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*Social security, income inequality and growth**

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Abstract

In most industrial countries, public pension systems redistribute from workers to retired people, not from high-income to low-income earners. They are close actuarial fairness. However, they are not all equivalent. In particular, some pension benefits are linked to full lifetime average earnings, while others are only linked to partial earnings history. In the latter case, we then show in this article that an actuarially fair pay-as-you-go pension system can both reduce lifetime income inequality and enhance economic growth. We also shed light on the dilemma between inequality and economic growth in retirement systems: greater progressivity results in less lifetime inequlity but also less growth.

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1 Introduction

For several years, industrial countries have simultaneously experienced an increase in their life expectancy and weakness of their fertility, two trends that characterize the aging process of population. Therefore, with an unchanged age of retirement, the ratio of pensioners to workers (the dependency ratio) should reach in France, for example, 70.1 % in 2040, whereas it was 35.8 % in 1990. The debate on the financing of our public retirement systems is crucially related to the dependency ratio. Indeed, these systems are financed on a pay-as-you-go (PAYG) basis, i.e. pension benefits are paid through contributions of contemporary workers. Hence, they must cope with the increasingly larger number of pensioners compared to the number of contributors. Changes are unavoidable. If we want to guarantee in the near future the current level of benefits within the same system, it will be necessary either to increase the contribution rate or the length of contribution (by delaying the age of retirement). However, this financing problem calls into question the role of PAYG retirement systems in our societies. For instance, by evaluating the real pre-tax return on

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non-financial corporate capital at 9.3%[1] and the growth rate over the same period (1960 to 1995) at 2.6%, Feldstein (1995*a*, *b*, 1996) unequivocally advocates the privatisation of retirement systems and the change to fully funded systems. He thus assesses the potential present-value gain to nearly $20 trillion for the United States.

Faced with these financial arguments, it is often sustained (see Cutler, 1998) that PAYG retirement systems are essential instruments for fighting poverty that privatization would end. One can indeed observe that most pension benefit formulas are progressive, especially in Anglo-Saxon countries where pensions are weakly related to earnings (see OECD, 2007). However, appearances can be misleading. When considering the American benefit formula, all empirical studies focusing on the redistributive aspect of the retirement system stressed that the progressivity of the system is only apparent because the formula does not take into account specificities related to gender, life expectancy or institutional features (Burkhauser and Walick, 1981; Garrett, 1995; Gustman and Steinmeier, 2001; Coronado *et al.*, 1999, 2000; Brown *et al.*, 2006). First, the redistribution within the system is carried out from men towards women. Second, redistribution within the system is to the advantage of people who live longer and, as noted by Deaton and Paxton (1998, 1999), differences in life expectancies are strongly related to social inequality: high-income earners live longer than low-income earners. Third, as argued by Lindbeck and Persson (2003) and Bozio and Piketty (2008), institutional features such as linking pensions to the best or last years tend to favor those with steep age-earnings profiles, i.e. again high-income earners. When considering these elements, Gustman and Steinmeier (2001) show that retirement system returns are almost identical whatever the household earnings. In the same line, Coronado *et al.* (2000) and Brown *et al.* (2006) show that the U.S. Social Security has no impact on the GINI index measuring income inequality. As the American pension benefit formula is one of the most progressive (see OECD, 2007), most retirement systems in the industrial world appear, in fact, close to actuarial fairness[2] (see Stahlberg, 1990, for the Swedish system).

In this article, focusing on the age-earnings profile feature highlighted by Lindbeck and Persson (2003) and Bozio and Piketty (2008), we analyze the extent to which public PAYG retirement systems can reduce income inequality whereas most of them are actuarially fair. In particular, we investigate the relation between pension benefits and earnings history. In some countries, the earnings-related part of pension benefits is linked to full lifetime average earnings. In others like Greece, Spain, Sweden (before the 1994 legislation), France and Norway it is linked to the best or last years. As pointed out by Lindbeck and Persson (2003) and Bozio and Piketty (2008), the way pension benefits are calculated when considering heterogenous work histories and age-earnings profiles can have important consequences in terms of income redistribution, even if the system as a whole stays actuarially fair. To study the impact of the calculation of pension benefits on income redistribution, the construction of a

[1] This return combines profits before all federal, state and local taxes with the net interest paid. The method of calculation is described in Feldstein *et al.* (1983).

[2] Or, as noted by Lindbeck and Persson (2003), quasi-actuarial fairness since the growth rate which corresponds to the retirement system return is lower than the interest rate.

framework, which accurately adresses the main features of work history and how earnings are determined, is necessary.

On one hand, earnings are strongly related to human capital (e.g. Mincer, 1997, Neal and Rosen, 2000). As learning activities and human capital formation are concentrated at young ages, a work history can be summarized by two different periods. During the first period, people invest time to be trained, including higher education and job training. This first period is characterized by relatively low earnings. In the second period, they benefit from their human capital investment and then earn more. Age-earnings profiles are thus increased, except for high-school dropouts whose age-earnings profiles are almost flat (e.g., Andolfatto *et al*., 2000). Lillard (1977) then highlighted that the age-earnings profile of a worker is increased with his learning ability and the time he has spent in training. In order to replicate these facts, the Ben-Porath (1967) human capital model has been widely used (e.g., Mincer, 1997, Neal and Rosen, 2000). In this model, individuals maximize the present value of their lifetime earnings by allocating their time between training activities and work. It predicts accurately that individuals with higher abilities will invest more in human capital and therefore will have steeper age-earnings profiles than their counterparts with lower abilities.

On the other hand, earnings are very significantly linked to the earnings of their parents (Bowles and Gintis, 2002; d'Addio, 2007). The easiest way to replicate this fact in the Ben-Porath (1967) model consists in assuming that children inherit part of the human capital of their parents. However, as shown by Huggett *et al*. (2006), when individuals differ only in their initial human capital endowment, the model generates a counterfactual pattern regarding the US earnings distribution. By contrast, they show that the US earnings distribution can be replicated quite well when considering differences in learning ability across individuals. This suggests that intergenerational earnings persistence is based on the inheritability of learning ability within families. Supporting such a view, education appears as a major contributor to intergenerational earnings mobility and educational differences tend to persist across generations (d'Addio, 2007). However, as shown by Bowles and Gintis (2002), it does not imply that intergenerational earnings are based only on genetic transmission. Learning ability also reflects non-cognitive personality traits such as, a taste for learning at school, which can be influenced by the family background as much as by the genes.

