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NOTES

NEWS ABOUT NEWS: INFORMATION ARRIVAL AND IRREVERSIBLE INVESTMENT

ALLAN DRAZEN
University of Maryland
and
NBER

PLUTARCHOS SAKELLARIS
University of Maryland

We analyze how uncertainty about *when* information about future returns to a project may be revealed affects investment. Whereas good news about future returns boosts investment, good news about news (that is, news that information may arrive sooner) is shown to depress investment. We show that early revelation increases the value of an irreversible investment project to a risk-neutral investor. Our framework allows us to study irreversible investment projects whose value has a time-variable volatility. We also consider how heterogeneity of revelation information across firms may induce a better-informed firm to share its information with competitors.

Keywords: Investment, Irreversibility, Revelation, Uncertainty

1. INTRODUCTION

When decisions have an irreversible component, uncertainty about future outcomes plays a key role in the decision to commit to a course of action. Because it is costly to reverse a decision, waiting to commit until some of the uncertainty is resolved may yield benefits that more than outweigh the forgone short-run returns. The possible arrival of significant new information about outcomes thus can make the option of waiting to commit quite valuable.

If news about future possible outcomes is valuable when decisions are costly to reverse, then information about *when* such news might arrive (news about news) should be valuable as well. The likelihood of receiving new information should affect the timing of irreversible decisions, as should changes in that likelihood,

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even if these changes convey no new information about what the outcomes may be. Trading on asset exchanges, for example, often slows in anticipation of release of new economic data; political decisions often are delayed if it is believed that relevant new information soon will become available.

The most complete discussion of the implications of uncertainty about future returns when decisions are irreversible is in the theory of irreversible investment, where it is shown that the option value of waiting to invest may lead firms to avoid investing in projects that have a positive expected present discounted value. This literature demonstrates that uncertainty about future returns to a project may in itself depress investment, and that positive information about future returns will increase investment.[1]

Although the theory of irreversible investment yields a general framework for studying uncertainty about the value of an installed project, uncertainty about when information about outcomes may be revealed has not been treated explicitly. We present a model that separates the effect on investment of uncertainty about the value of an installed project into the effects of uncertainty about eventual returns (outcome uncertainty) and the effects of uncertainty about when outcome uncertainty itself may be resolved (revelation uncertainty).

A higher probability of knowing outcomes sooner (good news about revelation) can decrease *current* investment as a firm waits to learn about outcomes but, in general, increases the firm's value. We show how our results on uncertainty about the arrival of new information are related to the time-varying volatility in the value of a new project. (In contrast, most of the literature has studied the impact of uncertainty on investment when the variance of value, or some underlying fundamental, stays constant over time.) A common form of behavior under uncertainty—wait a prespecified length of time, then act if information has still not arrived—is *inconsistent* with constant volatility of returns over time. We also show that when firms compete for a project, a firm better informed about when news may arrive will share this information costlessly with the less well-informed firm.

The process of government decision making often gives rise to revelation uncertainty, which, in turn, may affect investment decisions. One example is the uncertainty as to when negotiations over NAFTA would end and the outcome would be revealed, distinct from outcome uncertainty on the probability of NAFTA ratification. Another example concerns large-scale reform, as in several Eastern European countries during the period of large-scale privatization and market reform. When large-scale reforms begin to succeed, so that expected future returns increase, the rate at which information about future returns flows in also becomes faster. If the effect of waiting for more information dominates the effect of newly acquired positive information, successful reform may temporarily decrease investment.

