LONGEVITY RISK MANAGEMENT AND SHAREHOLDER VALUE FOR A LIFE ANNUITY BUSINESS

BY

CRAIG BLACKBURN, KATJA HANEWALD, ANNAMARIA OLIVIERI AND MICHAEL SHERRIS

ABSTRACT

The life annuity business is heavily exposed to longevity risk. Risk transfer solutions are not yet fully developed, and when available they are expensive. A significant part of the risk must therefore be retained by the life insurer. So far, most of the research work on longevity risk has been mainly concerned with capital requirements and specific risk transfer solutions. However, the impact of longevity risk on shareholder value also deserves attention. While it is commonly accepted that a market-consistent valuation should be performed in this respect, the definition of a fair shareholder value for a life insurance business is not trivial. In this paper, we develop a multi-period market-consistent shareholder value model for a life annuity business. The model allows for systematic and idiosyncratic longevity risk and includes the most significant variables affecting shareholder value: the cost of capital (which in a market-consistent setting must be quantified in terms of frictional and agency costs, net of the value of the limited liability put option), policyholder demand elasticity and the cost of alternative longevity risk management solutions, namely indemnity-based and index-based solutions. We show how the model can be used for assessing the impact of different longevity risk management strategies on life insurer shareholder value and solvency.

KEYWORDS

Economic value, market-consistent embedded value, capital management, Solvency, reinsurance, securitization, limited liability put option, cost of capital.

1. INTRODUCTION

Life insurers writing products that guarantee a retirement income, including life annuities, are increasingly recognizing the need to manage the risk of unanticipated improvements in longevity. This risk results from the uncertain mortality

Astin Bulletin 47(1), 43-77. doi: 10.1017/asb.2016.32

downward trend, impacting all lives in the portfolio to a greater or lesser extent. Traditionally, life insurers have been concerned mainly with idiosyncratic longevity risk, which is reduced through pooling lives in an insurer's portfolio. Many years of mortality improvements and increased uncertainty about future mortality developments require a change in the management of longevity risk. The decision about the risk management strategy needs to account for the impact on the value of the business.

Basically, the value of the business consists of the present value of current and future profits taking into account cost of capital. Assessing the impact of risk management on business value is a complex task, due to several conflicting effects. For example, the costs of risk transfer solutions clearly reduce potential profits; on the other hand, they improve the level of solvency and reduce frictional costs, thus reducing the cost of capital. Zanjani (2002); Krvavych and Sherris (2006); Froot (2007); Yow and Sherris (2008) suggest that risk management strategies for insurers result in an increase of shareholder value when reducing efficiently frictional costs.

Profits mainly come from the loadings charged to policyholders in excess of the actuarially fair premium rate. Higher premium loadings, and hence higher prices, reduce demand depending on policyholder price sensitivity. On the other hand, higher levels of solvency should keep the level of demand high. This trade-off between solvency, price and demand is an important factor in determining value maximizing risk management strategies. This aspect is not well understood in the assessment of longevity risk management.

A number of previous studies have recognized the impact on insurer value of product pricing and consumers' preferences for an insurer's solvency. Zanjani (2002); Froot (2007); Yow and Sherris (2008); Gründl *et al.* (2006); Zimmer *et al.* (2011); Nirmalendran *et al.* (2013) incorporate consumer preferences for solvency in an insurer's value maximization model. Zimmer *et al.* (2009) and Zimmer *et al.* (2011) are the first to provide estimates of consumers' reactions to insurance default risk. Zimmer *et al.* (2011) incorporate the demand curve into a single-period shareholder value maximization model for a non-life insurance company. Nirmalendran *et al.* (2013) incorporate consumers' preferences for an insurer's solvency in annuity demand and use a shareholder value maximization model for an annuity provider to assess optimal product pricing and capitalization strategies under different solvency capital requirements. Risk management and its impact on solvency and shareholder value has not been considered in this setting.

A life insurer writing annuity business will most often use reinsurance to manage its longevity risk, although capital market securitization is also of increasing interest. Blake and Burrows (2001) proposed survivor bonds as a hedge for longevity risk, where the coupon payment each year is proportional to the number of survivors in a cohort. Dowd *et al.* (2006) proposed survivor swaps, as an exchange of cash flows based on the outcome of a survivor index. Reinsurance is indemnity-based, whereas securitization is index-based, which includes basis risk between the index and the annuity portfolio of the insurer. Basis risk is higher for higher levels of idiosyncratic mortality risk since smaller portfolios of lives produce more variability between the actual experience of an insurer's portfolio of lives and the survivor index used. This increases the risk of life insurer insolvency, especially in the older ages of the annuitants, often referred to as tail risk.

The market for longevity risk securitization is growing (Tan *et al.*, 2015). The first longevity bond was announced by the European Investment Bank, BNP Paribas and PartnerRe in November 2004, but failed to attract sufficient investor interest (Blake *et al.*, 2006). Survivor, or longevity, swaps have been more successful than securitization of longevity risk through survivor, or longevity, bonds. The first survivor swap took place between Swiss Re and the UK life office Friends' Provident in 2007. Although legally an insurance contract, this was a pure longevity risk transfer of $1.7 billion on a closed portfolio of annuitants. In 2008, the first derivative transaction, based on a "q-forward", took place between JPMorgan and Lucidia (Coughlan *et al.*, 2007). Also in 2008, the first capital market survivor swap was executed. Canada Life hedged $500 million of its UK annuity book, with JPMorgan acting as the intermediary (Blake *et al.*, 2010). Between 2007 and 2014, 29 survivor swaps were completed in the UK. The largest to date was a $16 billion survivor swap arranged for the British Telecom Pension Scheme by the Prudential Insurance Co of America in July 2014 (Tan *et al.*, 2015).

Longevity risk management using securitization is considered in several studies, including Cowley and Cummins (2005); Wills and Sherris (2010); Biffis and Blake (2010); Gupta and Wang (2011), along with a small number of studies on the reinsurance of longevity risk in Olivieri (2005); Olivieri and Pitacco (2008); Levantesi and Menzietti (2008). Risk management solutions using reinsurance and securitization have been compared for risks other than longevity, including mortality risks (MacMinn and Richter, 2011), insurable risks in general (Cummins and Trainar, 2009) and catastrophe risks (Lakdawalla and Zanjani, 2012). Gupta and Wang (2011) assess securitization and natural hedging strategies for the management of longevity risk in a multi-period shareholder maximization framework. MacMinn and Richter (2011) compare index-based and indemnity-based hedging for the mortality risk inherent in a life book in a two-period shareholder value framework. The use of survivor swaps and bonds has not been compared in a multi-period stochastic shareholder value model.

In this paper, we investigate the impact of longevity risk management on life insurer shareholder value and solvency for a life annuity portfolio, using reinsurance and securitization. Capital management is also considered based on a recapitalization and dividend strategy that maintains regulatory capital requirements as defined under Solvency II. Frictional and agency costs are included, as well as the limited liability put option and the policyholder demand. We design a rather comprehensive stochastic valuation model, using a multi-period framework that allows us to examine the impact on profit volatility as well as solvency over the full time-horizon of the business. The framework is market-consistent, and then all risk margins are assessed based on fair value principles. Future

https://doi.org/10.1017/asb.2016.32 Published online by Cambridge University Press

mortality rates are modeled using the multi-factor affine term structure mortality model developed by Blackburn and Sherris (2013), a framework which allows efficient simulation of future scenarios.

To keep the complexity of the investigation at a reasonable level, we disregard risks other than the longevity risk. Thus, in particular, interest rates are assumed to be deterministic. The impact of both systematic and idiosyncratic longevity risk over the full term of the life annuity portfolio is included.

We assess shareholder value in terms of Economic Value (EV) and Market-Consistent Embedded Value (MCEV). The main difference between these two valuation frameworks relates to when profit is reported: completely at the time of policy issue for the assessment of the EV, and gradually over time according to the MCEV assumptions. While in a fair valuation setting, the EV seems to be the natural way for assessing the business value, the MCEV extends, along fair value principles, a traditional actuarial valuation structure of the insurance business, still popular in insurance practice. The different approaches to profit reporting have significant implications for the volatility of shareholder value, as we show in our discussion.

This paper makes two main contributions. (i) We describe a multi-period stochastic shareholder value model for a life annuity business that can be used to assess the impact of different longevity risk management strategies on life insurer shareholder value and solvency. Indeed, despite the practical importance of such an assessment, the required formal setting has not yet been adequately discussed in the existing literature. (ii) We demonstrate how longevity risk management strategies significantly reduce the volatility of shareholder value, mainly through the reduction of the probability of insolvency. Important new insights into the effective management of longevity risk are provided.

The structure of the paper is as follows. Section 2 presents the stochastic shareholder valuation model, along with the longevity risk management solutions, including the survivor swap and bond. Section 3 presents the multi-period stochastic mortality model used for systematic and idiosyncratic longevity risks, and the interest rate model used for valuation of cash flows. Section 4 presents the results of the numerical investigation, while Section 5 concludes.

2. Cash flows, liability reserve, capital management and shareholder value

2.1. Shareholder value

In this paper, we assess the shareholder value for a life insurer writing life annuity business, and hence exposed to longevity risk. We consider alternative longevity risk management solutions, so to identify which one is more beneficial to shareholder value.

We adopt a market-consistent valuation approach, and assess shareholder value alternatively using an EV and an MCEV approach. The EV structure is usual in a fair valuation setting, while the MCEV extends (in a market-consistent manner) a traditional actuarial valuation approach, popular in insurance practice. Basically, the main difference between the EV and MCEV structures relates to when profit is reported: While the EV is based on an asset-liability logic, so that profit is fully reported at policy issue, the MCEV is based on a deferral-and-matching logic, according to which profit is gradually released in time. These different logics affect the volatility of the shareholder value and the determination of insurer solvency.

Whatever the valuation structure, shareholder value results from the contribution of several quantities: portfolio cashflows (premiums, benefits, expenses), net of the cashflows from risk transfer arrangements (either reinsurance or securitization), capital allocated and the related cost. Such quantities are specified in detail in the following sections. Then, we will formally define the shareholder value, alternatively in terms of EV and MCEV.

2.2. The portfolio

We refer to a life annuity portfolio consisting of a single cohort of n_0 individuals from a homogeneous population aged $x = 65$ at time-0. The case of multiple cohorts is not addressed in this paper, to keep the complexity of the overall model at a manageable level and to obtain results easier to understand. The portfolio is examined until run-off. A single premium π (as defined in Section 2.7) is paid at time-0 by all individuals and the annuity payments are in arrears. Each annuity is for an annual payment of $b = \$1,000$ as long as the annuitant is alive. The ultimate age at which the contract terminates is age 100.

The number of annuitants at time-t is denoted as $\widetilde{I}(t; x)$, and then the total annual payment for the insurer at time-t is $b \cdot \widetilde{I}(t; x)$. (Here and in the following, we use a tilde to indicate when a variable is random.) We assume that the number $\widetilde{I}(t; x)$, and then the total annual payment $b \cdot \widetilde{I}(t; x)$, are affected by systematic and idiosyncratic longevity risk. To this purpose, the numbers $\widetilde{I}(t; x)$ are generated according to the stochastic mortality model described in Section 3.

2.3. Survivor swaps and bonds

We consider the transfer of the insurer's longevity risk through either a survivor (or longevity) swap or a survivor (or longevity) bond as a static hedge.

