The mathematics lover's companion by Edward Scheinerman, pp. 274, £12.99 (paper), ISBN 978-0-30025-539-3, Yale University Press (2021)

This book was first published in hardback in 2017 but was not reviewed here then. It aims to convey the excitement and pleasure of mathematics through a large number of topics that are generally unfamiliar to the mathematically inexperienced (a little of it appears in A-level) but without requiring any difficult techniques. I think it succeeds very well. Scheinerman starts with what he says epitomizes his choice of topics, the proof that there are infinitely many primes; he says that this (i) is not well known to non-mathematicians, (ii) highlights the idea of proof, (iii) relies only on school-level mathematics, (iv) has an element of surprise, and (v) has a real-life application (cryptography). Each of these aspects is discussed in this context, and then there are 22 fairly short chapters on other topics. Many are indeed the sort of thing that any mathematician would like to see in school courses, if only there were time for them.

Scheinerman is not afraid of equations; there is plenty of genuine mathematics here, and it should not frighten either enthusiastic teenagers or adults who have not forgotten their mathematical notation, while teachers should welcome so many really attractive extensions of standard material.

The chapter on Fibonacci is a good deal more sophisticated than is usual in popular books, covering for example identities obtained by double counting and the surprisingly under-appreciated fact that $\phi^n/\sqrt{5}$ rounds off to F_n to the nearest integer for any $n \geqslant 0$. There is a good, concise, explanation of musical tuning systems, a nice limiting process involving dice that tends to $6/\pi^2$, and a brilliantly short and clear explanation of Benford's law, based on invariance under scaling. Other topics discussed include triangle geometry (mentioning four of the standard centres but not the collinearity of three of them), the use of i to produce Pythagorean triples, Euler's theorem, the "average gap" between primes, the derangement problem, Russell's antinomy, algorithms, chaos, non-transitive dice, Arrow's theorem and Newcomb's paradox. Several of these topics are rarely mentioned in popular books. On the cover is the startling heart-shaped locus of $(x^2 + y^2 - 1)^3 = x^2y^3$.

Inevitably any teacher will think of omissions, or believe that some explanations could be improved, but I felt that there are remarkably few of the latter here. It is good to see a chapter on 0.999999999…, but it does not address what usually worries learners in this topic, that the sequence 0.9, 0.99, 0.999, … never actually reaches 1. Naturally an American writer is likely to skate over the concept of limits in a pre-calculus context. Throughout, however, the diagrams are plentiful, and the equations clear and accurate, although there are several ugly line breaks after an equals sign. Explanations are first-class, and I was pleased to see Scheinerman using "alternative" and not "alternate", but I do think that, although the twin-prime conjecture and Goldbach's conjecture may be notorious, they are not infamous.

This is undoubtedly one of the best popular mathematics books of recent years and it is strongly recommended, above all for teachers who would like to rekindle their enthusiasm for mathematics and as a gift for anyone—but especially, I think, any teenager—who loves the subject, as the title suggests.

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