2. SIMPLE EXAMPLE WITH TIME-VARYING VOLATILITY

We begin with a simple example in which high variance in returns may create an incentive to wait before committing to an irreversible project. This example

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allows us to draw a connection between revelation uncertainty and time-varying volatility of returns.[2] Consider a three-period model in which information arrives in the second and third periods, $t=1$ and $t=2$. A risk-neutral firm can invest in a factory that produces one widget per year forever, with zero operating cost, where the investment cost, I, is sunk. The initial price of a widget is P_0, and may rise to $(1+u)P_0$ in the second period with probability $\pi/2$ or fall to $(1-d)P_0$ with probability $\pi/2$, while with probability $1-\pi$, it stays the same. In the third period, the price may rise to $(1+u)P_1$ with probability $\rho/2$ or fall to $(1-d)P_1$ with probability $\rho/2$, while with probability $1-\rho$, it remains equal to P_1. The probabilities π and ρ may be unequal, implying time-varying volatility of returns. To see this, note that the variance of the value of the installed project (where $u=d$) is $\sigma^2=\pi[u\beta P_0/(1-\beta)]^2$ at $t=0$ and is $\sigma^2=\rho[u\beta P_1/(1-\beta)]^2$ at $t=1$.

For given values of the four parameters, P_0/I, u, d, and β, we can calculate combinations of π and ρ such that the firm is indifferent between committing to investment in the first period or remaining uncommitted. More generally, we could think of an indifference surface in these six parameters. A higher value of either of the probabilities π or ρ, holding the remaining five parameters constant implies that the firm prefers to remain uncommitted. We employ backward induction to produce such a surface in π and ρ, the solid line in Figure 1. (The values for the remaining parameters are $P_0/I=0.15$, $u=d=1/2$, and $\beta=0.91$.) In this figure, combinations of π and ρ below the solid curve imply that the firm commits to the project at $t=0$. Note that these could involve situations in which the variance is high in the first period but low in the second, or vice versa. In the case that π and ρ lie above the curve *and* there is no change in the price between the first and the second period, the firm will commit at $t=1$ for $\rho<\rho^*=0.19$, represented by the dashed line in Figure 1.

Let us first consider the standard model of constant volatility of returns over time, which corresponds to points along the diagonal, where $\pi=\rho$. A standard result from the literature is that higher uncertainty (i.e., a higher σ) implies a higher value to remaining uncommitted. Hence, using the parameter values of the figure, a firm that would be indifferent at $\pi=\rho=0.55$ would prefer to remain uncommitted at $t=0$ for any value above 0.55. Note that if there is no price change between the two periods (that is, $P_0=P_1$), constant volatility of returns implies that the firm chooses either to commit at $t=0$ or wait until $t=2$, after which time no more information will be revealed. This characteristic can be shown to be more general: *With constant volatility of returns, a firm that finds it optimal not to commit in the beginning will remain uncommitted as long as the price is unchanged but may change in the future.*

In contrast, consider allowing the volatility of returns to vary over time, corresponding to points off the diagonal, where $\pi\neq\rho$. As the solid-line indifference surface in Figure 1 makes clear, higher variance of returns in the first period (corresponding to $\pi>0.55$) can be offset by lower variance in the second period (corresponding to $\rho<0.55$). Committing at $t=1$ (corresponding to combinations of π and ρ in the northwest part of the figure) is possible even if there has been no change in price.

