Laser and Particle Beams (2009), *27*, 57–68. Printed in the USA.

doi:10.1017/S0263034609000081

Focusing of dark hollow Gaussian electromagnetic beams in a plasma

M.S. SODHA,[1] S.K. MISHRA,[2] AND S. MISRA[2]
[1]Disha Academy of Research and Education, Disha Crown, Katchna Road, Shankarnagar, Raipur, India
[2]Ramanna Fellowship Program, Department of Education Building, Lucknow University, Lucknow, India

(RECEIVED 24 October 2008; ACCEPTED 11 November 2008)

Abstract

This paper presents an investigation of the focusing of dark hollow Gaussian electromagnetic beams (HGB) in plasma, considering collisional, ponderomotive, and relativistic nonlinearities. A paraxial like approach, in which the parameters are expanded, in terms of radial distance from the maximum of irradiance rather than that from the axis, has been adopted. To highlight the nature of focusing, both critical curves and the divider curves have been obtained as a plot of dimensionless radius vs. power of the beam. The effect of the order of HGB (n), and nature of nonlinearity on self focusing of the beam has also been explored.

Keywords: Hollow Gaussian beam; Nonlinear plasma physics; Self focusing; paraxial

INTRODUCTION

The propagation dynamics of laser beams through nonlinear media has been extensively investigated; among many of the non-linear phenomena, the one which has attracted most attention is the self-focusing of the laser beams (Chioa *et al.*, 1964; Kelley, 1965; Sodha *et al.*, 1976; Rasmussen & Rypdal, 1986; Silberbarg, 1990; Kothari & Abbi, 1990; Snyder *et al.*, 1990; Hora, 1991; Sprangle & Esarey, 1991; Desaix *et al.*, 1991; Karlsson & Anderson, 1992; Milchberg *et al.*, 1995; Berge, 1998; Upadhyaya *et al.*, 2002, Saini & Gill, 2006; Gill & Saini, 2007; Yu *et al.*, 2007), because the nonlinear effects are highly sensitive to the irradiance distribution along the wavefront of the beam, which gets significantly affected by self focusing. The interest in self focusing can also be appreciated in the context of promising applications in laser-plasma interaction, specifically optical harmonic generation (Sprangle & Esarey, 1991; Milchberg *et al.*, 1995; Zhou *et al.*, 1996), X-ray generation (Eder *et al.*, 1994), inertial confinement fusion (Tabak *et al.*, 1994; Deutsch *et al.*, 1996), and laser driven accelerators and particle beams (Sprangle *et al.*, 1988; Umstadter *et al.*, 1996; Mora & Antonsen, 1996; Andreev *et al.*, 1997, 1998; Amiranoff *et al.*, 1998; Sari *et al.*, 2005; Neff *et al.*, 2006; Zhou *et al.*, 2007; Chen *et al.*, 2008; Niu *et al.*, 2008).

Address correspondence and reprint requests to: M.S. Sodha, Disha Academy of Research and Education, Disha Crown, Katchna Road, Shankarnagar, Raipur - 492 007, India. E-mail: msodha@rediffmail.com

So far the main thrust of the theoretical and experimental investigations on self focusing of a laser beam in a plasma has been directed toward the study of the propagation characteristics of a Gaussian beam (Akhmanov *et al.*, 1968; Hora, 1969; Sodha *et al.*, 1974*a*, 1976; Hora, 1975; Jones *et al.*, 1982; Hauser *et al.*, 1992; Esarey *et al.*, 1997; Osman *et al.*, 1999; Umstadter, 2001; Sharma *et al.*, 2003, 2004). Nevertheless, a few papers have been published on the self focusing of super Gaussian beams (Nayyar, 1986; Grow *et al.*, 2006; Fibich, 2007), self trapping of degenerate modes of laser beams (Karlsson, 1992), and self trapping of Bessel beams (Johannisson *et al.*, 2003). Apart from these, great interest has recently been evinced in optical beams with central shadow, usually known as dark hollow beams (DHB) on account of their wide and attractive applications in the field of modern optics, atomic optics, and plasmas (Soding *et al.*, 1995; Kuga *et al.*, 1997; Ovchinnikov *et al.*, 1997; Yin *et al.*, 1998; Song *et al.*, 1999; Xu *et al.*, 2002; Cai *et al.*, 2003; Yin *et al.*, 2003; York *et al.*, 2008). To describe the DHBs (the beam with zero central intensity), several theoretical models like TEM_{01} mode doughnut beam, some higher order Bessel beams, superposition of off-axis Gaussian beams, and dark-hollow Gaussian beams etc. have been introduced (Arlt & Dholakiyia, 2000; Zhu *et al.*, 2002; Ganic *et al.*, 2003; Cai & Lin, 2004; Deng *et al.*, 2005; Mei & Zhao, 2005); a number of experimental methods have been developed for the production of hollow laser beams (Herman & Wiggins, 1991; Wang & Littman, 1993; Lee *et al.*, 1994).

In the relatively recent studies the propagation of various DHBs in paraxial optical systems and turbulent atmosphere, has been investigated in detail (Cai & He, 2006; Cai & Zhang, 2006*a*, 2006*b*; Gao & Lu, 2006; Mei & Zhao, 2006). A review of the literature highlights the fact that the propagation characteristics of DHBs in a plasma or other nonlinear media have not been studied to a significant extent; as an exception, the beam propagation in the TEM_{10} mode has been studied to some extent, in a plasma for regions around the axis and the maximum of irradiance, in the geometrical optics approximation (Sodha *et al.*, 1974b; Sharma *et al.*, 2005). Prakash *et al.* (2006) modified the theory by taking the saturating nature of nonlinearity and diffraction into account and considering a doughnut (TEM_{01}) beam.

In this communication, the authors have investigated the self focusing of dark cylindrical hollow Gaussian beams (HGBs), in which the irradiance along the axis is zero, and the maximum is away from the axis. It should however be realized that some interesting effects (Feit & Fleck, 1988; Vidal & Johnston, 1996; Johnston *et al.*, 1997) predicted by detailed numerical simulation like breaking up into a number of beams can not be recovered in the cylindrical geometry; hence the theory has some limitations, particularly for beams with powers above the critical value. However, since cylindrical beams are commonly used, a theory for cylindrical beams (even approximate) is in order. A paraxial like approach, similar to the one given by *Akhmanov et al.* (1968) and developed by *Sodha et al.* (1974*a*, 1976) has been used in the present analysis; the nonlinear dielectric function has been expanded in terms of radial distance from the maximum of irradiance, rather than that from the axis, as is the case with Gaussian beams.

Before proceeding further, it is instructive to consider the nonlinearities, responsible for focusing in plasma. In collisional plasmas, the electrons get heated to different temperatures in the transverse plane on account of the radial distribution of the field of the beams; in the steady-state the electron temperature distribution is determined by the balance of the Ohmic heating and the power loss by collisions with heavier particles (ions, molecules etc.), and thermal conduction. The radial redistribution of electron density (and thereby the dielectric function) is determined by balancing the gradient of the partial pressure of electrons and ions by the space charge field and making use of the charge neutrality condition. The collisional nonlinearity sets up in periods on the order of $1/\delta_c \nu_e$, where δ_c is the fraction of excess energy lost by an electron in a collision, and ν_e is the electron collision frequency. The role of conduction is significant when $(\delta_c r_0^2/l^2) \leq 1$, where l is the mean free path of electrons; thermal conduction has not been considered in this paper. In this paper, the ions have been considered to be at constant temperature, which is justified when $\nu_{im} > \delta_{c,ei} \nu_{ei}$ and the neutral atoms are abundant enough to provide a constant temperature sink; here ν_{im} refers to the frequency of ion-neutral atom collisions and the subscripts *ei* refer to electron ion collisions. The process of Ohmic heating of electrons by the wave and subsequent loss of energy to ions/atoms is also some times referred to as inverse bremsstrahlung.

For collisionless plasma, the ponderomotive force on electrons, proportional to the gradient of irradiance at a point causes a redistribution of electron density, and thereby the dielectric function; this nonlinearity sets in a period on the order of r_0/c_s, where r_0 is the width of the beam, and c_s is the ion sound speed.

For very high powers of the beams, the quiver velocity of the electrons is comparable to the speed of light in vacuum, and it causes a change in the electron mass, and thereby in the plasma frequency and the dielectric function. Thus the radial distribution of the irradiance of the beam causes a corresponding redistribution of the electron mass and hence of the dielectric function. This nonlinearity sets in periods on the order of ω_{pe}^{-1}, where ω_{pe} is the plasma frequency. Situations, when ions play a significant role have not been considered.