The rest of the paper is organized as follows. In Section 2, we present the basic assumptions related to the age-earnings profiles and the calculation of pension benefits. In Section 3, we analyze optimal behaviors of individuals and firms considering the basic assumptions. Following Huggett *et al*. (2006), we assume, in particular, that individuals differ only in their learning abilities. In Section 4, we specify the equilibrium features when retirement systems are actuarially fair. In Section 5, we then show that actuarially fair retirement systems, depending on the calculation of pension benefits, can both enhance economic growth and reduce lifetime income inequality. We also shed light on the dilemma between income inequality and economic growth in retirement systems: greater progressivity results in less lifetime income inequality but also less growth. In the last section, we briefly conclude.

2 Age-earnings profile and pension benefits: basic assumptions

2.1 Human capital and age-earnings profiles

The model is a version of the Ben-Porath model (1967). Individuals live for three periods: they are young, adult and old. When young, they go to school. During this period, which corresponds to primary and secondary education (compulsory schooling), individuals born in $t-1$ learn basic knowledge represented by the average knowledge $\overline{Z}_{t-1}$ of the contemporary working generation. In addition, they can choose to make an effort e_{t-1} in learning (where $e_{t-1}=0$ or 1) to pass the final secondary school examination, qualifying for university entrance. In the second period, those who have made the effort can then complement their basic knowledge by pursuing training during a period h_t instead of entering directly the labor market[3]. At the end of their complementary training, their human capital is characterized by:

$$Z_t^s = Bh_t^{\delta}\,\overline{Z}_{t-1}, \quad B>0, \quad \delta>0, \tag{1}$$

where δ represents the return to complementary training in terms of human capital.

Skilled workers, those who have completed their training before entering the labor market, are thus characterized by a first period h_t with no earnings. Afterwards, they earn $Z_t^s w_t$, where w_t is the wage rate per unit of effective labor. Earnings of skilled workers W_t^s over their whole active period are thus:

$$W_t^s = (1-h_t)Z_t^s w_t, \tag{2}$$

and are then characterized by a steep profile. By contrast, unskilled workers are characterized by the basic human capital during all their working period:

$$Z_t^u = \overline{Z}_{t-1}, \tag{3}$$

and are then characterized by flat age-earnings profiles:

$$W_t^u = Z_t^u w_t. \tag{4}$$

In a simple and efficient way, we then specify an economy in the line of Lillard (1977) and Andolfatto *et al.* (2000) where age-earnings profiles of workers increased with the time spent in training and where high-school dropouts have flat age-earnings profiles.

2.2 Pension benefits

The calculation of pension benefits is specific to each country, and can sometimes be very complex. Nevertheless, it has been summarized (see Casamatta *et al.*, 2000; Docquier and Paddison, 2003; Sommacal, 2006) by considering two different parts, a redistributive part (the Beveridgean part), characterized by a basic flat-rate benefit,

[3] In that case training is a full-time activity that can be assimilated to higher education. We could have assumed alternatively that training is a part-time activity without qualitatively changing the results (see Le Garrec, 2005).

and an insurance part (the Bismarckian part) characterized by earnings-related benefits. The latter is not necessarily proportional to all contributions and then based on full lifetime average earnings (see OECD, 2007). It is particularly the case in Greece and Spain where benefits are only linked to final salary. It also used to be the case in Sweden before the 1994 legislation. In France, before the Balladur reform of 1993, earnings-related benefits were linked to the ten best years, then gradually to the 25 best years after the reform. In the United States, the 35 best years are considered to calculate the benefits, 20 in Norway.

Let us define $\tilde{W}_t^i$, $i = s, u$, as the representative earnings on which benefits are based. It does not matter which period is used to calculate the unskilled representative earnings because the age-earnings profile is consistently flat. It follows that:

$$\tilde{W}_t^u = W_t^u. \tag{5}$$

For the skilled workers, if the reference earnings $\tilde{W}_t^s$ corresponds to full lifetime average earnings, we have then W_t^s, otherwise, if it corresponds to the best or last years, we have $Z_t^s w_t$. The representative earnings of skilled workers can then be specified as

$$\tilde{W}_t^s = \mu Z_t^s w_t + (1-\mu) W_t^s, \tag{6}$$

where $\mu = 0$ if pension benefits are linked to full lifetime average earnings or $\mu = 1$ if they are linked to the best or last years.

Assuming that the basic flat-rate benefit $\bar{p}_{t+1}$ is based on the contemporary wage of unskilled workers[4], the calculation of pension benefits for any worker in t is then given by

$$p_{t+1} = \theta_{t+1} \tilde{W}_t + \nu_{t+1} W_{t+1}^u, \tag{7}$$

where ν_{t+1} represents the size of the flat-rate component of pension benefits and θ_{t+1} the size of the earnings-related component.

As noted in the introduction, most retirement systems of industrialized economies are close to actuarial fairness. In terms of the retirement system implicit return, i.e. the ratio of the pension benefits received by an individual to the amount of his contributions, this means that

$$\frac{p_t^u}{\tau W_{t-1}^u} \approx \frac{p_t^s}{\tau W_{t-1}^s}, \tag{8}$$

where τ is the public pension system contribution rate and p_t^i is the pension in t of a worker of type i in $t-1$, $i = u, s$. If $(p_t^u/\tau W_{t-1}^u) > (p_t^s/\tau W_{t-1}^s)$, then the retirement system is fiscally to the advantage of low-income earners. In this case, the system is progressive. In the opposite case, $(p_t^u/\tau W_{t-1}^u) < (p_t^s/\tau W_{t-1}^s)$, it is regressive. By definition, pure contributory retirement systems, i.e., with pension benefits proportional to all contributions and based on full lifetime average earnings, are actuarially fair. It is the case in our framework if $\bar{p} = 0$ and $\mu = 0$.

[4] It is designed to ensure that pensioners achieve some minimum standard of living.

