Application of the Least Squares Method for Determining Magnetic Compass Deviation

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This paper describes an algorithm for evaluation of magnetic compass deviation based on the least squares method. An automatic system was built, coupled with a magnetic compass recording device, for fine estimation of the deviation curve on the basis of an incomplete compass swing circulation.

1. THE HEART OF THE PROBLEM. Present-day classical magnetic compasses have the ability to transmit their readings. Therefore, it is possible to incorporate the magnetic compass as one of the sensors of an integrated navigation system. That raises the problem of taking into account compass deviation during computation in the system. For this purpose, it is convenient to use the well-known formula:

$$\delta = A + B\sin KM + C\cos KM + D\sin 2KM + E\cos 2KM, \tag{1}$$

where KM is the magnetic course, and A, B, C, D, E are the coefficients of deviation.

The formula, in fact, is a particular case of a Fourier series of the second degree, called the Archibald Smith formula in navigation handbooks (Hobs, 1990; Piecaev *et al.*, 1969). It was the author's aim to create an application software for automatic evaluation of the coefficients A, B, C, D and E for a system consisting of a computer coupled with an inductive probe for reading the position of the compass rose. As a result, computer software was created that can be a part of an integrated system. Moreover, the application software was used for execution of a simple deviation monitor comprising: a magnetic probe mounted during measurement on the magnetic compass, and a laptop computer. Immediately after carrying out a compass swing, a table or a graph of the deviation can be printed out together with the parameters describing the curve. Experiments showed that, for evaluation of the coefficients of deviation, there is no need to perform the full compass swing circulation of 360°. Satisfactory results were obtained after completing a turn of about 100°.

2. DEVIATION EVALUATION WITH THE LEAST SQUARES METHOD. The most popular method for evaluating deviation is the execution of a complete compass swing circulation and a comparison between compass and magnetic courses, or compass and magnetic bearings, on many courses. Usually, this is done by readings on 36 courses, with steps of 10°. Another popular method is deviation evaluation on eight courses, being the multiple of the angle of 45° (Heine, 1990; Maloney, 1985; and Piecaev *et al.*, 1969). Then, applying the formulae given in every navigation handbook, the coefficients of formula (1) are calculated. Finally, knowing these coefficients, a smooth deviation curve can be estimated from (1).

Both methods are significant simplifications, for they do not take into account the errors of each measurement taken on the circulation, particularly in the case of comparing the measurements with the gyrocompass readings, where differences can be of 1°. For values of the deviations that may occur in practice, these errors can deform the results significantly. During circulation, it is possible to take more readings than the minimum required for the evaluation of the five coefficients in (1). Therefore, the idea of using one of the estimation methods can be suggested.

The assumption was made that, during circulation, it is possible to take discreet measurements of compass course *KK* and to record them at a determined frequency. It will be assumed that the values of magnetic courses *KM* are known at these moments. The course, in the program, is calculated using the assumption that the circulation is steady; that means the turn speed is constant, and the initial and the final courses are known. For that purpose, the author uses the DGPS receiver and evaluates current bearings on two chosen points coded in the receiver as WayPoints. In Gdansk Bay, it is convenient to choose the lighthouses of Nowy Port and Hel (Figure 1). It is also convenient to start manoeuvring from the Southerly course and

