*Macroeconomic Dynamics*, **16**, 2012, 240–277. Printed in the United States of America.
doi:10.1017/S1365100510000106

# OIL SHOCKS AND OPTIMAL MONETARY POLICY

**CARLOS MONTORO**
*Banco Central de Reserva del Perú*
*Bank for International Settlements*
*and*
*CENTRUM Católica*

This paper studies how monetary policy should react to oil shocks in a microfounded model with staggered price-setting and oil as an input in a CES production function. In particular, we extend Benigno and Woodford [*Journal of the European Economic Association* 3 (6) (2005), 1–52] to obtain a second-order approximation to the expected utility of the representative household when the steady state is distorted and the economy is hit by oil price shocks. The main result is that oil price shocks generate an endogenous trade-off between inflation and output stabilization when oil has low substitutability in production. We also find, in contrast to Benigno and Woodford, that this trade-off is reduced, but not eliminated, when we get rid of the effects of monopolistic distortions in the steady state. Moreover, the size of the endogenous "cost-push" shock generated by fluctuations in the oil price increases when it is more difficult to substitute other factors for oil.

**Keywords:** Optimal Monetary Policy, Welfare, Second-Order Solution, Oil Price Shocks, Endogenous Trade-Off

## 1. INTRODUCTION

Oil is an important production factor in economic activity because every industry uses it to some extent. Moreover, because oil cannot be easily substituted with other production factors, economic activity is heavily dependent on its use. Furthermore, the oil price is determined in a weakly competitive market; there are few large oil producers dominating the world market, setting its price above a perfect competition level. Also, its price fluctuates considerably due to the effects of supply and demand shocks in this market.[1]

This paper was started while I was working at the Banco Central de Reserva del Perú. I would like to thank Chris Pissarides, Gianluca Benigno, Pierpaolo Benigno, John Driffill, Paul Castillo, Marco Vega, Vicente Tuesta, Jean-Marc Natal, participants at the Macroeconomics Student Seminar at LSE and the BCRP, and two anonymous referees for their valuable comments and suggestions. The views expressed herein are those of the author and do not necessarily reflect those of the Banco Central de Reserva del Perú nor those of the Bank for International Settlements. Any errors are my own responsibility. Address correspondence to: Carlos Montoro, Office for the Americas, Bank for International Settlements, Torre Chapultepec—Rubén Darío 281–1703, Col. Bosque de Chapultepec 11580, México DF, México; e-mail: carlos.montoro@bis.org, carlos.montoro@pucp.edu.pe.

 1365-1005/11 

The heavy dependence on oil and the high volatility of its price generate a concern among policymakers on how to react to oil shocks. Oil shocks have serious effects on the economy because they raise prices for an important production input and for important consumer goods (gasoline and heating oil). This causes an increase in inflation and subsequently a decrease in output, generating a dilemma for policymaking. On one hand, if monetary policy makers focused exclusively on the recessive effects of oil shocks and try to stabilize output, this would generate inflation. On the other hand, if monetary policy makers focused exclusively on neutralizing the impact of the shock on inflation through a contractive monetary policy, some sluggishness in the response of prices to changes in output would imply large reductions in output. In practice, when dealing with rising oil prices, policymakers have been confronted with a trade-off between stabilizing inflation and output. But, what exactly should be the optimal stabilization of inflation and output? Which factors affect this trade-off? To our knowledge the formal study of this topic is limited.[2]

However, the behavior of central banks in practice contrasts with the result in the standard New Keynesian model that ensuring complete price stability is the optimal thing to do, even when an oil shock leads to large drops in output. To deal with this apparent contradiction and to answer the questions presented above, we extend the literature on optimal monetary policy including oil in the production process in a standard New Keynesian model. In doing so, we extend Benigno and Woodford (2005) to obtain a second-order approximation to the expected utility of the representative household when the steady state is distorted and the economy is hit by oil price shocks. We include oil as a nonproduced input as in Blanchard and Galí (2007), but differently from those authors, we use a constant-elasticity-of-substitution (CES) production function to capture the low substitutability characteristic of oil. Thus, a low elasticity of substitution between labor and oil indicates a high dependence on oil.[3]

The analysis of optimal monetary policy in microfounded models with staggered price setting using a quadratic welfare approximation was first introduced by Rotemberg and Woodford (1997) and expounded by Woodford (2003) and by Benigno and Woodford (2005). This method allows us to obtain a linear policy rule derived from maximizing the quadratic approximation of the welfare objective subject to the linear constraints that are first-order approximations of the structural equations. This methodology is called linear–quadratic (LQ). The advantage of this approach is that it makes it possible to characterize analytically how changes in the production function and in the oil shock process affect the monetary policy problem. Moreover, in contrast to the Ramsey policy methodology, which also allows a correct calculation of a linear approximation of the optimal policy rule, the LQ approach is useful not only to evaluate the optimal rules, but also to evaluate and rank suboptimal monetary policy rules.