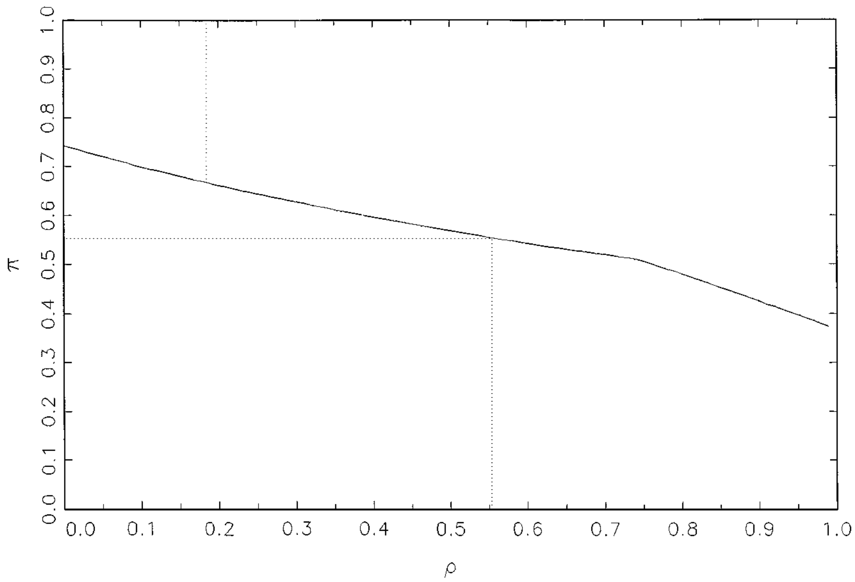

FIGURE 1. Indifference curve for commitment at $t = 0$.

3. UNCERTAINTY ABOUT TIME OF INFORMATION ARRIVAL

In the above model, information arrived in each period but the volatility of returns was allowed to vary over time, in contrast to the standard model with constant volatility. In many situations involving irreversible investment, there is a *single* important piece of information, which may be revealed at an unknown point in time. Prior to the revelation of this piece of information, uncertainty about returns is large; subsequent to its revelation, this uncertainty is significantly reduced (or perhaps eliminated).

Toward this end, we consider a multiperiod model of optimal timing of irreversible investment in a single risky asset. The asset yields a known return r each period until a known time $\bar{T}$, after which it yields a net return of either R^h with probability p or R^1 with probability $1 - p$. Assume $R^h > 0 > R^1$ and $pR^h + (1 - p)R^1 > 0$. A risk-neutral firm discounts returns by a factor β per period. The return to no investment is normalized to zero.[3] Conceptually, the return to the risky asset will be affected by the realization of some future event that is known to occur at time $\bar{T}$,[4] where the realization may be known with certainty at some time T before $\bar{T}$.[5] (One may call $\bar{T}$ the outcome date and T the revelation date.) Uncertainty about the value of an installed project stems from two sources: uncertainty about the eventual returns to the installed project (outcome uncertainty) and uncertainty about when information about outcomes may be revealed (revelation uncertainty). The return structure of the currently risky asset is time invariant, so that the probability that information will be revealed in each period will depend on time, but the nature of the information that will be revealed does not change.

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We further assume that bad outcomes matter, in the sense of ruling out parameter values such that investment is undertaken immediately even though it is known that the bad outcome will occur.[6]

The revelation date $T\,(\leq \bar{T})$ (the first date at which the outcome will be known with certainty) is stochastic, with a subjective probability distribution represented by the cumulative distribution function $H(T)$. The distribution $H(T)$ implies probabilities of revelation in each period, conditional on uncertainty not having been resolved previously. Looking at the problem from period 0, the probability π_t of uncertainty being resolved in period t, conditional on no previous resolution, is

$$\pi_t = \frac{h(t)}{1 - H(t-1)}. \tag{1}$$

The timing of decisions and events if uncertainty has not been resolved and the firm has not committed to investment is as follows. At the beginning of period t, the firm decides whether to commit to investment in the risky asset or remain uncommitted. If it commits (irreversibly), the firm earns r in every period from t to $\bar{T} - 1$ and R^i at $\bar{T}$. Uncertainty then is resolved with probability π_t. If the firm is uncommitted and the news is good ($R^i = R^h$ with probability p), the firm undertakes the project at $t + 1$. However, if the news is bad, the firm decides never to undertake the project. Because returns are stationary and the distribution $H(T)$ is known, the firm's decision may be described as choosing a date T^* to commit conditional on uncertainty not having been previously resolved. Equivalently, the firm chooses a maximum number of periods to wait before committing to investment.

One way to find the optimal solution is first to calculate the expected value as of t of waiting j periods to commit to investment in excess of the expected value of committing at t. Call this excess value $V_{t+j}(t)$. The optimal length of time to wait before committing to investment then is found by simply choosing the maximum value $V_{t+j^*}(t)$ in the set $\{V_{t+j}(t)\}$ and waiting j^* periods to invest. We formalize this as follows.