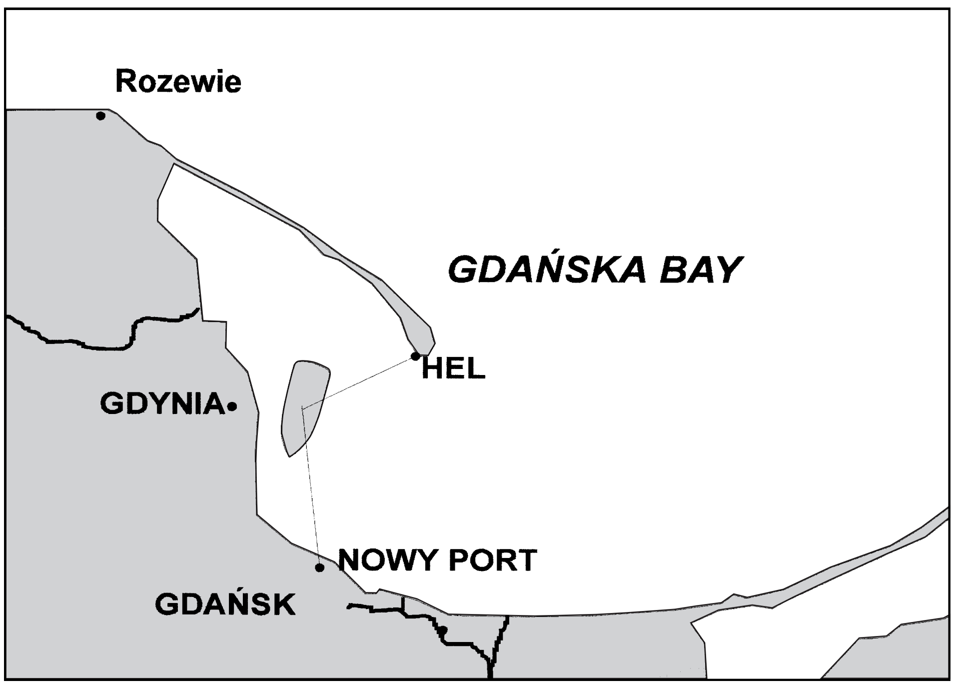

Figure 1. The diagram of manoeuvring.

turn to the left. The moment of crossing the vessel head on the first direction is a suitable time for starting the measurement, and the moment of crossing the direction on the lighthouse of Hel – finishing them. Since an error of the DGPS system in Gdansk Bay is below 2 metres (Reference Station Rozewie), and the distance to the lighthouse about 10 nautical miles, the error of computed direction can be assessed as a fraction of one degree.

The deviation is recorded every 10° as the difference between the recorded compass course and the magnetic course obtained from interpolation of the angle between the initial and final courses on both lighthouses. The deviation on individual courses is calculated from (2).

$$\Delta_i = KM_i - KK_i, i \in \overline{1, n}, \tag{2}$$

The condition of correct deviation evaluation is to cross the reference directions twice by the vessel diameter. The directions do not need to be identical, that means, there is no need to carry out the full circulation, but only a part of it. From the assumption of a steady turn, the change of the course should be made evenly, so during interpolation the computer's internal clock can be used to collect the following compass and magnetic (reference) courses. By this means, the vectors of the measurements of magnetic courses KM_i and compass courses KK_i, where $i = 1$ to n, can be created. Using (2), the difference of these values for each i gives the vector of the measured deviations.

Each of the vector's values is described by the same equation, with constant coefficients A, B, C, D and E. However, because of the errors of the measurement process these values differ from the real ones.

$$\left.\begin{aligned} \Delta_1 &= A + B\sin KM_1 + C\cos KM_1 + D\sin 2KM_1 + E\cos 2KM_1, \\ \Delta_n &= A + B\sin KM_n + C\cos KM_n + D\sin 2KM_n + E\cos 2KM_n. \end{aligned}\right] \tag{3}$$

To improve the estimates is a classical task of middle measurements levelling. Interesting is the case when the function (4) is a minimum. That means, an attempt is being made to determine such values of A, B, C, D and E where the square root of the differences between the values of the deviation obtained from (1) and the measurements is the least.

$$J(A, B, C, D, E) = \left(\sum_{i=1}^{n} (A + B\sin KM_i + C\cos KM_i + D\sin 2KM_i + E\cos 2KM_i - \Delta_i)^2\right). \tag{4}$$