The survivor swap takes the form of a reinsurance contract with no counterparty default risk. Each party agrees to make periodic payments until the maturity of the swap at time-T, or until the insurer defaults. Similar to an interest rate swap, there is a fixed and a floating leg. The fixed leg are payments based on an agreed survivor curve at time-0, $\overline{S}(0, t; x)$, while the floating leg payments are based on the actual survivors $\widetilde{I}(t; x)$ in the annuity portfolio for each period. The survivor curve $\overline{S}(0, t; x)$ is defined in Section 3; we assume that it is based on the best-estimate assumption about the longevity of the cohort. We assume that a swap premium is included in the annual swap payments, and we denote

https://doi.org/10.1017/asb.2016.32 Published online by Cambridge University Press

by γ^R the relevant coefficient. The net swap payment at time-t cashed by the insurer is

$$\widetilde{NSP}(t) = b \cdot \Big(\widetilde{I}(t; x) - (1 + \gamma^R) \cdot \overline{S}(0, t; x)\Big). \tag{1}$$

For the survivor bond, we adopt an arrangement similar to Blake and Burrows (2001), who propose a bond structured as an interest bearing bond with an initial purchase price at time-0 and regular interest payments proportional to the population survivor index. We design the longevity bond as an annuity bond with floating rate payments similar to the cash flows for the survivor swap. The main difference between these is that the survivor bond has floating payments based on a population survivor index $I(t; x)$ which is free from idiosyncratic longevity risk; see Section 3 for details in this respect. To allow comparison between the longevity swap and the longevity bond, we assume that $I(t; x)$ is obtained in a population of n_0 individuals aged $x = 65$ at time-0. Further, we assume that the same premium coefficient is adopted for the longevity swap and the longevity bond and that the size of the bond is proportional to the annual amount b. The net bond payment cashed by the insurer at time-t is then

$$\widetilde{NBP}(t) = b \cdot \Big(I(t; x) - (1 + \gamma^R) \cdot \overline{S}(0, t; x)\Big). \tag{2}$$

The longevity bond involves basis risk; this basis risk is greater for smaller portfolio sizes.

The hedging strategy consists of underwriting a proportion ω_h, $0 \leq \omega_h \leq 1$, of the survivor swap ($h = S$) or a proportion ω_h, $0 \leq \omega_h \leq 1$, of the survivor bond ($h = B$) payments. Since the hedge is static, the proportion ω_h is stated at time-0 and is kept until run-off. A strategy involving a mix of survivor swap and bond will not be considered (so that $\omega_S = 0$ if $\omega_B > 0$, and vice versa).

2.4. Annuity portfolio cash flows

The annuity portfolio cash flow at time-0, $CF(0)$, is the premiums less the initial expenses, $E(0)$:

$$CF(0) = n_0 \cdot \pi - E(0). \tag{3}$$

Clearly, $CF(0)$ is a flow certain.

Conversely, the annuity portfolio cash flow $\widetilde{CF}(t)$ at time-t, $t > 0$, is random, and is an outflow. It is the annuity payment plus the (random) recurrent expenses $\widetilde{E}(t)$, net of the payments received under the underwritten proportion of the survivor swaps or bonds. We have

$$\widetilde{CF}(t) = -b \cdot \widetilde{I}(t; x) - \widetilde{E}(t) + \omega_S \cdot \widetilde{NSP}(t) + \omega_B \cdot \widetilde{NBP}(t). \tag{4}$$

The insurer's expenses consist of acquisition costs, asset management costs, overhead and other general expenses. Acquisition costs are assumed to be pro-

https://doi.org/10.1017/asb.2016.32 Published online by Cambridge University Press

portional to the annuity single premium and are paid at time-0:

$$E(0) = e^{[i]} \cdot n_0 \cdot \pi, \tag{5}$$

where $e^{[i]}$ is the proportion of initial acquisition costs.

Recurrent expenses are asset management costs, overhead and other general expenses, and are charged to the portfolio in each period. We express such expenses as a proportion of the technical provision, $\widetilde{V}(t)$ (which is defined in Section 2.5). Therefore, the expenses incurred at time-t, $t > 0$, are defined as follows:

$$\widetilde{E}(t) = e^{[r]} \cdot \widetilde{V}(t), \tag{6}$$

where $e^{[r]}$ denotes the proportion of recurrent expenses.

2.5. Technical provisions and required capital

Here, we describe the calculation of the technical provisions and capital that are required to back the insurer's obligations.

The amount of the technical provision must correspond to the value of liabilities, including a margin for longevity risk. We aim at performing a market-consistent valuation of the liabilities, and then adopt a market approach for the assessment of the risk margin. This means, in particular, that the risk margin accounts for systematic, while disregarding idiosyncratic, longevity risk.

There have been a number of approaches proposed to price the longevity risk. Milevsky *et al.* (2005) values a pure endowment contract using an instantaneous Sharpe ratio. Bauer *et al.* (2010) use a forward mortality framework, as presented in Bauer *et al.* (2008), for pricing a zero coupon longevity bond and show how the Sharpe ratio coincides with a change of probability measure assuming a constant market price of longevity risk. Biffis *et al.* (2010) use a change of measure approach to generate risk-adjusted survivor curves based on a generalized Lee–Carter model.

In this paper, we calibrate the market prices of risk in the mortality model for liability valuation to be consistent with the reinsurance loading; we assume that there is no explicit profit loading in the reinsurance contract. See Section 3 for details and Section 4.8 for a discussion of the impact on the results.

We denote with $a(t; x)$ the market value at time-t of a life annuity with annual benefit $b = \$1$. Then,

$$\widetilde{V}_p(t) = b \cdot \widetilde{l}(t; x) \cdot a(t; x) \tag{7}$$

is the technical provision required at time-t for the portfolio for which no hedging has been underwritten.

For a fully hedged portfolio, using either the survivor swap or survivor bond, the liability of the insurer is in respect of the agreed survivor curve $\overline{S}(0, t; x)$, set at time-0 for defining the fixed leg payments. Then, the technical provision that

https://doi.org/10.1017/asb.2016.32 Published online by Cambridge University Press

the insurer must hold at time-t is defined as follows:

$$V_h(t) = b \cdot n_0 \cdot \overline{S}(0, t; x) \cdot a(0, t; x), \tag{8}$$

where $n_0 \cdot \overline{S}(0, t; x)$ represents the expected number of survivors based on the agreed survivor curve, while $a(0, t; x)$ represents the forward market value of a life annuity with annual benefit $b = \$1$, also based on the survivor curve $\overline{S}(0, t; x)$. The quantities $a(t; x)$ and $a(0, t; x)$ are defined in detail in Section 3. We note that $V_h(t)$ is a value certain, as for a fully hedged portfolio the longevity risk is fully transferred. However, if hedging is realized through the longevity bond, idiosyncratic risk remains with the insurer, and this is not accounted for in the assessment of $V_h(t)$. This is in line with the fair valuation principles: We should include a margin for idiosyncratic risk but, as we have already noted, the market approach only accounts for systematic risks.

Considering that a proportion ω_h, $0 \leq \omega_h \leq 1$, of the portfolio is hedged, the technical provision that the insurer must hold at time-t is

$$\widetilde{V}(t) = (1 - \omega_h) \cdot \widetilde{V}_p(t) + \omega_h \cdot V_h(t). \tag{9}$$

A technical provision must be set up also in respect of future expenses. The expense reserve at time-t is defined as follows:

$$\widetilde{V}_e(t) = \sum_{s>t} e^{[r]} \cdot \widetilde{V}(s) \cdot \nu(t, s), \tag{10}$$

where $\nu(t, s)$ is the discount factor at time-t for a payment of \$1 at time-s from the forward interest rate curve; see Section 3 for a detailed definition. Note that all technical provisions are deterministic at time-0 (and then their value will be denoted without the tilde on the top).

As to the capital required (or solvency capital reserve), we adopt the Solvency II standard (see QIS5[1]). According to this standard, the required capital corresponds to the decrease that would be recorded by the Net Asset Value of the insurer in face of a 20% longevity shock, i.e. a permanent 20% decrease in mortality rates for each age. It is possible to show (see, for example, Olivieri and Pitacco (2003)) that this reduces to the difference between a technical provision based on the shocked mortality rates and the actual technical provision. For a portfolio with no hedging, we then assess the capital required at time-t as follows:

$$\widetilde{M}_p(t) = b \cdot \widetilde{I}(t; x) \cdot \left[a^{[-0.2]}(t; x) - a(t; x) \right], \tag{11}$$

where $a^{[-0.2]}(t; x)$ is the market value of the life annuity based on the shocked mortality rates.

Similarly, to the technical provision, we assume that if the portfolio is hedged, the required capital is reduced in the same proportion of the hedging. Further, it is possible that a capital relief is admitted when the longevity risk is hedged. Assuming that for a fully hedged portfolio, a reduction ω_c of the required capital is admitted, the capital required for a fully hedged portfolio is

$$\widetilde{M}_h(t) = \widetilde{M}_p(t) \cdot (1 - \omega_c). \tag{12}$$

The solvency capital reserve reflecting the proportion hedged and the extent of capital relief is then

$$\widetilde{M}(t) = (1 - \omega_h) \cdot \widetilde{M}_p(t) + \omega_h \cdot \widetilde{M}_h(t) \tag{13}$$

$$= (1 - \omega_h \cdot \omega_c)\, \widetilde{M}_p(t). \tag{14}$$

Finally, we refer to the following quantity as the total liability reserves:

$$\widetilde{V}_l(t) = \widetilde{V}(t) + \widetilde{V}_e(t) + \widetilde{M}(t). \tag{15}$$

We note that, similarly to the technical provision, also the liability reserve is deterministic at time-0.

2.6. Dividend and recapitalization strategy

When considering a multi-period time-horizon, a dividend and recapitalization strategy should not be disregarded. Indeed, such a strategy has an impact on the insurer solvency and shareholder value.

We assume that the insurer strategy is to always meet the Solvency II capital requirement. However, the insurer will not subscribe new capital if the available assets are less than the technical provision; in this case, a situation of unfunded liabilities emerges, and shareholders accept the default. Otherwise, the insurer will either subscribe capital to restore Solvency II requirements or withdraw a dividend if capital exceeds the Solvency II requirements (even if, for brevity, we use the terms capital subscriptions and dividends, these capital flows can be meant simply as transfers of capital between the excess capital of the insurance company and the assets of the portfolio; thus, not necessarily they correspond to money received from or paid to shareholders).

The insurer starts with assets at time-0 from the premiums less initial expenses, i.e. with $CF(0)$ (see (3)). The initial assets must be sufficient to meet the total liability reserve at time-0, i.e. $V_l(0)$ (see (15)). In our setting (one cohort, entering at time-0), if the premium loading is higher than the required capital, then $CF(0) > V_l(0)$, and some profit can be released immediately. Vice versa, if $CF(0) < V_l(0)$, then an additional capital (in respect of the regulatory requirement) must be subscribed for the portfolio. Thus, at time-0, we have

$$A(0) = V_l(0) = CF(0) + R(0) - D(0), \tag{16}$$

where $A(0)$ is the amount of assets at time-0, $R(0)$ is any initial shareholder capital subscribed and $D(0)$ is any excess capital paid as dividends to shareholders. We note that all the quantities in (16) are certain, as they are observed at time-0.

The asset value at time-t, $t > 0$, is random, and is given by

$$\widetilde{A}(t) = \widetilde{A}(t-1) \cdot (1 + i(t)) + \widetilde{CF}(t) + \widetilde{R}(t) - \widetilde{D}(t), \tag{17}$$

where $\widetilde{R}(t)$ is any additional capital required at time-t from shareholders for the insurer to remain solvent and meet reserving requirement, $\widetilde{D}(t)$ represents the capital released at time-t to shareholders as dividends and $i(t)$ is the investment return in year $(t-1, t)$.

At time-t, $t > 0$, the dividend and recapitalization strategy is determined by the financial position of the insurer as follows:

- $\widetilde{A}(t) < \widetilde{V}(t)$: There are insufficient assets to cover the technical provision at time-t, and the insurer defaults.
- $\widetilde{A}(t) \geq \widetilde{V}(t)$, but $\widetilde{A}(t) - \widetilde{V}_l(t) < 0$: The insurer is not in default, but does not have enough capital to meet regulatory obligations. The shortfall, $\widetilde{R}(t)$, is recapitalized from shareholders, $\widetilde{R}(t) = \widetilde{V}_l(t) - \widetilde{A}(t)$.
- $\widetilde{A}(t) - \widetilde{V}_l(t) \geq 0$: The insurer is solvent and has enough capital to meet regulatory requirements. The excess capital is distributed to shareholders as a dividend, $\widetilde{D}(t) = \widetilde{A}(t) - \widetilde{V}_l(t)$.