PROPOSITION 1. *The optimal waiting time is given by*

$$T^* = \underset{0 \leq j \leq \bar{T}}{\operatorname{argmax}} \{V_j(0)\}. \tag{2}$$

(The proof of all propositions is in the Appendix.) An implication of Proposition 1 is Corollary 1.

COROLLARY 1. *A necessary and sufficient condition for it to be optimal to postpone investment at time* 0 *is that some* $V_j(0)\,(j > 0)$ *be positive.*

We now derive some basic results on the optimal length of time to wait to invest as a function of the π_i. A first result is that a higher likelihood of knowing early makes waiting more attractive.

PROPOSITION 2. *An increase in the probability of revelation in any future period i will increase the value of waiting to invest at least i periods, that is,* $\partial V_j(0)/\partial \pi_i > 0$, *for all* $j \geq i$.

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Note that, in contrast to the effect of good outcome news, good news about revelation (in the sense of uncertainty being resolved sooner) may depress current investment and can never increase it. We formalize this result as follows.

PROPOSITION 3. *Suppose that the optimal action is to invest immediately. An increase in revelation probabilities can lead to a postponement of investment; an increase in the probability of the good outcome cannot.*

More generally, one can characterize the optimal length of postponement in terms of revelation probabilities as follows: *Postpone investment as long as the probability of revelation in some future period is sufficiently high. A necessary condition for the optimality of investing at time T^* is that all future revelation probabilities are sufficiently low.*

In the previous model, in which information arrived each period, constant volatility of returns implied that a firm would either commit immediately or wait until there was no uncertainty, if price was unchanged. There is an intuitive analogue to this in this model. If the revelation probability, π_i, is the same in each period, then the firm either commits immediately or waits until all information has been revealed.[7]

The results that we have presented up to now have stressed that earlier revelation of uncertainty makes initial investment less likely by raising the value of the option to wait. Hence, although good revelation news may reduce investment, it will always increase the value of the project.[8] A more interesting question is whether early revelation depresses investment. Is the accumulation of capital higher or lower, in the sense of the number of projects being undertaken ex post, when information is more likely to be revealed earlier? Because our model is one of irreversible investment in a single project of exogenous size, the best measure to address this issue is the probability that the project will be undertaken over the firm's horizon.

Let us denote the probability that a project will be undertaken as $P(I)$. When $T^* = 0$, then $P(I) = 1$. However, when the optimal decision is to postpone investment, that is, when $T^* > 0$, the probability of investment is less than unity. The probability that the project will not be undertaken is equal to the probability that there is revelation before the decision date [call this probability $\Lambda(T^*)$] multiplied by the probability that the revelation is bad $(1 - p)$. Thus $P(I) = 1 - (1 - p)\Lambda(T^*)$. One can show that the probability of investment depends (strictly) negatively on π_k for $k < T^*$ and (weakly) negatively on π_k for $k \geq T^*$. Thus, *early revelation that leads the firm to postpone undertaking a project reduces the probability of investment because postponement allows the firm the possibility of learning that the project will be loss-making.*

It is instructive to establish the relation between changes in revelation uncertainty and the variability of the value of an installed project. In a discrete-time framework, it is convenient to characterize the variability of installed value V_t in terms of the one-period-ahead variance $\sigma_t^2 = E_{t-1}(V_t - E_{t-1}V_t)^2$. In our model, this variance is $\beta^{2(\bar{T}-t)}\pi_t$ var(R), if there has not been revelation before t, and zero otherwise, where var$(R) = p(R^h)^2 + (1-p)\,(R^l)^2 - [pR^h + (1-p)R^l]^2$ is the variance of returns. For simplicity of exposition, we set $\beta = 1$ for the rest of the discussion. The variance of the value is affected by revelation uncertainty (through π_t) and by outcome uncertainty [through var(R)]. It is clear that this variance is variable over time as long as the revelation probabilities are themselves time varying. Good revelation news through an increase in the π_t increases the (one-period-ahead) variance of installed value at time t while leaving all other variances the same. On the other hand, good outcome news through a decrease in var(R) decreases the variance of installed value in *all* time periods by the same proportion. The optimal length of postponement of investment can be characterized in terms of variances: *Postpone investment as long as the variance of installed value in some future period is sufficiently high. A necessary condition for the optimality of investing at time* T^* *is that all subsequent variances are sufficiently low.*