In the present paper, to illustrate the nature of focusing/defocusing, the critical curves and the divider curves have been given as a plot of dimensionless radius of the beam ρ_0 and the power of the beam Π_0 (Figs 1, 2a, and 2b). The regions, (1) above the critical curve, (2) between the critical and divider curves and (3) below the divider curve, characterize (1) focusing, (2) oscillatory divergence, and (3) steady divergence of the beam, respectively. The dependence of the focusing parameter on the distance of propagation has also been illustrated for typical points in the three regions of the beam power-radius space.

FOCUSING OF HOLLOW GAUSSIAN BEAM (HGB)

Propagation

Consider the propagation of a linearly polarized hollow Gaussian beam with its electric vector polarized along the

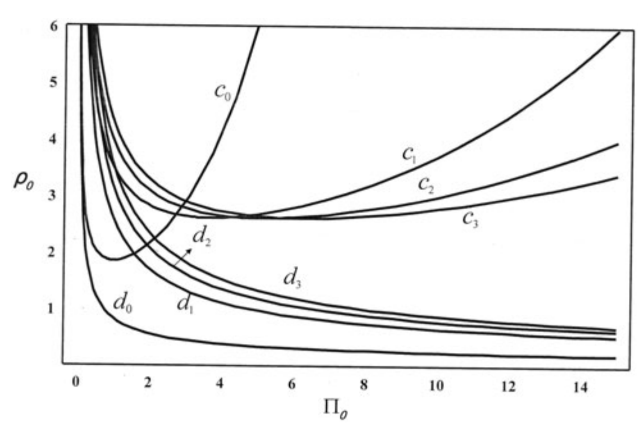

Fig. 1. Variation of the initial beam width ρ_0 $(=r_0\omega/c)$ with the initial power Π_0, for the propagation of various order HGBs in a collisionless plasma with dominant ponderomotive nonlinearity, for the parameter $\Omega^2 = 0.8$; the curves c and d refer to the critical power curve and divider curve respectively while the numerical subscripts 0, 1, 2, and 3 correspond to the order of the HGB n.

https://doi.org/10.1017/S0263034609000081 Published online by Cambridge University Press

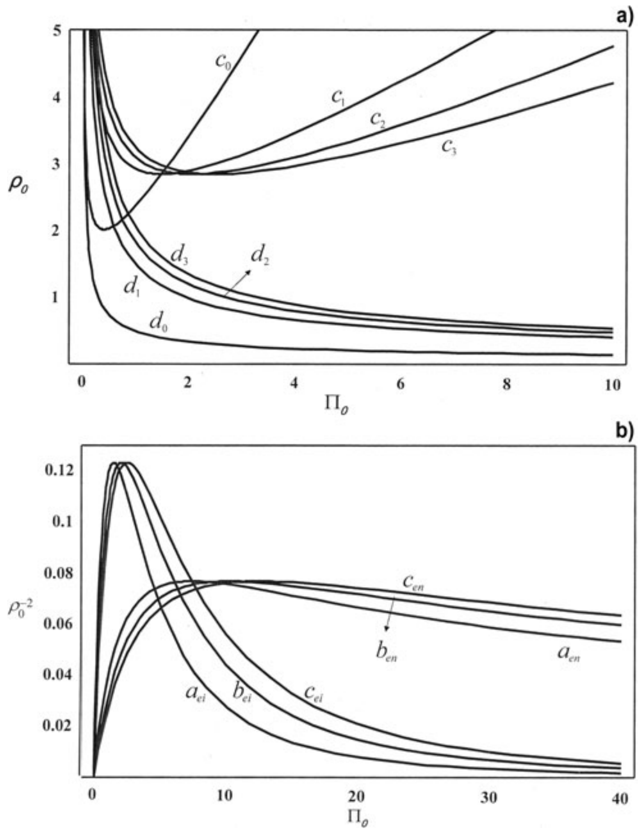

Fig. 2. (**a**) Variation of the initial beam width ρ_0 ($=r_0\omega/c$) with the initial power Π_0, for the propagation of various order HGBs in an electron-ion collision dominant plasma $s=-3$ for $\Omega^2 = 0.8$. The curves c and d refer to the critical power curve and divider curve respectively while the numerical subscripts 0, 1, 2, and 3 correspond to the order of the HGB n. (**b**) Critical power curves: The variation of the initial beam width $1/\rho_0^2$ with the initial power Π_0 for the propagation of HGBs of various orders in a collisional plasma for $\Omega^2 = 0.8$. The curves a, b and c refer to the order of the HGB $n = 1$, 2 and 3 respectively while the subscripts *ei* and *en* correspond to $s = -3$ and $s = 1$, respectively.

y-axis, propagating in a homogeneous plasma along the z-axis. In the steady-state, the electric field vector $\boldsymbol{E}$ for such a beam may be expressed in a cylindrical coordinate system with azimuthal symmetry as

$$\boldsymbol{E} = \hat{j}E_0(r, z)\exp i\omega t, \tag{1}$$

where

$$(E_0)_{z=0} = E_{00}\left(\frac{r^2}{2r_0^2}\right)^n \exp\left(-\frac{r^2}{2r_0^2}\right), \tag{2}$$

E_0 refers to the complex amplitude of the hollow Gaussian beam of initial beam width r_0, E_{00} is a real constant characterizing the amplitude of the HGB, n is the order of the HGB and a positive integer, characterizing the shape of the HGB and position of its maximum, ω is the wave frequency, $\hat{j}$ is the unit vector along the y-axis, and $|E_0|$ describes the electric field maximum at $r = r_{\max} = r_0\sqrt{2n}$. For $n = 0$, Eq. (2) represents a fundamental Gaussian beam of width r_0; however the interest of the present investigation is in higher order HGBs (i.e., $n > 0$).

The electric field vector $\boldsymbol{E}$ satisfies the wave equation (stationary frame),

$$\nabla^2\boldsymbol{E} - \nabla(\nabla\cdot\boldsymbol{E}) + \frac{\varepsilon(r, z)}{c^2}\frac{\partial^2 E}{\partial t^2} = 0, \tag{3}$$

where ε is the effective dielectric function of the plasma, and c is the speed of light in free space.

For transverse beams, the second term on Eq. (3) is zero. One can thus write the wave equation for the electromagnetic beam, as

$$\nabla^2\boldsymbol{E}_0 + (\omega^2/c^2)\varepsilon(r, z)E_0 = 0. \tag{4}$$

Following *Akhmanov et al.* (1968) and *Sodha et al.* (1974*a*, 1976), the solution of Eq. (4) can be chosen as

$$\boldsymbol{E}_0(r, z) = \boldsymbol{A}(r, z)\exp\left(-i\int k(z)dz\right), \tag{5}$$

where $\boldsymbol{A}(r, z)$ is the complex amplitude of the electric field $\boldsymbol{E}_0$, $k(z) = (\omega/c)\sqrt{\varepsilon_0(z)}$, $\varepsilon_0(z)$ is the dielectric function, corresponding to the maximum electric field on the wavefront of the HGB (see Eq. (11)).

Substituting $\boldsymbol{E}_0(r, z)$ from Eq. (5) in Eq. (4) and neglecting the term $\partial^2 A/\partial z^2$ (assuming $A(r, z)$ to be a slowly varying function of z), one obtains

$$2ik\frac{\partial A}{\partial z} + iA\frac{\partial k}{\partial z} = \left(\frac{\partial^2 A}{\partial r^2} + \frac{1}{r}\frac{\partial A}{\partial r}\right) + \frac{\omega^2}{kc^2}(\varepsilon - \varepsilon_0). \tag{6}$$

The complex amplitude $A(r, z)$ may be expressed as,

$$A(r, z) = A_0(r, z)\exp(-ik(z)S(r, z)), \tag{7}$$

where $S(r, z)$ is termed the eikonal associated with the hollow Gaussian beam. Substitution for $A(r, z)$ from Eq. (7) in Eq. (6) and the separation of the real and imaginary parts, yields

$$\frac{2S}{k}\frac{\partial k}{\partial z} + 2\frac{\partial S}{\partial z} + \left(\frac{\partial S}{\partial r}\right)^2 = \frac{1}{k^2 A_0}\left(\frac{\partial^2 A_0}{\partial r^2} + \frac{1}{r}\frac{\partial A_0}{\partial r}\right) + \frac{\omega^2}{k^2c^2}(\varepsilon - \varepsilon_0) \tag{8}$$

and

$$\frac{\partial A_0^2}{\partial z} + A_0^2\left(\frac{\partial^2 S}{\partial r^2} + \frac{1}{r}\frac{\partial S}{\partial r}\right) + \frac{\partial A_0^2}{\partial r}\frac{\partial S}{\partial r} + \frac{A_0^2}{k}\frac{\partial k}{\partial z} = 0. \tag{9}$$

To proceed further, one can adopt an approach, analogous to the paraxial approximation. Thus one may start by expressing Eqs (8) and (9) in terms of variables η and z, where

$$\eta = \left[(r/r_0 f) - \sqrt{2n}\right], \tag{10}$$

$r_0 f(z)$ is the width of the beam, and $r = r_0 f\sqrt{2n}$ is the position of the maximum irradiance for the propagating beam; it is shown later that in the paraxial like approximation, i.e., when $\eta << \sqrt{2n}$, Eqs (8) and (9) lead to the maintenance of the HGB character during propagation. Since the irradiance of the beam is a function of r and z only, expansions of expressions for relevant parameters made along r, near the irradiance maximum *via* $r = r_0 f(z)\sqrt{2n}$, are certainly justified in the paraxial like approximation; for $n = 0$ (Gaussian beam), the expansion is made (likewise) around $r = 0$ (as usual). Like the paraxial theory, the present analysis is strictly applicable when $\eta << \sqrt{2n}$.