A property of standard New Keynesian models is that stabilizing inflation is equivalent to stabilizing output around some desired level, unless some exogenous cost-push shock disturbances are taken into account. Blanchard and Galí (2007) called this feature the "divine coincidence." These authors argue that this special feature comes from the absence of nontrivial real imperfections, such as real wage rigidities. Similarly, Benigno and Woodford (2004, 2005) show that this trade-off also arises when the steady state of the model is distorted and there are government purchases in the model.

We found that, when oil is introduced as a poorly-substitutable input in a New Keynesian model, a trade-off arises between stabilizing inflation and the gap between output and some desired level. We call this desired level the "target level." In this case, because output fluctuates less at the target level than it does at the natural level, it becomes optimal to the monetary authority to react partially to oil shocks and, therefore, some inflation is desirable.

The intuition of this result is that when oil is considered a gross complement to labor in production in a CES technology, the divine coincidence disappears. This result is similar to the case of real wage rigidities explained in Blanchard and Galí (2007), where stabilizing inflation is no longer equivalent to stabilizing the welfare-relevant output gap. However, the mechanism here is different. This trade-off is generated by the convexity of real marginal costs with respect to the real oil price, which produces a time-varying wedge between the marginal rate of substitution and the marginal productivity of labor that impedes replication of the first-best equilibrium. Moreover, eliminating the distortions in steady state reduces the trade-off, because this wedge becomes less sensitive with respect to the oil price. However, in contrast to Benigno and Woodford (2005), making the steady state efficient cannot eliminate this trade-off.

Also, substitutability among production factors affects both the weights on the two stabilization objectives and the definition of the welfare-relevant output gap. The lower the elasticity of substitution, the higher the cost-push shock generated by oil shocks and the lower the weight on output stabilization relative to inflation stabilization. Moreover, when the share of oil in the production function is higher, or the steady-state oil price is higher, the size of the cost-push shock increases.

Section 2 presents our New Keynesian model with oil prices in the production function. Section 3 includes an LQ approximation to the policy problem. Section 4 uses the LQ approximation to the problem to solve for the different rules of monetary policy and derive some comparative statics to the parameters related to oil. The last section concludes.

## 2. A NEW KEYNESIAN MODEL WITH OIL PRICES

The model economy corresponds to the standard New Keynesian model in the line of Clarida et al. (1999). To capture oil shocks, we follow Blanchard and Galí (2007, 2010) by introducing a nonproduced input $M$, in this case representing oil. $Q$ will be the real price of oil, which is assumed to be exogenous. This model is similar to the one used by Castillo et al. (2007), except that we additionally include taxes on sales of intermediate goods to analyze the distortions in the steady state.

## 2.1. Households

We assume the following utility function on consumption and labor of the representative consumer:

$$U_{t_o} = E_{t_o} \sum_{t=t_o}^{\infty} \beta^{t-t_o} \left( \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+v}}{1+v} \right), \tag{1}$$

where $\sigma$ represents the coefficient of risk aversion and $v$ captures the inverse of the elasticity of labor supply. The optimizing consumer makes decisions subject to a standard budget constraint, which is given by

$$C_t = \frac{W_t L_t}{P_t} + \frac{B_{t-1}}{P_t} - \frac{1}{R_t} \frac{B_t}{P_t} + \frac{\Gamma_t}{P_t} + \frac{T_t}{P_t}, \tag{2}$$

where $W_t$ is the nominal wage, $P_t$ is the price of the consumption good, $B_t$ is the end of period nominal bond holdings, $R_t$ is the riskless nominal gross interest rate, $\Gamma_t$ is the share of the representative household in total nominal profits, and $T_t$ is net transfers from the government. The first-order conditions for the optimizing consumer's problem are

$$1 = \beta E_t \left[ R_t \left( \frac{P_t}{P_{t+1}} \right) \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \right], \tag{3}$$

$$\frac{W_t}{P_t} = C_t^{\sigma} L_t^{v} = \mathrm{MRS}_t. \tag{4}$$

Equation (3) is the standard Euler equation, which determines the optimal path of consumption. At the optimum the representative consumer is indifferent between consuming today or tomorrow, whereas equation (4) describes the optimal labor supply decision. $\mathrm{MRS}_t$ denotes the marginal rate of substitution between labor and consumption. We assume that labor markets are competitive and also that individuals work in each sector $z \in [0, 1]$. Therefore, $L$ corresponds to the aggregate labor supply:

$$L = \int_0^1 L_t(z) dz. \tag{5}$$

## 2.2. Firms

*Final good producers.* There is a continuum of final good producers of mass one, indexed by $f \in [0, 1]$, that operate in an environment of perfect competition. They use intermediate goods as inputs, indexed by $z \in [0, 1]$, to produce final consumption goods using the technology

$$Y_t^f = \left[ \int_0^1 Y_t(z)^{\frac{\varepsilon-1}{\varepsilon}} dz \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{6}$$

