute value as the long-run growth rate corresponding to price and wage flexibility. Nominal wage revision will not be necessary for recovering productivity growth, being a situation totally equivalent to flexibility. Any other value of trend inflation introduces a distortion that is greater the greater the difference.

In the human capital model, wage rigidity does not affect the productivity component of labor contracts, only the wage per unit of human capital because wage contracts consider the skill aspects separately and revise them with flexibility. Steady nominal wages grow then at the same rate as trend inflation and the maximum long-run growth rate is reached for null inflation, again a situation equivalent to price and wage flexibility.

The attainment of a situation equivalent to wage flexibility is the mechanism behind the different results in both cases. This finding is a clear contribution of this paper that provides a clear insight of the implications and costs of the nominal wage rigidity in the long run when trend inflation is different of the optimal one.

The other main difference between the models with stickiness in wage per hour and the model with stickiness in wage per unit of human capital is the magnitude of the effect of trend inflation on long-run growth. This effect is negligible in the first kind of models. Specifically, in the Schumpeterian model this effect is less than one hundredth of an annual percentage point for a change of four percentage points in the annual inflation rate. In contrast, in the human capital model this effect is much more significant given that, for a change of around two percentage points in the annual inflation rate, the effect on the growth rate is also a decline of around two percentage points, the sensibility being greater with deflation than with inflation.

The reason for this difference is the distortion that the average wage introduces not only in the demand for labor, but also in the accumulation of human capital and, hence, the long-run growth rate. This important effect suggests the convenience of considering labor skills in the analysis of the effects of nominal wage rigidity, at least from the economic growth perspective, given the role played by human capital in the wage settlements.

The pervasive presence of job categories or occupations in the remuneration settlements reflects the practical relevance of the wage per unit of human capital. Homogeneity in the job categories of the firms is not common. We can consider in general relative wages between categories as constant, and the periodic nominal wage revisions usually take this relative wage as given. Other remuneration aspects linked to the workers quality (or human capital) and their achievements, as bonus payments, are not warranted by the general wage settlements because they depend on the individual productivity performance.

NOTES

1. In Laguna and Sanso (2016), using a calibration for annual rates in the Schumpeterian and spillover effects models [Romer (1986)], it is possible to capture non-neutrality with only price

rigidity with a very low effect (almost negligible) of inflation on growth, which is maximum for zero inflation.

2. The result was also confirmed in Laguna and Sanso (2016) for the Schumpeterian and spillover effects models using a calibration for annual rates.

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APPENDIX A: OPTIMAL CONTROL PROBLEM IN THE HUMAN CAPITAL MODEL. STEADY-STATE IMPLICATIONS

A.1. WAGE FLEXIBILITY

The wage is the same for all types of labor services.

The Hamiltonian for this problem is

$$\begin{aligned} H_{t+\tau} &= \beta^{\tau}\left[\log\left(C_{t+\tau}\right) - \frac{1}{1+\nu}\int_0^1 \left(N_{st+\tau}\right)^{1+\nu} ds\right] \\ &+ \lambda_{1,t+\tau}\left[D_{t+\tau} + \int_0^1 \left(\frac{W^*_{st+\tau}}{P_{t+\tau}}\right) L_{st+\tau}\left(\frac{W^*_{st+\tau}}{P^i_{t+\tau}}\right) ds + \left(R_{t+\tau} - 1 - \delta\right) K_{t+\tau} - C_{t+\tau}\right] \\ &+ \lambda_{2,t+\tau}\left\{\int_0^1 \xi\left(1 - u_{st+\tau}\right) N_{st+\tau} h_{st+\tau} ds\right\}, \end{aligned}$$

subject to (28) and (38). The first-order conditions are the followings:

$$\frac{\beta^{\tau}}{C_{t+\tau}} = \lambda_{1,t+\tau}, \tag{A.1}$$

$$\beta^{\tau} N_{st+\tau}{}^{\upsilon} = \lambda_{1,t+\tau}\left(\frac{W^*_{st+\tau}}{P_{t+\tau}}\right) u_{st+\tau} h_{st+\tau} + \lambda_{2,t+\tau}\xi\left(1 - u_{st+\tau}\right) h_{st+\tau} \quad \forall s \in [0, 1], \tag{A.2}$$

$$\lambda_{2,t+\tau} = \frac{\lambda_{1,t+\tau}}{\xi}\frac{W^*_{st+\tau}}{P_{t+\tau}} \qquad \forall s \in [0, 1], \tag{A.3}$$

$$\begin{aligned} \lambda_{1,t+\tau+1} - \lambda_{1,t+\tau} &= -\lambda_{1,t+\tau}\left(R_{t+\tau} - 1 - \delta\right) \\ &- \lambda_{1,t+\tau}\int_0^1 \left(\frac{W^*_{st+\tau}}{P_{t+\tau}}\right)\left(\frac{W^*_{st+\tau}}{P^i_{t+\tau}}\right)^{-\sigma}\left[\frac{(1-\alpha) A}{\left(\Delta^i_{w,t+\tau}\right)^{1-\sigma\alpha}}\right]^{\frac{1}{\alpha}} ds, \end{aligned} \tag{A.4}$$

$$\lambda_{2,t+\tau+1} - \lambda_{2,t+\tau} = -\lambda_{1,t+\tau}\left(\frac{W^*_{st+\tau}}{P_{t+\tau}}\right) u_{st+\tau} N_{st+\tau} - \lambda_{2,t+\tau}\,\xi\left(1 - u_{st+\tau}\right) N_{st+\tau} \quad \forall s \in [0, 1], \tag{A.5}$$

$$K_{t+\tau+1} = D_{t+\tau} + \int_0^1 \left(\frac{W^*_{st+\tau}}{P_{t+\tau}}\right) L_{i,t+\tau}\left(W^*_{i,t+\tau}\right) ds + \left(R_{t+\tau} - \delta\right) K_{t+\tau} - C_{t+\tau}, \tag{A.6}$$

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$$h_{t+\tau+1} = \left\{ \int_0^1 \left[1 + \xi \left(1 - u_{st+\tau}\right) N_{st+\tau}\right] \frac{h_{st+\tau}}{h_{t+\tau}} ds \right\} h_{t+\tau}. \qquad \textbf{(A.7)}$$

In the steady state, from A1

$$\frac{\lambda_{1,t+\tau+1}}{\lambda_{1,t+\tau}} = 1 + g\left(\lambda_1\right) = \frac{\beta^{\tau+1}/C_{t+\tau+1}}{\beta^{\tau}/C_{t+\tau}} = \frac{\beta}{1+g}.$$

From (A.3)

$$\frac{\lambda_{2,t+\tau+1}}{\lambda_{2,t+\tau}} = \frac{\lambda_{1,t+\tau+1}}{\lambda_{1,t+\tau}} = 1 + g\left(\lambda_2\right) = \frac{\beta}{1+g}.$$

From (A.4) and (A.1)

$$1 + g = \frac{\beta}{1 + \delta - \left[A\left(\frac{\varepsilon-1}{\varepsilon}\right)\right]^{\frac{1}{\alpha}} \left(\frac{1-\alpha}{\Delta_W}\right)^{\frac{1-\alpha}{\alpha}}}.$$

From (A.5) and (A.3)

$$\frac{\lambda_{2,t+\tau+1}}{\lambda_{2,t+\tau}} = 1 - \zeta N_{st+\tau} = \frac{\beta}{1+g}.$$