3 Optimal behaviors

3.1 Individuals

Preferences of an individual of type x born in $t-1$ are described by the following utility function:

$$U_x = \ln c_t + \beta \ln d_{t+1} + \varepsilon \ln(1-x) e_{t-1}, \quad (9)$$

where c_t and d_{t+1} are, respectively, his consumption when adult and when old, and β is the discount factor; $\varepsilon \ln(1-x)$ is the utility cost of schooling effort, where $x \in [0, 1]$ represents learning ability. In this setting, a talented child characterized by $x=0$ endures no cost in making the effort. By contrast, a lazy or untalented child characterized by $x=1$ will endure an infinite cost and will then always choose not to make the effort, i.e. $e_{t-1}=0$. As explained in the introduction, x can be considered as an inherited (perfectly here) trait that represents family background and genetic transmission. For simplicity, we will assume that x is uniformly distributed over the population.

During the second life period, individuals consume a part of their disposable income, and save such as

$$c_t + s_t = W_t(1-\tau), \quad (10)$$

where s_t is the savings.

In the third life period, individuals are retired. They get back the savings lent to firms with interest, receive their pension from the public retirement system and consume their wealth. The budget constraint is then

$$d_{t+1} = R_{t+1} s_t + p_{t+1}, \quad (11)$$

where R_{t+1} is the real interest factor.

Let $\Omega_t^i = W_t^i(1-\tau) + \frac{p_{t+1}^i}{R_{t+1}}$ be the lifetime income of a worker of type i, $i=u$, s. Considering the reference earnings (6) of a skilled worker in the calculation of his pension benefits, an individual who has chosen to make the effort at school will maximize his lifetime income by spending the following time in training during his second life period:

$$h_t = \inf\left\{ h^0 \left[1 + \frac{\mu}{1-\tau+(1-\mu)\frac{\theta_{t+1}}{R_{t+1}}} \frac{\theta_{t+1}}{R_{t+1}} \right];\ 1 \right\}, \quad (12)$$

where $h^0 = \delta/(1+\delta)$ is the training length with no retirement system.

Proposition 1. *Linking pension benefits to partial earnings history generates an incentive to be trained longer.*

Retirement systems whose pension benefits are based, even partially, on the best or last years generate an incentive for longer training. Initially, the lengthening of training has a negative effect on income. During this period individuals have indeed no earning capacity. However, they earn more afterwards. In addition, as pensions are linked to the best or last years they also benefit, all things being equal, from an increase in their benefits. Following equation (12), individuals who undertake training may find it profitable to be trained longer as an investment in their pension benefits. Note that this incentive disappears completely if pension benefits are based on full lifetime average earnings ($\tilde{W}_t^s = W_t^s$) or if the system is totally flat-rate ($\theta_{t+1} = 0$). Note also that this incentive is weaker as the interest rate increases. Indeed, the higher the interest rate, the lower the present actuarial value of pension benefits.

To summarize, the incentive to be trained longer, generated by the retirement system, is due to the interaction of two factors:

- pension benefits are linked to the best or last years
- training results in steeper age-earnings profiles

The utility maximization of an individual subject to budgetary constraints (10) and (11) leads to the following saving function:

$$s_t = \frac{\beta}{1+\beta} W_t(1-\tau) - \frac{1}{1+\beta} \frac{p_{t+1}}{R_{t+1}}. \tag{13}$$

By reducing simultaneously the disposable income and the need for a future income, a retirement system reduces private savings. This result holds irrespective of the calculation of pension benefits and their financing.

Last, an individual will choose to make the effort at school if the opportunity of complementary training entails a monetary benefit higher than the utility cost associated with the effort, i.e. if $(1+\beta)\ \ln\ \Omega_t^s + \varepsilon\ \ln(1-x) \geqslant (1+\beta)\ \ln \Omega_t^u$. Given the uniform distribution of the parameter x over the population, the proportion of individuals x_t^* who choose to be trained in t (and then to make the schooling effort in $t-1$) to become skilled workers is defined by:

$$x_t^* = 1 - (I_t)^{-(1+\beta)/\varepsilon}, \tag{14}$$

where $I_t = \Omega_t^s / \Omega_t^u$ represents the lifetime income inequality between skilled and unskilled workers in t. Following (14), the higher this inequality, the larger the proportion of individuals incited to be trained: $\frac{\mathrm{d}x_t^*}{\mathrm{d}I_t} > 0$.

3.2 Firms

We consider a competitive sector characterized by a representative firm producing a good, which can be either consumed or invested, according to a Cobb–Douglas technology with constant return to scale:

$$Y_t = F\left(K_t, L_t^u, L_t^s\right) = AK_t^{\alpha}\left(Z_t^u L_t^u + (1-h_t) Z_t^s L_t^s\right)^{1-\alpha},\ 0 < \alpha < 1, \tag{15}$$

where Y_t is the output, K_t the capital physical stock, L_t^i the number of workers of type i in t, $i = u$, s, and A the total factor productivity. Assuming for simplicity as in Docquier and Paddison (2003), that skilled and unskilled labors are perfect substitutes[5], $L_t = Z_t^u L_t^u + (1-h_t) Z_t^s L_t^s$ represents the labor supply in efficiency units.

[5] Assuming alternatively that they are imperfect substitutes would not qualitatively alter the results. First, it would not change the training length as defined in equation (12) at all. By introducing a wage premium

Denoting per capita efficient capital by $k_t = \frac{K_t}{L_t}$ and assuming a total capital depreciation, the optimal conditions resulting from the maximization of the profit are:

$$R_t = A\alpha k_t^{\alpha-1}, \tag{16}$$

$$w_t = A(1-\alpha)k_t^{\alpha}. \tag{17}$$

Before studying the impact of retirement systems and the calculation of pension benefits on growth and income inequality, we have to characterize the equilibrium and its properties.

