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CLOSING THE MODEL? THE HARROD–MARSCHAK CORRESPONDENCE ON THE DRAFT OF THE "ESSAY IN DYNAMIC THEORY"

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During August and September 1938, Harrod discussed a first draft of his "Essay in Dynamic Theory" with Keynes and Marschak. The aim of this paper is to discuss Harrod's correspondence with Marschak in the light of the subsequent interpretations of the "Essay." Firstly, we show how Marschak focused on some of the features of Harrod's theory that would later give rise to widely different interpretations and some misunderstandings. Three issues in particular have been broadly debated in the literature: the concept of warranted rate of growth; instability; and non-linearity. Secondly, we show that a possible reason why Harrod neglected Marschak's remarks lies in their respective methodological approaches. While Marschak tried to frame Harrod's theory by means of a closed model, Harrod thought that the analysis of equilibrium and of the succession of states belonged to separate stages; this made building a complete model difficult at this stage, when he was presenting a "new tool of analysis." In spite of their respective conflicting approaches, Marschak's comments remain of great importance, and if Harrod had followed some of his advice, his theory would probably have improved in several respects, and certainly it would have been clearer and less ambiguous.

I. INTRODUCTION

In 1939, Roy Harrod published his "Essay in Dynamic Theory," which he described as an extension and development of his 1936 book *The Trade Cycle*. In the former,

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Harrod's intention was not to work out a complete model, but to provide "a tentative and preliminary attempt to give the outline of a 'dynamic' theory" (Harrod 2003b, p. 1188; 1939, p. 14). However, like Harrod's subsequent contributions, this essay was instead seen by most of his readers as the model of steady growth that, together with Domar's contribution (Domar 1946), gave rise to modern growth theory.[1] The construction of a growth theory was, however, not Harrod's intention, which was instead to explain the trade cycle using what he considered a new tool of analysis. Harrod's failure to convey his message of the necessity of a 'mental revolution,' which had to change the approach to economic theory, has two aspects. On the one hand, it arose from the incapacity of some of his readers to understand exactly what he was saying. On the other hand, Harrod himself contributed to misleading his readers, since many of the features of his theory were unclear and were discussed only in private correspondence. As well, some characteristics of Harrod's theory were not easy to formalize with the analytical tools available at the time. As we will see, Harrod's correspondence with Marschak sheds some light on the latter aspect.

The correspondence with Marschak is not voluminous. Most of it concerns administrative affairs related to Oxford and Harrod's request to frequent the Oxford Institute of Statistics once a week to become familiar with its empirical investigation. The part of correspondence that deals specifically with the draft of the "Essay" consists of five letters. On 5 July 1938, Harrod sent Marschak a first letter announcing he had been working on the theory propounded in *The Trade Cycle,* stating he now had it "in [his] head in much better shape" (Harrod to Marschak, 5 July 1938, p. 798), but needed to discuss means of statistical verification. In the second letter was enclosed the draft, in which Harrod reiterated the need for empirical verification. Marschak then sent Harrod his comments, which will be discussed at length in this paper. Harrod answered with two short letters, which took up only some minor points while ignoring the most valuable of Marschak's suggestions.

Jacob Marschak was one of a handful of Harrod's contemporaries who understood the innovative features of Harrod's proposal for a new dynamic theory. At the beginning of August 1938, a first draft of the "Essay" was sent to Keynes and to Marschak, who afterwards discussed it with the author by correspondence.[2] The discussion with Keynes was protracted over several weeks and gave rise to many changes to the draft of the "Essay."[3] The exchange with Marschak was much briefer and, more significantly, fruitless, in the sense that most of his remarks were either not understood or ignored—or both—by Harrod. Furthermore, in the published version of the "Essay," some aspects that Marschak believed were equivocal and for which he required clarification became even more obscure, justifying partially (but only

[1]For a detailed explanation of the origins of this misinterpretation of Harrod's theory and the role of the different exegetic traditions in this connexion, see Kregel (1980), and Besomi (2001; 1998a).

[2]The draft was first edited by Daniele Besomi and published in 1996 (Harrod 1996) with an editor's introduction and a extremely useful appendix (Besomi 1996b). Later, the draft was included in Harrod (2003a), by the same editor. All the correspondence cited in this paper can be found in these volumes. Extensive excerpts are reproduced or paraphrased in Young (1989).

[3]This paper will not deal with this correspondence, which has been studied exhaustively in Kregel (1980), Besomi (1995), and Besomi (1996a).

partially) the aforementioned misinterpretation of Harrod's essay as one providing a model of growth.

It is difficult to understand Harrod's indifference to Marschak's remarks, since it was he who solicited them, and had he followed at least some of the suggestions, many aspects of the ''Essay'' would have been much clearer and better appreciated. As for the causes of Harrod's lack of reaction, we can only speculate.

The aim of this paper is, firstly, to show how Marschak's criticism pointed out some of the features of Harrod's theory that would later give rise to several misinterpretations, especially on three issues: the concept of warranted rate of growth; instability; and non-linearity. Secondly, it is to show that a possible explanation of what, on the surface, may seem to be Harrod's obstinate attachment to his original theory can be better understood by exploring his methodological approach. Harrod's methodology consisted in distinguishing three successive stages, which had different epistemic status, and, as will be explained in section VI, made the building of a complete model inappropriate at this stage where Harrod was only presenting a ''new tool of analysis.'' Marschak instead tried to make Harrod's theory fit in a complete model, which would have as solutions the variables involved as functions of time, given the initial conditions. In these two approaches, there seems to be a conflict that makes it difficult, if not impossible, to conciliate both.

I will first describe the circumstances in which the draft was written and sent to Marschak, and provide a brief summary of the parts of Harrod's theory necessary to bring the debate in context (section II). Next, I will discuss the main problems arising from the deduction of the fundamental equation (section III) and two important features of Harrod's theory, which, even though understood by Marschak, later gave rise to many misinterpretations: instability (section IV); and non-linearity (section V). In section VI, I will examine Harrod's views on methodology and the difference between his approach and Marschak's. Finally, in section VII, I will draw some conclusions.

II. THE CONTENT OF THE DRAFT

Jacob Marschak was a Russian-born econometrician, who, after a brief experience as Minister of Labour in the Cossack–Menshevik Republic of Terek, emigrated to Germany. There, he studied in the universities of Berlin and Heidelberg. In 1933 he moved to Oxford. At the time Harrod wrote his ''Essay,'' Marschak was the director of the Oxford Institute of Statistics, established in 1935. He participated also in the Oxford Economists' Research Group, put in place by Harrod in 1936 with the aim of conducting empirical investigations, particularly on the trade cycle.[4] Since the beginning of 1937, Harrod's correspondence with him reveals a considerable interest in the empirical aspect of research.[5] In fact, Harrod felt he had to gain experience in the statistical field, and in January 1937 he asked Marschak if he could join the

[4]For a detailed history of the establishment of the Oxford Institute of Statistics and the Oxford Economists' Research Group, see Young and Lee (1993), especially Chapter 5.

[5]In his correspondence with Keynes, Harrod said that he considered Marschak as a ''minor Tinbergen'' (Harrod to Keynes, 6 July 1938), in Harrod (2003a,, p. 802).

institute one morning a week to participate in some projects and thus become familiar with this area of research (Harrod to Marschak, 11 January 1937, p. 609).

Accordingly, Harrod's request to Marschak in the letter enclosing the draft of the ''Essay'' was very specific: he asked him ''if you could read through this and consider it from the point of view of verification'' (Harrod to Marschak, 6 August 1938, p. 811). When he talked about verification, Harrod meant getting an empirical base for the formulation of general laws and maxims. He thought that relying exclusively on the deductive method was pointless because the variety of possible mutual implications is infinite. The deductive method was useful only if applied to hypotheses coming from empirical generalizations (cf. Harrod 1938, p. 404). This is the status Harrod attributed to the three axioms from which he deduced his fundamental equation.

Marschak, who also had interests in methodology,[6] read Harrod's draft very carefully and appreciated many of the innovative aspects of Harrod's approach that would have been neglected by later readers. In his long commentary, Marschak expressed doubts, suggestions, and criticisms of different nature. These ranged from stylistic remarks to criticism pointing directly to the heart of the ''axiomatic basis of the theory'' (Harrod 2003b, p. 1188; 1939, p. 14), and are accompanied in most of the cases by suggested emendations.