Thus from Eqs (8), (9), and (10) one obtains,

$$\begin{aligned}\frac{2S}{k}\frac{\partial k}{\partial z} + 2\frac{\partial S}{\partial z} + \frac{1}{r_0^2 f^2}\left(\frac{\partial S}{\partial \eta}\right)^2 = \frac{1}{k^2 A_0 r_0^2 f^2} \\ \times \left(\frac{\partial^2 A_0}{\partial \eta^2} + \frac{1}{(\sqrt{2n}+\eta)}\frac{\partial A_0}{\partial \eta}\right) + \frac{\omega^2}{k^2 c^2}(\varepsilon - \varepsilon_0)\end{aligned} \tag{11}$$

and

$$\begin{aligned}\frac{\partial A_0^2}{\partial z} + \frac{A_0^2}{r_0^2 f^2}\left(\frac{\partial^2 S}{\partial \eta^2} + \frac{1}{(\sqrt{2n}+\eta)}\frac{\partial S}{\partial \eta}\right) \\ + \frac{1}{r_0^2 f^2}\frac{\partial A_0^2}{\partial \eta}\frac{\partial S}{\partial \eta} + \frac{A_0^2}{k}\frac{\partial k}{\partial z} = 0.\end{aligned} \tag{12}$$

In the paraxial like approximation the relevant parameters (i.e., the dielectric function $\varepsilon(r, z)$, eikonal and irradiance) may be expanded around the maximum of the HGB, i.e., around $\eta = 0$. Thus, one can express the dielectric function $\varepsilon(\eta, z)$ around the maximum ($\eta = 0$) of the HGB as

$$\varepsilon(\eta, z) = \varepsilon_0(z) - \eta^2 \varepsilon_2(z), \tag{13}$$

where $\varepsilon_0(z)$ and $\varepsilon_2(z)$ are the coefficients associated with η^0 and η^2 in the expansion of $\varepsilon(\eta, z)$ around $\eta = 0$. The expressions for these coefficients have been derived later.

Substitution for $\varepsilon(\eta, z)$ from Eq. (13) in Eqs (11) and (12) leads to

$$\begin{aligned}\frac{2S}{k}\frac{\partial k}{\partial z} + 2\frac{\partial S}{\partial z} + \frac{1}{r_0^2 f^2}\left(\frac{\partial S}{\partial \eta}\right)^2 = \frac{1}{k^2 A_0 r_0^2 f^2} \\ \times \left(\frac{\partial^2 A_0}{\partial \eta^2} + \frac{1}{(\sqrt{2n}+\eta)}\frac{\partial A_0}{\partial \eta}\right) - \eta^2\frac{\omega^2}{k^2 c^2}\varepsilon_2\end{aligned} \tag{14}$$

and

$$\begin{aligned}\frac{\partial A_0^2}{\partial z} + \frac{A_0^2}{r_0^2 f^2}\left(\frac{\partial^2 S}{\partial \eta^2} + \frac{1}{(\sqrt{2n}+\eta)}\frac{\partial S}{\partial \eta}\right) \\ + \frac{1}{r_0^2 f^2}\frac{\partial A_0^2}{\partial \eta}\frac{\partial S}{\partial \eta} + \frac{A_0^2}{k}\frac{\partial k}{\partial z} = 0.\end{aligned} \tag{15}$$

One can express the solution of Eq. (15) following the paraxial like approximation $\eta << \sqrt{2n}$ as

$$A_0^2 = \frac{E_0^2}{2^{2n} f^2}\left(\sqrt{2n}+\eta\right)^{4n}\exp\left(-(\sqrt{2n}+\eta)^2\right), \tag{16}$$

with

$$S(\eta, z) = \frac{(\sqrt{2n}+\eta)^2}{2}\beta(z) + \varphi(z), \tag{17}$$

where

$$\beta(z) = r_0^2 f\frac{df}{dz},$$

$$E_0^2 = E_{00}^2\left(\frac{k(0)}{k(z)}\right) = E_{00}^2\left(\frac{\varepsilon_0(0)}{\varepsilon_0(z)}\right)^{1/2},$$

$\varphi(z)$ is an arbitrary function of z, and $f(z)$ is the beam width parameter for the HGB.

Most of the power of the beam is concentrated in the region around $\eta = 0$. There is certainly some power of the beam beyond this limitation, which is accounted for in an approximate manner by Eq. (16), which in common with the variational and moment approaches, assures that the nature of r dependence of irradiance does not change with propagation. Eq. (16) also ensures conservation of power as the beam propagates.

Substituting from Eqs (16) and (17) for A_0^2 and S into Eq. (14) and equating the coefficients of η^0 and η^2 on both sides of the resulting equation, one obtains

$$\varepsilon_0 f\frac{d^2 f}{d\xi^2} = \left(\frac{4}{f^2} - \rho_0^2\varepsilon_2\right) - \frac{1}{2}f\frac{df}{d\xi}\frac{d\varepsilon_0}{d\xi}, \tag{18}$$

and

$$\frac{1}{\varepsilon_0}\left(\left(nf\frac{df}{d\xi} + \Phi\right)\frac{d\varepsilon_0}{d\xi} + \frac{2}{f^2}\right) + 2nf\frac{d^2 f}{d\xi^2} + 2\frac{d\Phi}{d\xi} = 0, \tag{19}$$

where

$\xi = (c/r_0^2\omega)z$ is the dimensionless distance of propagation,
$\rho_0 = (r_0\omega/c)$ is the dimensionless initial beam width,

and

$\Phi = (\omega/c)\varphi$ is the dimensionless function associated with the eikonal.

The dependence of the beam width parameter f on the dimensionless distance of propagation ξ can be obtained by the numerical integration of Eq. (18) after putting suitable expressions for ε_0 and ε_2, and using the initial boundary conditions $f = 1$, $df/d\xi = 0$ at $\xi = 0$; Φ is obtained by simultaneous solution of Eqs 18 and 19, taking the additional boundary condition $\Phi = 0$, at $\xi = 0$ into account.

The Dielectric Function

Following *Sodha et al.* (1974*a*), the effective dielectric function of the plasma can be expressed as

$$\varepsilon(r, z) = 1 - \Omega^2(N_{0e}/N_0), \tag{20}$$

where $\Omega = (\omega_{pe}/\omega)$, $\omega_{pe} = (4\pi N_0 e^2/m)^{1/2}$ is the electron plasma frequency, N_0 is the undisturbed electron density of the plasma, N_{0e} is the electron density of the plasma in the presence of the electromagnetic field, m is the mass of the electron, and e is the electron charge.

Following the paraxial like approximation (*i.e.* $\eta << \sqrt{2n}$) one can expand the dielectric function $\varepsilon(\eta, z)$ in axial and radial parts around the maximum of $\boldsymbol{E}$ ($\eta = 0$). Thus one obtains from Eq. (13) and Eq. (20),

$$\varepsilon_0(z) = \varepsilon(\eta = 0, z), \tag{21}$$

and

$$\varepsilon_2 = -\left(\frac{\partial\varepsilon(\eta, z)}{\partial\eta^2}\right)_{\eta=0}. \tag{22}$$

To obtain the above coefficients, one should expand $\boldsymbol{E}\cdot\boldsymbol{E}^*$ in powers of η^2; thus

$$\begin{aligned}\boldsymbol{E}\cdot\boldsymbol{E}^* = A_0^2 &= \frac{E_0^2}{2^{2n}f^2}\left(\sqrt{2n}+\eta\right)^{4n}\exp\left(-(\sqrt{2n}+\eta)^2\right)\\ &\approx F_1(z) - \eta^2 F_2(z),\end{aligned} \tag{23}$$

where

$$F_1(z) = \frac{E_0^2}{f^2}n^{2n}\exp(-2n) \tag{24}$$

and

$$F_2(z) = \frac{2E_0^2}{f^2}n^{2n}\exp(-2n). \tag{25}$$

With the help of Eqs (24) and (25) one can easily obtain ε_0 and ε_2 for a specific nature of the nonlinearity.