It is convenient to write (3) in the matrix form:

$$\mathrm{Z} = \mathrm{GK}, \tag{5}$$

where G has the following form:

$$G = \begin{bmatrix} 1 & \sin KM_1 & \cos KM_1 & \sin 2KM_1 & \cos 2KM_1 \\ 1 & \sin KM_2 & \cos KM_2 & \sin 2KM_2 & \cos 2KM_2 \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ 1 & \sin KM_n & \cos KM_n & \sin 2KM_n & \cos 2KM_n \end{bmatrix}. \tag{6}$$

The measurements vector:

$$\mathrm{Z}^{\mathrm{T}} = [\Delta_1\, \Delta_2, \ldots, \Delta_{\mathrm{n}}], \tag{7}$$

and the result vector:

$$\mathrm{K}^{\mathrm{T}} = [A, B, C, D, E]. \tag{8}$$

From the assumption on validity of the theory of vessel magnetism, it can be assumed that the coefficients K do not change significantly over months; this means that the influence of time on their value during measurements can be ignored.

Table 1. The true values δ and approximated values Δ of the deviation

KM	0	10	20	30	40	50	60	70	80
δ	−0·50	0·24	1·01	1·76	2·43	2·97	3·36	3·57	3·60
Δ	−0·5	0·0	1·0	1·5	2·5	3·0	3·0	3·5	3·5
KM	90	100	110	120	130	140	150	160	170
δ	3·49	3·26	2·94	2·59	2·55	1·96	1·73	1·59	1·52
Δ	3·5	3·0	3·0	2·5	2·5	2·0	1·5	1·5	1·5
KM	180	190	200	210	220	230	240	250	260
δ	1·50	1·51	1·51	1·47	1·36	1·15	0·84	0·43	−0·05
Δ	1·5	1·5	1·5	1·5	1·5	1·0	1·0	0·5	0·0
KM	270	280	290	300	310	320	330	340	350
δ	−0·57	−1·09	−1·54	−1·89	−2·09	−2·11	−1·94	−1·57	−1·04
Δ	−0·5	−1·0	−1·5	−2·0	−2·0	−2·0	−2·0	−1·5	−1·0

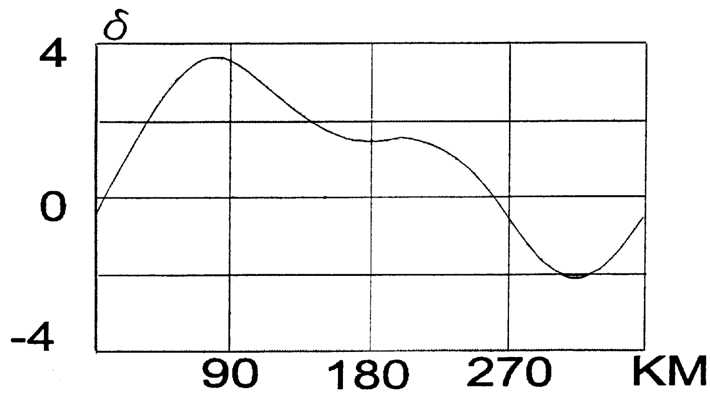

Figure 2. The diagram of the measured deviation.

Here we can use the well-known theorem:

If Z is a n-dimensional vector and G is a n by m matrix of linear independent columns, there is exactly one m-dimensional vector K that will minimize the norm $|Z-GK|$ for all K (taken in the n-dimensional Euclid space).

Moreover,

$$\hat{K} = (G^TG)^{-1} G^T Z. \tag{9}$$

Therefore, the problem of finding the optimal coefficients A, B, C, D and E that minimize the function (4), results in the solution of (9).

3. COMPUTATIONAL EXAMPLE. To illustrate the effectiveness of the method, the computation results of the deviation obtained on the basis of the simulated deviation given in the Table 1 and Figure 2 are used. Let us assume that the coefficients of the deviation curve are as follows:

$$A = 1, \quad B = 2, \quad C = -1, \quad D = 1, \quad E = -0{\cdot}5.$$

Therefore, the deviation curve has the values as in Table 1. Enclosed are also the approximated values of Δ that are used in computation discussed in the next section.