4 Equilibrium

The economy is composed of four markets corresponding to the unskilled labor, the skilled labor, the physical capital and the good. In a closed-economy setting, the general equilibrium can be obtained by considering only the clearing of three markets, as according to the Walras law, the fourth is necessarily cleared. In our case, we consider the clearing of the following markets:

unskilled labor:

$$L_t^u = (1 - x_t^*)N_{t-1}. \tag{18}$$

skilled labor:

$$L_t^s = x_t^* N_{t-1}. \tag{19}$$

physical capital:

$$K_{t+1} = N_{t-1}\left[x_t^* s_t^s + (1 - x_t^*)s_t^u\right]. \tag{20}$$

4.1 PAYG social security and the capital accumulation

Retirement systems have PAYG features, i.e. within a period, pension benefits are financed by contributions of workers of the same period. In other words, retirement systems transfer workers' income towards pensioners. Since workers are either skilled or unskilled, the social security balanced budget is defined as follows:

$$L_{t-1}^u p_t^u + L_{t-1}^s p_t^s = \tau\left[L_t^u W_t^u + L_t^s W_t^s\right]. \tag{21}$$

Assume that in each period population is growing at a constant rate n:

$$N_t = (1+n)N_{t-1}. \tag{22}$$

Since at date t there is a proportion x_t^* and $1 - x_t^*$ of respectively skilled and unskilled workers as specified in equations (18) and (19), the balanced budget of the retirement

for human capital, it would change the skill choice. Nevertheless, it would still allow one to express the latter as a function of the training length in the case of actuarial fairness. The forthcoming propositions would then not be affected.

system (21), with equations (1)–(7) and (22), is rewritten as:

$$\theta_t = \frac{\left[(1+n)\tau\left[x_t^*(1-h_t)Bh_t^\delta + 1 - x_t^*\right] - \nu_t\right]\left(x_{t-1}^* Bh_{t-1}^\delta + 1 - x_{t-1}^*\right)}{x_{t-1}^*\left[\mu + (1-\mu)(1-h_{t-1})\right]Bh_{t-1}^\delta + 1 - x_{t-1}^*} \frac{w_t}{w_{t-1}}. \tag{23}$$

As is obvious, a decreasing population growth, which corresponds to an aging population in the model, is associated with lower pension benefits, everything else being equal. Considering the social security balanced budget (23) and the capital market clearing (20), with equations (1)–(7), (13), (16), (17) and (22), the dynamics of capital accumulation in the model can be expressed as

$$\begin{aligned} &k_{t+1}\left[x_{t+1}^*(1-h_{t+1})Bh_{t+1}^\delta + 1 - x_{t+1}^*\right] \\ &\quad = \frac{A\alpha\beta(1-\alpha)(1-\tau)}{[\alpha(1+\beta)+\tau(1-\alpha)](1+n)} \frac{x_t^*(1-h_t)Bh_t^\delta + 1 - x_t^*}{x_t^* Bh_t^\delta + 1 - x_t^*} k_t^\alpha. \end{aligned} \tag{24}$$

As retirement systems reduce private savings (equation 13), all things being equal, PAYG retirement systems are harmful to the accumulation of physical capital: $\frac{\partial \frac{1-\tau}{\alpha(1+\beta)+\tau(1-\alpha)}}{\partial \tau} < 0$ (equation 24). In addition, as x^* and h are both forward-looking variables, their specification is crucial to determine the dynamic properties of the model and the convergence towards its steady-state (balanced growth) path.

4.2 Human capital and actuarial fairness

As noted in the introduction and characterized by equation (8), most retirement systems in the industrial world are close to actuarial fairness. It is obviously the case if pension benefits are based on full lifetime average earnings ($\bar{p}=0$ and $\mu=0$). It is less obvious that it is also possible if pension benefits are linked only to partial earnings history.

Proposition 2. *Retirement systems whose pensions are linked to the best or last years can be actuarially fair if they include a flat-rate component indexed on the unskilled earnings,* $\bar{p}_t = \nu_t W_t^u$, *such as* $\nu_t = \tilde{\nu}_t = \frac{(1+n)Bh_{t-1}^{1+\delta}}{Bh_{t-1}^\delta - 1}\tau \frac{x_t^*(1-h_t)Bh_t^\delta + 1 - x_t^*}{x_{t-1}^*(1-h_{t-1})Bh_{t-1}^\delta + 1 - x_{t-1}^*}$.

If $\nu_t > \tilde{\nu}_t$, the retirement system is fiscally to the advantage of low-income earners, $\frac{p_t^u}{\tau W_{t-1}^u} > \frac{p_t^s}{\tau W_{t-1}^s}$, and is then progressive. In the opposite case, $\nu_t < \tilde{\nu}_t$, it is regressive. This feature is easily understandable. On one hand, the flat-rate part of pension benefits is clearly to the advantage of low-income earners: they receive as much as high-income earners whereas they have contributed less. A flat-rate system is obviously progressive. On the other hand, the pension part that is linked to the best or last years, characterized by θ_t, favors high-income earners as they have steeper age-earnings profiles, as explained by Lindbeck and Persson (2003) and Bozio and Piketty (2008). If there is no flat-rate part then the system is regressive. Therefore, there is only one combination of the flat-rate and earnings-related parts, the one specified in Proposition 2, that characterizes actuarial fairness.

Consider an actuarially fair retirement system, i.e. either $\mu=0$ and $\bar{p}_{t+1}=0$ or $\mu=1$ and $\nu_{t+1}=\tilde{\nu}_{t+1}$. In that case, the lifetime income inequality $I_t = \frac{\Omega_t^s}{\Omega_t^u}$ becomes

$I_t = \frac{W_t^s}{W_t^u}$. With equations (1)–(4), the proportion of skilled workers in t defined by (14) becomes

$$x_t^* = 1 - \frac{1}{\left[(1-h_t)Bh_t^{\delta}\right]^{(1+\beta)/\varepsilon}}. \tag{25}$$

In this configuration, the choice for a young individual to make the effort at school in $t-1$ to become a skilled worker in t only depends on his learning ability and the length of the training he anticipates to complete. As h^0 corresponds to $\max\{(1-h)Bh^{\delta}\}$, we can deduce from (25) that any increase in the training length compared to the basic level h^0 will lead to a decrease in the skilled workers proportion: $\left.\frac{\partial x_t^*}{\partial h_t}\right|_{h_t \geqslant h^0} \leqslant 0$. Following Proposition 1, we can then expect that actuarially fair retirement systems whose pension benefits are based on partial earnings history reduce the proportion of skilled workers.