where $\varepsilon$ is the elasticity of substitution between intermediate goods. The demand function of each type of differentiated good is obtained by aggregating the input demand of final good producers,

$$Y_t(z) = \left[\frac{P_t(z)}{P_t}\right]^{-\varepsilon} Y_t, \qquad \textbf{(7)}$$

where the price level is equal to the marginal cost of the final good producers and is given by

$$P_t = \left[\int_0^1 P_t(z)^{1-\varepsilon} dz\right]^{\frac{1}{1-\varepsilon}}, \qquad \textbf{(8)}$$

and $Y_t$ represents the aggregate level of output,

$$Y_t = \int_0^1 Y_t^f df. \qquad \textbf{(9)}$$

*Intermediate goods producers.* There is a continuum of intermediate good producers indexed by $z \in [0, 1]$. All of them have the CES production function

$$Y_t(z) = \left\{(1-\alpha)\left[L_t(z)\right]^{\frac{\psi-1}{\psi}} + \alpha\left[M_t(z)\right]^{\frac{\psi-1}{\psi}}\right\}^{\frac{\psi}{\psi-1}}, \qquad \textbf{(10)}$$

where $M$ is oil that enters as a nonproduced input, $\psi$ represents the intratemporal elasticity of substitution between labor input and oil, and $\alpha$ denotes the quasi-share of oil in the production function. We use this generic production function to capture the fact that oil has few substitutes. In general, we assume that $\psi$ is lower than one. The real oil price, $Q_t$, is assumed to follow an AR(1) process in logs,

$$\log Q_t = (1-\rho)\log\overline{Q} + \rho \log Q_{t-1} + \xi_t, \qquad \textbf{(11)}$$

where $\overline{Q}$ is the steady state level of oil price and $\xi_t$ is an i.i.d. shock. From the cost minimization problem of the firm, we obtain an expression for the real marginal cost given by

$$\mathrm{MC}_t(z) = \left[(1-\alpha)^{\psi}\left(\frac{W_t}{P_t}\right)^{1-\psi} + \alpha^{\psi}\left(Q_t\right)^{1-\psi}\right]^{\frac{1}{1-\psi}}, \qquad \textbf{(12)}$$

where $\mathrm{MC}_t(z)$ represents the real marginal cost and $W_t$ nominal wages. Notice that marginal costs are the same for all intermediate firms, because technology has constant returns to scale and factor markets are competitive; that is, $\mathrm{MC}_t(z) = \mathrm{MC}_t$. On the other hand, the first-order condition for intermediate goods producers with respect to labor implies that the marginal product of labor, $\mathrm{MPL}_t$, satisfies

$$\mathrm{MPL}_t(z) = (1-\alpha)\left[\frac{Y_t(z)}{L_t(z)}\right]^{1/\psi} = \frac{W_t/P_t}{\mathrm{MC}_t}. \qquad \textbf{(13)}$$

Equation (13) implies the following labor demand for the individual firm:

$$L_t^d(z) = \left(\frac{1}{1-\alpha}\frac{W_t/P_t}{\mathrm{MC}_t}\right)^{-\psi} Y_t(z). \tag{14}$$

Intermediate producers set prices following a staggered pricing mechanism a la Calvo. Each firm faces an exogenous probability of changing prices given by $(1-\theta)$. A firm that changes its price in period $t$ chooses its new price $P_t(z)$ to maximize

$$E_t \sum_{k=0}^{\infty} \theta^k \zeta_{t,t+k} \Gamma\left[P_t(z), P_{t+k}, \mathrm{MC}_{t+k}, Y_{t+k}\right],$$

where $\zeta_{t,t+k} = \beta^k (C_{t+k}/C_t)^{-\sigma} P_t/P_{t+k}$ is the stochastic discount factor. The function $\Gamma(P_t(z), P_t, \mathrm{MC}_t, Y_t) = [(1-\tau)P_t(z) - P_t\,\mathrm{MC}_t][P_t(z)/P_t]^{-\varepsilon} Y_t$ is the after-tax nominal profits of the supplier of good $z$ with price $P_t(z)$, where the aggregate demand and aggregate marginal costs are equal to $Y_t$ and $\mathrm{MC}_t$, respectively. $\tau$ is the proportional tax on sale revenues, which we assume constant. The optimal price that solves the firm's problem is given by

$$\left[\frac{P_t^*(z)}{P_t}\right] = \frac{\mu E_t\left(\sum\limits_{k=0}^{\infty} \theta^k \zeta_{t,t+k} \mathrm{MC}_{t,t+k} F_{t,t+k}^{\varepsilon+1} Y_{t+k}\right)}{E_t\left(\sum\limits_{k=0}^{\infty} \theta^k \zeta_{t,t+k} F_{t,t+k}^{\varepsilon} Y_{t+k}\right)}, \tag{15}$$