We recall that the only risk that we are addressing is longevity risk; hence, the possible financial positions mentioned above originates only because of longevity losses or longevity profits.

2.7. Annuity demand

Before moving to the definition of the shareholder value, we describe the model we adopt for setting the initial portfolio size.

The number of policies initially sold is the result of a demand whose main determinants are the price, or loading, and the insolvency risk of the insurer. These are quantities that contribute to shareholder value not only indirectly, through the demand, but also directly. Instead of setting exogenously the initial portfolio size, it is then appropriate to model a demand function explicitly depending on them. We base our demand function on Zimmer *et al.* (2011) and Nirmalendran *et al.* (2013).

Zimmer *et al.* (2011) use experimental data and find that an exponential demand function provides an overall best fit with functional form:

$$\phi(\pi, d_1) = e^{(\alpha \cdot d_1 + \beta \cdot \pi + \theta)}, \tag{18}$$

where $\phi(\pi, d_1)$ represents the percentage of individuals, in respect of the maximum potential market size, willing to purchase at price π from an insurer with 1-year default probability d_1; α is the default sensitivity parameter ($\alpha < 0$), β is the price sensitivity parameter ($\beta < 0$) and θ is a constant.

The exponential demand function developed by Zimmer *et al.* (2011) was modified in Nirmalendran *et al.* (2013) to reflect price and default risk preferences in the annuity market. In particular, based on the results of Babbel

https://doi.org/10.1017/asb.2016.32 Published online by Cambridge University Press

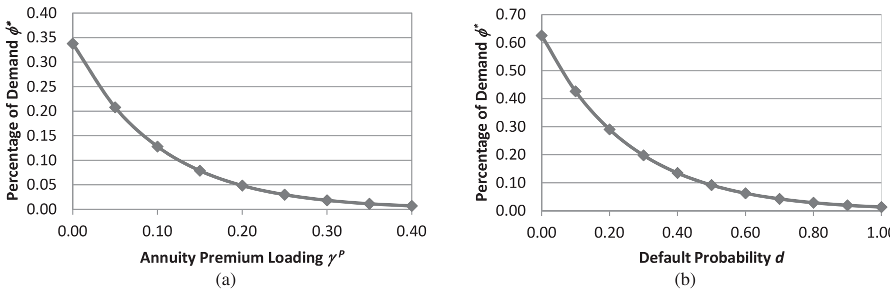


FIGURE 1: Price and default sensitivity of the demand for annuities. (a) Price sensitivity of demand. (b) Default sensitivity of demand.

and Merrill (2006), Nirmalendran *et al.* (2013) assume that annuity demand is not very sensitive to increases in premium loadings of up to 30%, but very sensitive to increase in the default risk of the annuity provider. Also, annuity demand is assumed to be less than 100% even when both the premium loading and the default probability are zero to account for other factors driving individuals' annuity demand such as bequest motives. Nirmalendran *et al.* (2013) calibrate the parameters of the demand function to reflect the Australian life annuity market using premium loading estimates provided by Ganegoda and Bateman (2008).[2]

We use a similar annuity demand function as in Nirmalendran *et al.* (2013). This is modified so that policyholders' price sensitivity is a function of the premium loading factor, γ^P, rather than the premium rate π, and the cumulative default probability over the full run-off of a cohort, d, instead of the 1-year probability, d_1. These minor modifications are introduced to ease computations and to reflect the long-term nature of annuity liabilities. We calibrate the price-default annuity demand curve to reflect the Australian annuity demand using a similar approach as Nirmalendran *et al.* (2013), and both specifications give comparable results.[3] We use the following demand function:

$$\phi^*(\gamma^P, d) = e^{(\alpha \cdot d + \beta \cdot \gamma^P + \theta)} \tag{19}$$

$$= e^{(-3.8328 \cdot d - 9.7089 \cdot \gamma^P - 0.4689)}, \tag{20}$$

where $\phi^*(\gamma^P, d)$ represents the percentage of individuals willing to buy the annuity.

Figure 1 shows the sensitivity of demand due to changes in price and changes in default risk in our model.

A maximum potential market size of $n_m = 25,000$ is assumed for the representative life annuity provider. This reflects the number of males aged 65 and the market share of Australia's largest life insurer. The number n_0 of annuities sold at time 0 is determined by multiplying the level of demand with the assumed

https://doi.org/10.1017/asb.2016.32 Published online by Cambridge University Press

maximum potential market size, n_m:

$$n_0 = n_m \cdot \phi^*(\gamma^P, d). \tag{21}$$

The single premium π paid at time-0 by each individual (see Section 2.2) is based on the best-estimate survivor curve at time-0, $\overline{S}(0, t; x)$, and a premium loading γ^P is applied. Thus, the single premium π is defined as follows:

$$\pi = b \cdot \left(1 + \gamma^P\right) \cdot \overline{a}(0; x), \tag{22}$$

where $\overline{a}(0; x)$ is the actuarial value at time-0 of a life annuity, with annual benefit $b = \$1$, based on the best-estimate survivor curve $\overline{S}(0, t; x)$; see Section 3.2 for details.

2.8. Frictional costs, agency costs and limited liability put option

In a fair value setting, frictional and agency costs, as well as the so-called limited liability put option must be accounted for.

Frictional costs arise from a variety of sources, including taxation and agency costs, as well as the costs of raising capital in the market to recapitalize the insurer. We address two types of frictional costs in the model: frictional cost on shareholder capital arising from the principal-agent problem in the shareholder-management relationship (Yow and Sherris, 2008) and frictional costs in the event of a recapitalization.

The annual frictional cost on shareholder capital is defined as a proportion ρ of the capital held over and above the technical provision. The insurer holds no capital above $\widetilde{V}_l(t)$, since we assume that the excess is distributed to shareholders (see Section 2.6). Thus,

$$\begin{aligned}\widetilde{FC}(t) &= \rho \cdot [\widetilde{V}_l(t) - \widetilde{V}(t)] \\ &= \rho \cdot [\widetilde{M}(t) + \widetilde{V}_e(t)].\end{aligned} \tag{23}$$

The present value at time t of frictional costs is

$$\widetilde{PV}_{FC}(t) = \sum_{s>t} \widetilde{FC}(s) \cdot v(t, s). \tag{24}$$

In the event of recapitalization, additional frictional costs arise, which we refer to as recapitalization costs. These costs are assumed to be proportional to the additional capital $\widetilde{R}(t)$ subscribed at time-t. Thus, recapitalization frictional costs are defined as follows:

$$\widetilde{FC}_R(t) = \psi \cdot \widetilde{R}(t). \tag{25}$$

https://doi.org/10.1017/asb.2016.32 Published online by Cambridge University Press

Their present value at time t is given by

$$\widetilde{PV}_{FC^R}(t) = \sum_{s>t} \widetilde{FC}_R(s) \cdot v(t,s). \tag{26}$$

In the event of insolvency at time-s, i.e. if $\widetilde{A}(s) < \widetilde{V}(s)$, the shareholders are not required to cover the shortfall between the assets of the company and its liability. The annuitants receive only the residual assets, namely less than the market value of the future guaranteed annuity benefits. The pay-off of this Limited Liability Put Option at the insolvency time-s is therefore $\max\{0, \widetilde{V}(s) - \widetilde{A}(s)\}$. We denote the value at time-t, $t < s$, of this pay-off with $\widetilde{LLPO}(t)$. This value is assessed through the simulation procedure described in Section 3.2. In particular, the pay-off of the option is discounted back to time-t using the forward interest rate curve and its value is assessed counting the number of trajectories in which there is a default.

2.9. Shareholder value according to the economic valuation approach

According to the economic valuation approach, the shareholder value is obtained comparing the value of assets to that of liabilities net of the cost of capital, in a fair value setting. In such a setting, assets are those accumulated with premiums net of the benefits and expenses, while liabilities are represented by the technical provision (assessed on a fair value basis) and expenses. Frictional and agency costs, net of the limited liability put option, measure the cost of capital.

In our setting (immediate life annuity, one cohort), the shareholder EV at time-0 is defined as follows:

$$\widetilde{EV}(0) = CF(0) - V(0) - V_e(0) - \widetilde{PV}_{FC}(0) - \widetilde{PV}_{FC^R}(0) + \widetilde{LLPO}(0). \tag{27}$$

2.10. Shareholder value according to the market-consistent embedded value approach

In the MCEV approach, shareholder value is assessed as the present value of future profits, net of the cost of capital, plus any capital held in excess of the regulatory requirement. This follows a deferral-and-matching logic for profit reporting, with total profit released over time. The timing of the emergence of profit is determined by the total liability reserve, as the annual profit is defined as follows:

$$\widetilde{AP}(t) = \widetilde{CF}(t) - \left(\widetilde{V}(t) - \widetilde{V}(t-1)\right) + i(t)\widetilde{V}(t-1). \tag{28}$$

The present value at time-t of future profits is then

$$\widetilde{FP}(t) = \sum_{s>t} \widetilde{AP}(s) \cdot v(t,s). \tag{29}$$

We note that in the traditional Embedded Value structure, the present value of future profits is based on industrial profits only, i.e. assessed considering the technical provisions (and not the total liability reserve, as we do in (28)). Referring to the total liability reserve is consistent with the market approach we have adopted for the assessment of the technical provision; indeed, part of the risk margin which in market practice must be included in the technical provision, in our setting is included in the required capital.

In a market-consistent setting, the cost of capital is measured by the frictional and agency costs, net of the value of the limited liability put option. We define the Value of the In-Force business (VIF) at time t as

$$\widetilde{VIF}(t) = \widetilde{FP}(t) - \widetilde{PV}_{FC}(t) - \widetilde{PV}_{FC^R}(t) + \widetilde{LLPO}(t). \tag{30}$$

We note that according to definition (28) for the annual profit, the VIF accounts also for the required capital. We then define the MCEV of the business at time-t as

$$\widetilde{MCEV}(t) = \widetilde{VIF}(t) + E^Q(t), \tag{31}$$

where $E^Q(t)$ is the time-t total equity of the insurer held in excess of the required capital.

In our case, at time-0, there are no carried forward profits and no current equity in excess of the required capital, so $E^Q(0) = 0$. At any future time-t, $t > 0$, no capital is held in excess of the total liability reserving requirement $\widetilde{V}_l(t)$; then, $E^Q(t) = 0$ at any time-t, $t \geq 0$. The shareholder value at time-t, $t \geq 0$ is then simply given by $\widetilde{VIF}(t)$.

Comparing $\widetilde{EV}(0)$, as defined in (27), with $\widetilde{VIF}(0)$, as defined in (30), the difference between the two valuation approaches emerges clearly: While under the EV structure, the present value of future profits (namely, $CF(0) - V(0) - V_e(0)$) is fully reported at time-0, under the MCEV structure this total profit is progressively released in time, where its timing is driven by the total liability reserve. In a fair value setting, this different timing of the profit reporting does not affect significantly the expected shareholder value, while having an impact on its volatility, as we comment in Section 4.

3. The longevity risk multi-period model

3.1. Stochastic mortality model, interest rate model, market value of survivor benefits

The mortality model we adopt is in the framework of the forward rate models proposed by Heath *et al.* (1992) (HJM) for interest rates. This framework, adapted to mortality rates, is well suited for the multi-period analysis of an insurer's liability and regulatory capital requirements. In contrast to short rate

https://doi.org/10.1017/asb.2016.32 Published online by Cambridge University Press

mortality models, the forward rate structure allows us to determine the distribution of each annuitant's uncertain lifetime at all-time points in the future and to value future liabilities along any simulated path of future mortality rates. This approach to mortality modeling has been considered, among others, by Plat (2011), Dahl (2004), Miltersen and Persson (2005), Cairns *et al.* (2006) and Bauer *et al.* (2008).