The steady-state supply of labor is the same for all s and is constant over time. From this expression, we obtain the constant value N_{ss} in the steady state

$$N_{ss} = \frac{1}{\zeta}\left(1 - \frac{\beta}{1+g}\right).$$

From (A.2) and (A.3)

$$\frac{\beta^{\tau+1} N_{st+\tau+1}{}^{\upsilon}}{\beta^{\tau} N_{st+\tau}{}^{\upsilon}} = \frac{\lambda_{2,t+\tau+1}}{\lambda_{2,t+\tau}} \frac{h_{st+\tau+1}}{h_{st+\tau}},$$

$$\beta = \frac{h_{st+\tau+1}}{h_{st+\tau}} \frac{\beta}{1+g} => g\left(h_s\right) = g.$$

The growth rate of human capital is the same as the output growth rate and the same for all s. We obtain the steady-state value of u from the accumulation process of human capital.

$$h_{st+\tau+1} = h_{st+\tau} + \xi\left(1 - u_{st+\tau}\right) N_{st+\tau} h_{st+\tau}.$$

The steady-state growth rate of human capital is

$$g\left(h_s\right) = g = \xi\left(1 - u_{st+\tau}\right) N_{st+\tau} = \xi\left(1 - u_{ss}\right) N_{ss},$$

where u_{ss} is the steady-state value for any s. From this expression, we can deduce that the value of u is also the same for all types of labor services and is constant over time.

$$u_{ss} = 1 - \frac{g}{\xi N_{ss}}.$$

We close the system of equations in steady state with the expressions obtained in this subsection.

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A.2. STICKY WAGES

Note that the first-order condition for $u_{st+\tau}$ in (A.3) implies that the real wage at time $t+\tau$ has to be the same for all individuals. However, since the nominal wage correspond to the effective labor, the re-optimized real wage with rigidity should be constant in the steady state and, therefore, the nominal re-optimized wage grows at the same rate as the aggregate price. This implies that, when the trend inflation is different from zero, there will be variations in the real wage across individuals. Obviously, this contradicts (A.3). So, the previous problem is not valid with wage rigidity.

The Hamiltonian for this situation is

$$H_{t+\tau} = \beta^{\tau}\left[\log\left(C_{t+\tau}\right) - \frac{1}{1+\nu}\int_0^1 \left(N_{st+\tau}\right)^{1+\nu} ds\right]$$

$$+\lambda_{1,t+\tau}\left[D_{t+\tau} + \int_0^1 \left(\frac{W^*_{st+\tau}}{P_{t+\tau}}\right) L_{i,t+\tau}\left(\frac{W^*_{st+\tau}}{P^i_{t+\tau}}\right) ds + \left(R_{t+\tau}-1-\delta\right) K_{t+\tau} - C_{t+\tau}\right]$$

$$+\sum_{q=0}^{J-1}\lambda^q_{2,t+\tau}\left\{\int_{\frac{q}{J}}^{\frac{q+1}{J}} \xi\left(1-u^q_{s,t+\tau}\right) N^q_{st+\tau} h^q_{st+\tau} ds\right\},$$

subject to (26), (28), (29), (31), (32), (33), (35), (36), (39), and (41). The first-order conditions are the followings:

$$\frac{\beta^{\tau}}{C_{t+\tau}} = \lambda_{1,t+\tau}, \tag{A.8}$$

$$\beta^{\tau} N_{st+\tau}{}^{\upsilon} = \lambda_{1,t+\tau}\left(\frac{W^*_{st+\tau}}{P_{t+\tau}}\right) u_{st+\tau} h_{st+\tau} + \lambda_{2,t+\tau}\xi\left(1-u_{st+\tau}\right) h_{st+\tau} \;\forall s \in [0,1], \tag{A.9}$$

$$\lambda^q_{2,t+\tau} = \frac{\lambda_{1,t+\tau}}{\xi}\frac{W^{1q}_{st+\tau}}{P_{t+\tau}} \quad \forall s \in \left[\frac{q}{J}, \frac{q+1}{J}\right], \quad q = 0, 1, 2, \ldots, J-1, \tag{A.10.1)–(A.10.J}$$