Ponderomotive Nonlinearity

For a collisionless plasma, the ponderomotive force on the electrons is proportional to the gradient of the irradiance, which causes a redistribution of the electron density, and thereby the dielectric function; this nonlinearity sets in a period on the order (r_0/c_s), where r_0 is the width of the beam, and c_s is the ion sound speed.

Hence, for a collisionless plasma at moderate fields (when the quiver speed of the electron is much smaller than the speed of light in vacuum), the modified electron density function N_{0e} is given (*Akhmanov et al.*, 1968; *Sodha et al.*, 1976),

$$N_{0e} = N_0\exp(-\beta EE^*), \tag{26}$$

where,

$$\beta = (e^2/8k_B T_0\omega^2 m),$$

k_B is the Boltzmann constant, and T_0 is the temperature of the atoms/ions.

Substituting for N_{0e} from Eq. (26) in Eq. (20) and using Eqs (21), (22), (24), and (25), one can easily obtain $\varepsilon_0(z)$ and $\varepsilon_2(z)$ as

$$\varepsilon_0(z) = 1 - \Omega^2\exp(-pn^{2n}\exp(-2n)), \tag{27}$$

and

$$\varepsilon_2(z) = \Omega^2(2pn^{2n}\exp(-2n))\exp(-pn^{2n}\exp(-2n)), \tag{28}$$

with $p = \beta E_0^2/f^2$ being proportional to the irradiance of the HGB at $\eta = 0$.

Collisional Nonlinearity

In collisional plasmas, the distribution of electron and ion temperatures takes place in the transverse plane on account of the nonuniform distribution of the electron temperature, caused by the radial dependence of the irradiance of the beam; this nonuniformity in temperature creates the pressure gradients of the electron and ion gases. In the steady state with plasma neutrality these pressure gradients are balanced by the space charge field, and lead to a redistribution of the electron density and hence the modified dielectric function. The collisional nonlinearity sets in a period $1/\delta_c\nu_e$, where δ_c is the fractional loss of excess energy by an electron in a collision with heavier species (ions and neutral particles), and ν_e is the electron collision frequency. For collisional plasmas, the modified electron density N_{0e} may thus be

https://doi.org/10.1017/S0263034609000081 Published online by Cambridge University Press

expressed as (*Akhmanov et al.*, 1968; *Sodha et al.*, 1976)

$$N_{0e} = N_0(1 + \alpha EE^*)^{(s/2)-1}, \quad (29)$$

where

$$\alpha = \left(e^2/6k_B T_0 \omega^2 m \delta_c\right)$$

and the collision frequency ν_e is proportional to the s^{th} power of the random electron speed. For electron-ion collision dominated plasma, one has $s = -3$ and for electron-neutral collision dominated plasma, it is $s = 1$. Eq. (21) is based on the fact that thermal conduction does not play a significant part in the energy balance of electrons, which is justified when $(\delta_c r_0^2/l^2) \gg 1$ (see *Sodha et al.*, 1976). It is also assumed that the heavier particles are abundant enough to provide a heat sink at almost constant temperature for energy loss by the electrons.

Substituting for N_{0e} from Eq. (29) in Eq. (20) and by using Eq. (21), (22), (24), and (25), one can obtain expressions for $\varepsilon_0(z)$ and $\varepsilon_2(z)$ as,

$$\varepsilon_0(z) = 1 - \Omega^2\left(1 + pn^{2n}\exp(-2n)\right)^{(s-2)/2}, \quad (30)$$

and

$$\varepsilon_2(z) = \Omega^2(2-s)\left(pn^{2n}\exp(-2n)\right) \times \left(1 + pn^{2n}\exp(-2n)\right)^{(s-4)/2}, \quad (31)$$

where $p = \alpha E_0^2/f^2$.

Relativistic Nonlinearity

For very high powers of the beams, the quiver speed of the electrons is comparable to the speed of light in vacuum, causing a change in the mass of the electron, and hence a change in the plasma frequency leads the modified dielectric function. Thus, the relativistic variation of the electron mass may also cause nonlinearity (Hora, 1975; Kane & Hora, 1977). This nonlinearity sets in a period on the order ω_{pe}^{-1}. The dielectric function for a circularly polarized beam can be expressed as

$$\varepsilon = 1 - \Omega^2(1 + \gamma EE^*)^{-1/2}, \quad (32)$$

where $\gamma = (e^2/m_0^2\omega^2 c^2)$, m_0 is the rest mass of the electron and E is the amplitude of the beam.

From Eqs (21), (22), (24) (25), and (32) one can easily express $\varepsilon_0(z)$ and $\varepsilon_2(z)$ as

$$\varepsilon_0(z) = \left(1 - \Omega^2(1 + pn^{2n}\exp(-2n)\right)^{-1/2} \quad (33)$$

and

$$\varepsilon_2(z) = \Omega^2\left(pn^{2n}\exp(-2n)\right)\left(1 + pn^{2n}\exp(-2n)\right)^{-3/2}, \quad (34)$$

where $p = \gamma E_0^2/f^2$.

Nature of Self Focusing: Critical and Divider Curves

Ponderomotive Nonlinearity

Using the expression for $\varepsilon_2(z)$ from Eq. (28), Eq. (18) ensures vanishing of $d^2f/d\xi^2$ for a value of p (say p_c), corresponding to a beam width $\rho_0 f$, when

$$p\rho_0^2 f^2 = \frac{2\exp\left(pn^{2n}\exp(-2n)\right)}{\Omega^2 n^{2n}\exp(-2n)}, \quad (35)$$

which at $\xi = 0$ reduces to

$$p_0\rho_0^2 = \frac{2\exp\left(p_0 n^{2n}\exp(-2n)\right)}{\Omega^2 n^{2n}\exp(-2n)}, \quad (36)$$

where $p_0 = \beta E_{00}^2$.

Eq (36) represents the critical power curve plotted as ρ_0 *versus* p_0 and separates the self focusing region from the rest. For the points lying above the critical power curve, the beam undergoes oscillatory convergence (self focusing), while for the points below this curve, the beam executes oscillatory divergence or steady divergence. The points on the curve lead to self trapped mode propagation of the HGB.

One should notice that Eqs (35) and (36) have the same algebraic form; this suggests that if during its propagation, the beam power and the beam width $(p, \rho_0 f)$ satisfy Eq. (35), the corresponding point in the $f - \xi$ curve will be a point of inflection ($d^2f/d\xi^2 = 0$), a necessary condition for oscillatory convergence/divergence. However, all the points below the critical curve do not lead to a point of inflection, and hence lead to a steady divergence of the beam. The condition for any point $(p, \rho_0 f)$ to be a point of inflection is that it must satisfy Eq. (35). One can write Eq. (27) as

$$\exp\left(pn^{2n}\exp(-2n)\right) = \frac{\Omega^2}{(1-\varepsilon_0(z))}. \quad (37)$$

Therefore Eq. (35) reduces to

$$p\rho_0^2 f^2 = \frac{2}{n^{2n}\exp(-2n)(1-\varepsilon_0(\xi))}$$

or

$$p_0\rho_0^2 = \frac{2(\varepsilon_0(\xi)/\varepsilon_0(0))^{1/2}}{n^{2n}\exp(-2n)(1-\varepsilon_0(\xi))}. \quad (38)$$

https://doi.org/10.1017/S0263034609000081 Published online by Cambridge University Press

For $p > 0$, from Eq. (36) one has

$$\Omega^2 > (1 - \varepsilon_0(z))$$

or

$$\varepsilon_0(z) > (1 - \Omega^2).$$

These relations lead to

$$\frac{\sqrt{\varepsilon_0(z)}}{(1 - \varepsilon_0(z))} > \frac{\sqrt{1 - \Omega^2}}{\Omega^2}.$$

Therefore from Eq. (38) one obtains

$$p_0\rho_0^2 > \frac{2\sqrt{1 - \Omega^2}}{n^{2n}\exp(-2n)\Omega^2(\varepsilon_0(0))^{1/2}}. \qquad (39)$$

Thus, for the point $(p, \rho_0 f)$ to arrive at a point of inflection $(d^2f/d\xi^2 = 0)$, one must have the beam having initial point (p_0, ρ_0) lying above the curve

$$p_0\rho_0^2 = \frac{2}{n^{2n}\exp(-2n)\Omega^2}\left(\frac{1 - \Omega^2}{\varepsilon_0(0)}\right)^{1/2}. \qquad (40)$$

Eq. (40) represents a curve (ρ_0 *versus* p_0) that further divides the region below the critical power curve in two regions, and has hence been termed the divider curve by *Sharma et al.* (2003). The area between the divider curve and the critical power curve represents the region of oscillatory divergence, while the area below the divider curve describes the region of steady divergence.