We use, in particular, the affine mortality framework presented in Blackburn and Sherris (2013), which avoids the need for simulations within simulations at future time periods when valuing the future liabilities. We use the estimation and forecasting results from Blackburn (2013) for the Australian male population. The parameters of the mortality term structure are estimated from historical mortality rates available in the Human Mortality Database. The model specification is relatively simple and allows multiple mortality risk factors to be either non-mean reverting or mean reverting processes under a risk-neutral measure. The non-mean reverting process corresponds to an exponentially increasing mortality rate with age and a simple HJM volatility function. Our risk-neutral measure is defined as the best-estimate cohort survivor curve used to value the fair value of annuity cash flows, i.e. the annuity value without loadings. We also estimate a market pricing measure that is used for market valuation that is consistent with the assumed survivor swap premiums.

Following Milevsky and Promislow (2001) and Biffis *et al.* (2010), we assume a continuous-time framework that defines mortality rates equivalent to a credit risk defaultable intensity process. The approach is similar to that of Lando (1998), Schönbucher (1998) and Duffie and Singleton (1999) for pricing defaultable bonds. We use the two-factor mortality model calibrated in Blackburn (2013) with a deterministic volatility function and Gaussian dynamics.

We define a filtered probability space $(\Omega, \mathcal{F}, \mathbb{F}, \overline{Q})$, where $\mathbb{F} = (\mathcal{F}_t)_{t>0}$ and $\overline{Q}$ is a martingale measure and is based on the best-estimate survivor probability. We define two sub-filtrations $\mathbb{G}$ and $\mathbb{H}$ such that $\mathbb{F} = \mathbb{G} \vee \mathbb{H}$. The sub-filtration $\mathbb{G} = (\mathcal{G}_t)_{t>0}$ contains all financial and actuarial information, while the sub-filtration $\mathbb{H} = (\mathcal{H}_t)_{t>0}$ captures the occurrence of death. A counting process, $\overline{N}(t; x)$, counts the number of deaths in a given cohort, $\overline{N}(t; x) = \sum_{i=1}^{n_0} 1_{\tau_i<t}$, where τ_i is a $\mathbb{F}$-stopping time and admits an intensity process $\mu(t; x)$, where $\mu(t; x)$ is a predictable process with $\int_0^t \mu(s; x) ds < \infty$.

The survivor index for the cohort of n_0 individual in the portfolio at time-0, initial age x using population mortality rates is the proportion of survivors at time-t and is denoted by $\bar{S}(t; x)$. The survivor index at time t is given by

$$\bar{S}(t; x) = \frac{n_0 - \overline{N}(t; x)}{n_0}. \tag{32}$$

https://doi.org/10.1017/asb.2016.32 Published online by Cambridge University Press

The stochastic forward interest and mortality rates are given by

$$f(t,s) = f(0,s) + \int_0^s \nu_f(u,s)du + \int_0^s \sigma_f(u,s)d\overline{W}_r(u), \tag{33}$$

$$\mu(t,s;x) = \bar{\mu}(0,s;x) + \int_0^s \nu_\mu(u,s;x)du + \int_0^s \sigma_\mu(u,s;x)d\overline{W}_\mu(u), \tag{34}$$

where $f(t,s)$ and $\mu(t,s;x)$ are the $\mathbb{F}$-adapted interest and mortality forward processes at time-t, and $\bar{\mu}(0,t;x)$ is the best-estimate initial forward mortality curve. With the meaning of the other parameters in Equations (33) and (34) as commonly used in these models.

The martingale measure is not unique, hence we define an equivalent $\mathbb{Q}$-measure with Radon–Nikodym density:

$$\left.\frac{dQ}{d\overline{Q}}\right|_{\mathbb{F}_t} = e^{-\int_0^t \lambda(s)dW_\mu(s) - \frac{1}{2}\int_0^t |\lambda(s)|^2 ds}, \tag{35}$$

where $\lambda(s)$ are the market prices of longevity risk. We assume no change to the mortality hazard rate process under the measure change. This new measure is the pricing and market valuation measure. We restrict $\lambda(s)$ to a constant price of longevity risk; Milevsky and Promislow (2001) defines λ as the instantaneous Sharpe ratio. A constant price of longevity risk in the forward mortality model does not affect the volatility function, but scales the initial forward mortality curve. The stochastic forward interest and mortality rates under the market measure are given by

$$f(t,s) = f(0,s) + \int_0^s \nu_f(u,s)du + \int_0^s \sigma_f(u,s)dW_r(u), \tag{36}$$

$$\mu(t,s;x) = \mu(0,s;x) + \int_0^s \nu_\mu(u,s;x)du + \int_0^s \sigma_\mu(u,s;x)dW_\mu(u), \tag{37}$$

where $\mu(0,t;x)$ is the risk-adjusted initial forward mortality curve. There is no change to the interest rate process under this measure change.

The time-t market value of \$1 paid at time-T in case of survival is given by

$$P(t,T;x) = \mathbb{E}^Q\left[e^{-\int_t^T r(s)ds} S(T;x) \middle| \mathcal{F}_t\right] = S(t;x)\mathbb{E}^Q\left[e^{-\int_t^T [r(s)+\mu(s;x)]ds} \middle| \mathcal{G}_t\right], \tag{38}$$

where the survivor index under the market measure is

$$S(t;x) = \frac{n_0 - N(t;x)}{n_0}, \tag{39}$$

with a clear meaning of the quantities $N(t;x)$ and $r(s)$ in Equations (39) and (38), respectively.

https://doi.org/10.1017/asb.2016.32 Published online by Cambridge University Press

Assuming that interest rates and mortality rates are independent, and using the forward mortality and interest rate model, the market value can also be written as

$$P(t, T; x) = S(t; x)e^{-\int_t^T [f(t,s)+\mu(t,s;x)]ds}. \quad (40)$$

At time-0, we define the forward market value of \$1 paid at time-T in case of survival, for $0 \le t \le T$, as

$$P(0, t, T; x) = \frac{P(0, T; x)}{P(0, t; x)} = e^{-\int_t^T [f(0,s)+\mu(0,s;x)]ds}, \quad (41)$$

and the forward survivor probability as

$$S(0, t, T; x) = e^{\int_t^T \mu(0,s;x)ds}. \quad (42)$$

The forward survivor probability is the probability of surviving from t to T, unconditional on surviving to t, and based on the cohort information at time 0. We assume interest rates are deterministic by setting $\sigma_f(t, s) = 0$. Blackburn (2013) provides a more extensive coverage of the model.

Given the processes in Equations (36) and (37), the discounted value of \$1 payable on survival for an individual aged x at time-t is a $\mathbb{Q}$-martingale for all T and given by

$$P^*(t, T; x) = \frac{P(t, T; x)}{B_t} = \frac{S(t; x)e^{-\int_t^T [f(t,s)+\mu(t,s;x)]ds}}{B_t}, \quad (43)$$

where B_t is the money market account value, defined as $dB_t = B_t r(t)dt$.

For $P^*(t, T; x)$ to be a $\mathbb{Q}$-martingale, the interest rate and mortality rate drift conditions must satisfy

$$\nu_f(t, T) = \sigma_f(t, T) \int_t^T \sigma_f(t, s)' ds, \quad (44)$$

$$\nu_\mu(t, T; x) = \sigma_\mu(t, T; x) \int_t^T \sigma_\mu(t, s; x)' ds, \quad (45)$$

where $\sigma_\mu(t, T; x)$ is the deterministic volatility function defined in Equation (46), and $\sigma_f(t, T)$ is the volatility function of the interest rate process (which is set to 0 in this paper).

Using the forward modeling framework, we specify, under the $\mathbb{Q}$-measure, a multi-factor stochastic mortality model for each cohort. Each initial forward mortality curve is a risk-adjusted version of the best-estimate mortality curve, and the volatility function for the two-factor model is

$$\sigma_\mu(t, T; x) = \left[\sigma_1 e^{-\delta_1(T-t+(x-x_0))}, \quad \sigma_2 e^{-\delta_2(T-t+(x-x_0))}\right], \quad (46)$$

https://doi.org/10.1017/asb.2016.32 Published online by Cambridge University Press

TABLE 1

FITTED RISK-NEUTRAL PARAMETERS; YEARS 1965–2009, AGES 50–99.

δ_1	δ_2	σ_1	σ_2
−0.1014	−0.1307	1.923e-4	5.742e-5

where x is the cohort age at time-0 and x_0 is the lowest assumed age in the affine model. The term $e^{-\delta_i(x-x_0)}$ scales the volatility function by the initial cohort age.

The estimated parameters of the two-factor mortality model are shown in Table 1. The mortality model is estimated from historical Australian male population data obtained from the Human Mortality Database, for the years 1965 to 2009 and ages 50 to 99.

3.2. Model implementation

The mortality model is implemented as a discrete time version of the HJM model using Monte Carlo simulation based on Glasserman (2003). The model uses discrete time points $t_0 = 0 < t_1 < \cdots < t_n$, where $t_n = T$ is the time corresponding to the oldest age when all the annuity contracts have terminated. We generate the mortality rates to give a survivor index for each simulation path, each simulated path includes systematic longevity risk. From these mortality rates, the actual deaths in the portfolio are generated by sampling from an exponential distribution. The number of simulations, M, is set to 10,000.

The time-0 forward interest rates for these discrete time points are denoted by $\widehat{f}(0,0), \widehat{f}(0,t_1), \ldots, \widehat{f}(0,t_{n-1})$. These are the discrete time values of the initial forward curve $f(0,t)$ given by

$$\widehat{f}(0,t_i) = \frac{1}{t_{i+1}-t_i}\int_{t_i}^{t_{i+1}} f(0,s)ds \tag{47}$$

(here and in the following, the discrete-time version of the quantities used in the assessment of the shareholder value are denoted with a hat on the top).

Similarly, the initial forward mortality rates are denoted by $\widehat{\mu}(0,0;x)$, $\widehat{\mu}(0,t_1;x), \ldots, \widehat{\mu}(0,t_{n-1};x)$, and these discrete forward mortality rates are given by

$$\widehat{\mu}(0,t_i;x) = \frac{1}{t_{i+1}-t_i}\int_{t_i}^{t_{i+1}} \mu(0,s;x)ds. \tag{48}$$

Parameters for the forward interest rate curve are based on Nirmalendran *et al.* (2013), who calibrated the models to Australian market data. The same interest rate term structure is used to price annuities, determine investment returns, value liabilities and discount cash flows. The fitted yield curve assumes the following interest rates: 1-year maturity: 3.0%, 5-years: 3.4%, 10-years: 3.8% and 30-years: 4.7%.

https://doi.org/10.1017/asb.2016.32 Published online by Cambridge University Press

The forward mortality curve evolves according to the dynamics

$$\widehat{\mu}(t_i, t_j; x) = \widehat{\mu}(t_{i-1}, t_j; x) + \widehat{\nu}_\mu(t_{i-1}, t_j; x)[t_i - t_{i-1}] + \widehat{\sigma}_\mu(t_{i-1}, t_j; x)\sqrt{t_i - t_{i-1}}Z_i, \quad j = i, \ldots, T, \tag{49}$$

where Z_i are normal $N(0, 1)$ random variables. The drift term is given by

$$\widehat{\nu}_\mu(t_{i-1}, t_j; x)[t_{j+1} - t_j] = \frac{1}{2}\left(\sum_{k=i}^{j}\widehat{\sigma}_\mu(t_{i-1}, t_k; x)[t_{k+1} - t_k]\right)^2 - \frac{1}{2}\left(\sum_{k=i}^{j-1}\widehat{\sigma}_\mu(t_{i-1}, t_k; x)[t_{k+1} - t_k]\right)^2, \tag{50}$$

where $\widehat{\sigma}_\mu(t_i, t_j; x)$ is the volatility function defined in Equation (46) evaluated at discrete times t_i and t_j.