$$\lambda_{1,t+\tau+1} - \lambda_{1,t+\tau} = -\lambda_{1,t+\tau}\left(R_{t+\tau}-1-\delta\right)$$

$$-\lambda_{1,t+\tau}\int_0^1 \left(\frac{W^*_{st+\tau}}{P_{t+\tau}}\right)\left(\frac{W^*_{st+\tau}}{P^i_{t+\tau}}\right)^{-\sigma}\left[\frac{(1-\alpha)A}{\left(\Delta^i_{w,t+\tau}\right)^{1-\sigma\alpha}}\right]^{\frac{1}{\alpha}} ds \qquad \forall s \in [0,1], \tag{A.11}$$

$$\lambda^{q+1}_{2,t+\tau+1} - \lambda^q_{2,t+\tau} = -\lambda_{1,t+\tau}\left(\frac{W^q_{st+\tau}}{P_{t+\tau}}\right) u^q_{st+\tau} N^q_{st+\tau} - \lambda^q_{2,t+\tau}\,\xi\left(1-u^q_{st+\tau}\right) N^q_{st+\tau} \quad \forall s$$

$$\in \left[\frac{q}{J}, \frac{q+1}{J}\right] \qquad q = 0, 1, 2, \ldots, J-1, \tag{A.12.1)–(A.12.J}$$

$$K_{t+\tau+1} = \mathrm{D}_{t+\tau} + \int_0^1 \left(\frac{W^*_{st+\tau}}{P_{t+\tau}}\right) L_{st+\tau}\left(\frac{W^*_{st+\tau}}{P^i_{t+\tau}}\right) di + \left(R_{t+\tau}-\delta\right) K_{t+\tau} - C_{t+\tau}, \tag{A.13}$$

$$h_{t+\tau+1} = \left\{\int_0^1 \left[1+\xi\left(1-u_{st+\tau}\right) N_{st+\tau}\right]\frac{h_{st+\tau}}{h_{t+\tau}} ds\right\} h_{t+\tau}, \tag{A.14}$$

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In the steady state, from (A.8)

$$\frac{\lambda_{1,t+\tau+1}}{\lambda_{1,t+\tau}} = 1 + g\left(\lambda_1\right) = \frac{\beta^{\tau+1}/C_{t+\tau+1}}{\beta^{\tau}/C_{t+\tau}} = \frac{\beta}{1+g}.$$

From (A.11) and (A.8)

$$1 + g = \frac{\beta}{1 + \delta - \left[A\left(\frac{\varepsilon-1}{\varepsilon}\right)\right]^{\frac{1}{\alpha}}\left(\frac{1-\alpha}{\Delta_W}\right)^{\frac{1-\alpha}{\alpha}}}.$$

From A.10.2–A.10.J (which represents labor services which do not change wages in $t + \tau + 1$)

$$\frac{\lambda^{q+1}_{2,t+\tau+1}}{\lambda^{q}_{2,t+\tau}} = \frac{\lambda_{1,t+\tau+1}}{\lambda_{1,t+\tau}}\frac{1}{\Pi}$$

$$= 1 + g\left(\lambda^{q}_{2}\right) = \frac{\beta/\Pi}{1+g\left(C\right)} \quad q = 0,\ 1,\ 2,\ \ldots,\ J-2 \quad \forall s \in \left[\frac{q}{J}, \frac{q+1}{J}\right] \text{in t} + \tau.$$

From A.10.1 (which represents labor services which change wages in $t + \tau + 1$)

$$\frac{\lambda^{q+1}_{2,t+\tau+1}}{\lambda^{q}_{2,t+\tau}} = \frac{\lambda_{1,t+\tau+1}}{\lambda_{1,t+\tau}}\Pi^{J-1} = 1 + g(\lambda^{0}_{2}) = \frac{\beta\Pi^{J-1}}{1+g}\ q = J-1 \quad \forall s \in [\frac{J-1}{J}, 1] \text{ in t+}\tau.$$