Collisional Nonlinearity

Using Eqs (30) and (31) for $\varepsilon_0(z)$ and $\varepsilon_2(z)$ with Eq. (18) and following a similar treatment (as in the preceding case) one can obtain the critical power curve and the divider curve as,

Critical power curve

$$p_0\rho_0^2 = \frac{4\left(1 + p_0 n^{2n}\exp(-2n)\right)^{(4-s)/2}}{\Omega^2(2 - s)n^{2n}\exp(-2n)} \qquad (41)$$

Divider curve

$$p_0\rho_0^2 = \frac{4}{\Omega^2(2 - s)n^{2n}\exp(-2n)}\left(\frac{1 - \Omega^2}{\varepsilon_0(0)}\right)^{1/2}, \qquad (42)$$

with $p_0 = \alpha E_{00}^2$.

Relativistic Nonlinearity (Circularly Polarized Beam)

Using the expression for $\varepsilon_0(z)$ and $\varepsilon_2(z)$ from Eqs (33) and (34) with Eq. (18), and following similar algebraic treatment as above, the expressions for the critical power curve and the divider curve can be written as,

Critical power curve

$$p_0\rho_0^2 = \frac{4\left(1 + p_0 n^{2n}\exp(-2n)\right)^{-3/2}}{\Omega^2 n^{2n}\exp(-2n)}, \qquad (43)$$

Divider curve

$$p_0\rho_0^2 = \frac{4}{\Omega^2 n^{2n}\exp(-2n)}\left(\frac{1 - \Omega^2}{\varepsilon_0(0)}\right)^{1/2}, \qquad (44)$$

with $p_0 = \gamma E_{00}^2$.

Power of the Hollow Gaussian Beam

The power of the hollow Gaussian beam with the irradiance distribution

$$EE^* = \frac{E_0^2}{2^{2n}f^2}\left(\frac{r^2}{r_0^2 f^2}\right)^{2n}\exp\left(-\frac{r^2}{r_0^2 f^2}\right),$$

can be expressed as

$$P = \frac{c}{8\pi}\int_0^\infty \varepsilon^{1/2} EE^* 2\pi r dr, \qquad (45)$$

where $\varepsilon(r, z)$ is the dielectric function of the plasma. For the circularly polarized beams, the right-hand-side gets multiplied by two. One can obtain the power of the HGB by using an appropriate dielectric function for all three kinds of nonlinearities.

Ponderomotive Nonlinearity

Using the dielectric function $\varepsilon(r, z)$ from Eq. (20) and Eq. (26), Eq. (45) reduces to

$$P = \frac{cr_0^2}{8\beta}\frac{p}{2^{2n}}\int_0^\infty \lambda^{2n}\exp(-\lambda)$$
$$\times\left[1 - \Omega^2\exp\left(-(p/2^{2n})\lambda^{2n}\exp(-\lambda)\right)\right]^{1/2} d\lambda,$$

where $\lambda = (r^2/r_0^2 f^2)$. The dimensionless power of the beam

https://doi.org/10.1017/S0263034609000081 Published online by Cambridge University Press

may be expressed as

$$\Pi = \frac{8\beta}{cr_0^2}P = \frac{p}{2^{2n}}\int_0^\infty \lambda^{2n}\exp(-\lambda) \times \left[1-\Omega^2\exp\left(-(p/2^{2n})\lambda^{2n}\exp(-\lambda)\right)\right]^{1/2}d\lambda, \quad (46)$$

with $p = \beta E_0^2/f^2$. Further at $z = 0$, $f = 1$ the initial power is given as

$$\Pi_0 = \frac{p_0}{2^{2n}}\int_0^\infty \lambda^{2n}\exp(-\lambda) \times \left[1-\Omega^2\exp\left(-(p_0/2^{2n})\lambda^{2n}\exp(-\lambda)\right)\right]^{1/2}d\lambda, \quad (47)$$

with $p_0 = \beta E_{00}^2$.

Similarly the initial power of the HBG for collisional and relativistic nonlinearities may be expressed as

For Collisional Nonlinearity

$$\Pi_0 = \frac{p_0}{2^{2n}}\int_0^\infty \lambda^{2n}\exp(-\lambda) \times \left[1-\Omega^2\left(1+(p_0/2^{2n})\lambda^{2n}\exp(-\lambda)\right)^{(s-2)/2}\right]^{1/2}d\lambda \quad (48)$$

with $p_0 = \alpha E_{00}^2$.

For Relativistic Nonlinearity

$$\Pi_0 = \frac{p_0}{2^{2n}}\int_0^\infty \lambda^{2n}\exp(-\lambda) \times \left[1-\Omega^2\left(1+(p_0/2^{2n})\lambda^{2n}\exp(-\lambda)\right)^{-1/2}\right]^{1/2}d\lambda \quad (49)$$

with $p_0 = \gamma E_{00}^2$.

SCHEME OF COMPUTATION

To have a numerical appreciation of the results, the critical power curve, the divider curve, and the dependence of the beam width parameter f of the HGB on ξ have been computed for a chosen set of parameters and different kinds of nonlinearities.

To obtain the critical curve between initial power Π_0 and initial beam width ρ_0, one first computes the critical curve with ρ_0 and p_0, as parameters using the appropriate equations (Eqs (36), (40), (41), (42), (43), and (44)) obtained herein and also computes the power Π_0, corresponding to a set of values (p_0, ρ_0) on the critical curve, using Eq. (45). Thus one can obtain the critical curve with ρ_0 and Π_0, as the parameters. The critical power curve and the divider curve ρ_0 *versus* Π_0 have been plotted for different kinds of nonlinearities for chosen sets of parameters Ω, s and n. The computations have also been made to investigate the dependence of the beam width parameter f, associated with the propagation of the hollow Gaussian beam on the dimensionless distance of propagation ξ in homogeneous plasmas. Starting with a combination of the parameters Π_0, ρ_0, n, and Ω one can obtain the solution for the beam width parameter f by simultaneous numerical integration of Eqs (18) and (19) using suitable expressions for ε_0 and ε_2 under appropriate boundary conditions *viz* $f = 1$, $df/d\xi = 0$ and $\Phi = 0$ at $\xi = 0$.

NUMERICAL RESULTS AND DISCUSSION

In the present study, the propagation characteristics of a hollow Gaussian beam in plasma have been investigated; these are determined by the modified dielectric function around the maximum of the irradiance on the wavefront of the HGB. One can see from the irradiance distribution profile for HGB (Eq. (2)) that the radius of the bright ring increases when the order of the HGB n increases, which means that the area of the dark region across the HGB increases as n increases. It is instructive to remember that (1) the collisional nonlinearity sets in a period $1/\delta\nu$, (2) the ponderomotive nonlinearity sets in a period r_0/c_s where c_s is the ionic sound speed, and (3) the relativistic nonlinearity sets in a period ω_{pe}^{-1}. The present steady state theory is valid when the duration of the laser radiation is longer than these characteristic times.

To have a better understanding of the phenomena and numerical appreciation of the results, the critical curves and the beam width parameter f as a function of dimensionless distance of propagation ξ have been computed for a set of parameters Π_0, ρ_0, n, and for all the three kinds of nonlinearities. Further, these results for higher order HGB have been compared with corresponding results for a fundamental Gaussian beam ($n = 0$).