At time-0, the market value of \$1 survivor benefit in the discrete time model is

$$\widehat{P}(0, t_i; x) = \exp\left(-\sum_{t_u=t_0}^{t_{i-1}}\left[\widehat{f}(0, t_u) + \widehat{\mu}(0, t_u; x)\right]\cdot\left[t_{u+1} - t_u\right]\right), \tag{51}$$

and the survivor curve is

$$\widehat{S}(0, t_i; x) = \exp\Big(-\sum_{t_u=t_0}^{t_{i-1}}\widehat{\mu}(0, t_u; x)\cdot[t_{u+1} - t_u]\Big), \tag{52}$$

with the forward market price of \$1 survivor benefit, for $t_0 \le t_i \le t_s \le t_n$, given by

$$\widehat{P}(0, t_i, t_s; x) = \exp\Big(-\sum_{t_u=t_i}^{t_{s-1}}[\widehat{f}(0, t_u) + \widehat{\mu}(0, t_u; x)]\cdot[t_{u+1} - t_u]\Big) \tag{53}$$

and the forward survivor curve assessed as

$$\widehat{S}(0, t_i, t_s; x) = \exp\Big(-\sum_{t_u=t_i}^{t_{s-1}}\widehat{\mu}(0, t_u; x)\cdot[t_{u+1} - t_u]\Big). \tag{54}$$

We generate M forward mortality curves at each discrete time point t_i. We define the expected number of survivors in the portfolio at time-t_i (disregarding idiosyncratic risk) as

$$\widehat{I}(t_i; x) = n_0\cdot\exp\Big(-\sum_{t_s=t_0}^{t_i}\mu^{(m)}(t_s, t_s; x)[t_{s+1} - t_s]\Big), \tag{55}$$

where $m = 1, 2, \ldots, M$, and where n_0 is the initial portfolio size.

https://doi.org/10.1017/asb.2016.32 Published online by Cambridge University Press

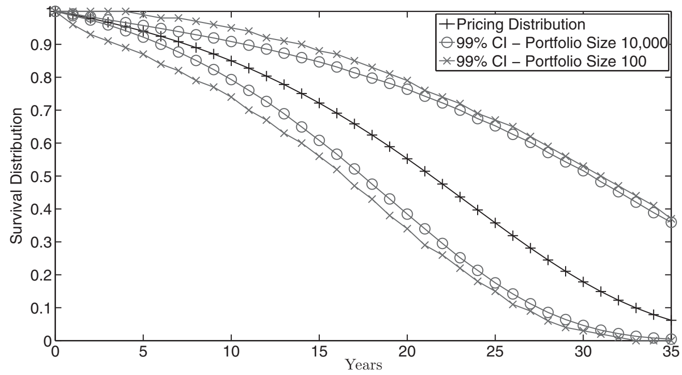


FIGURE 2: Portfolio survivors $\widetilde{I}^{(m)}(t;x)$ for portfolios of 65-year olds.

To generate idiosyncratic longevity risk, random death times for individuals are determined by the first time the mortality hazard rate is above a random level ϱ. The random death time is determined as

$$\tau_i = \inf\left\{t_u : \sum_{t_s=t_0}^{t_u} \mu^{(m)}(t_s;x) \geq \varrho\right\}, \tag{56}$$

where ϱ is an exponential random variable with parameter 1.

The number of survivors at time-t_i for path m, is $\widetilde{I}^{(m)}(t_i;x) = n_0 - \widetilde{N}^{(m)}(t_i;x)$, where

$$\widetilde{N}^{(m)}(t_i;x) = \sum_{i=1}^{n_0} 1_{\{\tau_i \leq t_i\}}, \tag{57}$$

and $\widetilde{I}^{(m)}(0;x) = n_0$. For a large portfolio, the idiosyncratic risk will be low and $\widetilde{I}^{(m)}(t_i;x) \approx \widehat{I}^{(m)}(t_i;x)$, and from our model definition, the law of large numbers gives us

$$\frac{1}{M}\sum_{m=1}^{M} \widehat{I}^{(m)}(t_j;x) \to \mathbb{E}[\widehat{I}(t_j;x)] = \widehat{S}(0,t_j;x) = S(0,t_j;x). \tag{58}$$

Figure 2 plots the distribution of $\widetilde{I}^{(m)}(t;x)$ for two different portfolio sizes of 65-year old policyholders. Smaller portfolio sizes generate much more uncertainty even in the early years.

Using the simulated mortality paths, we can determine the market value of an annuity that pays \$$b$ per year to surviving annuitants in a cohort. For those aged x at time-0, this is given by

$$\widehat{a}(0; x) = \sum_{t_s=t_1}^{t_n} b \cdot \exp\left(- \sum_{t_j=t_0}^{t_{s-1}} \left(\widehat{f}(0, t_j) + \widehat{\mu}(0, t_j; x) \cdot [t_{j+1} - t_j] \right) \right). \tag{59}$$

At time-0, we can also determine forward market values of the annuity given by

$$\widehat{a}(0, t_i; x) = \sum_{t_s=t_{i+1}}^{t_n} b \cdot \exp\left(- \sum_{t_j=t_0}^{t_{s-1}} \left(\widehat{f}(0, t_j) + \widehat{\mu}(0, t_j; x) \cdot [t_{j+1} - t_j] \right) \right). \tag{60}$$

At future times, for a given simulation path, m, the market value of the annuity at time-t is

$$\widehat{a}^{(m)}(t_i; x) = \sum_{t_s=t_{i+1}}^{t_n} b \cdot \exp\left(- \sum_{t_j=t_i}^{t_{s-1}} \left(\widehat{f}^{(m)}(t_j, t_i) + \widehat{\mu}^{(m)}(t_j, t_i; x) \cdot [t_{j+1} - t_j] \right) \right). \tag{61}$$

The annuity premium paid by annuitants at time 0 is based on the best-estimate survivor curves under the $\overline{\mathbb{Q}}$-measure and not the pricing and market valuation measure. The actuarial value of an annuity that pays \$$b$ per year to each annuitant in a cohort age x at time-0 is given by

$$\widehat{\widehat{a}}(0; x) = \sum_{t_s=t_1}^{t_n} b \cdot \exp\left(- \sum_{t_j=t_0}^{t_{s-1}} \left(\widehat{f}(0, t_j) + \widehat{\widehat{\mu}}(0, t_j; x) \right) \cdot [t_{j+1} - t_j] \right), \tag{62}$$

where $\widehat{\widehat{\mu}}(0, t_j; x)$ is the best-estimate cohort forward survivor curve.

Swap payments to the reinsurer are fixed at time-0 and are based on the best-estimate forward survivor curve, given as

$$\widehat{\widehat{S}}(0, t_i; x) = \exp\left(- \sum_{t_j=t_0}^{t_{i-1}} \widehat{\widehat{\mu}}(0, t_j; x) \cdot [t_{j+1} - t_j] \right). \tag{63}$$

The value of the fixed payments for the survivor swap value at time-0 are equated under the actuarial value with the reinsurance loading and the market valuation measure to give

$$\sum_{t_i=t_0}^{t_{n-1}} P(0, t_i; x) = \sum_{t_i=t_0}^{t_{n-1}} (1 + \gamma^R) \overline{P}(0, t_i; x). \tag{64}$$

https://doi.org/10.1017/asb.2016.32 Published online by Cambridge University Press

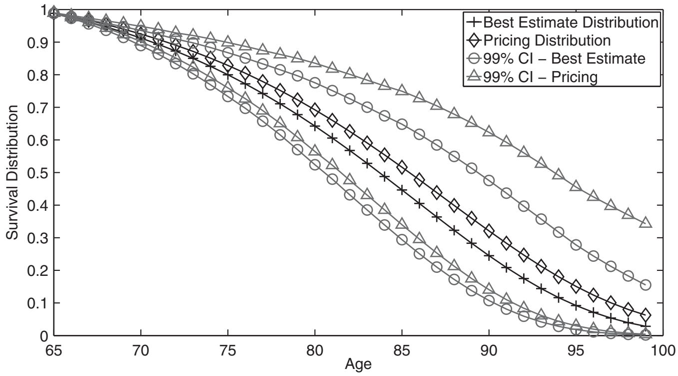


FIGURE 3: Cohort survival distribution aged 65 in 2010.

There is no change in the interest rate process under this measure change, and Equation (64) can be reduced to

$$\sum_{t_i=t_0}^{t_{n-1}} S(0, t_i; x) = \sum_{t_i=0}^{t_{n-1}} e^{-\int_0^{t_i}\int_0^u ||\lambda(s)\sigma_\mu(s,u;x)||dsdu}\bar{S}(0, t_i; x)$$
$$= \sum_{t_i=t_0}^{t_{n-1}} e^{-\lambda\int_0^{t_i}\int_0^u ||\sigma_\mu(s,u;x)||dsdu}\bar{S}(0, t_i; x). \quad (65)$$

We have assumed λ to be a constant price of longevity risk, the same value for each factor in the mortality model. With $\lambda = [\lambda_1, \lambda_2]$ set so that $\lambda_1 = \lambda_2$, we solve Equation (65) for λ using a method of least squares. This gives an instantaneous Sharpe ratio of $\lambda = 0.1555$.[4] We assume that the maturity of the swap and longevity bond corresponds to the maximum possible duration of the portfolio.

Figure 3 shows the market pricing and best-estimate survivor curves, $S(t_0, t_{n-1}; 65)$ and $\bar{S}(t_0, t_{n-1}; 65)$, respectively, with 99% confidence intervals for a cohort aged 65 at time-0. The market valuation survivor curve is shifted upwards along with the confidence intervals.

The forward survivor curves, for different ages of the cohort, are shown in Figure 4. Each curve shows the forward distribution for the survivor index. As the future age increases, the uncertainty also increases substantially. This highlights the extent to which systematic longevity risk is prevalent at the older ages.

4. RESULTS

4.1. Economic value and market-consistent embedded value

Tables 2 and 3 illustrate the shareholder values assessed following the EV and the MCEV approach, with a 15% loading on policyholder annuity premiums

https://doi.org/10.1017/asb.2016.32 Published online by Cambridge University Press

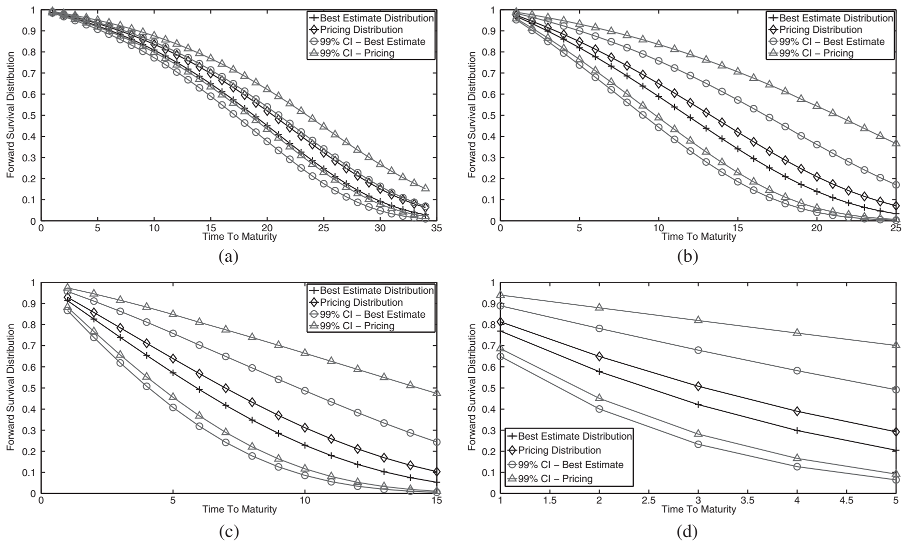


FIGURE 4: Forward survival distributions, cohort aged 65 in 2010. (a) Distribution at age 66. (b) Distribution at age 75. (c) Distribution at age 85. (d) Distribution at age 95.

and a run-off solvency equivalent to a 1-year default probability of 0.79%, resulting in a portfolio size of 1,399 policies based on the annuity demand function specified in Section 2.7. The mortality model is calibrated as described in Section 3. Referring to a report by Swiss Re (2005) suggesting frictional costs of holding capital of 2%, we assume a proportion of the annual frictional cost on shareholder capital $\rho = 1\%$, while the proportion of the annual recapitalization frictional cost is set to $\psi = 3\%$. No hedging is adopted in Tables 2 and 3.