4. MANY FIRMS

So far we have implicitly assumed that a single firm has sole access to the project. In many cases, however, an investment or project may be available to more than one potential investor. For an individual firm, this means that there is some probability that, by waiting, the opportunity to invest in a future period will be lost, implying an incentive to commit earlier. In this section we enrich the framework in order to study the interaction between the possibility of early revelation and of investment being preempted. We show that the possibility of being preempted implies not only that the firm may commit earlier, but also that a firm with a superior ability to process information and hence benefit from early revelation will find it optimal to share its information *costlessly* with a firm with an inferior information processing ability.

To make these ideas more precise, suppose that, at the beginning of each period t, there is an exogenous probability $1-\theta_t$ that the investment opportunity will disappear if the firm remains uncommitted. (If the firm has committed earlier, the investment is locked in and cannot disappear.) If the investment opportunity is still available (this occurring with probability θ_t), there is a probability π_t that uncertainty will be resolved. One then can show that the disappearance probabilities θ_t have an effect similar to that of revelation probabilities π_t, as follows.

PROPOSITION 4. *A decrease in any of the survival probabilities* θ_i *will decrease the value of waiting to invest at least i periods and will decrease (or leave unchanged) the optimal waiting time.*

If the firm takes the θ_t as parametric and exogenous to its decisions, then it is irrelevant for its decisions whether the disappearance of the investment possibility comes from an act of nature or from a competitor grabbing the project; in either case the result in Proposition 4 will hold. In this sense, under the assumption that

θ_t is taken as exogenous, the simple framework presented above can capture the interaction of many firms competing for the same project.

In the case where the firm takes account of the influence its own actions may have on θ_t, the analysis is more complicated. Each firm will take account of other firms' strategies in deciding when to commit. In a two-player game, for example, one can derive optimal strategies and the critical levels of the π_i consistent with commitment not taking place immediately,[9] but that is not our interest here. Rather, we want to point out an interesting implication of heterogeneity in information, namely that *a firm with a better ability to process information may find it optimal to share some of its information costlessly with a less well-informed firm.*

Suppose that two firms are asymmetric in their ability to *process* information in the following sense: Some events or pieces of information that would reveal the ultimate outcome to the first firm will not reveal it to the second. Formally, the first firm, which has better ability to process information (firm B), perceives higher revelation probabilities $\{\pi_i\}$ than the second, which has worse information processing ability (firm W). We argue that if there is a possibility that firm B may be preempted by firm W, it will want to share its ability to process information with its competitor to induce the competitor not to commit to investment. (When both firms move simultaneously, they split the returns from the project.)

To make this more specific, suppose that firm B perceives a high enough chance of early revelation that it is optimal for it to wait (say, until period j). Its competitor, firm W, perceives such a low chance of early revelation that it would invest immediately. In other words, firm W's optimal behavior given low revelation probabilities implies that firm B faces $\theta_1 = 0$. Firm B therefore would find it optimal to choose to commit immediately as well, and they would split the expected value of the project at time 0. If firm B can induce firm W to wait until j, it can do no worse than split the project at j. Because the expected value of waiting till j exceeds that of committing at 0, it will be optimal for firm B to try to induce firm W to wait. It could do this by sharing its knowledge on how to process information (that is, how to learn about early revelation), thus raising the $\{\pi_i\}$ that firm W perceives. Hence, costlessly sharing its ability to process information may be welfare improving for a firm.