Figure 1 illustrates the dependence of the dimensionless initial power Π_0 on the dimensionless initial beam width ρ_0 for self trapping, corresponding to the ponderomotive nonlinearity for various orders of the HGB. The figures show the critical as well as divider curves; thus the $\rho_0 - \Pi_0$ space can be divided into three regions namely those corresponding to oscillatory focusing, oscillatory divergence, and steady divergence. If any initial point (Π_0, ρ_0) lies on the critical power curve then $d^2f/d\xi^2$ vanishes at $\xi = 0$; since $df/d\xi$ is initially zero (for a plane wavefront) it continues to be zero and f remains equal to one as the HGB propagates through the plasma. Such propagation is termed as uniform waveguide propagation. For the initial point (Π_0, ρ_0) lying above the critical power curve, $d^2f/d\xi^2 < 0$ and hence when the beam propagates through the plasma the power Π increases and the beam width $\rho_0 f$ decreases. Therefore, when the beam propagates the point (Π, ρ) will move in the $\rho_0 - \Pi_0$ space in the right downward direction and at some value of f it will reach the critical power curve (i.e., the point of inflection); beyond this $df/d\xi$ starts decreasing until $df/d\xi$ vanishes, i.e., f reaches a minimum. Thus, during the propagation of the beam, the beam width parameter f corresponding to such a point (lying above the critical power curve) oscillates between the initial value unity and a minimum. Similarly, for the initial point (Π_0, ρ_0) lying between the critical power curve and the divider curve, the beam width parameter f oscillates as the beam propagates in the plasma between a maximum and the initial value unity. The points lying below the divider curve will never be able to attain a point of inflection and hence the beam having an initial point (Π_0, ρ_0) in this region steadily diverges. The curves in Figure 1 indicate that the region for oscillatory divergence becomes narrower while self focusing and steady divergence occupy larger areas in the $\rho_0 - \Pi_0$ space as the order of the HGB increases. The trend of the critical power curve gets reversed beyond some value of the critical power. This can be understood in terms of $\varepsilon_2(z)$, which at first rises rapidly to a maximum value (i.e., minimum self trapping width) and then falls sharply with increasing power of the beam for lower order HGBs. The divider curve shows that the beam width associated with the region of steady divergence increases with increasing order of the HGB. Further, a comparative study has been made for fundamental Gaussian beam, which follows a similar variation. The figure also suggests that the minimum self trapping width is lowest for $n = 0$ and increases with increasing order of the HGB.

Figure 2a describes the curves corresponding to critical conditions for focusing and defocusing of the beam in an electron-ion collision dominant plasma ($s = -3$) for various orders of the HGBs; the curves display a trend like Figure 1. Further, the self focusing region occupies a larger area and a higher value of absolute minimum of the beam width for self trapping to occur for an electron-neutral collision dominant plasma ($s = 1$), in comparison with that for an electron-ion collision dominated ($s = -3$) plasma, in the $\rho_0 - \Pi_0$ space; this comparison has been demonstrated in Figure 2b. The figure also suggests that for the same initial beam width ρ_0, self focusing is stronger for $s = -3$ for low power HGBs, it gets more pronounced for $s = 1$ with increasing power of the beam.

Figure 3a expresses the dependence of the beam width parameter f on the dimensionless distance of propagation ξ for a collisionless plasma with dominant ponderomotive nonlinearity. The figure describes the characteristic propagation of the HGB in the three regions namely self focusing, oscillatory divergence, and steady divergence for chosen points (Π_0, ρ_0) from Figure 1 for $n = 1$; the focusing is more pronounced for the beam having high initial power and larger initial beam width. The effect of higher order of HGBs on the propagation has been depicted in Figure 3b, starting with fixed (Π_0, ρ_0) for all the beams. It is seen that for low power beams that the self focusing character of the HGB decreases and leads to oscillatory divergence as n increases, but as the power of the beam increases the trend is seen to get reversed i.e., self focusing is more pronounced for higher order HGBs; this has been shown in Figure 3c.

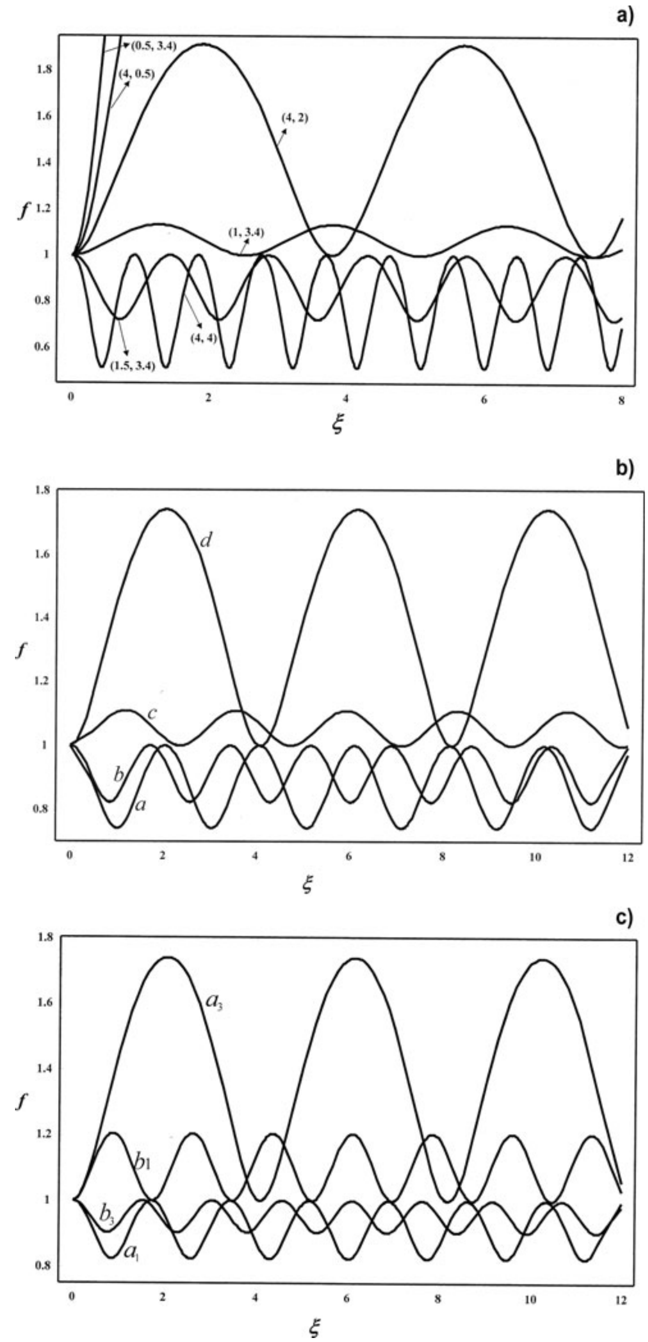

Fig. 3. (**a**) Dependence of the dimensionless beam width parameter f on the dimensionless distance of propagation ξ, in a collisionless plasma with dominant ponderomotive nonlinearity, for the parameters $\Omega^2 = 0.8$ and first order HGB ($n = 1$); the curves refer to an arbitrarily chosen set of initial power and initial beam width (Π_0, ρ_0) as indicated over the curves. (**b**) Variation of the dimensionless beam width parameter f on the dimensionless distance of propagation ξ, in a collisionless plasma with dominant ponderomotive nonlinearity for various order HGBs for $\Omega^2 = 0.8$, $\Pi_0 = 2$ and $\rho_0 = 4$; the curves a, b, c and d refer to the order of the HGB $n = 0$, 1, 2 and 3 respectively. (**c**) Dependence of the dimensionless beam width parameter f on the dimensionless distance of propagation ξ, in a collisionless plasma with dominant ponderomotive nonlinearity for various order HGBs, for the parameters $\Omega^2 = 0.8$ and $\rho_0 = 3$; the curves a and b refer to the initial power of the beam $\Pi_0 = 2$ and 10 respectively, while subscripts 1 and 3 correspond to the order of the HGB n.

https://doi.org/10.1017/S0263034609000081 Published online by Cambridge University Press

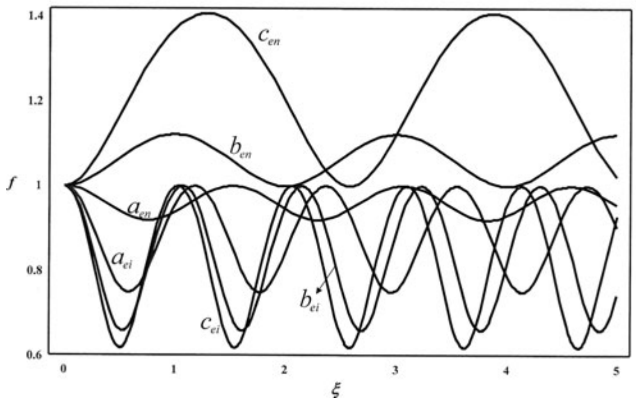

Fig. 4. Variation of the dimensionless beam width parameter f on the dimensionless distance of propagation ξ, in a collisional plasma for various order HGBs, for the parameters $\Omega^2 = 0.8$, $\Pi_0 = 3$ and $\rho_0 = 4$; the curves a, b and c refer to the order of the HGB $n = 1$, 2 and 3 respectively, while the subscripts *ei* and *en* correspond to $s = -3$ and $s = 1$ respectively.