where $\mu \equiv \frac{\varepsilon}{\varepsilon-1}/(1-\tau)$ is the price markup net of taxes, $P_t^*(z)$ is the optimal price level chosen by the firm, and $F_{t,t+k} = \frac{P_{t+k}}{P_t}$ is the cumulative level of inflation. The optimal price solves equation (15) and is determined by the average of expected future marginal costs as

$$\left[\frac{P_t^*(z)}{P_t}\right] = \mu E_t\left(\sum_{k=0}^{\infty} \varphi_{t,t+k} \mathrm{MC}_{t,t+k}\right), \tag{16}$$

where

$$\varphi_{t,t+k} \equiv \frac{\theta^k \zeta_{t,t+k} F_{t,t+k}^{\varepsilon+1} Y_{t+k}}{E_t\left(\sum\limits_{k=0}^{\infty} \theta^k \zeta_{t,t+k} F_{t,t+k}^{\varepsilon} Y_{t+k}\right)}.$$

Because only a fraction $(1-\theta)$ of firms change prices every period and the remaining one keeps its price fixed, the aggregate price level, defined as the price of the final good that minimizes the cost of the final goods producers, is given by the following equation:

$$P_t^{1-\varepsilon} = \theta P_{t-1}^{1-\varepsilon} + (1-\theta)\left[P_t^*(z)\right]^{1-\varepsilon}. \tag{17}$$

Following Benigno and Woodford (2005), equations (15) and (17) can be written recursively, introducing the auxiliary variables $N_t$ and $D_t$ (see Appendix B for details on the derivation),

$$\theta\left(\Pi_t\right)^{\varepsilon-1}=1-(1-\theta)\left(\frac{N_t}{D_t}\right)^{1-\epsilon}, \tag{18}$$

$$D_t=Y_t\left(C_t\right)^{-\sigma}+\theta\beta E_t\left[\left(\Pi_{t+1}\right)^{\epsilon-1}D_{t+1}\right], \tag{19}$$

$$N_t=\mu Y_t\left(C_t\right)^{-\sigma}\mathrm{MC}_t+\theta\beta E_t\left[\left(\Pi_{t+1}\right)^{\epsilon}N_{t+1}\right], \tag{20}$$

where $\Pi_t = P_t/P_{t-1}$ is the gross inflation rate. Equation (18) comes from the aggregation of individual firms' prices. The ratio $N_t/D_t$ represents the optimal relative price $P_t^*(z)/P_t$. These last three equations summarize the recursive representation of the nonlinear Phillips curve.

## 2.3. Government and Monetary Policy

In the model we assume that the government owns the oil endowment. Oil is produced in the economy at zero cost and sold to the firms at the exogenous price $Q_t$. The government transfers all the revenues generated by oil to consumers, represented by $P_t Q_t M_t$. There is also a proportional tax on sale revenues ($\tau$). We abstract from any other role for the government and assume that it runs a balanced budget every period. Thus, the budget constraint implies that total net transfers in real terms are

$$\frac{T_t}{P_t}=Q_t M_t+\tau Y_t.$$

Moreover, we abstract from any monetary frictions, assuming that the central bank can control the riskless nominal short-term interest rate $R_t$ directly.

## 2.4. Market Clearing

In equilibrium, labor and intermediate and final goods markets clear. Because of the assumption on the government transfers, the economywide resource constraint is given by

$$Y_t=C_t. \tag{21}$$

The labor market–clearing condition is given by

$$L_t=L_t^d, \tag{22}$$

where the demand for labor comes from the aggregation of individual intermediate producers in the same way as for the labor supply,

$$\begin{aligned} L_t^d=\int_0^1 L_t^d(z)dz&=\left(\frac{1}{1-\alpha}\frac{W_t/P_t}{\mathrm{MC}_t}\right)^{-\psi}\int_0^1 Y_t(z)dz \\ &=\left(\frac{1}{1-\alpha}\frac{W_t/P_t}{\mathrm{MC}_t}\right)^{-\psi}Y_t\Delta_t, \end{aligned} \tag{23}$$

where $\Delta_t = \int_0^1 [P_t(z)/P_t]^{-\varepsilon} dz$ is a measure of price dispersion. Because relative prices differ across firms due to staggered price setting, input usage will differ as well, implying that it is not possible to use the usual representative firm assumption. Therefore, the price dispersion factor, $\Delta_t$, appears in the aggregate labor demand equation. We can also use (17) to derive the law of motion of $\Delta_t$:

$$\Delta_t = (1-\theta)\left[\frac{1-\theta\,(\Pi_t)^{\varepsilon-1}}{1-\theta}\right]^{\varepsilon/(\varepsilon-1)} + \theta\Delta_{t-1}\,(\Pi_t)^{\varepsilon}\,. \tag{24}$$

Note that inflation affects the welfare of the representative agent through the labor market. We can see, from (24), that higher inflation increases price dispersion and, from (23), that higher price dispersion increases the labor amount necessary to produce a certain level of output, implying more disutility on (1).