From these measured deviations, as examples, two ten-element vectors comprising different measurement sectors (changes of the course) can be formed. One vector with the measurements of the sector from 0° to 90° (measurements taken every 10°):

$$Z^T = [-0{\cdot}5,\ 0,\ 1{\cdot}0,\ 1{\cdot}5,\ 2{\cdot}5,\ 3{\cdot}0,\ 3{\cdot}5,\ 3{\cdot}5,\ 3{\cdot}5],$$

and the other from 0° to 360° (measurements every 40°):

$$Z^T = [-0{\cdot}5,\ 2{\cdot}5,\ 3{\cdot}5,\ 2{\cdot}5,\ 1{\cdot}5,\ 1{\cdot}5,\ 1,\ -1,\ -2,\ -0{\cdot}5].$$

On the basis of these vectors, using (9), the respective vectors K were evaluated, and using these, the deviation curves were plotted and are shown at Figures 3 and 4. The

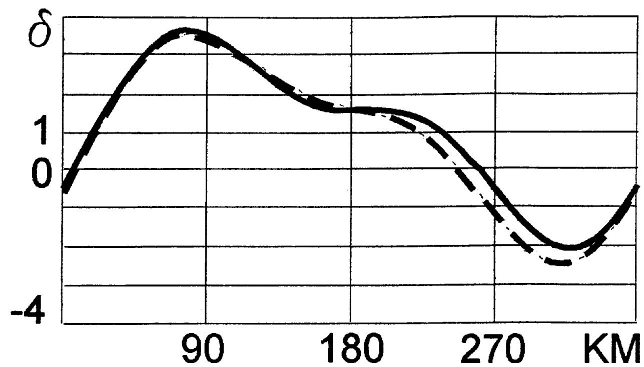

Figure 3. The deviation curve for A, B, C, D and E parameters computed from measurements in the sector of 90°. $A = 0{\cdot}83$, $B = 2{\cdot}31$, $C = -1{\cdot}1$, $D = 0{\cdot}91$ and $E = -0{\cdot}32$.

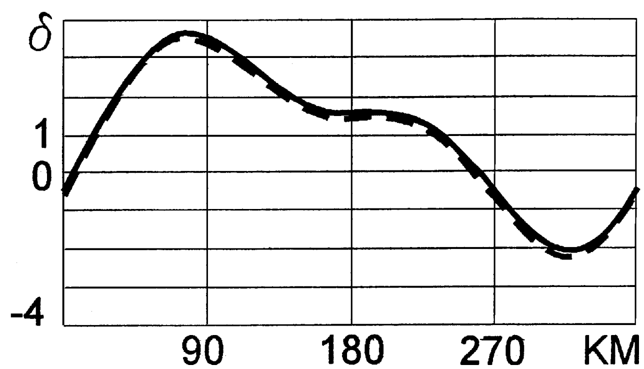

Figure 4. The deviation curve for A, B, C, D and E parameters computed from measurements in the sector of 360°. $A = 0{\cdot}99$, $B = 1{\cdot}92$, $C = -0{\cdot}95$, $D = 1{\cdot}04$ and $E = -0{\cdot}5$.

measurements are plotted using the dashed line, and the calculated deviation using the solid line. The results from the evaluation of the coefficients of (1), and the curves computed from them, show that obtained results are identical.

4. DISCUSSION. The method presented can be perceived as a modification of the well-known technique for the deviation curve evaluation on the basis of the measurements taken on eight courses. However, the method is not suitable for manual calculations; it requires an appropriate computing device, supported with

the inductive probe for transmission of the course readings. The method allows levelling of some part of the errors arising during reading of the magnetic course, and allows saving of the time devoted to taking measurements.

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KEY WORDS

1. Compasses. 2. Corrections. 3. Automation.