Before entering into the details of Marschak's criticism, it is necessary to briefly examine Harrod's outline of the dynamic theory in the first draft of the ''Essay.'' Even if this may seem a repetition of an argument well known, in order to understand Marschak's remarks it is important not only to consider the ''fundamental equation'' but also to illustrate the method Harrod used to deduce it, since this is a crucial point from which many later misunderstandings arose.

In the draft, Harrod presented his theory as a ''marriage between the 'acceleration principle' and the 'multiplier' theory'' (Harrod 2003b, p. 1188; 1939, p. 14), based on three axioms:

(i) the level of a community's income is the most important determinant of its supply of saving;
(ii) the rate of the increase of its income is an important determinant of its demand for saving; and
(iii) demand is equal to supply.

Then he defined the geometric rate of growth $G=\frac{x_1-x_0}{x_0}$, where x is output. This is the actual rate of growth between time 0 and time 1, taking intervals short enough so that it is indifferent if one puts x_0 or x_1 in the denominator.[7] After defining G, Harrod defined the ''warranted rate of growth'' G_w, as the rate of growth ''which if it occurs

[6]Marschak (1941) discussed the methodological problems of ''causal vs. stochastic'' and ''induction vs. deduction.''. He also commented on statics and dynamics. In Marschak (1942), he dealt with the difference between identities and equilibrium conditions (this will be dealt with in section III of this paper), with the characteristics of equilibrium in static and dynamic models, and stability. In both papers he made reference to several examples taken from the economic literature, but he never mentioned Harrod.

[7]This means Harrod was taking intervals of infinitesimal duration, which is why the fundamental equation represents equilibrium *at one point in time*.

will leave all parties satisfied that they have produced the right amount."[8] The warranted rate of growth is, according to Harrod, a highly unstable moving equilibrium.

He next designated s as the fraction of income "individuals and corporate bodies choose to save," and C as the value of capital goods required for the production of a unit increment of output,[9] which depends on "technology and other fundamental conditions."[10] Both s and C vary in the different phases of the trade cycle.[11] The value of increment of capital per unit increment of output actually produced may differ from C, and Harrod called it C_p.

From the definition of C_p, and from the fact that demand is equal to supply, Harrod deduced a formula for G:

(1) $C_p = \frac{I}{x_1 - x_0} \Rightarrow I = C_p(x_1 - x_0)$
(2) $S = sx_0$

where I is investment and S total savings, and from $S = I$:

(3) $\frac{s}{C_p} = \frac{x_1 - x_0}{x_0} = G$

Harrod called $G = s/C_p$ a truism, because it was derived only from the definition of C_p and axiom (iii). At this point, Harrod proceeded to deduce the fundamental equation: if C_p coincides with C, the increase of output that actually occurs will be equal to the one "justified by the circumstances"; that is to say $G = G_w = s/C$.[12] Given *in each instant* the propensity to save s and the technological and other conditions represented by the coefficient C, and given the fact that, in equilibrium, demand is equal to supply, the rate of growth G_w remains determined as the only one compatible with all these fundamental conditions. If the actual increase in capital goods per increment unit of output is not the one "justified" by the "fundamental conditions," the actual rate of growth G will be different from G_w.

[8]Harrod (2003b, p. 1189). This definition was modified in the published version of the "Essay," to "the rate of growth which, if it occurs, will leave all parties satisfied that they produced neither more nor less than the right amount. Or, to state the matter otherwise, it will put them into a frame of mind which will cause them to give such orders as will maintain the same rate of growth." Harrod gave different definitions of G_w not necessarily equivalent to each other, which gave rise to problems in subsequent interpretations. This will be discussed in the next section.

[9]Also the definition of C gave also rise to many interpretative problems (section III).

[10]In the published version, on Keynes' suggestion, he included among the "other conditions" the state of confidence and the rate of interest.

[11]This was stated explicitly, even if some later interpreters took these coefficients as fixed (section V).

[12]In paragraphs 10 and 11 of the draft, and paragraphs 12, 13, and 14 of Harrod (1939), the fundamental equation was modified to reduce the influence of the accelerator, including long-run investments and foreign trade. The final equation is $G_w = \frac{s + i - k - K/x - E/x}{C}$, where i is the fraction of current income spent on imports, E is the value of exports in absolute terms (the value of exports is considered independent of the level of activity, but imports are a fraction of income), x is the output, and $kx + K$ the long-range capital outlay, constituting a part depending on current output and a part independent of it. This form of the fundamental equation will be relevant in the discussion of the turning point of the depression and for the recommendations of economic policy.

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In Harrod's view, the crucial point was that the dynamic equilibrium thus defined was highly unstable.[13] In fact, after having derived the fundamental equation, he studied the stability of equilibrium. According to Harrod, when there is a departure from the warranted rate of growth, this departure will be self-aggravating instead of being self-righting, contrary to what happens in the static field. If, for example, there is an excessive output, so that G exceeds G_w, then the actual increase in capital goods C_p will be less than C, due to an excessive depletion of stocks or shortage of equipment. Entrepreneurs will interpret this shortage of stocks as an insufficient investment (stocks are included in investment) and accordingly increase their orders for stocks or for new equipment, stimulating in this way expansion and departing further from equilibrium.[14]

After having discussed instability, Harrod affirmed that "the strict part of the theory ceases here,"[15] and proceeded to study how the forces that take the economy away from the moving equilibrium can be checked. In the long run, the actual rate of growth can neither increase in an explosive way nor fall uninterruptedly. The divergence of G from G_w is limited by the fact that growth itself triggers changes in C and s that, in the end, cause the direction of movement to reverse. These changes in the coefficients s and C make Harrod's model non-linear, and account for the turning points of the cycle. Without changes in the coefficients, the behavior of G would be explosive, and so the theory would not describe a cycle. This essential point was not taken into account by most of the commentators, who rather tried to formalize Harrod's theory in terms of linear equations.

Even if, as Marschak pointed out, the explanation of the turning points was not exhaustive, Harrod explained verbally how the coefficients are expected to change in the boom and in the slump, causing the tide to turn. In the case of the boom, the condition to get a reversal of the movement was that G_w overcame G. This happened because the actual rate of growth G is limited by the productive resources of the system, while G_w is not. Harrod called the natural rate of growth (G_n) the maximum rate of growth attainable, given the state of technology and population. When $G > G_w$, income is increasing, producing changes in C and s that cause, in turn, an increase in G_w. As G is limited by G_n, while G_w is not, the increasing in income will last until G_w overcomes G, initiating a cumulative process in the opposite direction.

In the case of a slump, the condition for recovery is that the warranted rate G_w must fall below the actual rate G. According to Harrod, there are forces that ensure this point will eventually be reached. First of all, the long-range capital outlay K and exports E (both independent from current income) serve as cushions mitigating the depression. Secondly, the fraction of income saved s will fall as income falls, contributing to the fall in G_w. Thirdly, changes in C also contribute to the reduction of the warranted rate. When describing the behavior of C in the descending phase,

[13]This was later mitigated, and when the debate on the "knife-edge" took place, Harrod proposed to replace the metaphor with an analogy to a "shallow dome" or to "a ball rolling on a grassy slope" (Besomi, 1998a, p. 58–59, and esp. note 24). In a reply to an article written by Joan Robinson (1970), he even said that "I hope we shall hear no more of the 'Harrod knife-edge' " (Harrod 1970, p. 741).

[14]This assumption regarding entrepreneurs' behavior later provoked several criticisms, and will be discussed in section IV.

[15]This sentence was omitted in the published version.

Harrod began a foggy discussion about the meaning of C when the rate of growth is negative. The coefficient C is always positive, meaning the required accumulation when growth rate is positive and the necessary decumulation when it is negative, while C_p can have a negative value. The negative value of C_p means there is an accumulation of capital when a decumulation is required. To arrest the downward pressure, the value of C_p must rise above the value of C. If we think of C as being constituted of circulating and fixed capital, and if we assume for simplicity that fixed capital cannot be liquidated, in the slump C (the required decumulation) will be abnormally depressed. This will help C_p to rise above C[16], and so begin an ascending phase.

III. MARSCHAK'S COMMENTS ON THE FUNDAMENTAL EQUATION

The description in the previous section of the way Harrod deduced his fundamental equation is useful to underline the fact that the proof was not made through a direct deduction from the three initial axioms, but by establishing a comparison with the equation for the actual rate G. In fact, Harrod claimed that $G = s/C_p$ was a truism, and that if C_p had that particular value C that left all parties satisfied,[17] then G would coincide with G_w. This methodology generated misunderstandings for reasons we will examine in this section. But the problem was not only the method: the very definitions of the variables were unclear, and sometimes, as we will see, even contradictory.