## 2.5. The Steady State

Variables in the steady state are denoted overlined (i.e., $\overline{X}$). The details of the steady state of the variables are in Appendix A. We depart from a steady state where gross inflation $\overline{\Pi} = 1$. Output in steady state is given by

$$\overline{Y} = \left[(1-\overline{\alpha})\,\overline{MC}\right]^{\frac{1}{\sigma+v}} \left(\frac{1-\overline{\alpha}}{1-\alpha}\right)^{\frac{1+\psi v}{\sigma+v}\frac{1}{1-\psi}},$$

where real marginal costs in steady state are

$$\overline{MC} = \frac{1-\tau}{\varepsilon/(\varepsilon-1)} \le 1, \tag{25}$$

where $\overline{\alpha} \equiv \alpha^{\psi}(\overline{Q}/\overline{MC})^{1-\psi}$ is the share of oil in total costs in steady state. Note that, from the definition of $\overline{\alpha}$, the steady state value of output depends on the steady state ratio of the real oil price to real marginal costs. This implies that a permanent increase in the real oil price will generate a permanent increase in $\overline{\alpha}$, given $\psi < 1$. Also, as in standard New Keynesian models, the real marginal costs in steady state are equal to the inverse of the mark-up. Because monopolistic competition and taxes affect the steady state of the model, output in steady state can be below the efficient level (the steady state is distorted). In the special case where $\tau = -1/(\varepsilon-1) < 0$, distortions are eliminated and the steady state is efficient. Let us denote the steady state distortion by

$$\Phi = 1 - \frac{1-\tau}{\varepsilon/(\varepsilon-1)}.$$

We have that $\Phi = 0$ when a subsidy on sales make the steady state undistorted.

## 2.6. The Log Linear Economy and the Natural Equilibrium

To illustrate the effects of oil on the dynamic equilibrium of the economy, we take a log linear approximation to equations (3), (4), (11), (12), (18), (19), (20), and (23) around the deterministic steady state. We denote variables in their log deviations around the steady state with lower case letters [i.e., $x_t = \log(X_t/\overline{X})$]. After the goods and labor market–clearing conditions are imposed to eliminate real wages from the system, the dynamics of the economy is determined by the equations

$$l_t = y_t - \delta\left[(\sigma + v)\, y_t - q_t\right], \tag{26}$$

$$mc_t = \chi\,(v + \sigma)\, y_t + (1 - \chi)\, q_t, \tag{27}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa mc_t, \tag{28}$$

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}\left(r_t - E_t \pi_{t+1}\right), \tag{29}$$

$$q_t = \rho q_{t-1} + \xi_t, \tag{30}$$

where $\delta \equiv \psi\chi\frac{\overline{\alpha}}{1-\overline{\alpha}}$, $\chi \equiv \frac{1-\overline{\alpha}}{1+v\psi\overline{\alpha}}$, and $\kappa \equiv \frac{1-\theta}{\theta}(1-\theta\beta)$. $\delta$ and $(1-\chi)$ account for the effects of oil prices on labor and marginal costs, respectively. $\kappa$ is the elasticity of inflation respect to marginal costs.

Interestingly, the effects of oil prices on marginal costs, given by $(1-\chi)$ in equation (27), depend crucially on the quasi-share of oil in the production function, $\alpha$, and on the elasticity of substitution between oil and labor, $\psi$. Thus, when $\alpha$ is larger, $\chi$ is smaller, making marginal costs more responsive to oil prices. Also, when $\psi$ is lower, the impact of oil on marginal costs is larger. It is important to note that even though the quasi-share of oil in the production function, $\alpha$, can be small, its impact on marginal cost, $\overline{\alpha}$, can be magnified when oil has few substitutes (that is, when $\psi$ is low). Moreover, a permanent increase in the real oil price or in the distortions in steady state (that is, an increase in $\overline{Q}$ or a decrease in $\overline{MC}$) would make the marginal costs of firms more sensitive to oil price shocks because it would increase $\overline{\alpha}$. In the case where $\alpha = 0$, the model collapses to a standard closed economy New Keynesian model without oil.

The natural equilibrium corresponds to the case where nominal rigidities are absent and prices are flexible. We denote variables in this equilibrium with the supra-index "$n$." Under flexible prices, real marginal costs satisfy $mc_t^n = 0$ and the equilibrium can be expressed as

$$\left(y_t - y_t^n\right) = E_t\left(y_{t+1} - y_{t+1}^n\right) - \frac{1}{\sigma}\left(r_t - E_t\pi_{t+1}\right), \tag{31}$$

$$\pi_t = \kappa_y\left(y_t - y_t^n\right) + \beta E_t \pi_{t+1}, \tag{32}$$

where $\kappa_y \equiv \kappa\chi(v + \sigma)$. Equations (31) and (32) are the dynamic IS and the Phillips curve, respectively, in terms of the output gap $(y_t - y_t^n)$. The natural level

https://doi.org/10.1017/S1365100510000106 Published online by Cambridge University Press

of output depends negatively on deviations of the oil price from its steady state:

$$y_t^n = -\left(\frac{1+\psi v}{\sigma + v}\right)\left(\frac{\overline{\alpha}}{1-\overline{\alpha}}\right) q_t. \tag{33}$$

The natural output depends, among other parameters, on the share of oil in total costs in steady state. The higher $\overline{\alpha}$, the more important the impact of oil price shocks on the natural level. Also, note from equation (33) that the response of the natural output to oil shocks is qualitatively similar to the reaction to productivity shocks in the standard New Keynesian model with the opposite sign. However, as we will see in the next section, the assumption of low substitutability of oil has important effects on the design of optimal monetary policy.