As a consequence, there will be two values of N. From (A.12)

$$\frac{\lambda^{q+1}_{2,t+\tau+1}}{\lambda^{q}_{2,t+\tau}} = 1 - \xi N^{q}_{st+\tau} = 1 - \xi N^{1q}_{st+\tau} = \frac{\beta/\Pi}{1+g} => N^{q} = N^{1}$$

$$= \frac{1}{\xi}\left(1 - \frac{\beta/\Pi}{1+g}\right) \quad q = 0, 1, 2, \ldots, \text{J} - 2 \text{ in,}$$

$$\times \text{t} + \tau \quad \forall s \in \left[\frac{q}{J}, \frac{q+1}{J}\right] \quad \text{in t} + \tau$$

$$\frac{\lambda^{0}_{2,t+\tau+1}}{\lambda^{J-1}_{2,t+\tau}} = 1 - \xi N^{0}_{st+\tau} = \frac{\beta\Pi^{J-1}}{1+g} => N^{0} = \frac{1}{\xi}\left(1 - \frac{\beta\Pi^{J-1}}{1+g}\right) \quad \forall s \in \left[\frac{J-1}{J}1\right],$$

From (A.9)

$$\frac{\beta^{\tau+1}N^{\upsilon}_{st+\tau+1}}{\beta^{\tau}N^{\upsilon}_{st+\tau}} = \frac{\lambda^{q+1}_{2,t+\tau+1}}{\lambda^{q}_{2,t+\tau}}\frac{h_{st+\tau+1}}{h_{st+\tau}} \qquad q = 0, 1, 2, \ldots, \text{J}-1 \quad \forall s \in \left[\frac{q}{J}, \frac{q+1}{J}\right] \quad \text{in t+}\tau,$$

$$\beta = \frac{h_{st+\tau+1}}{h_{st+\tau}}\frac{\beta/\Pi}{1+g} => g = \frac{1+g\left(h^{1}\right)}{\Pi} - 1 \qquad q = 0,\ 1,\ 2,\ 3,\ \ldots,\ J-3$$

$$\times \forall s \in \left[\frac{q}{J}, \frac{q+1}{J}\right] \quad \text{in t+}\tau,$$

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$$\beta\left(\frac{N^1}{N^0}\right)^v = \frac{h_{st+\tau+1}}{h_{st+\tau}}\frac{\beta\Pi^{J-1}}{1+g} => g = \left(1+g\left(h^0\right)\right)\frac{\Pi^{I-1}}{\left(\frac{N^1}{N^0}\right)^v} - 1 \quad \forall s \in \left[\frac{J-1}{J}, 1\right] \text{ in } \ t{+}\tau,$$

$$\beta\left(\frac{N^0}{N^1}\right)^v = \frac{h_{st+\tau+1}}{h_{st+\tau}}\frac{\beta/\Pi}{1+g} => g = \frac{\left(1+g\left(h^{01}\right)\right)}{\Pi\left(\frac{N^0}{N^1}\right)^v} - 1 \quad \forall s \in \left[\frac{J-2}{J}, \frac{J-1}{J}\right] \text{in } t{+}\tau \ ,$$

As a consequence, there will also be three expressions of u in the steady state

$$u^1 = \frac{\left[2-(1+g\left(C\right))\,\Pi\right] - \frac{\beta/\Pi}{1+g}}{1-\frac{\beta/\Pi}{1+g}} \qquad q = 0,\ 1,\ 2,\ 3,\ \ldots,\ J-3$$

$$\times\,\forall s \in \left[\frac{q}{J}, \frac{q+1}{J}\right] \qquad \text{in } t+t \quad \text{in } \ t+\tau,$$

$$u^0 = \frac{\left[2-\frac{(1+g)}{\Pi^{J-1}}\left(\frac{N^1}{N^0}\right)^v\right] - \frac{\beta\Pi^{J-1}}{1+g}}{1-\frac{\beta\Pi^{J-1}}{1+g}} \qquad \forall s \in \left[\frac{J-1}{J}, 1\right] \text{ in } \ t+\tau,$$

$$u^{01} = \frac{\left[2-(1+g)\,\Pi\left(\frac{N^0}{N^1}\right)^v\right] - \frac{\beta/\Pi}{1+g}}{1-\frac{\beta/\Pi}{1+g}} \qquad \forall s \in \left[\frac{J-2}{J}, \frac{J-1}{J}\right] \text{ in } t+\tau \ ,$$