When the retirement system is purely contributory, i.e. if $\mu = 0$ and $\bar{p}_{t+1} = 0$, it has no impact on the training length in the second period of life (equation 14). By contrast, as characterized by equation (12), linking pension benefits to partial earnings history generates an incentive to be trained longer, which depends crucially on the actualized Bismarckian component $\frac{\theta_{t+1}}{R_{t+1}}$. Using equations (16), (17) and (23), (24), we have $\frac{\theta_{t+1}}{R_{t+1}} = \left[\frac{Bh_t^{\delta}(1-h_t)-1}{Bh_t^{\delta}-1}\right]\frac{\beta(1-\alpha)(1-\tau)\tau}{\alpha(1+\beta)+\tau(1-\alpha)}$. Thus, the training length according to the social security features can be summarized as:

$$h_t = \begin{cases} h^0, & \text{if } \mu = 0 \text{ and } \bar{p}_{t+1} = 0, \\ h^0\left[1 + \frac{\beta(1-\alpha)\tau}{\alpha(1+\beta)+\tau(1-\alpha)} \frac{Bh_t^{\delta}(1-h_t)-1}{Bh_t^{\delta}-1}\right], & \text{if } \mu = 1 \text{ and } \nu_{t+1} = \tilde{\nu}_{t+1}. \end{cases} \tag{26}$$

If $\mu = 1$ and $\nu_{t+1} = \tilde{\nu}_{t+1}$, we deduce from (26) that $\lim_{h \to h^0} \text{RHS} > h^0$ and $\lim_{h \to 1} \text{RHS} < h^0$. In this case, the training is expressed as a function $h_t = h(\tau)$ such as $h^0 \leqslant h(\tau) \leqslant 1$. In the case of a pure contributory system, as the latter has no impact on the training length, we will note conveniently that $h_t = h(\mu\tau)$, where $\mu = 0$, i.e. $h(0) = h^0$. Thereafter, as the skill choice depends only on the training length (25), it can also be expressed as $x_t^* = x^*(\mu\tau)$, where $\mu = 0$ or $\mu = 1$. In that case, $x_t^* = x^*(0)$ corresponds to an unchanged proportion of skilled workers in t compared to a situation with no retirement system: $x^*(0) = 1 - \frac{1}{\left[(1-h^0)Bh^{0\delta}\right]^{\frac{1+\beta}{\varepsilon}}}$.

4.3 Dynamic properties

As underlined by equations (25) and (26), human capital variables are in their steady-state values independently of the calculation of pension benefits. Therefore, considering an actuarially fair retirement system, the physical capital accumulation dynamics (24) can be rewritten as:

$$k_{t+1} = \frac{A\alpha\beta(1-\alpha)(1-\tau)}{[\alpha(1+\beta)+\tau(1-\alpha)](1+n)} \frac{1}{x^*(\mu\tau)Bh^{\delta}(\mu\tau) + 1 - x^*(\mu\tau)} k_t^{\alpha}. \tag{27}$$

Since $\alpha < 1$, given $k_0 > 0$, the model has good dynamic properties and converges to its steady-state (balanced growth) path characterized by $h(\mu\tau)$, $x^*(\mu\tau)$ and $k = \left[\frac{A\alpha\beta(1-\alpha)(1-\tau)}{[\alpha(1+\beta)+\tau(1-\alpha)](1+n)} \frac{1}{x^*(\mu\tau)Bh^{\delta}(\mu\tau)+1-x^*(\mu\tau)}\right]^{1/(1-\alpha)}$, where $\mu = 0$ or $\mu = 1$.

As the convergence is verified, the impact of retirement systems and the calculation of pension benefits on growth and income inequality can now be discussed.

5 Social security, income inequality and growth

On the balanced growth path, we deduce from the labor market clearing relations (18) and (19), as well as from equations (1), (3), (15) and (22), the economic growth rate g:

$$1+g=\frac{Y}{Y_{-1}}=(1+n)\frac{\bar{Z}}{\bar{Z}_{-1}}=(1+n)\left[x^*Bh^{\delta}+1-x^*\right]. \tag{28}$$

In addition to the population growth, equation (28) stresses that economic growth depends on the rate of knowledge accumulation which is driven by both the proportion of skilled workers in the economy and the length of training.

A pure contributory system ($\nu=0$ and $\mu=0$) has no impact on the training length (equation 14). As a consequence, considering equation (25), it has also no impact on the proportion of skilled workers. Indeed, in that case, as pension benefits are proportional to all contributions the retirement system can no longer alter the skill choice. Pure contributory systems are then characterized by an unchanged investment in human capital, i.e. h^0 and x^* (0), and it follows that:

Proposition 3. *Pure contributory retirement systems have no impact on either economic growth or on lifetime income inequality.*

Consider alternatively retirement systems whose pensions are linked to the best or last years, i.e. $\mu=1$. If they are actuarially fair, following equation (26), the training is determined by $h=h^0\left[1+\frac{\beta(1-\alpha)\tau}{\alpha(1+\beta)+\tau(1-\alpha)}\frac{Bh^{\delta}(1-h)-1}{Bh^{\delta}-1}\right]$, where $\lim_{h\to h^0}\text{RHS}>h^0$ and $\lim_{h\to 1}\text{RHS}<h^0$. This equation thus defines a relation between the training and the contribution rate such as $h=h(\tau)<1$ and $\frac{\partial h}{\partial \tau}>0$. In addition, as the skill choice is specified by $x^*=1-\frac{1}{[(1-h)Bh^{\delta}]^{(1+\beta)/\varepsilon}}$, it follows that:

Proposition 4. *Compared with no retirement system or with pure contributory systems, actuarially fair retirement systems whose pensions are linked to the best or last years enhance economic growth* (*at least if the contribution rate is sufficiently low*) *and lower lifetime income inequality.*

At first glance, the result with regard to income inequality appears counter-intuitive. Indeed, the reduction in lifetime income inequality follows the lengthening of the skilled workers' training. This lengthening raises the difference between earnings of the skilled workers and those of the unskilled ones. Such retirement systems then increase earnings inequality. However, from a life cycle perspective, with no retirement system or with a pure contributory system (Proposition 1), individuals who decide to undertake training choose the duration h^0, which maximizes their lifetime income; h^0 thus maximizes the lifetime income inequality between skilled and unskilled workers. A lengthening of the training thus raises lifetime income inequality when $h<h^0$. Conversely, when $h>h^0$, a lengthening of the training reduces lifetime income inequality because we move away from the individually optimal training length. Therefore, even if the retirement system does not carry out transfers from high-income to low-income earners, we know from Proposition 1 that such an earnings-related pension benefit formula generates an incentive for longer training. Skilled workers are then encouraged to train themselves more than their individually optimal level. Consequently, actuarially fair retirement systems, whose pensions are linked to the best or last years, reduce lifetime income inequality compared to a situation with no retirement system (or purely contributory).