To interpret the results, note that the market value of the annuity is $12,790 and the best-estimate actuarial value of the annuity is $12,180. The premium charged to the policyholder with a 15% loading is $14,708. Initial expenses are 3% of the premium and the value of recurrent expenses is $640 per policy, which is 5% of the total liability reserve. The frictional costs are 0.8% of the technical provision, and the LLPO almost zero. This reflects the high level of solvency of the life insurer resulting from a premium loading of 15%, even without risk transfer.

The most striking result is the volatility of the present value of future profits under the MCEV. Because the initial premium loading, which has zero volatility in present value terms, is re-spread and accounted for as part of the annual insurer profit in the MCEV, this gives rise to volatility in accounting results. Risk management will be shown to be a very effective way of reducing this volatility.

We point out that with deterministic interest rates, the two approaches produce almost the same shareholder values, assuming the insurer does not default. This is because $CF(0) - V(0) \approx FP(0)$, i.e. the reserve does not affect total profit, but just the timing of its emergence.

https://doi.org/10.1017/asb.2016.32 Published online by Cambridge University Press

TABLE 2

ECONOMIC VALUE WITHOUT HEDGING.

Economic Value			
Portfolio Size	1,399		
1-Year Default Probability	0.79%		
	Expected Value	*CoV*	*Expected Value per Policy*
Total Assets	$19,597,515		$14,008
Technical Provisions	$17,893,499		$12,790.00
Expenses	$896,093	5.7%	$640.52
Frictional Costs	$142,612	12.7%	$101.94
Recapitalization Costs	$27,164	69.1%	$19.42
LLPO	$4,131	961%	$2.95
EV	**$642,277**	**16.42%**	**$459.10**
Total Liabilities	**$19,597,515**		**$14,008.23**

TABLE 3

VIF WITHOUT HEDGING.

MCEV			
Portfolio Size	1,399		
1-Year Default Probability	0.79%		
	Expected Value	*CoV*	*Expected Value per Policy*
Future Profits	$807,598	116.5%	$577.27
Frictional Costs	$142,612	12.7%	$101.94
Recapitalization Costs	$27,164	69.1%	$19.42
LLPO	$4,131	961%	$2.95
VIF	**$641,952**	**148.24%**	**$458.87**

In the following, we consider the situation where policyholder demand is based on the insurer always meeting the Solvency II requirement of a 1-year default probability of 0.5%. We determine the optimal premium loading. We consider different combinations of longevity risk transfer and relief from solvency capital requirements. Levels of reinsurance transfer, ω_h, are set either to 50% or 100%. Levels of capital relief, ω_c, are set either to 50% or 100%. The findings are compared to the case of no hedging. This allows us to assess the impact of both risk management and capital relief on shareholder value.

4.2. Demand function for a solvency II default probability

We use a default probability in the demand function equivalent to 0.5% per year. The demand function in this case is given in Figure 5. Using the market

https://doi.org/10.1017/asb.2016.32 Published online by Cambridge University Press

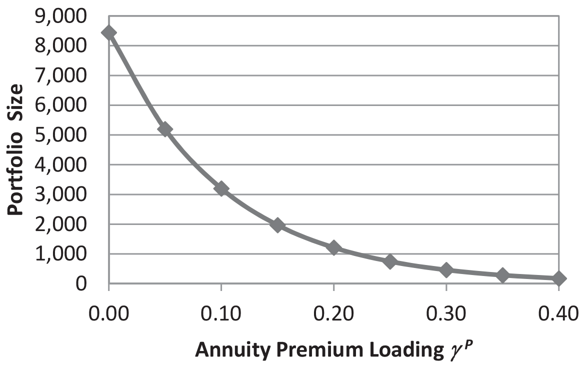


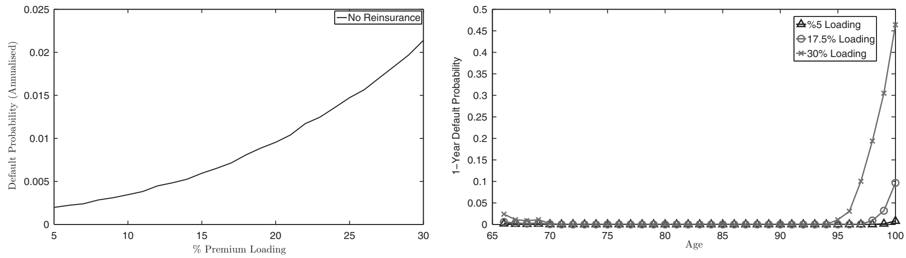
FIGURE 5: Portfolio size vs. premium loading.

size of 25, 000 annuitants, a premium loading of 5% results in a portfolio of 5,195 annuitants based on the annuity demand function. A premium loading of 30% reduces the size of the portfolio to 459. Because of this reduction in demand and the resulting increase in idiosyncratic longevity risk, premium loadings have significant impacts on solvency.

Figure 6a shows the sensitivity of the insurer's actual annualized 1-year default probability to the premium loading for the no hedging case. The default probability is determined from the simulation results; the number of defaulting paths divided by the total number of paths. Default can occur at any time during the annuity contract; thus, we annualize the default probability for easier comparison. The increasing default probability with premium loading results only from the reduced portfolio size. When the premium loading reaches 15%, or a portfolio size of 1,968, the default probability is above the Solvency II requirement of 0.5%.

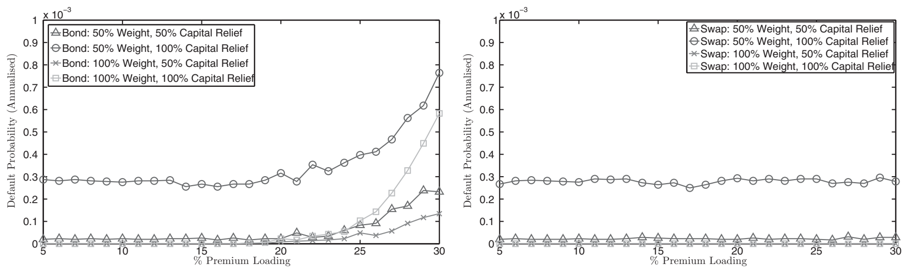
Figure 6b shows the default probabilities with age for a number of premium loadings. Without hedging, defaults occur in the early years and also the later years of the annuity contract, especially for the higher premium loadings. For most years, the insurer holds sufficient capital to avoid insolvency. For the 30% premium loading case, the default probability is above the 0.5% Solvency II requirement until the age of 70. In the 17.5% loading case, the default probability is above 0.5% in the first year only, while the 5% loading case is below the required 0.5% for all years except the final year of the contract. This is determined by the portfolio size and the resulting idiosyncratic risk, especially at the older ages. For an insurer charging higher premiums, a swap agreement will be more desirable because it is indemnity-based and hedges this insolvency risk.

Figures 7a and 7b show the annualized 1-year default probability with the survivor bond and survivor swap, respectively. For the bond, a higher premium loading also corresponds to having higher default probabilities, reflecting the smaller portfolio sizes. For the survivor swap, this effect does not occur since the idiosyncratic risk is hedged. In all cases, the default probability is below the



(a) (b)

FIGURE 6: 1-year default probability. (a) No reinsurance. (b) Default probability by age.



(a) (b)

FIGURE 7: 1-year default probability. (a) Bond. (b) Swap.

Solvency II requirement for all premium loadings with hedging. Capital relief has no effect when the insurer is fully hedged.

Capital relief can have an adverse effect on solvency with less than full hedging of longevity risk. This is because capital relief reduces the amount of capital too much and a simple one-to-one offset for the hedged risk is not optimal.

4.3. Shareholder value and volatility

Figures 8a and 8b show the expected VIF and EV, respectively, with alternative hedging solutions and capital relief assumptions. A premium loading of 11% or greater is required to generate a positive expected value for shareholders on a risk-adjusted basis. The shareholder value is increasing until a 20% premium loading, based on the price elasticity of the demand function.[5] For any fixed premium loading, there are small gains to the expected VIF and EV values when the insurer transfers longevity risk. This reflects the low level of frictional costs for the life insurer. The VIF is increasing in reinsurance weight and capital relief, with the survivor swap and survivor bond producing similar results. Although risk management can reduce frictional costs, the value of this for a life insurer with long-term business is not major.

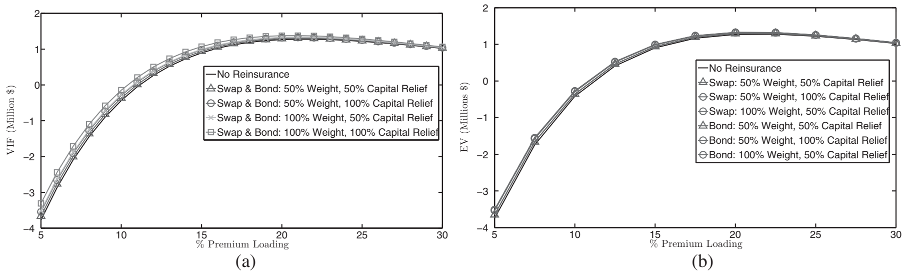


FIGURE 8: VIF and EV, expected value. (a) VIF, expected value. (b) EV, expected value.

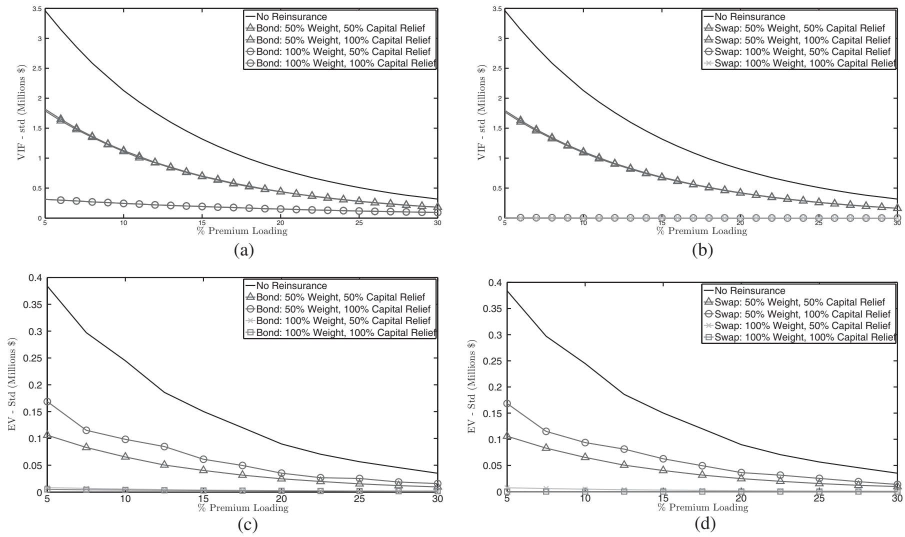


FIGURE 9: VIF and EV, volatility. (a) VIF, voltality — Bond. (b) VIF, voltality — Swap. (c) EV, voltality — Bond, (d) EV, voltality — Swap.

Figure 9 shows the impact of hedging on the VIF and the EV volatility. Figures 9a and 9b show the significant benefits of transferring longevity risk in the reduced volatility of the VIF. Once again, capital relief at 100% of the hedged risk is not optimal when only partially hedged. There is little difference between the two for the EV. However, volatility is very different as shown in Figures 9c and 9d. Note the much smaller scale used for EV in the figures.