APPENDIX B: STEADY-STATE SYSTEMS OF EQUATIONS

B.1. SCHUMPETERIAN MODEL

$$\frac{P^*}{P} = \frac{1}{\alpha}\frac{\sum_{\tau=0}^{I-1}\left(\beta\Pi^{1}/1-\alpha\right)^\tau}{\sum_{\tau=0}^{I-1}\left(\beta\Pi^{\alpha}/1-\alpha\right)^\tau}, \tag{B.1}$$

$$\frac{P^*_{-s}}{P} = \frac{1}{\Pi^s}\frac{P^*}{P} \qquad s = 1,\ 2,\ldots,\ I-1, \tag{B.2}$$

$$g = \left[\chi\alpha^{\frac{1}{1-\alpha}}L\frac{1}{I}\sum_{s=0}^{I-1}\left(\frac{P^*_{-s}}{P}\right)^{-\frac{1}{1-\alpha}}\left(\frac{P^*_{-s}}{P}-1\right)\right]^{\frac{\chi}{1-\chi}}(\gamma-1)+1, \tag{B.3}$$

$$L = \frac{(1-\alpha)}{\Delta_W^Y}, \tag{B.4}$$

$$\Delta_W^Y = \frac{W^*}{PY}\left[\frac{1}{J}\sum_{\tau=0}^{J-1}\left(\frac{1}{\Pi g}\right)^{\tau(1-\sigma)}\right]^{\frac{1}{1-\sigma}}, \tag{B.5}$$

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$$\frac{W^*}{PY}=\left(\frac{\sigma(1-\alpha)^{v}}{\sigma-1}\frac{C}{Y}\frac{\sum_{\tau=0}^{J-1}\beta^{\tau}\left(g^{\tau}\Delta_W^{Y}\right)^{(\sigma-1)(1+v)}\Pi^{\sigma(1+v)\tau}g^{(1+v)\tau}}{\sum_{\tau=0}^{J-1}\beta^{\tau}\left(g^{\tau}\Delta_W^{Y}\right)^{(\sigma-1)}\Pi^{(\sigma-1)\tau}}\right)^{\frac{1}{1+\sigma v}}, \tag{B.6}$$

$$\frac{W_{-s}^*}{PY}=\frac{1}{(\Pi g)^{s}}\frac{W^*}{YP}\qquad s=1,\ 2,\dots,\ J-1, \tag{B.7}$$

$$\frac{C}{Y}=1-\alpha^{\frac{1}{1-\alpha}}L\frac{1}{I}\sum_{s=0}^{I-1}\left(\frac{P_{-s}^*}{P}\right)^{-\frac{1}{1-\alpha}}\frac{A}{Y}-\left[\chi\alpha^{\frac{1}{1-\alpha}}L\frac{1}{I}\sum_{s=0}^{I-1}\left(\frac{P_{-s}^*}{P}\right)^{-\frac{1}{1-\alpha}}\left(\frac{P_{-s}^*}{P}-1\right)\right]^{\frac{1}{1-\chi}}\frac{A}{Y}, \tag{B.8}$$

$$\frac{A}{Y}=\frac{1}{\left(\alpha^{\frac{1}{1-\alpha}}\frac{1}{I}\sum_{s=0}^{I-1}\left(\frac{P_{-s}^*}{P}\right)^{-\frac{1}{1-\alpha}}\right)^{\alpha}L}, \tag{B.9}$$