### 2.7. Calibration

As benchmark calibration, we set a quarterly discount factor, $\beta$, equal to 0.99, which implies an annualized rate of interest of 4%. For the coefficient of the risk aversion parameter, $\sigma$, we choose a value of 1 and the inverse of the elasticity of labor supply, $v$, is calibrated to be equal to 0.5, similar to those values used in the RBC literature. The probability of the Calvo lottery is set equal to 0.66 which implies, that firms adjust prices, on the average, every three quarters. We choose a degree of monopolistic competition, $\varepsilon$, equal to 7.88, which implies a firm mark-up of 15% over the marginal cost assuming $\tau = 0$. We set the value of the elasticity of substitution between oil and labor at $\psi = 0.2$, equal to the average value reported by Hamilton (2009). We calibrate $\overline{\alpha} = 0.02895$ using information from the National Income Product accounts for the United States.[4] Finally, we assume a persistent AR(1) process for the logarithm of the real oil price ($\rho = 0.95$).

## 3. A LINEAR–QUADRATIC APPROXIMATE PROBLEM

In this section we characterize the sources of the trade-off between stabilizing inflation and economic activity that arise in this economy. Also, we present a second-order approximation of the welfare function of the representative household as a function of purely quadratic terms. This representation allows us to characterize the policy problem using only a linear approximation of the structural equations of the model and also to rank suboptimal monetary policy rules.

Because the model has an additional production input different from labor, a standard second-order Taylor approximation of the welfare function will include linear terms, which would lead to an inaccurate approximation of the optimal policy in a LQ approach. To deal with this issue, we use the methodology proposed by Benigno and Woodford (2005), which consists of eliminating the linear terms of the policy objective using a second-order approximation of the aggregate supply.

### 3.1. Sources of the Trade-Off

The efficient equilibrium is equivalent to the social planner problem of maximizing the utility of the representative agent, subject to the production functions for final goods and intermediate goods, the resources constraint, and the aggregation conditions for both production inputs. The efficiency conditions for this problem imply that the marginal rate of substitution is equal to the marginal productivity of labor,

$$MRS_t = \mathrm{MPL}_t(z), \qquad \textbf{(34)}$$

and a symmetric allocation in equilibrium, $C_t(z) = C_t$ and $L_t(z) = L_t$, for every $z$.

In the decentralized equilibrium of the model, the ratio between the marginal rate of substitution and the marginal productivity of labor equals the real marginal costs,

$$\frac{MRS_t}{\mathrm{MPL}_t(z)} = \mathrm{MC}_t \equiv 1 - \Phi_t, \qquad \textbf{(35)}$$

where $\Phi_t$ is the measure of the wedge between them. The optimality condition (34) implies that this wedge must be constant and equal to zero, that is, $\Phi_t = 0$, to be socially optimal. A second-order Taylor expansion of equation (35) in logarithms is

$$\Phi_t = \Phi - \chi\,(\sigma + v)\left(y_t - y_t^n\right) - \chi v \widehat{\Delta}_t$$
$$- \frac{1}{2}\frac{1-\psi}{1-\overline{\alpha}}\chi^2\,(1-\chi)\,\chi\,(\sigma+v)\left(y_t + \frac{\chi}{1-\chi}y_t^n\right)^2 + O\left(\|\xi_t\|^3\right), \qquad \textbf{(36)}$$

where $\|\xi_t\|$ denotes a bound on the size of the oil price shock. If monetary policy can be used to replicate the natural equilibrium, this wedge becomes

$$\Phi_t^{flex} = \Phi - \frac{1}{2}\frac{1-\psi}{1-\overline{\alpha}}\frac{1}{\sigma+v}\,(q_t)^2 + O\left(\|\xi_t\|^3\right), \qquad \textbf{(37)}$$

where we have used the definition of the natural output and evaluated the price dispersion term at zero. Note from equation (37) that when the flexible price allocation in the decentralized equilibrium is replicated, the wedge is time-varying and depends on the oil price. Because of this, a trade-off arises: it is not possible at the same time to stabilize inflation and to replicate the social planner equilibrium in the presence of oil shocks, unless $\psi = 1$, as in the Cobb–Douglas case.