The propagation of the various order HGBs in a collisional plasma has been illustrated in Figure 4 for $s = 1, -3$ starting with the same (Π_0, ρ_0) for all the beams. The figure suggests stronger self focusing for $s = -3$ than for $s = 1$ (due to stronger nonlinearity for $s = -3$). Further, self focusing is more pronounced for $s = -3$ with increasing order of the HGB, while the opposite trend is observed for $s = 1$. All the $f - \xi$ curves are in conformance with the above discussed critical curves.

One should notice that an expression for the dielectric function for relativistic nonlinearity (Eqs (33) and (34)), is exactly obtained by substituting $s = 1$ in the expression for the dielectric function for collisional nonlinearity (Eqs (30), (31)) with modified definition of $p(=\gamma E_0^2/f^2)$; therefore the results (in terms of p and Π) for relativistic nonlinearity are the same as those for $s = 1$ in collisional nonlinearity.

CONCLUSION

It is interesting to compare the propagation characteristics of the HGBs to those of the fundamental Gaussian beam. It is seen that the HGBs also exhibit three regions in the $\rho_0 - \Pi_0$ space which follow a nature similar to that in the case of the Gaussian beam. The regions for self focusing and steady divergence occupy larger areas while the oscillatory divergence becomes narrower with respect to the Gaussian beam in $\rho_0 - \Pi_0$ space for higher order HGBs. The nature of the critical curve involves that self focusing is more pronounced for lower orders for low power HGBs, while the trend just reverses for high power of the beam.

ACKNOWLEDGEMENT

The authors are grateful to the Department of Science and Technology, Government of India for financial support and to Prof. M. P. Verma for critical review of the manuscript.

REFERENCES

Akhmanov, S.A., Sukhorukov, A.P. & Khokhlov, R.V. (1968). Self focusing and diffraction of light in a nonlinear medium. *Sov. Phys. Usp*. **10**, 609–636.

Amiranoff, F., Baton, S., Bernard, D., Cros, B., Descamps, D., Derchies, F., Jaquet, F., Malka, V., Marques, J.R., Matthieussent, G., Mine, P., Modena, A., Mora, P., Manillo, J. & Nazmudin, Z. (1998). Observation of laser wakefield acceleration of electrons. *Phys. Rev. Lett*. **81**, 995–998.

Andreev, N.E., Gorbunov, L.M., Kirsanov, V.I., Nakajima, K. & Ogata, A. (1997). Structure of the wake field in plasma channels. *Phys. Plasma* **4**, 1145–1153.

Andreev, N.E., Gorbunov, L.M. & Frolov, A.A. (1998). On the laser wakefield acceleration in plasma channels. *Fiz. Plasmy* **24**, 888.

Arlt, J. & Dholakia, K. (2000). Generation of high order Bessel beams by use of an axicon. *Opt. Commun*. **177**, 297–301.

Berge, L. (1998). Wave collapse in physics: principles and applications in light and plasma waves. *Phys. Rep*. **303**, 259–370.

Cai, Y. & Lin, Q. (2004). Hollow elliptical Gaussian beam and its propagation through aligned and misaligned paraxial optical systems. *J. Opt. Soc. Am. A* **21**, 1058–1065.

Cai, Y., Lu, X. & Lin, Q. (2003). Hollow Gaussian beam and their propagation properties. *Opt. Lett*. **28**, 1084–1086.

Cai, Y. & He, S. (2006). Propagation of hollow Gaussian beams through apertured paraxial optical systems, *J. Opt. Soc. Am. A* **23**, 1410–1418.

Cai, Y. & Zhang, L. (2006*a*). Coherent and partially coherent dark hollow beams with rectangular symmetry and paraxial propagation properties. *J. Opt. Soc. Am. B* **23**, 1398–1407.

Cai, Y. & Zhang, L. (2006*b*). Propagation of various dark hollow beams in a turbulent atmosphere. *Opt. Express* **14**, 1353–1367.

Chen, Z.L., Unick, C., Vafaei-Najafabadi, N., Tsui, Y.Y., Fedosejevs, R., Naseri, N., Masson-Laborde, P.E. & Rozmus, W. (2008). Quasi-monoenergetic electron beams generated from 7 TW laser pulses in N_2 and He gas targets. *Laser Part. Beams* **26**, 147–155.

Chioa, R.Y., Garmire, E. & Townes, C.H. (1964). Self-trapping of optical beams. *Phys. Rev. Lett*. **13**, 479–482.

Deng, D., Fu, X., Wei, C., Shao, J. & Fan, Z. (2005). Far field intensity distribution and M^2 factor of hollow Gaussian beams. *Appl. Opt*. **44**, 7187–7190.

Desaix, M., Anderson, D. & Lisak, M.J. (1991). Variational approach to collapse of optical pulses. *J. Opt. Soc. Am. B*. **8**, 2082–2086.

Deutsch, C., Furukaw, H., Mima, K., Murakami, M. & Nishihara, K. (1996). Interaction physics of fast ignitor concept. *Phys. Rev. Lett*. **77**, 2483–2486.

Eder, D.C., Amendt, P., DaSilva, L.B., London, R.A., MacGowan, B.J., Matthaws, D.L., Penetrante, B.M., Rosen, M.D., Wilks, S.C., Donnelly, T.D., Falcone, R.W. & Strobel, G.L. (1994). Tabletop X-ray lasers. *Phys. Plasmas* **1**, 1744–1752.

Esarey, E., Sprangle, P., Krall, J. & Ting, A. (1997). Self-Focusing and Guiding of Short Laser Pulses in Ionizing Gases and Plasmas. *IEEE J. Quantum Electron*. **33**, 1879–1914.

Feit, M.D. & Fleck Jr., J.A. (1988). Beam non-paraxiality, filament formation and beam breakup in the self focusing of optical beams, *Opt. Soc. Am. B* **5**, 633–640.

Fibich, G. (2007). *Some Modern Aspects of Self-Focusing Theory, a chapter in Self-focusing: Past and Present* (Boyd R.W., Lukishova S.G. & Shen Y. R., eds.) New York: Springer Verlag.

Ganic, D., Gan, X. & Gu, M. (2003). Focusing of doughnut laser beams by high numerical aperture objective in free space. *Opt. Express* **11**, 2747–2752.

Gill, T.S. & Saini, N.S. (2007). Nonlinear interaction of a rippled laser beam with an electrostatic upper hybrid wave in collisional plasma. *Laser Part. Beams* **25**, 283–293.

Gao, Z. & Lu, B. (2006). Non-paraxial dark hollow Gaussian beams, *Chin. Phys. Lett*. **23**, 106–109.

Grow, T.D., Ishaaya, A.A., Vuong, L.T., Gaeta, A.L., Gavish, N. & Fibich, G. (2006). Collapse dynamics of super-Gaussian Beams. *Opt. Express*, **14**, 5468–5475.

Hauser, T., Scheid, W. & Hora, H. (1992). Theory of ions emitted from a plasma by relativistic self focusing of laser beams. *Phys. Rev. A* **45**, 1278–1281.

Herman, R.M. & Wiggins, T.A. (1991). Production and uses of diffractionless beams. *J. Opt. Soc. Am. A* **8**, 932–942.

Hora, H. (1969). Self focusing of laser beams in a plasma by Ponderomotive forces. *Z. Phys*. **226**, 156–159.

Hora, H. (1975). Theory of relativistic self focusing of laser radiations in plasmas. *J. Opt. Soc. Am*. **65**, 882–886.

Hora, H. (1991). *Plasmas at High Temperature and Density*, Heildelberg: Springer.

Johannisson, P., Anderson, D., Lisak, M. & Marklund, M. (2003). Nonlinear Bessel beams. *Opt. Commun*. **222**, 107–115.

Johnston, T.W., Vidal, F. & Fre'chette, D. (1997). Laser plasma filamentation and spatially periodic nonlinear Schrödinger equation approximation. *Phys. Plasmas* **4**, 1582–1588.

Jones, D.A., Kane, E.L., Lalousis, P., Wiles, P. & Hora, H. (1982). Density modification and energetic ion production at relativistic self focusing of laser beams in plasmas. *Phys. Fluids* **25**, 2295–2301.

Kane, E.L. & Hora, H. (1977). *Laser Interaction and Related Plasma Phenomena* (Schwarz H.J. & Hora H., Eds.). New York: Plenum.

Karlsson, M. (1992). Optical beams in saturable self focusing media. *Phys. Rev. A* **46**, 2726–2734.

Karlsson, M. & Anderson, D.J. (1992). Super-Gaussian approximation of the fundamental radial mode in nonlinear parabolic-index optical fibers. *Opt. Soc. Am. B* **9**, 1558–1562.