$$\frac{R}{\Pi}=g\left(\frac{1}{\beta}\right). \tag{B.10}$$

B.2. HUMAN CAPITAL MODEL

$$\frac{W^*}{P}=\left[\left(\frac{\sigma}{\sigma-1}\right)\left(\left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{\Delta_W{}^{1-\alpha\sigma}}{(1-\alpha)A}\right)^{\frac{1}{\alpha}}\frac{C}{K}\frac{\sum_{\tau=0}^{J-1}\beta^{\tau}N_{\tau}^{1+\upsilon}}{\sum_{\tau=0}^{J-1}\beta^{\tau}\Pi^{(\sigma-1)\tau}}\right]^{\frac{1}{1-\sigma}}, \tag{B.11}$$

$N_\tau = N^1$ for $\tau = 0, 1, 2, \dots, J-2$ $\quad N_\tau = N^0$ for $\tau = J-1$

$$\frac{W_{-s}^*}{P}=\frac{1}{(\Pi)^{s}}\frac{W^*}{P}\qquad s=1,\ 2,\dots,\ J-1, \tag{B.12}$$

$$\Delta_W=\frac{W^*}{P}\left[\frac{1}{J}\sum_{\tau=0}^{J-1}\left(\frac{1}{\Pi}\right)^{(1-\sigma)\tau}\right]^{\frac{1}{1-\sigma}}, \tag{B.13}$$

$$\frac{C}{K}=\frac{A^{\frac{1}{\alpha}}}{\Delta_P}\left[\left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{1-\alpha}{\Delta_W}\right]^{\frac{1-\alpha}{\alpha}}-g-\delta, \tag{B.14}$$

$$1+g=\frac{\beta}{(1+\delta)-\left[A\left(\frac{\varepsilon-1}{\varepsilon}\right)\right]^{\frac{1}{\alpha}}\left(\frac{1-\alpha}{\Delta_W}\right)^{\frac{1-\alpha}{\alpha}}}, \tag{B.15}$$

$$N^1=\frac{1}{\zeta}\left(1-\frac{\beta/\Pi}{1+g}\right), \tag{B.16}$$

$$N^0=\frac{1}{\zeta}\left(1-\frac{\beta\Pi^{J-1}}{1+g}\right), \tag{B.17}$$

$$u^1=\frac{[2-(1+g)\Pi]-\frac{\beta/\Pi}{1+g}}{1-\frac{\beta/\Pi}{1+g}}, \tag{B.18}$$

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$$u^0 = \frac{\left[2 - \frac{(1+g)}{\Pi^{J-1}}\left(\frac{N^1}{N^0}\right)^{v}\right] - \frac{\beta\Pi^{J-1}}{1+g}}{1 - \frac{\beta\Pi^{J-1}}{1+g}}, \tag{B.19}$$

$$u^{01} = \frac{\left[2 - (1+g)\,\Pi\left(\frac{N^0}{N^1}\right)^{v}\right] - \frac{\beta/\Pi}{1+g}}{1 - \frac{\beta/\Pi}{1+g}}, \tag{B.20}$$

$$\frac{P^*}{P} = \frac{\sum_{\tau=0}^{I-1}(\beta\Pi^{\varepsilon})^{\tau}}{\sum_{\tau=0}^{I-1}\left(\beta\Pi^{\varepsilon-1}\right)^{\tau}}, \tag{B.21}$$

$$\frac{P^*_{-s}}{P} = \frac{1}{\Pi^s}\frac{P^*}{P} \quad s = 1,\ 2, \ldots,\ I-1, \tag{B.22}$$

$$\Delta_P = \frac{P^*}{P}\frac{1}{I}\left[\sum_{\tau=0}^{I-1}\left(\frac{1}{\Pi^{\tau}}\right)^{-\varepsilon}\right], \tag{B.23}$$