Regarding method, Marschak pointed out some aspects that are not clear when G_w is defined in the way Harrod did. The first is that equations $G = s/C_p$ and $G_w = s/C$ do not stand on the same foot, because the former is a definition and the latter an equilibrium condition. In fact, Marschak argued that this was an imprecision, and in a footnote in his letter to Harrod, he said that "I felt it difficult to understand . . . until it became clear to me that the equation $C_p = s/G$ was a definition of C_p while the equation $C = s/G_w$ is the equilibrium equation for saving, viz. the equality of demand and supply. I wonder whether you agree" (Marschak to Harrod, August or early September 1938, p. 846). In a later work (Marschak 1942, p. 62), Marschak explained that the distinction was not always clear to economists, who very often did not use the identities they proclaimed and tried instead to prove stability conditions. An identity is something that, when not satisfied, makes the system logically inconsistent. An equilibrium condition is a condition that, if not satisfied, makes the system not persistent in time, but not necessarily logically inconsistent. To make this point clearer, Marschak suggested that Harrod should deduce the fundamental equation directly from the three axioms.

The second aspect that, according to Marschak, needed clarification was that G_w is not a particular value of G, but is another variable that interacts with G to originate the cyclical movements. The above definition of G_w made it look like the particular value of G corresponding to the case $C = C_p$. Although Marschak did not point this

[16]This explanation is not clear, and, in fact, Marschak will ask Harrod to make explicit his assumptions about the behaviour of C.

[17]According to Harrod, entrepreneurs are satisfied when there is not undesired variation of stocks.

out when discussing the deduction of the fundamental equation, he did it later. In fact, when Harrod explained his use of the expression "stimulus to expansion," he used a redaction style, which led to the same misunderstanding.[18] At that point Marschak observed that "the reader must remain clear that G and G_w are two variables (which may or may not have equal values), so that G_w is not a particular value of G."[19]

To make things clear, Marschak proposed a different way of deducing the fundamental equation. He suggested adding the subscript w to the equations associated with the initial axioms, so that it remained clear that the coefficients C and s were seen in *ex-ante* terms.[20] If the fundamental equation was deduced directly from the three axioms established at the outset, which were thought of in *ex-ante* terms, Harrod's three axioms would clearly perform their role as axioms. Marschak translated those three axioms into equations: the first to obtain a supply function for savings; the second to obtain a demand equation for savings; and the third, the equilibrium condition. A system of equations was obtained, which had as solutions the level and rate of growth of output as functions of time. He proposed the following equations:

$(\alpha)\ \ s_w x_w = f(x_w)$ (Supply function of *ex-ante* savings)

$(\beta)\ \ s_w x_w = \varphi\left(\frac{dx_w}{dt}\right)$ (Demand function of *ex-ante* savings)

$(\gamma)\ \ f(x_w) = \varphi\left(\frac{dx_w}{dt}\right)$ (Equilibrium equation)

where the subscript w stands for warranted, s is the propensity to save, and x the output. If the supply and demand equations remained constant, a definite path for G_w could be obtained from the three axioms. Later, Marschak completed this scheme with two more equations whose interaction with the warranted ones had to endogenously generate the cycle.

A problem closely related to Harrod's method of obtaining the fundamental equation was his unusual use of the *ex-ante* and *ex-post* concepts. In the first place, the very use of the term *ex-ante* was different from the usual, meaning the equilibrium values of the variables and not the planned decisions of entrepreneurs and consumers.[21] Secondly, the use of these terms seems to necessarily imply a lag,

[18]In fact, Harrod wrote, "If G is equal to warranted G, (G_w), it is to be supposed that the general level of enterprise undertaken ... is found to be satisfactory." That was the expression that, according to Marschak, could lead the reader to think that G_w was a particular value of G (Harrod 2003b, p. 1195). In the published version of the "Essay," he directly wrote, "If G is equal to G_w," (Harrod 1939, p. 23).

[19]Marschak to Harrod, August or early September 1938, in Harrod (2003a, p. 848). Harrod corrected the redaction of that particular sentence, but the main problem persisted because of the way the fundamental equation was deduced.

[20]Harrod did not follow this advice.

[21]Keynes pointed this out in a letter he sent to Harrod with his first comments on the draft. According to Keynes, C was not *ex-ante* if one used the standard definition of the term: C *ex-ante* would be the addition of capital entrepreneurs actually plan to make, but Harrod's C was the addition of capital entrepreneurs ought to plan to make to equate *ex-ante* and *ex-post* investment. Keynes suggested that Harrod call C the "warranted investment." Harrod changed the phrasing to make clear what his use of the expression *ex-ante* was, but he did continue to use the expression *ex-ante* with his own meaning. He further clarified what he meant in Harrod (1951, pp. 268–271). There he explained that for him, *ex-ante* investment is not planned investment, which may not be realized, but it is related to the actual out-turn. Intended investment will not happen, unless it leads to a growth in output that coincides with the warranted rate.

while Harrod presented his warranted rate as instantaneously determined. Harrod recognized the problem, admitting that ''if there were no lag there would always be an immediate adjustment to the new warranted position'' (Harrod 2003b, p. 1202), but he said that ''it is only in the formulation of the fundamental equation in its different forms for determining the warranted rate of growth that consideration of lags is rigidly excluded'' (Harrod 2003b, p. 1203). This is because if *ex-ante* meant the planned decisions, lags would have been necessary to explain why adjustment was not instantaneous. But since he used *ex-ante* to designate the equilibrium values of the variables, and the fundamental equation refers to equilibrium at *one instant*, lags were not necessary to deduce the equilibrium condition.

The role of the propensity to save *s* also led to misunderstandings, since Harrod thought of it indistinctly as *ex-ante* and *ex-post* saving. He justified this by saying he saw no reason why they should differ, and in the case they did, the argument remained valid, since the effect on the rate of growth was the same as the effect of a difference between C_p and C. This was underlined by McCord Wright, who claimed that ''Harrod is here confusing ex post and ex ante, and should also state whether he is dealing with 'full' or less than 'full' employment. Once we allow G and C to be determined ex ante and treat s as an ex post result, his relation need not follow.''[22] According to McCord Wright, Harrod is here failing to differentiate mathematical manipulation with real proof. If G and C were *ex-ante* and s was *ex-post*, then it could happen that in the case of an unwarranted increase in investment when there was unemployment, *s ex-post* rose to equal that unwarranted investment. According to McCord Wright, Harrod's claim is valid only if the propensity to consume is the long-run full employment one, but not in an expansion from a situation of unemployment. Alexander instead thought of this equality between *ex-ante* and *ex-post* saving as a substitute for axiom (iii) (Alexander 1950, p. 727).

The problem of the meaning of *ex-ante* and *ex-post* values also emerged from the definition of C. In the draft of the ''Essay,'' Harrod himself had expressed a doubt about this: ''C_p is an ex-post quantity, I am not clear if C should be regarded as its corresponding ex-ante'' (Harrod 2003b, p. 1192; 1939, p. 19). As a matter of fact, the definition of C in the draft was not clear. The first definition of C was ''the value of capital goods required for the production of a unit increment of output'' (Harrod 2003b, p. 1190; 1939, p. 16), which made C look like a purely technical coefficient. Then, he added that C ''may be somewhat dependent on the rate of interest'' (Harrod 2003b, p. 1190; 1939, p. 17). Later, Harrod wrote that C was ''the amount of capital required by technological and other conditions per unit increase of output'' (Harrod 2003b, p. 1191). These ''other conditions'' would be specified in the published version of the ''Essay,'' including (on Keynes' suggestion) ''the state of confidence, the rate of interest, etc'' (Harrod 1939, p. 18). Later on, Harrod asserted that ''C is rather that quantity of capital goods, which, if producers foresaw the total development during the period, they would produce, or that which, if they do produce it, makes them feel satisfied that they have neither exceeded nor fallen short of the mark'' (Harrod 2003b, p. 1192) The concept of C was then modified in the ''Essay''

[22]McCord Wright (1949, p. 325). In reality it is not G and C that were determined *ex-ante* for Harrod, but it was C and s (the fundamental conditions), and from them he derived G_w, which was determined by them.

to that ''which producers regard as ideally suited to the output which they are undertaking in that period,''[23] and also the increment of capital goods ''which is desired'' (Harrod 2003b, p. 1195; 1939, p. 22). Marschak thought C ought to represent ''expected technological conditions and nothing else.''[24]

The difference between all these definitions is more problematic than it may seem, because even if it is true that no matter which one of the above definitions is accepted, the fundamental equation remains always an equilibrium condition, the meaning of this equilibrium is quite different. If coefficient C is thought of as depending only on technological conditions, the fundamental equation would be an equilibrium condition, understanding equilibrium in the sense of reproduction of the system. The physical possibility of reproduction of the system has nothing to do with the satisfaction of the entrepreneurs or with their behavior. The problem arises because other definitions of C are related to entrepreneurs' satisfaction, as one of the definitions of C is the increment of capital goods that does not produce undesired variations of stocks (this is what Harrod meant for satisfaction of the entrepreneurs). Considering this definition of C, the fundamental equation is also an equilibrium condition, but equilibrium is now interpreted as a state in which individuals are satisfied, and this does not need to be compatible with the possibility of reproduction of the system.