As shown above, when oil is considered a gross complement to labor in production in a CES technology, the divine coincidence disappears. This result is similar to the case of real wage rigidities explained in Blanchard and Galí (2007), where stabilizing inflation is no longer equivalent to stabilizing the welfare-relevant output gap. However, the mechanism here is different. In this case, the flexible price allocation cannot replicate the social planner allocation because of the second-order effects of oil shocks in the wedge between the marginal rate of substitution and the marginal product of labor. When oil is difficult to substitute in production, real marginal costs become a convex function of the real oil price, because the participation of this input in marginal costs also increases with its price.

Interestingly, eliminating the distortions in steady state cannot eliminate the trade-off. In this case, after making $\Phi = 0$, the wedge becomes

$$\Phi_t^{flex,efss} = -\frac{1}{2}\frac{1-\psi}{1-\widetilde{\alpha}}\frac{1}{\sigma+v}(q_t)^2 + O\left(\|\xi_t\|^3\right) \qquad \textbf{(38)}$$

for $\widetilde{\alpha} \equiv \alpha^{\psi}(\overline{Q})^{1-\psi} \leq \overline{\alpha}$. In this case, eliminating the distortion in the steady state eliminates the constant and reduces the variability of the wedge with respect to the oil price. However, it is still not possible to replicate the social planner equilibrium in the presence of oil shocks. The intuition for this result is that when oil is considered a gross complement to labor in production in a CES technology, the share of oil in total costs in the steady state depends also on the steady state distortion. Eliminating the distortion (a more competitive economy) makes the wedge less sensitive to increases in the real oil price. However, making the steady state efficient cannot completely eliminate this sensitivity.

To measure this trade-off, in the next subsection we derive a quadratic loss function from the second-order Taylor expansion of the welfare function of the representative agent. We obtain a expression in terms of inflation and the deviations of output from a target level (the welfare-relevant output gap). This target level accounts for the effects of oil shocks in the wedge and maximizes the welfare of the representative agent when inflation is zero.

### 3.2. A Second-Order Approximation to Utility

A second-order Taylor series approximation to the utility function, expanding around the nonstochastic steady-state allocation, is

$$U_{t_o} = \overline{Y}\overline{u}_c \sum_{t=t_o}^{\infty} \beta^{t-t_o}\left(\Phi_L y_t + \frac{1}{2}u_{yy}y_t^2 + u_{yq}y_t q_t + u_{\Delta}\widehat{\Delta}_t\right) + \text{t.i.p.} + O\left(\|\xi_t\|^3\right), \qquad \textbf{(39)}$$

where $y_t \equiv \log(Y_t/\overline{Y})$ and $\widehat{\Delta}_t \equiv \log\Delta_t$ measure deviations of aggregate output and the price dispersion measure from their steady state levels, respectively. The term "t.i.p." collects terms that are independent of policy (constants and functions of exogenous disturbances) and hence irrelevant for ranking alternative policies. The coefficients $u_{yy}$, $u_{yq}$, and $u_{\Delta}$ are defined in the Appendix B. $\Phi_L$ is the wedge between consumption and labor in the utility function in the steady state, defined by

$$\begin{aligned}\Phi_L &= 1 - \frac{\overline{V}_L}{\overline{U}_C}\frac{d\overline{L}}{d\overline{Y}} \\ &= 1 - (1-\overline{\alpha})(1-\Phi)\left[1-\delta(\sigma+v)\right].\end{aligned} \qquad \textbf{(40)}$$

Note that in an economy with labor as the only input in the production function, as in Benigno and Woodford (2005), the wedge between consumption and labor in the utility function is equal to the distortion in steady state $\Phi$. In those models, a subsidy that eliminates this distortion also eliminates the linear term in the second-order Taylor expansion of the utility function. However, in an economy with other inputs different from labor, we have in general that $\Phi_L \neq \Phi$, and eliminating the monopolistic distortion does not eliminate the linear term in equation (39).

We substitute the second-order Taylor expansion of the price dispersion equation for $\widehat{\Delta}_t$ as a function of quadratic terms of inflation in our welfare approximation. Also, we use the second-order approximation of the Phillips curve to solve for the infinitely discounted sum of the expected level of output as a function of purely quadratic terms. Then, as in Benigno and Woodford (2005), we replace this last expression in (39) and rewrite it as

$$U_{t_o} = -\Omega \left\{ E_{t_o} \sum_{t=t_o}^{\infty} \beta^{t-t_o} \left[ \frac{1}{2}\lambda \left(y_t - y_t^*\right)^2 + \frac{1}{2}\pi_t^2 \right] - T_{t_o} \right\} + \text{t.i.p.} + O\left(\|\xi_t\|^3\right), \tag{41}$$

where $\Omega = \overline{Y}\overline{u}_c\lambda_\pi$ and $T_{t_o} = \frac{\Phi_L}{\kappa_y} v_{t_o}$; $\lambda_\pi$ and $v_{t_o}$ are defined in Appendix B.3. $\lambda$ measures the relative weight between a welfare-relevant output gap and inflation. $y_t^*$ is the target output, the level of output that maximizes our measure of welfare when inflation is zero. The values of $\lambda$ and $y_t^*$ are given by