5. CONCLUSIONS

If an event conveys good news both about the possibility of early revelation and about outcomes, the net effect on investment will depend on which effect dominates. This appears quite relevant in understanding investment dynamics during a multistage reform program, in which good progress at one stage suggests not only better ultimate outcomes, but also that residual uncertainty will be resolved faster. Many economies are undergoing long and difficult transitions, with investment remaining low in spite of what appear to be large profit opportunities. A crucial step in understanding such transitions is a framework to analyze how investment is affected by when it is known whether the transition will be successful.

More generally, an implication of this paper is that investors benefit from earlier resolution of uncertainty. As in the case of economic transitions, it is unavoidable that the political process creates uncertainty about when important information will arrive. Nonetheless, government policy should attempt to do nothing that needlessly increases this uncertainty, or increases the information differential between firms.

NOTES

1. Cukierman (1980) and Bernanke (1983) considered models in which the arrival of information makes future returns less uncertain, providing a channel for valuing the option to wait and gather more information. In McDonald and Siegel (1986), Pindyck (1988), and Bertola and Caballero (1994), among others, information arrives each period and updates the conditional distribution of future returns. An excellent treatment of much of this literature can be found in Dixit and Pindyck (1994).

2. In the standard framework, as represented by the discrete-time models of Dixit and Pindyck (1994) or the continuous-time models with geometric Brownian motion [as in McDonald and Siegel (1986)], the instantaneous variance is constant.

3. This modeling is equivalent to the existence of a second asset whose return is riskless and investment is reversible, rather than irreversible. In this case the return to the risky asset is defined as the excess over the return to the safe asset.

4. An example of such an event would be an election, where the new party takes office on a given day, but where the election's outcome may be known beforehand; or, a possible policy change scheduled to take effect on a given date, where it is known well beforehand whether or not it will take place.

5. We assume that the outcome date $\bar{T}$ is known and concentrate on uncertainty about the timing of revelation. We hold $\bar{T}$ independent of T so that changes in the distribution of T can be seen as pure changes in revelation uncertainty, not affecting the stream of returns. Alternatively, one can concentrate on uncertainty about the outcome date, with no possibility of early revelation, as in Drazen and Helpman (1990) and Calvo and Drazen (1998).

6. Formally, we assume that for $t = 0$, $\sum_{s=0}^{s=\bar{T}-t-1} \beta^s r + \beta^{\bar{T}-t} R^l < 0$. Note that the condition being satisfied at $t = 0$ implies that it will be satisfied for all $t > 0$ because $\beta < 1$. Intuitively, the condition is that the discounted flow of returns until $\bar{T}$ cannot be so high as to offset a certain bad outcome.

7. Formally, one can show that the critical value of π such that the firm is indifferent is monotonically nonincreasing over time.

8. The preference for early resolution of uncertainty can be related to an inherent convexity in utility aggregation, as in nonexpected utility preferences, but here it arises under risk neutrality due to irreversibility. See our working paper version for details.

9. Assume that, if both firms move simultaneously, the returns will be split, but if one firm moves first and preempts, it gets the entire project. Then the critical value of the $\{\pi_i\}$ must be larger than in the case of a sole potential investor in order to make it optimal to wait.

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APPENDIX

Proof of Proposition 1. Strategies as of time 0 are: Commit to investment immediately, wait one period to commit to investment, wait two periods to commit, etc. The optimal strategy is the one yielding the highest expected value. The expected return to each of these strategies corresponds to the associated value $V_j(0)$, so that the optimal strategy is to wait $T^* = \text{argmax}_j V_j(0)$ periods. ■