All the matters raised by the several definitions of the coefficient C have an analogous counterpart in the several definitions of G_w. Even if Marschak did not explicitly comment on the inconsistency of the different definitions,[25] he realized that additional assumptions had to be made to make everything fit into a coherent system. As will be discussed in the next section, one of these assumptions was about the entrepreneurs' behavior.

As a matter of fact, a difference of G from G_w is formally equivalent to a difference of C_p from C. Harrod provided several definitions of G_w[26]—the warranted rate of growth is that which is ''determined by certain 'fundamental conditions', namely the propensity to save, the state of technology etc.'' (Harrod 2003b, p. 1191; 1939, p. 17) and that which ''if it occurs will leave all parties satisfied that they have produced the right amount'' (Harrod 2003b, p. 1189). In a subsequent statement, he also claimed that ''of course what applies to the system in general may not apply to each individual separately. But if one feels he has over-produced or over-ordered, this will be counterbalanced by an opposite experience of an equal importance in some other part of the field'' (Harrod 2003b, p. 1194; 1939, p. 22). In the published version he added a clarifying paragraph, defining G_w as that rate of growth that ''if it occurs, will leave all parties satisfied that they have produced neither more nor less than the right amount. Or, to state the matter otherwise, it will put them into a frame of mind which

[23]Harrod (1939, p. 19), modified on Keynes' suggestion.

[24]Marschak to Harrod, August or early September 1938, in Harrod (2003, p. 846). Keynes commented on this, saying that C is not *ex-ante,* strictly speaking. C *ex-ante* would be, according to Keynes, the investment entrepreneurs actually plan to make, while C is the investment they ought to plan to make (Keynes to Harrod, 17 August 1938, in Harrod (2003, p.828).

[25]He said the definition of the moving equilibrium G_w was not clear to him, as mentioned above, but he did not refer explicitly to an inconsistency.

[26]This was noted by McCord Wright, who recorded six different definitions of G_w (McCord Wright 1949, p. 326).

will cause them to give such orders as will cause them to maintain the same rate of growth'' (Harrod 1939, p. 16). Of course, these propositions are not equivalent: even if the producers are satisfied, this does not imply they will desire to maintain the same rate of growth. At this stage, Marschak said the definition of the moving equilibrium G_w was not clear to him, because he realized there were two different definitions of G_w. The warranted rate had to satisfy the conditions ''that the rate 'will leave all parties satisfied' which is alternatively alluded to as 'certain fundamental conditions namely the propensity to save etc.''' (Marschak to Harrod, August or early September 1938, p. 845).

The inconsistency has been pointed out by Alexander and Schelling. Alexander thought there was a fourth axiom implicit in Harrod's theory, which he called axiom 3b), which was that ''If investment *ex-post* is justified in any period, entrepreneurs will (unless prevented by physical limitations) in the succeeding period increase production in the same proportion as it has just been increased. If in any period investment *ex post* is less than the justified investment, entrepreneurs will in the next period increase the rate of growth of production, and vice versa'' (Alexander 1950, p. 728).

Schelling also thought that one of Harrod's errors was to consider as synonymous the two propositions that ''on the one hand, they feel that they have done the right thing; on the other hand, they are induced to 'continue in the same line of advance'''' (Schelling 1947, p. 868).

Moreover, Schelling identified another problem: the link between the warranted rate of growth and the individual firms. In his words,

> We must relate, somehow, this warranted rate of growth to the individual firms which make up the economy.... Some industries advance, others decline. ... Harrod recognizes this. He speaks of the growth as being composed of the combined actions of many individuals, some disappointed, some elated, 'on balance' satisfied.
>
> When this is recognized, however, what happens to the continuity of the process? The same firms do not do the investing each year. Even firms which expand at roughly the same rate purchase capital in lumps other than regular annual increments. The investor who must this year be induced to invest is not the same one who is satisfied with last year's investment.... satisfaction is not transmitted with high fidelity throughout the economy when many firms compose it (Schelling 1947, p. 868–869).

The underlying problem was the relation between the micro and the macro levels, or, to state the matter otherwise, between two concepts of equilibrium that do not need to be compatible with each other. On the one side, there is equilibrium characterized as a condition for the reproduction of the system, given the technological conditions and the propensity to save. This is the macroeconomic aspect, which, in Harrod's theory, corresponds to the interpretation of G_w as the rate of growth that equals savings to *ex-ante* investment. On the other side, there is the condition that entrepreneurs are satisfied they have produced the right amount; that is, they did not suffer undesired variations of stocks. This means that at the microeconomic level, it was assumed that entrepreneurs behave as wanting to maximize profit, and, once they reached

equilibrium, kept their line of conduct. Moreover, keeping this line of conduct leaves them satisfied. This notion of equilibrium corresponds to the interpretation of G_w as the rate of growth that leaves all parties satisfied. The relation between the macroeconomic and the microeconomic levels was neglected in the "Essay," and so nothing ensures these notions of equilibrium are consistent which each other.[27] In *The Trade Cycle*, on the contrary, these two levels were connected by the price level, which transmitted the information from the macro level to the micro level:

> The total amount of output and consumable goods is the aggregate of those undertaken, by a large number of different individuals, each with no direct private interest in the equation of new capital construction to the volume of saving in the community in general. The action of each is the result of the impact of the general situation on him personally. The fluctuation of prices measures the resistance of individuals to change. It provides the force necessary to make them change their level of output and so behave that their resultant output is what is required to make capital construction equal to saving (Harrod 1936, p. 171).

However, the question of prices was not taken into consideration in the "Essay," thus leaving these two notions of equilibrium isolated from each other, and so not necessarily compatible. There was no mechanism playing the role prices had in *The Trade Cycle*. This was rightly pointed out by Alexander, who said that as G_w is entirely dependent on the behavior of entrepreneurs, there is no reason to suppose that it exists and is equal to "the ratio of two terms, one of which is a characteristic of the behaviour of savers, and the other is a technological characteristic of production" (Alexander 1950, p. 725). Also Bodenhorn (1956, pp. 608–609) noted that the coefficient C could be interpreted as a response coefficient or as a technical capital coefficient, but for Harrod's model to make sense, one must be able to interpret it in both ways. That is, Harrod's model only makes sense if the response coefficient equals the technical capital coefficient.

IV. INSTABILITY

In Harrod's view, the crucial feature of dynamic equilibrium, which distinguished it from static equilibrium, was instability. Harrod described how, when a difference between G and G_w arises, it tends to grow larger. For instance, the actual rate G may depart from G_w because of an excessive output or a rise in the propensity to save s, causing C_p to fall below C. In any case, the immediate effect will be an undesired variation of stocks, which induces the entrepreneurs to try to correct it. But in doing so, they will be amplifying the divergence between G and G_w.