As such systems reduce lifetime income inequality, from equation (14) it follows that they reduce the proportion of skilled workers in the economy. This has a negative effect on economic growth. On the other hand, they incite skilled workers to train longer. Since the latter effect dominates the former (at least for a sufficiently low size of the system) we can stress a positive impact of PAYG retirement systems on economic growth as empirically reported by Sala-i-Martin (1996) and Zhang and Zhang (2004). We then extend results obtained by Zhang (1995), Sala-i-Martin (1996), Kemnitz and Wigger (2000), Le Garrec (2001) and Zhang and Zhang (2003) with identical ability individuals. Initiated by the lengthening of training, our mechanism is directly related to Kemnitz and Wigger (2000) and Le Garrec (2001)[6]. It also explains the difference with Docquier and Paddison (2003). By assuming a fixed training length, they indeed show that retirement systems based on partial earnings history can enhance growth, but only if they are fully funded.

In the current framework, actuarially fair retirement systems whose pensions are linked to the best or last years appear to be to the advantage of low-income earners, not through direct fiscal redistribution, but by enhancing growth. However, are these systems and their features socially optimal? If adopting the view of low-income earners, should we not integrate greater progressivity? and with which consequence for growth?

Proposition 5. *For given contribution rates, compared to actuarially fair PAYG retirement systems whose pensions are linked to the best or last years, any marginal increase (decrease) of the flat-rate component reduces (increases) both economic growth and lifetime income inequality.*

The flat-rate versus partial earnings-related structure sheds light on the dilemma between income inequality and economic growth in retirement systems. In order to reduce (more) the level of lifetime income inequality, retirement systems must be more progressive. Nevertheless, it is harmful for growth. Indeed, an increase in the flat-rate part, which sustains a reduction of lifetime income inequality, also means, for a given contribution rate, a reduction of the earnings-related part and thus a reduction of the incentive to be trained longer. If a retirement system gets close to a pure flat-rate system, which is the case of Anglo-Saxon countries, it has no impact on the training length (Proposition 1). Consequently, such a system results in a reduction

[6] In Zhang (1995) and Zhang and Zhang (2003), PAYG retirement systems result in more growth by reducing fertility of altruistic parents who consequently invest more in the education of their children. In Sala-i-Martin (1996), old workers are associated with negative externalities in the average stock of human capital. By inducing earlier retirement, PAYG retirement systems then stimulate growth.

of economic growth while it also reduces lifetime income inequality significantly. Conversely, a pure earnings-related system where benefits are linked to the best or last years boosts economic growth but is regressive. The choice of the intragenerational redistribution degree consists of a lifetime income inequality/growth trade-off. In such a context, if policymakers want to favor future generations, they will opt for more growth and then for greater regressivity. By contrast, if they want to favor contemporary low-income earners, they will opt for greater progressivity.

6 Conclusion

In 1950, life expectancy at birth in Western Europe was 68 years. Nowadays, it is 80 years and should reach 85 years in 2050 (United Nations, 2009). The downside of this trend is the serious threat that is hanging over the financing of our public retirement systems. Financed on a PAYG basis, they must cope with the increasingly larger number of pensioners compared to the number of contributors. Changes are unavoidable and are of major importance in OECD countries. In 2005, the payment of pension benefits represented 38% of all their public social expenditures. As a matter of fact, retirement systems are the major program of industrial countries' redistributive policies and their importance should still grow with the aging of their population.

Claiming the broad inefficiency of PAYG retirement systems (being accused of low return and of distorting individual behaviors), some economists as Feldstein (1995*a*, *b*, 1996) stress that these financial difficulties give opportunities to move to fully funded systems. However, replacing conventional PAYG systems by financial – or funded – defined contribution (FDC) systems involves such a large cost of transition that it appears socially and politically difficult to implement such a reform in western democracies. That is why in recent years strong focus has been put on non-financial- or notional- defined contribution (NDC) systems as legislated in Sweden in 1994. As described by Palmer (2006), NDC systems are PAYG systems that mimic FDC systems. Individual contributions are noted on individual accounts. Accounts are credited with a rate of return that reflects demographic and productivity changes.

By basing benefits on individual accounts, NDC systems have undoubtedly desirable features in terms of transparency. However, as existing retirement systems (except in Anglo-Saxon countries) are already close to actuarial fairness, we cannot expect from NDC systems a significant decrease in negative incentive effects. In many respects, introducing a NDC system largely involves moving from a defined benefit to a defined contribution system aiming at the contribution rates' stabilization. From this perspective, one should note that this objective can be achieved similarly within the scope of more conventional defined benefit systems, as seen in the point system in France. In that case, the unit of pension rights is earnings points and not euros and can be adjusted according to demographic and productivity changes as in a NDC system. As stressed by Börsch-Supan (2006), cleverly designed conventional retirement systems can often do the same job as NDC systems. As shown in this article, they can even do better.

In particular, a conventional retirement system can allow pension benefits to be linked only to partial earnings history as observed in Greece, Spain, France, Norway and in Sweden before the 1994 legislation. In that case, associated with the appropriate flat-rate benefits, the latter can be actuarially fair as a NDC system while enhancing economic growth by promoting the accumulation of human capital. It does better in this matter than a NDC system which has no impact on economic growth.

Furthermore, an actuarially fair retirement system whose pensions are linked to the best or last years lowers lifetime income inequality, whereas a NDC system does not. More generally moving to a NDC system, by nature purely contributory, definitively closes the debate on the progressivity of the retirement system, which is an important one in democracy. Of course, compared to any actuarially fair system, greater progressivity results in negative incentive effects that lead to less economic growth. However, even if the current choice is actuarial fairness, at least a conventional system still allows one to ask whether greater progressivity involving less lifetime inequality is worth the cost in terms of economic growth. To contribute to the debate, as the latter can be enhanced through more investment in human capital, it is worth noting that the negative effect on economic growth associated with greater progressivity could be counterbalanced by any relevant educational policy. Besides, as suggested by our results, a properly designed progressive system with pension benefits linked to the best or last years could significantly decrease lifetime income inequality while being neutral in terms of economic growth.