4.4. Shareholder dividend and recapitalization Strategy

Figure 10 shows the present value and volatility of dividends distributed to shareholders over the life of the annuity contract for varying premium loadings, and for alternative hedging solutions. There is a point between a premium loading of 13% and 16%, where the value no longer reduces as shown in Figure

https://doi.org/10.1017/asb.2016.32 Published online by Cambridge University Press

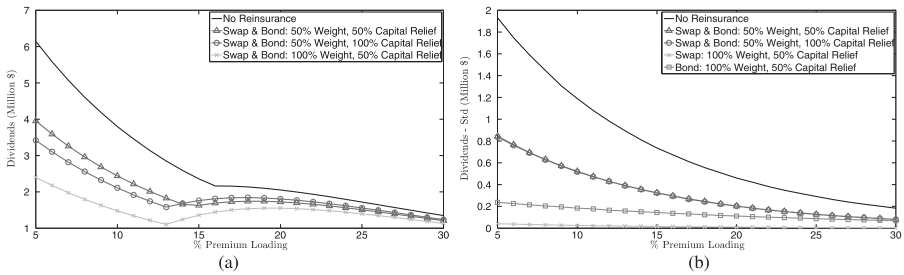


FIGURE 10: Dividend. (a) Dividend, expected value. (b) Dividend, volatility.

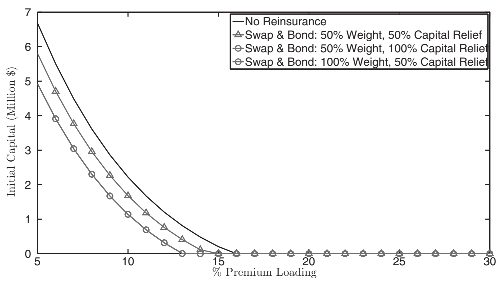


FIGURE 11: Initial shareholder capital.

10a. Figure 11 shows that this point corresponds to where the premium loading is sufficient to meet reserving and regulatory requirements, with no initial shareholder capital required. For lower premium loadings, the initial shareholder capital is returned as dividends in addition to the profits from the annuity premiums. The benefits of risk management on the volatility of dividends is shown in Figure 10b. Hedging longevity risk results in a significant reduction in dividend volatility.

The shareholder recapitalization amounts, excluding initial shareholder capital, are shown in Figure 12. Recapitalization is reduced by hedging longevity risk, regardless of premium loading. The volatility of recapitalization also reduces significantly. The extent of capital relief does not have a significant impact on recapitalization requirements.

4.5. Frictional costs

Figure 13 shows the reduction in expected value and volatility of frictional costs. Frictional costs are based on the difference between the total reserves, $\widetilde{V}_l(t)$, and the market value of the annuity liability, $\widetilde{V}(t)$. The expected value of frictional costs are reduced with hedging. Since insurer defaults occur mainly in the older ages, this has a minimal effect on the time-0 expected value. The benefits of hedging occur in the reduction in the volatility of frictional costs as seen in Figure 13b.

https://doi.org/10.1017/asb.2016.32 Published online by Cambridge University Press

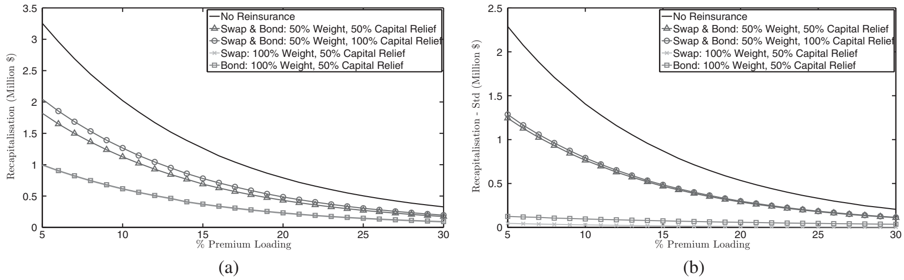


FIGURE 12: Recapitalization. (a) Recapitalization, expected value. (b) Recapitalization, volatility.

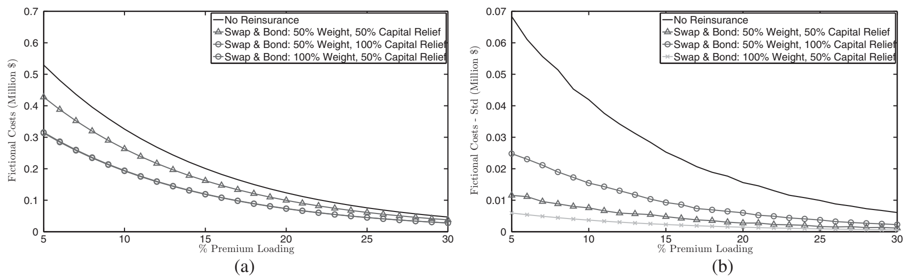


FIGURE 13: Frictional costs. (a) Frictional costs, expected value. (b) Frictional costs, volatility.

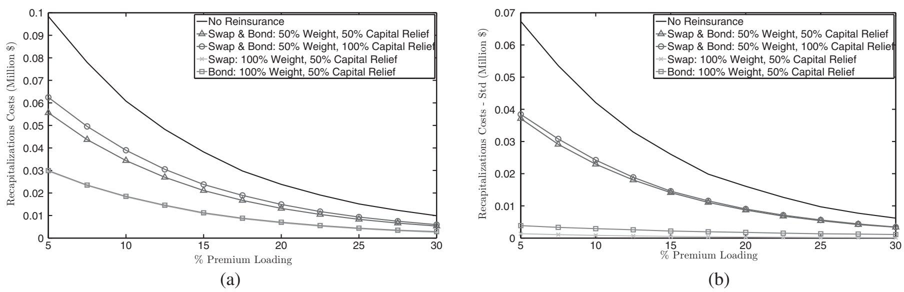


FIGURE 14: Recapitalization costs. (a) Recapitalization costs, expected value. (b) Recapitalization costs, volatility.

Figure 14 shows a reduction in the expected value and volatility of recapitalization costs with hedging. These figures do not include the time-0 initial shareholder contributions, but only ongoing recapitalization costs.

4.6. Expenses

The insurer's expenses are shown in Figure 15. Hedging does not reduce the expected value of expenses, but does reduce the volatility.

https://doi.org/10.1017/asb.2016.32 Published online by Cambridge University Press

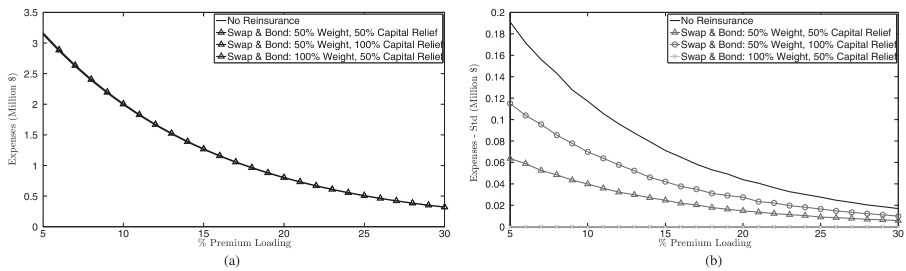


FIGURE 15: Expense. (a) Expenses, expected value. (b) Expenses, volatility.

4.7. Impact of solvency probability

Although the results assume that policyholder demand is determined by the Solvency II default probability, risk management using any amount of hedging will reduce the default probabilities. This increases demand. The results with the actual default probabilities are not shown here, but give the same conclusions as presented. In these cases, shareholder values as a percentage of the total assets do not change significantly.

4.8. Impact of hedging costs and financial risks

Our model uses the market price of longevity risk for a consistent valuation of insurance liabilities and of the two longevity risk transfer arrangements. We do not allow for additional profit loadings on the prices of the survivor swap and the survivor bond imposed by the counterparties involved in these transactions. Introducing such profit loadings would reduce the annuity provider's annual profits, resulting in a lower shareholder value (VIF and EV) and higher default probabilities. This would make longevity risk transfer less attractive to the shareholders of the annuity provider than the "No Reinsurance"-strategy where longevity risk is retained by the annuity provider. Loadings that differ across longevity risk transfer arrangements would of course determine the comparison between the survivor swap and the survivor bond. While this aspect must not be disregarded in practice, the main conclusions of our investigation, and in particular the valuation approach, remain important also in view of a practical assessment of longevity risk management strategies.

We disregard risks other than longevity risk to focus on this risk and to keep the model's complexity at a reasonable level. In practice, annuity providers need to identify, assess and manage all relevant risks, including financial risks such as interest rate risk, when developing their risk management strategy. In practice, they hedge interest rate risks through investment strategies such as immunization so that this risk is minimal leaving only the longevity risk to be hedged. But the market for this is limited and usually this is done with reinsurance using a longevity swap. There are papers that consider the hedging of longevity risk and interest rate risk including Luciano *et al.* (2012); Ngai and Sherris (2011); Liu and Sherris (2015) but do not consider the impact of pricing and solvency along with the modeling and hedging of these risks. Our aim has been to incorporate these pricing and solvency issues into the model with an emphasis on the risks that cannot readily be hedged with financial instruments. Hence, the focus on longevity risk and its hedging and impact on solvency and pricing.

5. Conclusions

We investigate the impact of longevity risk management on shareholder value for a life insurer issuing life annuities. We develop a rather comprehensive stochastic model, in order to address the main drivers of the solvency and value of a life annuity business. We focus on longevity risk only and we perform the assessment based on a single cohort. This allows us to make clearer a number of important aspects of the valuation of a business exposed to longevity risk.

We analyze how longevity risk management is successful in reducing the default probability of the insurer. We show that this results from a reduction in the volatility of cash flows. We use an EV and an MCEV approach; while the former is the natural approach in a fair value setting, the embedded value is a traditional and popular actuarial model for the valuation of the life insurance business. It is therefore important to develop it in a market-consistent manner (and in some respects, this is already performed in actuarial practice), and make appropriate comparisons. We show that the MCEV approach generates volatility in future profits in a stochastic model because of the re-spreading of the initial annuity premium to match future outgoes. This volatility can be significantly reduced when hedging longevity risk.

For the hedging solutions, we show that both survivor swaps and bonds reduce volatility. Survivor swaps provide an indemnity-based hedge and are most effective in reducing risk. The index-based survivor bond does not hedge the idiosyncratic risk. This is an important factor, especially in the older ages of a cohort, and has a significant impact on solvency. Capital relief for hedged risk should be carefully assessed. Taking too much capital relief reduces capital to the extent that it has an adverse impact on the solvency of the insurer.

We incorporate a dividend strategy that maintains the solvency capital requirements under Solvency II along with market consistent risk margins. We show that an important benefit of hedging is the reduction in the volatility of the dividends. Since shareholders will value the stability of dividends, this is a benefit of hedging not captured in standard shareholder valuation models. We also demonstrate how Solvency II capital requirements are inadequate at the older ages of a cohort because of idiosyncratic risk.

The multi-period stochastic shareholder value model developed in this paper deals with important practical aspects such as the trade-off between solvency and premium loading and the valuation of the limited liability put option which so far have not been considered appropriately in the longevity risk management literature.

Acknowledgements

The authors would like to acknowledge the financial support of ARC Linkage Grant Project LP0883398 Managing Risk with Insurance and Superannuation as Individuals Age with industry partners PwC, APRA and the World Bank as well as the support of the Australian Research Council Centre of Excellence in Population Ageing Research (CEPAR) (project number CE110001029). A. Olivieri also acknowledges partial funding from the Italian MUR.