The instability principle was postulated in section 9 of the draft, and thus it remained included in the "strict part of the theory," which consisted of the description of the system at one instant, that is, establishing the relations between the

[27]McCord Wright pointed out that "steady advance in reality requires only that the rate of expansion of growing firms and regions happens to offset the rate of decline of those contracting. Nobody need be satisfied" (McCord Wright 1949, p. 326).

different variables of the system *at a given point in time*. Together with the three axioms postulated at the beginning, the instability principle was, for Harrod, a premise necessary to get an explanation of the cycle independent of lags, frictions, or systematic waves of optimism and pessimism (cf. also Nevile and Kriesler 2008). Once the existence of an equilibrium rate of growth was postulated, it *had* to be unstable. Otherwise, the cycle wouldn't be generated, as Harrod thought, by the process of growth itself. After any deviation, the economy would return to the equilibrium rate, and thus one would have only momentary deviations from the equilibrium rate of growth. That is why Harrod discussed the instability principle in the first part of the draft, which refers to the strict part of the theory (par. 1–11). He thought the "instability of dynamic equilibrium . . . was 'logically prior' to the second part, the analysis of the cyclical fluctuations of the economy in reference to a stable rate of expansion" (Kregel 1980, p. 117).

As the following quotation shows, the instability was postulated beforehand and can be considered as one of those empirical generalizations discussed in the first section of this work. In fact, this was the way Harrod himself summarized his theory in section 9 of the "Essay":

> The dynamic theory so far stated may be summed up in two propositions. (i) A unique warranted line of growth is determined jointly by the propensity to save and the quantity of capital required by technological and other considerations per unit increment of total output. Only if producers keep to this line they will find that on balance their production in each period has been neither excessive nor deficient. (ii) On either side of this line there is a 'field' in which centrifugal forces operate, the magnitude of which varies directly as the distance of any point in it from the warranted line. The moving equilibrium of advance is thus a highly unstable one (Harrod 2003b, pp. 1195–1196; 1939, p. 23).

The instability principle was one of the most problematic aspects of the "Essay." One reason is that its status was not clear. In the draft Harrod treated the instability principle as a premise (par. 9), but then, in the published version of the "Essay," in response to Keynes' criticism, Harrod tried to prove it. He found a condition the coefficients had to satisfy if the instability principle was to remain valid. Since instability was postulated at the outset, the system obviously had to be unstable as a result. But this generated an ambiguity about the status of the instability principle. As well, his demonstration was not correct, generating a debate about what was the most general set of premises from which one could get the instability result.[28]

Another source of confusion was that Harrod discussed instability in a scheme referring to an instant, while the analysis of stability requires instead the comparison of different points in time. This means that Harrod should have discussed it in the second part of the draft, the one dealing with the "succession of events." When Harrod discussed disequilibrium situations, as Bodenhorn pointed out, he was "no longer assuming proposition (iii), 'that demand is equal to supply' . . ., nor is he assuming proposition (ii), 'that the rate of increase of . . . income is an important

[28]For example, Alexander (1950), Jorgenson (1960), and Nevile (1962).

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determinant of [the] demand for saving. . . ' " (Bodenhorn 1956, p. 610). However, Harrod did not formalize this.

Marschak was aware that stability analysis had to involve different points in time, and, therefore, proposed to use a differential equation formalizing the cumulative process of divergence from equilibrium (as showed below in this section). He also commented on the connection between the instability principle and the entrepreneurs' behavior, as he understood that it was not derived from the axioms, but involved an additional postulate.

The additional postulate about entrepreneurs' behavior was, in Marschak's words, "that the divergence $G-G_w$ induces the individuals to certain actions" (Marschak to Harrod, August or early September 1938, p. 846). In fact, the entrepreneurs were supposed to behave as wanting to maintain a certain value of C, outside which they would interpret to be a situation of undesired accumulation (or decumulation) of stocks, and then reduce (or increase) their orders. Marschak rightly pointed out that one could also assume the opposite behavior:

> Am I right in understanding this as an empirical assumption of the boom (and depression) rather than a conclusion following from the other equations. In fact, one could imagine the opposite case: a shortage of capital equipment being met by a reduction of output rather than leading to an increase. It seems that the definition of 'unstable equilibrium' implies certain empirical postulates regarding the psychology of entrepreneurs' reactions to current profits made in capital good production. Could this be stated explicitly? (Marschak to Harrod, August or early September 1938, pp. 846–847)

and again: "some sort of reaction of entrepreneur's expectations to their realizations is probably assumed: (e.g. a change of expected technological productivity?)."[29]

But Harrod did not admit[30] that his cumulative process depended on some specific behavior of entrepreneurs. About this crucial matter, Harrod just wrote that

> you ask the leading question whether my theory that there are centrifugal forces on each side of the equilibrium (warranted rate) is based on some empirical view as to entrepreneurs' probable reactions. I do not think so. On the other hand there must be some empirical basis for my theory. This may be summarized in 2 propositions. 1. The volume of saving supplied depends mainly on the size of income. 2. A considerable part of the demand for saving depends on the rate of growth. There, I think, empiricism ends and deduction begins (Harrod to Marschak, 7 September 1938).

Yet, it is from the particular behavior of entrepreneurs that the cumulative process was derived, and for which Marschak suggested a mathematical formalization in the form of a differential equation. The equation represented the cumulative divergence

[29]Marschak to Harrod, August or early September 1938, p. 847. Also, Alexander pointed out that this behavior was not the only one possible, and that "it is an important characteristic of Mr. Harrod's model that he assumes this is so" (Alexander 1950, p. 728).
[30]He did it later, in Harrod (1951), Section IV.

as $dG/dt = \lambda(G-G_w)$, where λ was some increasing function. Later, Marschak himself would modify the equation to include another aspect of Harrod's explanation, that of the different speeds of G and G_w, which originated the necessity of including further non-linearities in the model.

The matter of entrepreneurs' behavior was one of the aspects that had the utmost importance in later interpretations, which tried to formalize the cumulative process triggered by it using two different strategies: the introduction of behavioral equations; and the introduction of expectations equations. In fact, it is true that the connection between different states of the system was missing.

Alexander argued that Harrod was assuming a specific behavior of entrepreneurs. For him, the statement that "if $C = C_p$ investment was justified may be regarded as a definition of the term justified." (Alexander 1950, p.727). But nothing guarantees that $C = C_p$ implies $G = G_w$ (seeing G_w as the rate that will make entrepreneurs continue with the same line of advance). This implication depended on the behavioral pattern of the entrepreneur. This criticism is similar to Bodenhorn's, described in the previous section: the behavioral pattern would have to be one that made equal the technical and response coefficients.

In an article he wrote in 1951, Harrod admitted he was making a postulate about entrepreneurs' behavior, and he qualified it as an "as reasonable a postulate as any other in regard to the behaviour of an entrepreneur" (Harrod 1951, pp. 271–272).

Baumol (1948) pointed out that up to the point of Harrod's argument where the fundamental equation was deduced, everything followed from the axioms set at the outset, but when it came to the study of instability, Harrod implicitly introduced additional assumptions. Baumol remarked that instability was not incompatible with the initial axioms but did not follow from them, since it could happen that, for example, entrepreneurs thought that the undesired variation of stocks they saw was only temporary, and so decide not to adjust orders.[31] He formalized Harrod's model using expectations functions, as did Nevile (1962), for whom the minimum assumptions required to get Harrod's results included an assumption about entrepreneurs' expectations.

Hawtrey instead agreed with Harrod's formulation of the instability principle, since "'time lags' do not adequately express what is involved. A dynamic theory must take into account the respective rates of progress of the various tendencies set up by a state of disequilibrium" (Hawtrey 1939, p. 471), and he compared Harrod's principle of instability with his own principle of the inherent instability of credit.

V. NON-LINEARITY

After having discussed the stability of the moving equilibrium, Harrod inquired in paragraphs 12 to 21 of the draft how the forces that bring the system away from equilibrium could be checked. While in the previous part of the article Harrod confined himself to stage 1 of the analysis, where the coefficients s and C were fixed, in this part he studied how they vary to cause the turning points of the cycle. This part

[31] Or, as Hawtrey pointed out, it could happen that the banking system did not provide to entrepreneurs the required capital to do this (Hawtrey 1939, p. 473).

was substantially shrunk in the published version of the ''Essay'' where, immediately after ''the strict part of the theory,'' Harrod affirmed that ''the fundamental dynamic equation has been used to demonstrate[32] the inherent tendency of the system to instability. Space forbids an application of this method of analysis to the successive phases of the trade cycle'' (Harrod 1939, p. 28). Harrod only made a few observations about the interpretation of C and the negative rates of growth ''for the convenience of the reader who may be tempted to experiment with this tool'' (Harrod 1939, p. 29). Thus, all the central part of the draft, which dealt with the different phases of the cycle, remained in the published version as an exercise for the reader. This decision was taken to satisfy Keynes' editorial preference for ''something short'' (Keynes to Harrod, 19 September 1938, p. 867), but it had serious consequences for the subsequent interpretations of the ''Essay.''[33]

In Harrod's set-up, characterized by the cumulative amplification of a difference between G and G_w, in order to obtain a cyclical movement it was necessary to devise a mechanism enabling the warranted rate to overtake the actual rate so that the movement could be reversed instead of degenerating into explosive growth. Harrod did so by allowing changes in the coefficients s and C induced by the very process of growth they were governing. In fact, Harrod's point was that ''the trend of growth may itself generate forces making for oscillation'' (Harrod 2003b, p. 1188). This fact, that non-linearities are a premise for endogeneity, is essential; ''This is, I think, one of the most interesting points in the Essay'' (Marschak to Harrod, August or early September 1938, p. 847), asserted Marschak. But it was ignored in most of the subsequent interpretations.