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Appendix A – Proof of propositions

Proposition 1.

Following (12), $h^0 = \frac{\delta}{1+\delta} < 1$.

If $\tilde{W}_t^s = W_t^s$ ($\mu = 0$) or $\theta_{t+1} = 0$, we still have according to (12) $h_t = h^0$.

By contrast, if $\mu = 1$ and $\theta_{t+1} > 0$, $\frac{\delta}{1+\delta}\frac{\mu}{1-\tau+(1-\mu)\frac{\theta_{t+1}}{R_{t+1}}}\frac{\theta_{t+1}}{R_{t+1}} > 0$ and it follows that $h_t > h^0$.

Proposition 2.

Assuming that $\mu = 1$, actuarial fairness defined by (8) entails that:

$$\frac{\theta_t B h_{t-1}^\delta \overline{Z}_{t-2} w_{t-1} + \nu_t \overline{Z}_{t-1} w_t}{\tau(1-h_{t-1}) B h_{t-1}^\delta \overline{Z}_{t-2} w_{t-1}} = \frac{\theta_t \overline{Z}_{t-2} w_{t-1} + \nu_t \overline{Z}_{t-1} w_t}{\tau \overline{Z}_{t-2} w_{t-1}}$$

and then:

$$\frac{\theta_t B h_{t-1}^\delta w_{t-1} + \nu_t \left(x_{t-1}^* B h_{t-1}^\delta + 1 - x_{t-1}^*\right) w_t}{\tau(1-h_{t-1}) B h_{t-1}^\delta w_{t-1}} = \frac{\theta_t w_{t-1} + \nu_t \left(x_{t-1}^* B h_{t-1}^\delta + 1 - x_{t-1}^*\right) w_t}{\tau w_{t-1}}.$$

Introducing (23) with $\mu = 1$ entails that:

$$\frac{\left[(1+n)\tau\left[x_t^*(1-h_t)Bh_t^\delta + 1 - x_t^*\right] - \nu_t\right] B h_{t-1}^\delta + \nu_t \left(x_{t-1}^* B h_{t-1}^\delta + 1 - x_{t-1}^*\right)}{(1-h_{t-1}) B h_{t-1}^\delta}$$
$$= \left[(1+n)\tau\left[x_t^*(1-h_t)Bh_t^\delta + 1 - x_t^*\right] - \nu_t\right] + \nu_t \left(x_{t-1}^* B h_{t-1}^\delta + 1 - x_{t-1}^*\right).$$

Actuarial fairness with $\mu = 1$ is then obtained if:

$$\nu_t = \frac{(1+n) B h_{t-1}^{1+\delta}}{B h_{t-1}^\delta - 1} \tau \frac{x_t^*(1-h_t) B h_t^\delta + 1 - x_t^*}{x_{t-1}^*(1-h_{t-1}) B h_{t-1}^\delta + 1 - x_{t-1}^*}.$$

Proposition 4.

From equation (28) we derive $\frac{\mathrm{d}(1+g)}{\mathrm{d}\tau} = (1+n)\left[Bh^\delta - 1\right]\frac{\mathrm{d}x^*}{\mathrm{d}\tau} + x^* \delta B h^{\delta-1}\frac{\mathrm{d}h}{\mathrm{d}\tau}$.

From equation (25) we derive $\frac{\mathrm{d}x^*}{\mathrm{d}\tau} = \frac{1+\beta}{\varepsilon}\left[(1-h)Bh^\delta\right]^{-(1+\beta+\varepsilon)/\varepsilon}\frac{\partial\left[(1-h)Bh^\delta\right]}{\partial\tau}$. Close to $\tau = 0$, we show that $\left.\frac{\partial\left[(1-h)Bh^\delta\right]}{\partial\tau}\right|_{\tau=0} = 0$ and then $\left.\frac{\mathrm{d}x^*}{\mathrm{d}\tau}\right|_{\tau=0} = 0$. It follows $\left.\frac{\mathrm{d}(1+g)}{\mathrm{d}\tau}\right|_{\tau=0} = x^* \delta B h^{\delta-1}\frac{\mathrm{d}h}{\mathrm{d}\tau}$.

From (26), and since $h \geqslant h_0$, we show that $\operatorname{sign}\left[\frac{\mathrm{d}h}{\mathrm{d}\tau}\right] = \operatorname{sign}\left[\frac{\partial \frac{\tau}{[\alpha(1+\beta)+\tau(1-\alpha)]}}{\partial\tau}\right]$, where $\frac{\partial \frac{\tau}{[\alpha(1+\beta)+\tau(1-\alpha)]}}{\partial\tau} = \frac{\alpha(1+\beta)}{[\alpha(1+\beta)+\tau(1-\alpha)]^2} > 0$.

We then show that: $\left.\frac{\mathrm{d}(1+g)}{\mathrm{d}\tau}\right|_{\tau=0} > 0$.

from (14) we derive $\frac{\mathrm{d}I}{\mathrm{d}\tau} = \frac{\varepsilon}{1+\beta} I^{(1+\beta+\varepsilon)/\varepsilon}\frac{\mathrm{d}x^*}{\mathrm{d}\tau}$. It follows that $\operatorname{sign}\left[\frac{\mathrm{d}I}{\mathrm{d}\tau}\right] = \operatorname{sign}\left[\frac{\mathrm{d}x^*}{\mathrm{d}\tau}\right]$.

We know from equation (25) that $\frac{\mathrm{d}x^*}{\mathrm{d}\tau} = \frac{1+\beta}{\varepsilon}\left[(1-h)Bh^\delta\right]^{-(1+\beta+\varepsilon)/\varepsilon}\frac{\partial\left[(1-h)Bh^\delta\right]}{\partial\tau}$. As $h^0 = \arg\max\{(1-h)Bh^\delta\}$, it follows that $\left.\frac{\partial\left[(1-h)Bh^\delta\right]}{\partial h}\right|_{h \geqslant h^0} \leqslant 0$.