Notes

1. The European Insurance and Occupational Pensions Authority (EIOPA) publishes documents relating to the Quantitative Impact Study (QIS) at http://archive.eiopa.europa.eu/consultations/qis/insurance/quantitative-impact-study-5/index.html

2. The Australian market for lifetime annuities is dominated by a single provider, who reports lifetime annuity sales of AUD 389.4 million in 2015 and AUD 662.1 million in 2014 (http://www.challenger.com.au/group/1H16_Analyst_Pack.pdf).

3. We are not aware of an empirical study that provides estimates of the price and default risk sensitivity of the demand for life annuities.

4. Fama and French (2002) report Sharpe ratios of 0.15–0.44 (depending on whether dividend growth, earning growth or real returns are used to calculate the equity premium) for the S&P 500 index over the period 1951–2000 (see Table I in Fama and French, 2002). Bauer *et al.* (2010) compare different methods for estimating the market price of longevity risk and calculate Sharpe ratios of 0.0371–0.1209 based on UK pension annuity data (see Table 1 in Bauer *et al.*, 2010).

5. A similar hump-shaped relationship between premium loading and shareholder value was found by Nirmalendran *et al.* (2013) in a multi-period cash flow model for a life insurer offering lifetime guaranteed annuities calibrated using Australian market data. The estimated optimal range of premium loadings compares with premium loadings estimated for the Australian annuity market. For nominal life annuities sold to 65-year-old males, assuming general population mortality rates, the results of James and Vittas (2001) indicate a premium loading of 8.6%, while the findings of Doyle *et al.* (2004) suggest a premium loading of 12.1%, and Ganegoda and Bateman (2008) report a 24% premium loading.

References

Babbel, D.F. and Merrill, C.B. (2006) *Rational Decumulation.* Philadelphia: Wharton School, University of Pennsylvania.

Bauer, D., Börger, M. and Ruß, J. (2010) On the pricing of longevity-linked securities. *Insurance: Mathematics and Economics*, **46**(1), 139–149.

Bauer, D., Börger, M., Ruß, J. and Zwiesler, H.-J. (2008) The volatility of mortality. *Asia-Pacific Journal of Risk and Insurance*, **3**(1), 1–35.

Biffis, E. and Blake, D. (2010) Securitizing and tranching longevity exposures. *Insurance: Mathematics and Economics*, **46**(1), 186–197.

Biffis, E., Denuit, M. and Devolder, P. (2010) Stochastic mortality under measure changes. *Scandinavian Actuarial Journal*, **2010**(4), 284–311.

Blackburn, C. (2013) Longevity Risk Management and Securitisation in an Affine Mortality Modelling Framework. University of New South Wales, PhD Thesis.

Blackburn, C. and Sherris, M. (2013) Consistent dynamic affine mortality models for longevity risk applications. *Insurance: Mathematics and Economics*, **53**(1), 64–73.

Blake, D. and Burrows, W. (2001) Survivor bonds: Helping to hedge mortality risk. *Journal of Risk and Insurance*, **68**(2), 339–348.

BLAKE, D., CAIRNS, A., DOWD, K. and MACMINN, R. (2006) Longevity Bonds: Financial Engineering, Valuation, and Hedging. *Journal of Risk & Insurance*, **73**(4), 647–672.
BLAKE, D., DE WAEGENAERE, A., MACMINN, R. and NIJMAN, T. (2010) Longevity risk and capital markets: The 2008-2009 update. *Insurance: Mathematics and Economics*, **46**(1), 135–138.
CAIRNS, A., BLAKE, D. and DOWD, K. (2006) Pricing death: Frameworks for the valuation and securitization of mortality risk. *ASTIN Bulletin*, **36**(1), 79–120.
COUGHLAN, G., EPSTEIN, D., SINHA, A. and HONIG, P. (2007) q-forwards: Derivatives for transferring longevity and mortality risk. Tech. rep., JPMorgan Pension Advisory Group.
COWLEY, A. and CUMMINS, J. (2005) Securitization of life insurance assets and liabilities. *Journal of Risk and Insurance*, **72**(2), 193–226.
CUMMINS, J. D. and TRAINAR, P. (2009) Securitization, Insurance, and Reinsurance. *Journal of Risk and Insurance*, **76**(3), 463–492.
DAHL, M. (2004) Stochastic mortality in life insurance: Market reserves and mortality-linked insurance contracts. *Insurance: Mathematics and Economics*, **35**(1), 113–136.
DOWD, K., BLAKE, D., CAIRNS, A. and DAWSON, P. (2006) Survivor swaps. *Journal of Risk and Insurance*, **73**(1), 1–17.
DOYLE, S., MITCHELL, O. S. and PIGGOTT, J. (2004) Annuity values in defined contribution retirement systems: Australia and Singapore compared. *Australian Economic Review*, **37**(4), 402–416.
DUFFIE, D. and SINGLETON, K. J. (1999) Modeling term structures of defaultable bonds. *Review of Financial Studies*, **12**(4), 687–720.
FAMA, E. F. and FRENCH, K. R. (2002) The equity premium. *The Journal of Finance*, **57**(2), 637–659.
FROOT, K. A. (2007) Risk management, capital budgeting, and capital structure policy for insurers and reinsurers. *Journal of Risk and Insurance*, **74**(2), 273–299.
GANEGODA, A. and BATEMAN, H. (2008) *Australia's disappearing market for life annuities*. UNSW Centre for Pensions and Superannuation Discussion Paper 1 (08), Australian School of Business, University of New South Wales, Sydney.
GLASSERMAN, P. (2003) *Monte Carlo Methods in Financial Engineering*, 1st Edition. Springer, New York.
GRÜNDL, H., POST, T. and SCHULZE, R. N. (2006) To hedge or not to hedge: Managing demographic risk in life insurance companies. *Journal of Risk and Insurance*, **73**(1), 19–41.
GUPTA, A. and WANG, H. (2011) Assessing securitisation and hedging strategies for management of longevity risk. *International Journal of Banking, Accounting and Finance*, **3**(1), 47–72.
HEATH, D., JARROW, R. and MORTON, A. (1992) Bond pricing and the term structure of interest rates: A new methodology for contingent claims valuation. *Econometrica*, **60**(1), 77–105.
JAMES, E. and VITTAS, D. (2001) Annuity markets in comparative perspective: Do consumers get their money's wotrth? In: Private Pensions Series OECD 2000 Private Pensions Conference. Vol. 3. OECD Publishing, p. 313.
KRVAVYCH, Y. and SHERRIS, M. (2006) Enhancing insurer value through reinsurance optimization. *Insurance: Mathematics and Economics*, **38**(3), 495–517.
LAKDAWALLA, D. and ZANJANI, G. (2012) Catastrophe bonds, reinsurance, and the optimal collateralization of risk-transfer. *The Journal of Risk and Insurance*, **79**(2), 449–476.
LANDO, D. (1998) On cox processes and credit risky securities. *Review of Derivatives Research*, **2**(2–3), 99–120.
LEVANTESI, S. and MENZIETTI, M. (2008) Longevity risk and reinsurance strategies for enhanced pensions. In *MTISD 2008 - Methods, Models and Information Technologies for Decision Support Systems*, pp. 195–198. New York: Lecce. Available at: http://sibaese.unisalento.it/index.php/MTISD2008/issue/current.
LIU, C. E. and SHERRIS, M. (2015) *Immunization and Hedging of Longevity Risk*. UNSW Business School Research Paper (2015ACTL12), Australia: University of New South Wales.
LUCIANO, E., REGIS, L. and VIGNA, E. (2012) Delta–gamma hedging of mortality and interest rate risk. *Insurance: Mathematics and Economics*, **50**(3), 402–412.
MACMINN, R. and RICHTER, A. (2011) The Choice of Trigger in an Insurance Linked Security: The Mortality Risk Case. Tech. rep., Illinois State University.

MILEVSKY, M. and PROMISLOW, S. D. (2001) Mortality derivatives and the option to annuitise. *Insurance: Mathematics and Economics*, **29**(3), 299–318.
MILEVSKY, M. A., PROMISLOW, S. D. and YOUNG, V. R. (2005) Financial Valuation of Mortality Risk via the Instantaneous Sharpe Ratio: Applications to Pricing Pure Endowments. Quantitative finance papers, arXiv.org.
MILTERSEN, K. R. and PERSSON, S.-A. (2005) Is mortality dead? Stochastic forward force of mortality rate determined by no arbitrage. Tech. Rep. June 2005, Norwegian School of Economics.
NGAI, A. and SHERRIS, M. (2011) Longevity risk management for life and variable annuities: The effectiveness of static hedging using longevity bonds and derivatives. *Insurance: Mathematics and Economics*, **49**(1), 100–114.
NIRMALENDRAN, M., SHERRIS, M. and HANEWALD, K. (2013) Pricing and solvency of value-maximizing life annuity providers. *ASTIN Bulletin*, **44**(01), 39–61.
OLIVIERI, A. (2005) Designing longevity risk transfers: The point of view of the cedant. Giornale dell'Istituto Italiano degli Attuari LXVIII, 1–35, reprinted on: ICFAI Journal of Financial Risk Management Issue March 2007.
OLIVIERI, A. and PITACCO, E. (2003) Solvency requirements for pension annuities. *Journal of Pension Economics and Finance*, **2**(02), 127–157.
OLIVIERI, A. and PITACCO, E. (2008) Assessing the cost of capital for longevity risk. *Insurance: Mathematics and Economics*, **42**(3), 1013–1021.
PLAT, R. (2011) One-year value-at-risk for longevity and mortality. *Insurance: Mathematics and Economics*, **49**(3), 462–470.
SCHÖNBUCHER, P. J. (1998) Term structure modelling of defaultable bonds. *Review of Derivatives Research*, **2**(2–3), 161–192.
SWISS RE (2005) Insurer's cost of capital and economic value creation: Principles and practical implications. Swiss Re Technical Publishing Sigma 3.
TAN, K. S., BLAKE, D. and MACMINN, R. (2015) Longevity risk and capital markets: The 2013–2014 update. *Insurance: Mathematics and Economics*, **63**, 1–11.
WILLS, S. and SHERRIS, M. (2010) Securitization, structuring and pricing of longevity risk. *Insurance: Mathematics and Economics*, **46**(1), 173–185.
YOW, S. and SHERRIS, M. (2008) Enterprise risk management, insurer value maximisation, and market frictions. *ASTIN Bulletin*, **38**(1), 293–339.
ZANJANI, G. (2002) Pricing and capital allocation in catastrophe insurance. *Journal of Financial Economics*, **65**(2), 283–305.
ZIMMER, A., GRÜNDL, H. and SCHADE, C. (2011) Price-Default Risk-Demand-Curves and the Optimal Corporate Risk Strategy of Insurers: A Behavioral ApproachHumboldt-Universität zu Berlin and Goethe University Frankfurt.
ZIMMER, A., SCHADE, C. and GRÜNDL, H. (2009) Is default risk acceptable when purchasing insurance? Experimental evidence for different probability representations, reasons for default, and framings. *Journal of Economic Psychology*, **30**(1), 11–23.

CRAIG BLACKBURN
ARC Centre of Excellence in Population Ageing Research (CEPAR)
University of New South Wales
Sydney NSW 2052, Australia
E-Mail: c.blackburn@unsw.edu.au

KATJA HANEWALD (Corresponding author)
ARC Centre of Excellence in Population Ageing Research (CEPAR)
University of New South Wales
Sydney NSW 2052, Australia
E-Mail: k.hanewald@unsw.edu.au

https://doi.org/10.1017/asb.2016.32 Published online by Cambridge University Press

ANNAMARIA OLIVIERI
Dipartimento di Economia
University of Parma
Via Kennedy, 6
43100 Parma, Italy
E-Mail: annamaria.olivieri@unipr.it

MICHAEL SHERRIS
School of Risk and Actuarial Studies and ARC Centre of Excellence in Population Ageing Research (CEPAR)
University of New South Wales
Sydney NSW 2052, Australia
E-Mail: m.sherris@unsw.edu.au

https://doi.org/10.1017/asb.2016.32 Published online by Cambridge University Press