The best explanation of the variations of the coefficients can be found in *The Trade Cycle*. There, Harrod explained accurately how the propensity to save s increased with income for two reasons: because people tend to save a larger proportion of a larger income[34]; and because in the ascending phase of the cycle, profits increase, and since entrepreneurs save more than workers, this results in an increased propensity to save by the whole community. This last process is what Harrod called ''shift to profit.'' The coefficient C also varied in the different phases of the cycle. In the slump, the interest rate falls, and so capital is a relatively cheaper factor of production. Moreover, the prices of capital goods relative to those of consumable goods fall, and this reinforces the effect of the fall of the rate of interest. Moreover, the inventions and improvements in the productive methods also have an important role in the determination of C, even if it is difficult to say if these inventions are more likely to occur in the boom or in the slump.

In the ''Essay'' it was also clear that coefficients s and C vary during the different phases of the cycle, even if the explanation of their specific behavior was not worked out in full detail. Marschak thought that a formulation of their specific behavior was

[32]Really it is not a correct demonstration, as explained in section IV.

[33]The shrinking of the part dealing with the successive phases of the cycle led to putting the emphasis on the growth aspect. For an exhaustive discussion of Harrod's subsequent interpretations as providing a growth theory, see Besomi (2001).

[34]He even attached a ''Note on Saving'' to chapter 2, where he examined the different motives that induced people to save and concluded that the assumption that people save a larger proportion of a larger income was correct (Harrod 1936, pp.106–109).

necessary to obtain a periodic behavior of G_w: "I presume that you postulate a change in the form of these functions themselves [supply and demand for saving]; in fact, you did discuss changes in the parameter C; could the postulates as to the behaviour of C (viz. The reactions to changes in G) be stated explicitly?" (Marschak to Harrod, August or early September 1938, p. 847).

The explanation of the behavior of C is essential to let G_w reach G and thus allow for the cycle to be endogenously generated, which was what Harrod was trying to obtain. To get this, as Harrod wrote in the draft,

> it is supposed that when the actual rate has for some time been displaced from the warranted rate, the warranted rate itself may[35] chase the actual rate upwards or downwards. If the former eventually overtakes the latter a new equilibrium is achieved and if the former goes beyond the latter forces are generated setting up a reverse movement (Harrod 2003b, pp. 1198–1199).

Marschak appreciated the fact that, if Harrod could provide an accurate explanation of the variations of C and s—and their interaction—which produced variations in G_w, he could get an endogenous cycle, which was, in fact, Harrod's aim. Marschak was aware that Harrod was not trying to derive periodicity directly from the demand-supply scheme of the first part of the draft, which yielded only explosive solutions. He maintained that additional postulates were made to explain the cycle.

According to Marschak, Harrod implicitly resorted to two additional postulates when discussing the turning points of the cycle. These postulates regard the relative speeds of the rates G and G_w. To get the reversal of the movement, it was necessary that G_w grew faster than G in the ascending phase, and that it fell faster than G in the descending phase. In the case of the boom, an additional postulate is made when the natural rate of growth G_n is introduced. This is "the maximum rate allowed by the increase of population, accumulation of capital, technological improvement and the work/leisure preference schedule, supposing there is always full employment in some sense."[36] G could only be above the natural rate while there was unemployment, but once this was absorbed, it had to return to the natural rate. Harrod had explained that in the case of $G > G_w$, a progressive growth of the difference $G-G_w$ sets in, and the reverse movement would begin only when G_w rose above G. If these variations of G_w were triggered by the process of growth itself, then we would obtain an endogenous cycle. But to achieve this, an additional postulate was needed: "if it can be postulated that G_w continues to rise after intersecting the ceiling (some sort of 'inertia'), the turning of G does follow from the postulate of cumulative process.... This is, I think, one of the most interesting points in the Essay..." (Marschak to Harrod, August or early September 1938, p. 847).

The other additional postulate Marschak mentioned was related to the slump. In the draft there was an entire section, which disappeared in the published version, dealing with the slump and negative rates of growth. Marschak pointed out that

[35]This provoked a comment from Marschak: "'can'. But will it?" (Marschak to Harrod, August or early September 1938, p. 849).

[36]Harrod (2003b, pp. 1197–1198; 1939, p.30). Marschak mistook this for the ceiling, which could be higher than G_n if there was unemployment (but only until unemployment was absorbed).

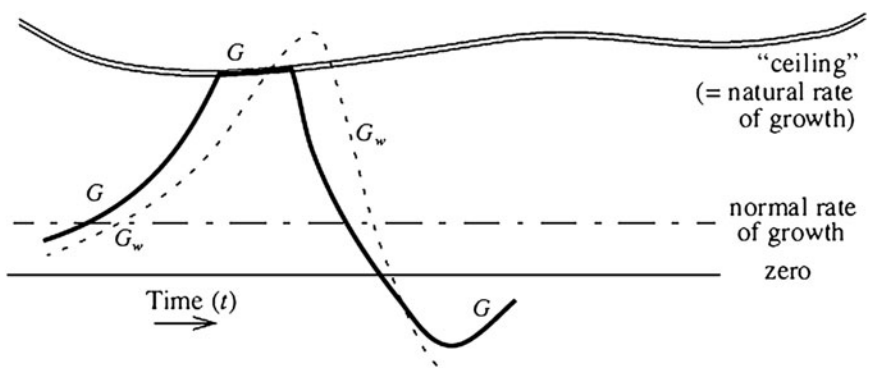
> It is proved in the Essay that the two rates [Marschak refers to G and G_w] do get below zero. This is necessary but not sufficient to produce an intersection between the two curves below the zero-line. It would be necessary to show (theoretically or on empirical grounds) that while G rises faster than G_w in the upswing, it falls slower than G_w in the downswing (Marschak to Harrod, August or early September 1938, p. 847).

The following diagram made by Marschak illustrates these different phases of the cycle.[37]

To formalize this, Marschak suggested a modification to the equation he had previously proposed to explain variations of G : $d(G-G_w)/dt = \lambda(G - G_w)$, so the difference $G-G_w$ grows progressively when $G > G_w$ and decreases when $G < G_w$.

Harrod did not answer to Marschak's remark about the speeds of G and G_w. In his reply to Marschak, he made two graphical examples involving the four rates of growth (actual rate G, warranted rate G_w, normal warranted rate,[38] and natural rate G_n), where he showed the behavior of the rate of growth with respect to the normal G_w.[39]

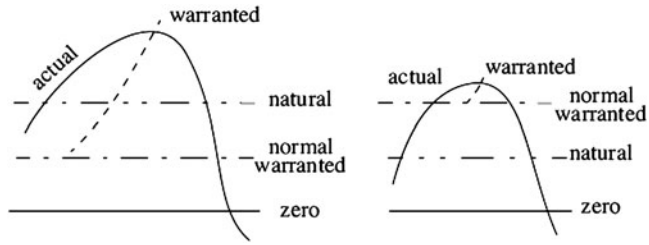
One of the examples was the case in which normal $G_w > G_n$ and the other example the case in which normal $G_w < G_n$. In both cases G_w overcame G, but the explanation of why this happened was still missing. Yet, Marschak considered it fundamental to the endogenous generation of a cyclical behavior. Harrod did not even attempt to answer the question why it was supposed that, in the ascending phase of the cycle, G_w overcame G, and in the descending phase, G_w fell below G. He just said that the increment of G_w "may be presumed to be due to inflation," "might be due to a shock to confidence," "it might be because, savings having risen back to a normal proportion, there is still an excess of unused capacity" and, in the case of the descending phase, he repeated what he already said in the draft, that "in the slump the



[37]The diagram is taken from Marschak to Harrod, August or early September 1938, in Harrod (2003a, p. 845).

