Forum

Rhumbline Distances

Captain Ivica Tijardović

This paper compares the differences between the rhumbline courses and distances calculated by three methods: the classical formula, precise formula and by software contained in a typical GPS receiver. It concludes that, while the calculations made by the GPS receiver are generally more accurate than the classical method, navigators must use Tables of Latitude Parts to calculate rhumbline distances accurately.

KEY WORDS

1. Planning. 2. Rhumblines. 3. Geodesy.

1. INTRODUCTION. One of the most important tasks for every marine navigator is to determine accurately the rhumbline course and distance between the positions of departure and destination, or the course and distance between two waypoints. This course and distance can be obtained in a number of different ways, but some of the most common methods are:

(i) by means of the navigation computations of a Global Positioning System (GPS) receiver;
(ii) by calculation using classical formula (difference of latitude in minutes multiplied by the secant of the rhumbline course);
(iii) by calculation using a precise formula (difference of latitude parts in minutes multiplied by the secant of the rhumbline course).

This paper compares the differences between the courses and distances calculated by these three methods. The PC used to achieve this had a numeric precision of 10 digits for the mantissa and 2 digits for the exponent. For the calculations at (i), the method employed in the Furuno–PS8000 GPS receiver was used as an example. This method determines the distance from a model of meridional parts for a sphere, which are then converted to nautical miles for a specified spheroid.

The use of GPS receivers is now well established in the marine community for oceanic positioning and navigation. Clearly, other manufacturers of GPS receivers may use other methods to those employed by Furuno in the PS8000, but it is difficult to obtain details of the software and methods they use; thus the accuracy and efficiency of their methods cannot be assessed.

2. EXAMPLE. The three methods were used to calculate the rhumbline distance between the following two points:

$$P_1(\phi = 10° \, 50' \text{ N}, \lambda = 126° \text{ E}) \text{ and } P_2(\phi = 34° \text{ N}, \lambda = 120° \, 50' \text{ W}).$$

While this is a rather extreme example because of the large change in longitude, it serves to illustrate the differences in the methods and calculations. More realistic applications can be derived from the tables given later.

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2.1. *Using the Calculations of a Example GPS Receiver*.

$$\delta\phi = 1390' \text{ (difference of latitude)}$$
$$\delta\lambda = 6790' \text{ (difference of longitude)}$$

$$M_2 = 3437{\cdot}746771 \times \ln \tan\left(45° + \frac{34°}{2}\right) = 2171{\cdot}48'$$

$$-M_1 = 3437{\cdot}746771 \times \ln \tan\left(45° + \frac{10°\ 50'}{2}\right) = -653{\cdot}91'$$

Difference of meridional parts for a sphere, $M = 1517{\cdot}57'$.

2.1.1. *Rhumbline Course*.

$$\tan C_L = \frac{\delta\lambda}{M} = \frac{6790}{1517{\cdot}57}.$$

Therefore the rhumbline course $C_L = 077°\ 24'\ 05''$. (When these positions were inserted as waypoints in the Furuno–PS8000, it indicated $C_L = 77°$.)
2.1.2. *Rhumbline Distance*.

$$D_L = \delta\phi \times \sec C_L = 1300 \times \sec 77°\ 24'\ 05'' = 6372{\cdot}65 \text{ geographical miles.}$$

For WGS 84, $\dfrac{\text{geographical mile}}{\text{nautical mile}} = 1{\cdot}001795274.$

Therefore the rhumbline distance $D_L = 6372{\cdot}65 \times 1{\cdot}001795274 = 6384{\cdot}1$ nautical miles. (Furuno–PS8000 indicates $D_L = 6384{\cdot}1$.)
2.2. *Using Calculations the Classical Way*. For WGS 84, e = 0·08181919034.

$$M_2 = 7915{\cdot}70447 \times \log\left(\tan\left(45° + \frac{34°}{2}\right) \times \left(\frac{1 - e \times \sin 34°}{1 + e \times \sin 34°}\right)^{\frac{e}{2}}\right) = 2158{\cdot}60'$$

$$-M_2 = 7915{\cdot}70447 \times \log\left(\tan\left(45° + \frac{10°\ 50'}{2}\right) \times \left(\frac{1 - e \times \sin 10°\ 50'}{1 + e \times \sin 10°\ 50'}\right)^{\frac{e}{2}}\right) = -649{\cdot}58'$$

Difference of meridional parts for the spheroid $M = 1509{\cdot}02'$.

2.2.1. *Rhumbline Course*.

$$\tan C_L = \frac{\delta\lambda}{M} = \frac{6790}{1509{\cdot}02}.$$

Therefore the rhumbline course $C_L = 077°\ 28'\ 12{\cdot}5''$.
2.2.2. *Rhumbline Distance*.

$$D_L = \delta\phi \times \sec C_L = 1390 \times \sec 77°\ 28'\ 12{\cdot}5'' = 6407{\cdot}1 \text{ nautical miles.}$$

2.3. *Using Calculations for the Precise Way*.

$$\text{Latitude parts of } \phi_2\ (\text{LP}_2) = 2028{\cdot}573$$
$$-\text{Latitude parts of } \phi_1\ (\text{LP}_1) = -645{\cdot}725$$
$$\text{Difference of latitude parts (DLP)} = 1382{\cdot}848.$$

2.3.1. *Rhumbline Course*.

$$\tan C_L = \frac{\delta\lambda}{M} = \frac{6790}{1509{\cdot}02}.$$

Therefore the rhumbline course $C_L = 077°\ 28'\ 12{\cdot}5''$.