Before proving Proposition 2, we need to specify $V_j(t)$ in terms of the parameters of the model. Let us denote by $A_{t+i}(t)$ the expected gain (as seen from t) of waiting from period $t+i-1$ to period $t+i$. This gain is discounted to period $t+i-1$. Then, $V_{t+j}(t)$ will be the present discounted value of the $A_{t+i}(t)$ from $i = 1$ to j, with each term also being multiplied by the probability of reaching that date with no resolution of uncertainty. With no resolution of uncertainty at t, we have

$$\begin{aligned} V_{t+j}(t) &= A_{t+1}(t) + \beta(1-\pi_{t+1})A_{t+2}(t) + \cdots \\ &\quad + \beta^{j-1}(1-\pi_{t+1})\cdots(1-\pi_{t+j-1})A_{t+j}(t) \end{aligned} \tag{A.1}$$

where $V_t(t) = 0$. The above equation implies a simple relation between the $V_{t+j}(t)$ in different time periods of the form

$$V_{t+j}(t) = V_{t+j-m}(t) + \beta^{j-m}\left[\prod_{s=t+1}^{t+j-m}(1-\pi_s)\right]V_{t+j}(t+j-m).$$

This says that the value of waiting until period $t+j$ may be thought of as the value of waiting until period $t+j-m$ plus the value of waiting another m periods. Thus, any $V_{T^*+k}(0)$ may be thought of as the value of waiting until T^* plus the value of waiting another k periods. If $V_{T^*}(0)$ is maximum, the value of waiting any longer once T^* has been reached must be negative.

Now we provide an expression for $A_i(t)$. Define $Q^U(t)$ as the expected return (as of $t-1$) from time t to $\overline{T}$ if uncertainty is resolved at t and the firm has not committed itself to investment before t. Define $Q^C(t)$ as the expected return (also as of $t-1$) from time t to $\overline{T}$ if uncertainty is resolved at t and the firm already has committed itself to investment.

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Both $Q^U(t)$ and $Q^C(t)$ are discounted to time t and may be written as

$$Q^U(t) = p\left[\sum_{s=t}^{\bar{T}-1} \beta^{(s-t)} r + \beta^{(\bar{T}-t)} R^h\right], \quad \textbf{(A.2)}$$

$$Q^C(t) = \sum_{s=t}^{\bar{T}-1} \beta^{(s-t)} r + \beta^{(\bar{T}-t)}[pR^h + (1-p)R^l], \quad \textbf{(A.3)}$$

where we have used the fact that if the firm has not committed prior to t, it will invest only if the realization is R^h and obtain a present value as of $\bar{T}$ of R^h, whereas if it has committed, the expected present discounted value of returns at $\bar{T}$ is $[pR^h + (1-p)R^l]$.

The net expected gain from waiting to invest until period $t+i$ rather than investing in period $t+i-1$, that is, $A_{t+i}(t)$, will be the excess of $Q^U(t+i)$ over $Q^C(t+i)$ multiplied by both the discount rate, β, and the probability of uncertainty being resolved in $t+i$ (i.e., π_{t+i}), net of the return r. We have, then,

$$A_{t+i}(t) = \beta\pi_{t+i}[Q^U(t+i) - Q^C(t+i)] - r, \quad \textbf{(A.4)}$$

which allows us to calculate each of the $V_j(t)$ in terms of underlying parameters. The excess value of remaining uncommitted, $Q^U(t+1) - Q^C(t+1)$, will be positive under the reasonable assumption that the bad outcome occurring with certainty leads to no investment.

Proof of Proposition 2. Using equations (A.1), (A.4) and differentiating with respect to π, we obtain

$$\begin{aligned}\frac{\partial V_j(0)}{\partial \pi_i} &= \beta^i(1-\pi_1)\cdots(1-\pi_{i-1})Q^U(i)\\ &- \beta^{i+1}(1-\pi_1)\cdots(1-\pi_{i-1})\pi_{i+1}Q^U(i+1) - \cdots\\ &- \beta^j(1-\pi_1)\cdots(1-\pi_{i-1})(1-\pi_{i+1})\cdots(1-\pi_{j-1})\pi_j Q^U(j)\\ &- \beta^j(1-\pi_1)\cdots(1-\pi_{i-1})(1-\pi_{i+1})\cdots(1-\pi_j)Q^C(j).\end{aligned}$$