[38]The normal rate is the rate "which would obtain if only the system advanced steadily along its warranted rate" (Harrod 2003b, p. 1198). It was eliminated in the published version of the "Essay," where the proper rate was introduced, which was the warranted rate corresponding to full employment (Harrod 1939, p.30).

[39]The diagram is taken from Harrod to Marschak, 7 September 1938, in Harrod (2003a, p. 860).



warranted is abnormally depressed owing to low saving and losses'' (Harrod to Marschak, 7 September 1938, p. 860).

The fourth equation proposed by Marschak was described only verbally and no explicit formula was propounded. It was an equation describing the changes in C during the different phases of the cycle as a function of G or $G-G_w$, making for a periodic shape of the G_w curve. When trying to introduce non-linearity in the form of a specific description of the behavior of C, Marschak put his finger on one of the issues that would have been misunderstood in later interpretations. In fact, the neoclassical tradition incorrectly attributed to Harrod the assumption of a fixed capital/output ratio, and the Post-Keynesian tradition (also incorrectly) attributed to him the assumption of rigidity of the propensity to save and the distribution of income.[40]

Hicks and Samuelson were among the few who recognized that non-linearity was essential in Harrod's approach. Samuleson admitted it was necessary to relax the assumption of linearity of both s and C to get the interaction of the multiplier and the accelerator to generate Harrod's behavior (Samuelson 1939). Even so, Harrod thought Samuelson's approach to be far from his, because he considered that the cause of the cycle was related to what he later called the '' 'basic antinomy''' (Harrod 1951, p. 262):

- The amount of saving depends primarily on the level of real income.
- The amount of saving required depends primarily on the rate of increase of income.

In Samuelson's model, instead, the instability result depended on the values of the coefficients, so Harrod thought the role of the ''basic antinomy'' was not clear (Harrod 1951). Hicks (1950) obtained non-linearity by bringing in a floor and a ceiling, but Harrod dismissed this approach because the turning points of the system had to be generated by the system itself, and not by extraneous constraints. In fact, commenting on Hicks, Harrod said that ''for me it is the basic antinomy which makes such movements inevitable; for Mr. Hicks it is the existence of certain lags combined with the assumption that the multiplier and acceleration coefficients have certain values'' (Harrod 1951, pp. 264–265).

[40]For a detailed survey, see Besomi (1998a, especially sections 3.3 and 3.4).

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VI. THE CONFLICT OF APPROACHES

In his correspondence with Marschak, Harrod did not take up any of the points described in the previous sections. A reason for this lack of reaction may perhaps be sought in the difference between their approaches to economic theory. Harrod's aim was not to construct a complete model, but to provide ''the outline of a dynamic theory.'' Instead, Marschak's remarks seemed directed to obtain a complete model. As a matter of fact, he summarized his comments as follows:

> the following set-up is implied:
> 1. ex-ante supply function of savings (= propensity to save).
> 2. ex-ante demand function of savings, involving a changing parameter C.
> 3. A postulate describing the changes of C as a function of (say) G, or of $G-G_w$, and making for a periodic shape of the G_w curve.
> 4. A postulate stating explicitly the reactions making for the progressive growth of the difference $|G-G_w|$.
>
> (1), (2) and (4) are given in the Essay more explicitly than (3), but perhaps not always explicitly enough (Marschak to Harrod, August or early September 1938, p. 847).

With this set-up, Harrod would have had a complete model where the cycle was endogenously generated and equilibrium unstable. The problem with this set-up was that it was not consistent with Harrod's conception of dynamics, which was outlined in his ''Scope and Method of Economics'' (Harrod 1938). Harrod thought that economic theory, whether static or dynamic, should proceed in three stages.[41] In the first, an instantaneous picture of the system is rendered by establishing the relationships between the different variables of the system *at a given point in time*. This stage constituted the ''comprehensive but simultaneous conspectus of the field as a whole, on which the validity of the prescriptions depends'' (Harrod 1938, p. 388). In the draft of the ''Essay,'' this corresponds to paragraphs 1 to 11, which Harrod called ''the strict part of the theory.'' The second stage had to deal with the ''succession of events,'' with the aim of establishing ''propositions concerning causal sequences not derived from the Law of Demand ... propositions, therefore, which are to some extent conjectural'' (Harrod 1938, p. 388). In the third stage, the preceding analysis is applied to the formulation of economic policies. These are, respectively, paragraphs 12 to 21 and 22 to 25.

Marschak's set-up was not consistent with this division of dynamics into different stages. In fact, the first two equations are the first two axioms of Harrod's theory, as they study the system at one instant in time, and are thus included in stage 1 of the analysis. The third and fourth points in Marschak's set-up have the role of linking different points in time (stage 2). These last two equations provided the mathematical possibility for getting periodical solutions not obtainable with the fundamental

[41]For a detailed explanation of Harrod's method, see Besomi (1999, especially chapter 5). See also Nevile and Kriesler (2008).

equation only, which concern just one instant. But, for Harrod, these two sets of equations belonged to different levels of his reasoning, so the formulation of a complete model including both was far from his ''tentative and preliminary attempt to give the outline of a 'dynamic' theory'' (Harrod 2003b, p. 1188; 1939, p. 14), and was not compatible with the division of dynamics into three stages.

In fact, Harrod claimed in 1951 that his choice not to complete the model was deliberate:

> Once the model is completed, oscillation may arise owing to the lags assumed; it would be beyond my power—and I believe it would give the best mathematicians great difficulty—to disentangle the oscillation-producing effect of the basic ''antinomy'' from that of the lags assumed. Thus to complete the model would, although allowing a more elegant presentation, introduce an element of confusion, from which rescue might prove impossible (Harrod 1951, p. 263).

Harrod was not against the construction of models, but he thought that this should be done at a later stage. In his ''Essay,'' he was just trying to provide a ''framework of thought, within which the significance of lags, the importance of which is not denied, may be fruitfully considered'' (Harrod 2003b, p. 1203). Lags were considered as frictions; they were important but were not the fundamental cause of the cycle. Since Harrod was trying to establish a general proposition, which was for him the scientific aspect of economics (cf. Harrod 1938, pp. 385–386), all aspects that weakened generality had to be excluded, and lags were between these. Once the general ''framework of thought'' was established, it was possible to consider other factors.[42]

It is worth quoting in full the following paragraph of his ''Supplement on Dynamic Theory,'' where he underlined this aspect, saying that his fundamental equation

> does not yield a complete account of the course of the cycle, and in that differs from certain so-called 'complete models' that have been set up for study. These complete models require special postulates and assumptions in regard to lags and coefficients, which can only be accepted subject to statistical verification. While it is interesting and satisfactory to have such complete models for comparison with the phenomena of the real world, it is clear that the 'theories' of the cycle based on them have a greatly inferior authority, since their logical status is precarious, while mine rests, like the 'law of demand' itself, on assumptions of the utmost simplicity and generality. But mine makes no pretension to giving a complete explanation of the cycle (Harrod 1952, p. 286).

Harrod thought at the time that all statements could be divided into tautologies and those subject to empirical tests (Harrod 1968, p. 184). He considered his fundamental equation $G_w = s/C$ could be regarded in two different ways. The first possibility was to interpret it as a definition of G_w, and so G_w would be the only rate of growth consistent with the coefficients C and s. The second possibility was seeing G_w as

[42]For a different interpretation, see Nevile (2003) and Nevile and Kriesler (2008). The authors think the exclusion of lags was not a fundamental point in Harrod's dynamics, but just a simplifying assumption.

a rate of growth that, if achieved, would be self-sustained because it left the entrepreneurs satisfied. Certainly, the rate of growth specified by *s/C* might not coincide with the entrepreneurial equilibrium. The first interpretation would consider the equation as a tautology,[43] and the second would imply empirical investigations about entrepreneurs' behavior. The equation for the actual rate of growth, $G = s/C_p$, was instead clearly a tautology.

Harrod concluded that "If we want to bring 'models' into economics, but to keep them meaningful, we might confine the term to a system of equations, not all of which are tautologies. Some at least might have adjustable parameters. It might be made a condition for the use of the word that some equations explicitly omit to take account of fringe influences" (Harrod 1968, p. 190). In the case of his own system, Harrod said it could be considered a model only when equations containing adjustable parameters, like behavioral ones, are included. But the equations alone could not be considered models.