Knowing from (26) that $h \geqslant h_0$ and $\text{sign}\left[\frac{dh}{d\tau}\right] > 0$, we then have $\frac{dx^*}{d\tau} \leqslant 0$ and then: $\frac{dI}{d\tau} \leqslant 0$.

Proposition 5.
For $\mu = 1$, with equations (12), (16), (17), (23) and with $\nu = \tilde{\nu}$ (Proposition 2), we show that training is determined by:

$$h = h^0 + \frac{\delta}{(1+\delta)(1-\tau)} \frac{(1+n)\tau\left[x^*(1-h)Bh^\delta + 1 - x^*\right] - \nu}{R}. \tag{29}$$

The proportion of skilled workers is determined by:

$$\frac{\varepsilon \log(1-x^*)}{1+\beta} = \log\left\{1 - \tau + \frac{(1+n)\tau\left[x^*(1-h)Bh^\delta + 1 - x^*\right] - \nu}{R} + \frac{\nu\left[x^* Bh^\delta + 1 - x^*\right]}{R}\right\} - \log\left\{(1-h)Bh^\delta(1-\tau) + \frac{(1+n)\tau\left[x^*(1-h)Bh^\delta + 1 - x^*\right] - \nu}{R} Bh^\delta + \frac{\nu\left[x^* Bh^\delta + 1 - x^*\right]}{R}\right\}. \tag{30}$$

The interest rate is

$$R = \frac{(1+\beta)\alpha + \tau(1-\alpha)}{\beta(1-\alpha)(1-\tau)}(1+n)\left[x^* Bh^\delta + 1 - x^*\right]. \tag{31}$$

Equation (29) can be rewritten as

$$\frac{(1+n)\tau\left[x^*(1-h)Bh^\delta + 1 - x^*\right] - \nu}{R} = \frac{(1+\delta)(1-\tau)}{\delta}(h - h^0). \tag{32}$$

By introducing this last equation in the skilled workers equation (2), we obtain:

$$\frac{\varepsilon \log(1-x^*)}{1+\beta} = \log\left\{1 - \tau + \frac{(1+\delta)(1-\tau)}{\delta}(h-h^0) + \nu\frac{\beta(1-\alpha)(1-\tau)}{[(1+\beta)\alpha + \tau(1-\alpha)](1+n)}\right\} - \log\left\{(1-h)Bh^\delta(1-\tau) + \frac{(1+\delta)(1-\tau)}{\delta}(h-h^0)Bh^\delta + \nu\frac{\beta(1-\alpha)(1-\tau)}{[(1+\beta)\alpha + \tau(1-\alpha)](1+n)}\right\}. \tag{33}$$

Let be $\frac{\varepsilon \log(1-x^*)}{1+\beta} = \log\{Y\} - \log\{Z\}$, where $Y = 1 - \tau + \frac{(1+\delta)(1-\tau)}{\delta}(h-h^0) + \nu\frac{\beta(1-\alpha)(1-\tau)}{[(1+\beta)\alpha + \tau(1-\alpha)](1+n)}$ and $Z = (1-h)Bh^\delta(1-\tau) + \frac{(1+\delta)(1-\tau)}{\delta}(h-h^0)Bh^\delta + \nu\frac{\beta(1-\alpha)(1-\tau)}{[(1+\beta)\alpha + \tau(1-\alpha)](1+n)}$.

We then have:

$$\frac{-\varepsilon}{(1+\beta)(1-x^*)}dx^* = \frac{Y_h dh + Y_\nu d\nu}{Y} - \frac{Z_h dh + Z_\nu d\nu}{Z}, \tag{34}$$

where $Y_h = \frac{(1+\delta)(1-\tau)}{\delta} \geqslant 0$, $Z_h = Y_h Bh^\delta = \frac{(1+\delta)(1-\tau)}{\delta}Bh^\delta \geqslant 0$ and $Y_\nu = Z_\nu = \frac{\beta(1-\alpha)(1-\tau)}{[(1+\beta)\alpha + \tau(1-\alpha)](1+n)} \geqslant 0$.

From an initial equilibrium $\nu = \tilde{\nu}$, $Z = (1-h)Bh^\delta Y$, we have

- $\frac{Y_h}{Y} - \frac{Z_h}{Z} = \frac{Y_h}{Y}\left(1 - \frac{1}{1-h}\right) \leqslant 0 \Rightarrow \frac{\partial x^*}{\partial h}\Big|_{\nu=\tilde{\nu}} \geqslant 0.$
- $\frac{Y_\nu}{Y} - \frac{Z_\nu}{Z} = \frac{Y_\nu}{Y}\left(1 - \frac{1}{(1-h)Bh^\delta}\right) \geqslant 0 \Rightarrow \frac{\partial x^*}{\partial \nu}\Big|_{\nu=\tilde{\nu}} \leqslant 0.$

The form of the differential (34) is then

$$\mathrm{d}x^* = a\mathrm{d}h - b\mathrm{d}\nu, \tag{i}$$

where $a \geqslant 0$ and $b \geqslant 0$.

By replacing the interest rate equation (31) in the formation equation (29), we have

$$h = h^0 + \frac{\beta(1-\alpha)\delta\left[(1+n)\tau\left[x^*(1-h)Bh^\delta + 1 - x^*\right] - \nu\right]}{(1+\delta)[(1+\beta)\alpha + \tau(1-\alpha)](1+n)[x^*Bh^\delta + 1 - x^*]}. \tag{35}$$

Differentiating this equation, we get:

$$\mathrm{d}h = -c\mathrm{d}h - e\mathrm{d}\nu + f[\nu - \tilde{\nu}]\mathrm{d}x^*, \tag{ii}$$

where $c \geqslant 0$, $e \geqslant 0$ and $f \geqslant 0$.

We then conclude, with (i) and (ii), for an initial equilibrium $\nu = \tilde{\nu}$, $\frac{\mathrm{d}h}{\mathrm{d}\nu} \leqslant 0$ and $\frac{\mathrm{d}x^*}{\mathrm{d}\nu} \leqslant 0$:

$$\frac{\mathrm{d}g}{\mathrm{d}\nu} \leqslant 0 \text{ and } \frac{\mathrm{d}I}{\mathrm{d}\nu} \leqslant 0.$$

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