When $i = j$, this reduces to $\partial V_j(0)/\partial\pi_j = \beta^j(1-\pi_1)\cdots(1-\pi_{j-1})[Q^U(j) - Q^C(j)] > 0$. When $i < j$, we simplify the expression for $\partial V_j(0)/\partial\pi_i$ by making repeated use of the relationships

$$\begin{aligned}\beta Q^U(k) &= Q^U(k-1) - pr,\\ \beta Q^C(k) &= Q^C(k-1) - r,\end{aligned}$$

for $k = i+1, \ldots, j$. This leads to

$$\frac{\partial V_j(0)}{\partial \pi_i} = \beta^i(1-\pi_1)\cdots(1-\pi_{i-1})(1-\pi_{i+1})\cdots(1-\pi_j)[Q^U(i) - Q^C(i)] + r\Omega,$$

where Ω is a positive constant and $Q^U(i) > Q^C(i)$ [from (A.2) and (A.3)]. ■

Proof of Proposition 3. The first part follows from Proposition 2 because an increase in π_i raises the excess value of waiting. To show the second part, we differentiate equation (A.1) with respect to p and obtain $\partial V_j(0)/\partial p = \partial A_1(0)/\partial p + \beta(1-\pi_1)\partial A_2(0)/\partial p + \cdots$. Note that $\partial A_j(0)/\partial p = \beta\pi_j[Q^U(j) - Q^C(j)]/(p-1) < 0$. This implies that $\partial V_j(0)/\partial p < 0$. ■

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When there are many firms considering the project, as in Section 6, the possibility of the opportunity disappearing, with probability $1-\theta_{t+i}$, reduces the expected gain of waiting from $t+i-1$ to $t+i$, $A_{t+i}(t)$. Then,

$$A_{t+i}(t) = \theta_{t+i}\beta\pi_{t+i}[Q^U(t+i) - Q^C(t+i)] - (1-\theta_{t+i})\beta Q^C(t+i) - r.$$

The $V_{t+j}(t)$, then, are formed as a discounted sum of the $A_{t+i}(t)$ where the terms also are discounted by the probability that the opportunity is still available at each date:

$$V_{t+j}(t) = A_{t+1}(t) + \beta\theta_{t+1}(1-\pi_{t+1})A_{t+2}(t) + \cdots$$
$$+ \beta^{j-1}\theta_{t+1}\cdots\theta_{t+j-1}(1-\pi_{t+1})\cdots(1-\pi_{t+j-1})A_{t+j}(t).$$

Proof of Proposition 4. The gain from waiting to invest may be written as

$$V_{t+j}(t) = \theta_{t+1}\beta\pi_{t+1}Q^U(t+1) + \theta_{t+1}\theta_{t+2}\beta^2(1-\pi_{t+1})\pi_{t+2}Q^U(t+2) + \cdots$$
$$+ \beta[\theta_{t+1}(1-\pi_{t+1}) - 1]Q^C(t+1) + \beta^2[\theta_{t+1}\theta_{t+2}(1-\pi_{t+1})(1-\pi_{t+2}) - 1]$$
$$\times Q^C(t+2) + \cdots - r - \beta r - \cdots.$$

Note that $Q^U(i) > 0$, $Q^C(i) > 0$, $0 < \theta_i < 1$, and $0 < \pi_i < 1$ for all i. Differentiating this expression with respect to any of the θ_i (where $i > t$) immediately implies that $\partial V_{t+j}(t)/\partial\theta_i > 0$ and that $|\partial V_{t+k}(t)/\partial\theta_i| < |\partial V_{t+h}(t)/\partial\theta_i|$ for $k < h$. These two results imply that a decrease in θ_i will cause the $V_{t+j}(t)$ to fall and that the value of j for which V_{t+j} is maximized will remain the same or fall. ■