The set-up Marschak proposed was a system of equations that would yield as a result the paths of the variables out of a set of initial conditions. This obscured the different epistemic status of the various equations, which was essential for Harrod in his presentation of a dynamic theory. It is important to keep in mind that he did not want to "profess to determine the course of events in detail but should provide a framework of concepts relevant to the study of change analogous to that provided by static theory for the study of rest" (Harrod 2003b, p. 1188; 1939, p. 14). If his concepts were to be as relevant as those of statics, they needed to have a high degree of probability. There was a logical difference between both sets of postulates that was omitted in Marschak's set-up.

Marschak's conception was near to Frisch's, which was quickly gaining acceptance when Harrod wrote the "Essay." According to Frisch, a dynamic theory explains "how one situation grows out of the foregoing" (Frisch 1933). A system is dynamic if at least one of the variables included relates to two different points in time. If n equations and n variables together with the initial conditions are given, the evolution of the system will, in general, be determined in a way that motion does not stop (that is the difference with static models [Frisch 1936]). Harrod was not trying to predict the evolution of a system given the initial conditions using a complete model, but he was trying to provide a new tool of analysis.

The main problem of Harrod's construction was that the link between different states of the system was missing, something quite odd for a dynamic theory. This led subsequent commentators on Harrod's theory to introduce lags, expectations, or behavior equations to describe how the system worked under disequilibrium. But Harrod wanted to stress the superiority of his theory with respect to the theories that used these mechanisms to generate the oscillations. The crucial point was that "the trend of growth may itself generate forces making for oscillation" (Harrod 2003b, p. 1188; 1939, p. 15).

[43]In this work he discussed the use of the term "tautology" and defined it as "deductions therefrom in accordance with arbitrary rules" (Harrod 1968, pp. 183–184).

Harrod did not deny the relevance of lags. On the contrary, he asserted that when decisions are made in a context of uncertainty, it was reasonable to introduce systematic lags and obtain an oscillatory behavior. However, ''the framework of equations within which the lag hypothesis should be applied are those of dynamic economics. This gives another reason for wishing an early precise formulation of these'' (Harrod 1938, p. 409). The explanation of the cycle had to be propaedeutic to any study about lags and other frictions (among these the absence of equilibrium between sectors pointed out by Keynes [Keynes to Harrod, 17 August 1938]). Thus, lags could be included in the second stage of dynamics, but not in the ''strict part of the theory.''

Harrod considered an equation as dynamic if the unknown variable was a rate of growth, and not just when lags between different points in time were involved. Yet, as Marschak correctly pointed out, these two things were not inconsistent, since a growth rate involves a time interval, even if of infinitesimal duration. In fact, Marschak did not agree with Harrod's assertion that ''those who define dynamic as having a cross reference to two points of time will not regard [the fundamental equation] as dynamic; that particular definition has its own interest and field of reference. I prefer to define dynamic as referring to propositions in which a rate of growth appears as an unknown variable'' (Harrod 2003b, p. 1191). Even if he agreed with Harrod in the fact that dynamics was not identical with the theory of lags, Marschak considered this paragraph to be ''too polemic,'' since velocity always involves two points in time, and so ''those who define dynamic as having a cross reference to two points of time do not necessarily require lags and will certainly agree that your [fundamental equation] is dynamic involving as it does a rate of growth (a velocity)'' (Marschak to Harrod, August or early September 1938, p. 848). In the published version of the article, the ''too polemic'' sentence remained with only one modification: instead of as ''will not regard [the fundamental equation] as dynamic,'' Harrod wrote ''may not regard [the fundamental equation] as dynamic'' (Harrod 1939, p. 17).

This definition of dynamics as concerning compatibility of different rates of growth was different from the definition propounded by some econometricians of the time, such as Kalecki, Frisch, Samuelson, and Tinbergen, whose models generated cycles around an equilibrium level of income. This would be the predominant vision until the mid-1950s, while Harrod instead was thinking in oscillations around a rate of growth (Nevile 2003, p. 101). Harrod's definition became widely accepted only after the 1950s.

VII. CONCLUSION

The Harrod–Marschak correspondence on the draft of the ''Essay'' is useful in understanding Harrod's own contribution to some of the later misinterpretations. Even for Marschak, who had the benefit of reading the draft, which in many aspects was clearer than the published version of the ''Essay'', inconsistencies arose. Marschak's proposed set-up was not consistent with Harrod's approach to economic theory, but, as we have seen, many of his suggestions and criticism were well founded.

Even so, Harrod decided to overlook most of the remarks. In fact, his first letter of response to Marschak's comments literally concerned a footnote matter.[44] In his second letter, Harrod also took up some minor points.[45] But Marschak's remarks pointed directly to the more problematic and misunderstood aspects of Harrod's theory: the concept of the warranted rate of growth, instability, and non-linearity.

Marschak thought that if G_w were deduced from the initial axioms, it would be clear that it was an equilibrium rate, and a variable different from G, not a particular value of it. In addition, Marschak thought that Harrod made three additional assumptions when he explained the interaction between G_w and G.

The first of these additional assumptions regarded the behavior of the entrepreneurs. As discussed in section IV, many interpreters realized that the instability result was somewhat dependent on this assumption. Marschak asked Harrod to make this assumption explicit, but Harrod denied that his theory depended on it. And when he tried to prove instability, he further contributed to the confusion, which led finally to the "knife-edge" interpretation of instability. But, as Kregel points out, this interpretation destroyed the meaning Harrod attached to the concept of warranted rate of growth (Kregel 1980, p. 120).

The other two additional assumptions Harrod implicitly made, according to Marschak, have to do with the turning points of the cycle. Since the three initial axioms didn't produce periodicity as a result, the changes in the coefficients C and s had to be made explicit. Marschak did not realize that the analysis of the moving equilibrium at a single point in time had, for Harrod, a different status from the analysis of the behavior of the system in disequilibrium conditions. While the fundamental equation was, for Harrod, the dynamic counterpart of the law of demand in static theory, the analysis of the system in disequilibrium was of a "more conjectural" character.

In the light of the subsequent interpretations of Harrod's work, it looks as if most of Marschak's remarks, if listened to, would have helped Harrod to pass on his message. It was true that a more accurate definition of the variables G_w and C was needed. If Harrod had answered to Marschak's doubts about the definition G_w, he would have probably realized he was giving several non-equivalent definitions. The same thing happened with the three additional assumptions Marschak attributed to

[44]The footnote was about C, where Harrod made an error trying to explain that the value of C was inversely proportional to the length of the period chosen. Marschak said that this was obvious and the discussion on the footnote "was not very easy to follow" (Marschak to Harrod, August or early September 1938, in Harrod [2003, p. 848]). Of course, this remark is negligible as compared with the rest of his comments, but to this Harrod devoted his first rejoinder to Marschak. He insisted in the correctness of his calculations—which were clearly incorrect—"I am now only taking up one small point. I am sure I am right! But it is the sort of pricking point about which a commentator's dissent makes one feel uncomfortable" (Harrod to Marschak, 7 September 1938, in Harrod [2003, p. 857]). In fact, the error persists in the published version of the "Essay," revealing Harrod's scarce mathematical knowledge.

[45]He commented on the graphical representation (reproduced here in section V) suggested by Marschak to represent the four rates of growth (G, G_n, G_w, and normal G_w). In doing so, Marschak mistook G_n for the ceiling (while the ceiling was G_n + unemployment). Harrod remarked on this mistake, and interpreted Marschak's diagram as using simultaneously two ordinate axes (output and rate of growth). He proposed then to represent the natural rate as constant and the ceiling of full employment as decreasing. Marschak could not have been using two ordinate axes, for the simple reason that, having mistaken the ceiling for the natural rate, he was representing only growth rates in his diagram.

Harrod. Even if Marschak made these remarks from the point of view of the building of a complete model, which was not compatible with Harrod's approach, the remarks are still valid. To accept them did not necessarily imply that he had to change his approach. If Harrod provided the elucidations required, with a clear explanation of the motives for not including new equations to obtain a complete system, he would have made his methodology clearer. If he made explicit the different stages of the new method he was propounding, not only would he have better explained it, but he would have also avoided the later misunderstandings arising from the fact that interpreters did not realize this was the application to a particular problem (the analysis of the trade cycle) of a new dynamic method.

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