Curves for the mathematically curious by Julian Havil, pp. 259, £14.99 (paper), ISBN 978-0-69120-613-4, Princeton University Press (2019)

Mathematicians often speak of the subject's beauty, but a frequent barrier to entry for the uninitiated is the fact that a lot of beautiful results (such as an ingenious isomorphism between seemingly unrelated groups, or the extraordinary properties of the Riemann zeta function) require a significant level of mathematical expertise to appreciate. Even complete mathematical laypeople, however, can recognise visual beauty when they see it, and it is perfectly possible to be awed by constructions such as the Mandelbrot set without having ever been introduced to the complex plane. That said, it becomes ever more fascinating the further you dig into the underlying mathematics, and so it goes with the collection of curves presented in this book. Some, such as the Euler spiral and the two space-filling curves, are beautiful enough in their own right to draw the eye of any onlooker, but it takes some understanding to appreciate the inclusion of comparatively pedestrian looking curves such as the rectangular hyperbola and the catenary. What is certain, however, is that even the most seasoned mathematicians are sure to find something to stimulate their curiosity amongst this anthology of two-dimensional Euclidean gems.

The curves in question are a rather eclectic mix, and each chapter can be taken completely in isolation. In fact, the author admits that the chapters themselves are simply ordered by word count, and there is no expectation that the order should be adhered to. That said, they all exhibit the same key features which gives a sense of structure to what otherwise might have become quite a meandering affair. First, the curve itself (or at least an example thereof) is displayed at the beginning of each chapter. Directly underneath, the author briefly explains why it was chosen, although this section is often used to pique the reader's interest and frequently raises more questions than it answers (such as when it is confidently stated that the catenary "makes a fascinating road surface"). Then the exploration of the curve begins in earnest, sometimes with a seemingly unrelated discussion, perhaps of mathematicians who never existed or a Scotsman who fought in the Russian army, and the connection to the curve at hand is revealed only slowly. Sometimes we dive right into the definition of the curve, as with the final chapter on elliptic curves, leaving the historical context for a later section. Either way, the result is never dry, as the author is an expert at injecting character and humour into every paragraph, managing to make even the most technical descriptions a pleasure to read. Furthermore, every curve that has been chosen comes with an intriguing narrative which helps to break up some of the more dense mathematics.

That said, it should be emphasised that the mathematics involved is challenging, and even the brightest A Level students (or equivalent) would struggle to appreciate many of the finer details. It is possible to read the chapters for the narratives alone and simply gloss over the mathematical explanations, but the book has so much more to offer if you have the time and ability to work through everything yourself, filling in the gaps with pencil and paper to hand. Given that the very first page jumps straight into the parametric equations for arc length and curvature and that the rest of the book touches topics such as real analysis, number theory and group theory with only the briefest of introductions, I think it's safe to say that undergraduate mathematics would be a reasonable prerequisite to be able to access all that this book has to offer. A younger or more inexperienced mathematician would certainly get a lot of enjoyment from the book thanks to the quality of its prose, but they certainly shouldn't expect to be led by the hand through any of the trickier sections; the author moves fast and has high expectations of the reader.

For those who can keep up, however, there are plentiful rewards. A personal favourite came in the chapter on the normal distribution. After some exploration of the historical circumstances that led to its discovery, the author takes a moment to examine an alternative derivation. We start from three natural assumptions about the

https://doi.org/10.1017/mag.2022.49 Published online by Cambridge University Press

distribution of errors in attempting to hit a target; namely, that the two perpendicular directions are independent, that the orientation shouldn't matter and that the probability densities should decrease as we move away from the target. There follow a series of mathematical sleights of hand that result in the almost miraculous appearance of that instantly recognisable e^{-kx^2} term, and which felt truly magical on first reading. While I'm sure many others are already familiar with this extraordinarily elegant derivation, the sheer breadth of mathematics being explored all but guarantees that every reader will experience similar moments of awe and wonder, provided they have the tools to recognise them.

Overall, the book was a delight to read. The writing is witty and entertaining, the history is at times peculiar and surprising, and the mathematics is rich and engaging. It would make a fine addition to a classroom bookcase or home coffee table, but while there are plenty of elegant diagrams and intriguing stories to give every curious reader the chance to glimpse mathematical beauty, only those with the ability to dig beneath the surface will understand just how much beauty this book has to offer.

10.1017/mag.2022.49

SAMUEL HEWITT
St Paul's School, Lonsdale Rd,
Barnes, London SW13 9JT
e-mail: *sh@stpaulsschool.org.uk*

Introduction to the theory of optimization in Euclidean space by Samia Challal, pp. 318, £73.59 (hard), ISBN 978-0-36719-557-1, also available as e-book, CRC Press (2019)

This book, part of the CRC *Series on Operations Research*, is designed for undergraduate courses in operations research and mathematics. It starts at a fairly basic level, with open and closed sets, functions of more than one variable, surfaces in three dimensions, and partial differentiation. The three main chapters are accurately described by their titles: Unconstrained optimisation; Constrained optimisation – equality constraints; and Constrained optimization – inequality constraints. The first of these introduces the ideas of convexity and also the Extreme Value Theorem. The second and third chapters are dominated by the use of Lagrangians. As the title suggests, the focus throughout is theoretical rather than practical, with consideration of regularity and the Karush-Kuhn-Tucker (KKT) conditions, and much is made of the Hessian matrix.

The main body of the text consists of theorems, proofs, remarks and worked examples. The proofs are written in quite heavily mathematical style, with little attempt to explain or to encourage conceptualisation, and references to some theorems (the Farkas-Minklowski Lemma is picked out by the author in the preface) are not explained at all, the reader being advised to consult other texts. There are some useful monochrome diagrams, particularly in the first chapter. The bulk of the learning is achieved through the large number of solved problems, with solutions presented in full detail, and anyone who works through them all will have a firm knowledge and detailed understanding of the content. There are not, however, any unsolved exercises; again the reader is referred to other texts, though no doubt the solved problems can be used by careful learners as problems-to-be-solved before the solutions are consulted.

If the book were not so expensive it might be worth the attention of school teachers preparing to teach, for instance, the OCR Additional Pure option in Further Mathematics; it gives good examples of 3D surfaces and functions of several variables, as well as further insights into the Hessian matrix. The Newton-Raphson method makes a rather unexpected appearance, together with a statement previously