Table 1. Differences in nautical miles (1852 m) (rounded) between distances calculated by GPS and the true rhumbline distances.

Latitudes (degrees)		Change of Longitude ($\delta\lambda$) (degrees)						
ϕ_1	ϕ_2	1	10	30	60	90	120	150
0	0	0	0	0	0	0	0	0
	10	4	3	1	1	0	0	0
	20	8	7	4	2	1	0	0
	30	11	10	7	4	2	1	0
	40	13	12	10	6	4	2	0
	50	14	13	12	8	5	3	1
	60	14	13	12	9	7	4	2
10	10	0	0	0	0	−1	−1	−1
	20	4	3	1	0	−1	−1	−2
	30	7	6	3	1	0	−2	−2
	40	9	8	6	3	0	−1	−3
	50	10	9	8	4	1	−1	−3
	60	10	9	8	5	3	0	−2
20	20	0	0	−1	−1	−2	−3	−3
	30	3	w	0	−1	−3	−4	−5
	40	5	4	2	−1	−2	−4	−6
	50	6	6	4	1	−2	−4	−6
	60	6	6	5	2	−1	−3	−5
30	30	0	0	−1	−3	−4	−5	−7
	40	2	1	−1	−3	−5	−6	−8
	50	3	3	1	−2	−4	−6	−8
	60	3	3	1	−1	−4	−6	−8
40	40	0	−1	−2	−4	−6	−8	−10
	50	1	0	−2	−4	−6	−8	−10
	60	1	1	−1	−3	−6	−8	−10
50	50	0	1	−2	−5	−7	−9	−11
	60	0	0	−2	−4	−7	−9	−11
55	55	0	1	−2	−5	−7	−9	−12
	60	0	1	−2	−5	−7	−9	−12

2.3.2. *Rhumbline Distance*.

$$D = \mathrm{DLP} \times \sec C_L = 1382{\cdot}848 \times \sec 77^\circ\ 28'\ 12{\cdot}5'' = 6374{\cdot}09 \text{ geographical miles.}$$

$$D = 6374{\cdot}09 \times 1{\cdot}001795274 = 6385{\cdot}5 \text{ nautical miles.}$$

2.4. *Comparison of Accuracy for this Example*. For this example, the methods used in the GPS receiver, under-reading the true distance by some 1·4 nm, appear to be more accurate than the classical method, which over-estimates the distance by 21·6 nm.

3. GENERAL APPLICATIONS. Tables 1 and 2 have been developed to show the differences in distance calculations for more general applications.

Table 1 gives the differences in nautical miles (rounded) between the calculation methods used in the Furuno GPS Receiver and the precise calculation (true distance) for a variety of latitudes and changes of longitude. Thus a negative difference (−) indicates that the GPS calculation of distance is shorter than the true distance.

Table 2. Differences in nautical miles (1852 m) (rounded) between distances (nm) calculated using classical methods and true rhumbline distances.

Latitudes (degrees)		Change of Longitude ($\delta\lambda$) (degrees)						
ϕ_1	ϕ_2	1	10	30	60	90	120	150
0	0	−0·11	−1	−3	−6	−10	−13	−16
	10	3	4	9	18	26	35	43
	20	5	6	10	17	25	33	40
	30	7	8	10	16	22	29	36
	40	8	8	10	14	19	24	30
	50	8	8	9	12	15	19	23
	60	7	7	8	9	11	13	16
10	10	−0·11	−1	−3	−7	−10	−13	−17
	20	3	4	8	15	22	30	37
	30	4	5	8	13	19	25	31
	40	5	6	7	11	15	20	25
	50	5	5	6	9	11	14	17
	60	4	4	5	6	7	9	11
20	20	−0·12	−1	−4	−7	−11	−15	−19
	30	2	3	5	10	15	20	26
	40	3	3	5	8	11	15	19
	50	3	3	4	5	7	9	11
	60	2	2	2	2	3	4	5
30	−0·14	−1	−4	−8	−12	−16	−21	
	40	1	1	3	5	7	10	12
	50	1	1	1	2	3	4	5
	60	0	0	0	0	0	−1	−1
40	40	−0·15	−1	−4	−9	−13	−18	−22
	50	0	0	0	0	0	−1	−1
	60	−1	−1	−2	−2	−3	−5	−6
50	−0.15	−1	−4	−9	−13	−17	−22	
	60	−1	−1	−2	−4	−6	−8	−9
55	55	−0·14	−1	−4	−8	−13	−17	−21
	60	−1	−1	−2	−4	−7	−9	−11

Table 2 gives the differences in nautical miles (rounded) between the precise calculation (true distance) and the classical methods for a variety of latitudes and changes of longitude. Again a negative difference (−) indicates that the distance obtained using the classical method of calculation is shorter than the true distance.

4. CONCLUSION.

From the tables of calculated differences, it can be concluded that:

(i) close to the equator, distances calculated by the Furuno GPS are almost identical to the precise distances (D = DLP × sec C_L) and are significantly more accurate than distances calculated using classical methods.

(ii) for relatively small changes of latitude and longitude (up to 20 degrees), distances calculated by the Furuno GPS are relatively accurate, but they fall away for larger changes of latitude, and at middle latitudes as the change in longitude increases.

(iii) at middle latitudes, distances calculated using classical methods ($D_L = \delta\phi \times \sec C_L$) are approaching the precise values, except where large changes in longitude are involved.

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(iv) in general, the distances calculated by the Furuno GPS are more accurate than those calculated using the classical method.

The overall conclusion is, that to calculate rhumbline distances accurately, navigators must use Tables of Latitude Parts, despite the advent of GPS receivers and their possible use for navigation calculations.

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