Kelley, P.L. (1965). Self-focusing of laser beams and stimulated Raman gain in liquids. *Phys. Rev. Lett*. **15**, 1010–1012.

Kothari, N.C. & Abbi, S.C. (1990). Instability growth and filamentation of very intense laser beams in self-focusing media. *Progr. Theor. Phys*. **83**, 414–442.

Kuga, T., Torii, Y., Shiokawa, N., Hirano, T., Shimizu, Y. & Sasada, H. (1997). Novel optical trap of atoms with a doughnut beam. *Phys. Rev. Lett*. **78**, 4713–4716.

Lee, H.S., Atewart, B.W., Choi, K. & Fenichel, H. (1994). Holographic non-diverging hollow beams. *Phys. Rev. A* **49**, 4922–4927.

Mei, Z. & Zhao, D. (2005). Controllable dark hollow beams and their propagation characteristics. *J. Opt. Soc. Am. A* **22**, 1898–1902.

Mei, Z. & Zhao, D. (2006). Controllable elliptical dark hollow beams. *J. Opt. Soc. Am. A* **23**, 919–925.

Milchberg, H.M., Durfee III., C.G. & Mcllrath, T.J. (1995). Highorder frequency conversion in the plasma waveguide. *Phys. Rev. Lett*. **75**, 2494–2497.

Mora, P. & Antonsen, T.M. (1996). Electron cavitation and acceleration in the wake of an ultraintense self focused laser pulse. *Phys. Rev. E* **53**, R2068–R2071.

Nayyar, V.P. (1986). Non-linear propagation of a mixture of degenerate modes of a laser cavity. *J. Opt. Soc. Am. B* **3**, 711–714.

Neff, S., Knobloch, R., Hoffmann, D.H.H., Tauschwitz, A. & Yu, S.S. (2006). Transport of heavy-ion beams in a 1 m free-standing plasma channel. *Laser Part. Beams* **24**, 71–80.

Niu, H.Y., He, X.T., Qiao, B. & Zhou, C.T. (2008). Resonant acceleration of electrons by intense circularly polarized Gaussian laser pulses. *Laser Part. Beams* **26**, 51–59.

Osman, F., Castillo, R. & Hora, H. (1999). Relativistic and ponderomotive self- focusing at laser plasma interaction. *J. Plasma Phys*. **61**, 263–273.

Ovchinnikov, Yu.B., Manek, I. & Grimm, R. (1997). Surface trap for Cs. Atoms based on evanescent-wave cooling. *Phys. Rev. Lett*. **79**, 2225.

Prakash, G., Sharma, A., Verma, M.P. & Sodha, M.S. (2006). *Proc. Nat. Acad. Sci. India* **76**, 257–263.

Rasmussen, J.J. & Rypdal, K. (1986). Blow-up in NLSE: A General Review. *Phys. Scripta* **33**, 481–497.

Saini, N.S. & Gill, T.S. (2006). Self-focusing and self-phase modulation of an elliptic Gaussian laser beam in collisionless magnetoplasma. *Laser Part. Beams* **24**, 447–453.

Sari, A.H., Osman, F., Doolan, K.R., Ghoranneviss, M., Hora, H., Hopfl, R, Benstetter, G. & Hantehzadehi, M.H. (2005). Application of laser driven fast high density plasma blocks for ion implantation. *Laser Part. Beams* **23**, 467–473.

Sharma, A., Prakash, G., Verma, M.P. & Sodha, M.S. (2003). Three regimes of intense laser propagation in plasmas. *Phys. Plasmas* **10**, 4079–4084.

Sharma, A., Verma, M.P. & Sodha, M.S. (2004). Self focusing of electromagnetic beams in a collisional plasmas with nonlinear absorption. *Phys. Plasmas* **11**, 4275–4279.

Sharma, A., Verma, M.P., Sodha, M.S. & Tripathi, V.K. (2005). Self focusing of TEM-10 mode laser beam in a plasma, *Indian J. Phys*. **79**, 393–399.

Silberbarg, Y. (1990). Collapse of optical pulses. *Opt. Lett*. **15**, 1282–1284.

Snyder, A.W., Chen, Y., Poladian, L. & Mitchell, D.J. (1990). Fundamental modes of highly nonlinear fibers. *Electron. Lett*. **26**, 643–644.

Sodha, M.S., Ghatak, A.K. & Tripathi, V.K. (1974*a*). *Self Focusing of Laser Beams in Dielectrics, Semiconductors and Plasmas*, Delhi: Tata-McGraw-Hill.

Sodha, M.S., Nayyar, V.P. & Tripathi, V.K. (1974*b*). Asymmetric focusing of the laser beam in a TEM-01 doughnut mode in dielectrics. *J. Opt. Soc. Am*. **64**, 941–943.

Sodha, M.S., Tripathi, V.K. & Ghatak, A.K. (1976). Seif focusing of laser beams in plasmas and semiconducters. *Prog. Opt*. **13**, 169–265.

Soding, J., Grimm, R. & Ovchinnikov, Yu.B. (1995). Gravitational laser trap for atoms with evanescent-wave cooling. *Opt. Commun*. **119**, 652–662.

Song, Y., Milam, D. & Hill, W.T. (1999). Long narrow all-light atom guide. *Opt. Lett*. **24**, 1805–1807.

Sprangle, P. & Esarey, E. (1991). Stimulated backscattered harmonic generation from intense laser interactions with beams and plasmas. *Phys. Rev. Lett*. **67**, 2021–2024.

Sprangle, P., Esarey, E., Ting, A. & Joyee, G. (1988). Laser wakefield acceleration and relativistic optical guiding. *Appl. Phys. Lett*. **53**, 2146–2148.

Tabak, M., Hammer, J., Glinisky, M.E., Kruer, W.L., Wilks, S.C., WoodWorth, J., Campbell, E.M., Perry, M.D. & Mason, R.J. (1994). Ignition and high gain with ultrapowerful lasers. *Phys. Plasmas* **1**, 1626–1634.

Umstadter, D., Chen, S.Y., Maksimchuk, A., Mourou, G. & Wagner, R. (1996). Nonlinear optics in relativistic plasmas and plasmas and laser wakefield acceleration of electrons. *Science* **273**, 472–475.

Umstadter, D. (2001). Review of physics and applications of relativistic plasmas driven by ultra-intense lasers. *Phys. Plasmas* **8**, 1774–1785.

Upadhyaya, A., Tripathi, V.K., Sharma, A.K. & Pant, H.C. (2002). Asymmetric self-focusing of a laser pulse in plasma. *J. Plasma Phys*. **68**, 75–80.

Vidal, F. & Johnston, T.W. (1996). Electromagnetic beam breakup: multi filaments, single beam equilbriya and radiation. *Phys. Rev. Lett*. **77**, 1282–1285.

Wang, X. & Littman, M.G. (1993). Laser cavity for generation of variable radius rings of light. *Opt. Lett*. **18**, 767–770.

Xu, X., Wang, Y. & Jhe, W. (2002). Theory of atom guidance in a hollow laser beam: dressed atom approach. *J. Opt. Soc. Am. B* **17**, 1039–1050.

Yin, J., Gao, W. & Zhu, Y. (2003). Propagation of various dark hollow beams in a turbulent atmosphere. *Prog. Opt*. **44**, 119–204.

Yin, J., Zhu, Y., Wang, W., Wang, Y. & Jhe, W. (1998). Optical potential for atom guidance in a hollow laser beam. *J. Opt. Soc. Am. B* **15**, 25–33.

York, A.G., Milchberg, H.M., Palastro, J.P. & Antonsen, T.M. (2008). Direct Acceleration of Electrons in a Corrugated Plasma Waveguide. *Phys. Rev. Lett*. **100**, 195001–7.

Yu, W., Yu, M.Y., Xu, H., Tian, Y.W., Chen, J. & Wong, A.Y. (2007). Intense local plasma heating by stopping of ultrashort ultraintense laser pulse in dense plasma. *Laser Part. Beams* **25**, 631–638.

Zhou, J., Peatross, J., Murnane, M.M., Kapteyn, H.C. & Christov, I.P. (1996). High-order frequency conversion in the plasma waveguide. *Phys. Rev. Lett*. **76**, 752–755.

Zhou, C.T., Yu, M.Y. & He, X.T. (2007). Electron acceleration by high current-density relativistic electron bunch in plasmas. *Laser Part. Beams* **25**, 313–319.

Zhu, K., Tang, H., Sun, X., Wang, X. & Liu, T. (2002). Flattened multi-Gaussian light beams with an axial shadow generated through superposing Gaussian beams. *Opt. Commun*. **207**, 29–34.