$$\lambda = \frac{\kappa_y}{\varepsilon}\left(1 - \sigma\psi\overline{\alpha}\right)\gamma, \tag{42}$$

$$y_t^* = -\left(\frac{1+\psi v}{\sigma + v}\right)\left(\frac{\alpha^*}{1-\alpha^*}\right) q_t, \tag{43}$$

where $\alpha^*$ accounts for the share of oil in total costs in the steady state that replicates the target level of output, given by

$$\alpha^* = \frac{\overline{\alpha}}{1+\eta}. \tag{44}$$

Both $\gamma$ and $\eta$ are functions of the deep parameters of the model; they are defined in Appendix B.3 and characterized in the next section. Note that the target level of output (44) is written in a way similar to the natural level of output in equation (33), for a different share of oil in total costs in steady state.

### 3.3. The Linear–Quadratic Policy Problem

The policy objective $U_{t_o}$ can be written in terms of inflation and the welfare-relevant output gap defined by $x_t$:

$$x_t \equiv y_t - y_t^*.$$

Benigno and Woodford (2005) showed that maximization of $U_{t_o}$ is equivalent to minimization of the lost function

$$L_{t_o} \equiv E_{t_o} \sum_{t=t_o}^{\infty} \beta^{t-t_o} \left( \frac{1}{2} \lambda x_t^2 + \frac{1}{2} \pi_t^2 \right), \quad (45)$$

subject to a predetermined value of $\pi_{t_o}$[5] and the Phillips curve for any date from $t_o$ onward:

$$\pi_t = \kappa_y x_t + \beta E_t \pi_{t+1} + u_t. \quad (46)$$

Note that we have expressed (46) in terms of the welfare-relevant output gap, $x_t$. $u_t$ is a "cost-push" shock, which is proportional to the deviations in the real oil price,

$$\begin{aligned} u_t &\equiv \kappa_y \left( y_t^* - y_t^n \right) \\ &= \varpi q_t, \end{aligned} \quad (47)$$

where

$$\varpi \equiv \kappa_y \left( \frac{1 + \psi v}{\sigma + v} \right) \left( \frac{\overline{\alpha}}{1 - \overline{\alpha}} - \frac{\alpha^*}{1 - \alpha^*} \right).$$

In this model a "cost-push" shock arises endogenously because oil generates a trade-off between stabilizing inflation and deviations of output from a target level different from the natural level. In the next section we characterize the conditions under which oil shocks preclude simultaneous stabilization of inflation and the welfare-relevant output gap.

If we are interested in evaluating monetary policy from a timeless perspective, that is, optimizing without regard to possible short-run effects and avoiding possible time inconsistency problems, the predetermined value of $\pi_{t_o}$ must equal $\pi_{t_o}^*$, the optimal value of inflation at $t_o$ consistent with the policy problem. Thus, the policy objective consists of minimizing (45) subject to the initial inflation rate

$$\pi_{t_o} = \pi_{t_o}^*. \quad (48)$$

## 4. OPTIMAL MONETARY RESPONSE TO OIL SHOCKS

In this section we use the LQ policy problem defined in the previous section to evaluate optimal and suboptimal monetary policy rules under oil shocks. This policy problem can be summarized to maximize the following Lagrangian:

$$\mathcal{L}_{t_o} \equiv -E_{t_o} \left\{ \begin{array}{c} \sum_{t=t_o}^{\infty} \beta^{t-t_o} \left[ \frac{1}{2} \lambda x_t^2 + \frac{1}{2} \pi_t^2 - \varphi_t \left( \pi_t - \kappa_y x_t - \beta E_t \pi_{t+1} - u_t \right) \right] \\ + \varphi_{t_o - 1} \left( \pi_{t_o} - \pi_{t_o}^* \right) \end{array} \right\}, \quad (49)$$

where $\beta^{t-t_o} \varphi_t$ is the Lagrange multiplier at period $t$.

The second-order conditions for this problem are well defined for $\lambda \geq 0$, which is the case for plausible parameters of the model.[6] Thus, as Benigno and Woodford (2005) show, because the loss function is convex, randomization of monetary policy is welfare-reducing and there are welfare gains when monetary policy rules are used.

Under certain circumstances the optimal policy involves complete stabilization of the inflation rate at zero for every period, that is, complete price stability. These conditions are related to how oil enters into the production function and are summarized in the following proposition:

PROPOSITION 1. *When the production function is Cobb–Douglas the efficient level of output is equivalent to the natural level of output.*

In the case of a Cobb–Douglas production function, the elasticity of substitution between labor and oil is unity (i.e., $\psi = 1$). In this case $\eta = 0$ and the share of oil in the marginal costs at the efficient level is equal to the share in the distorted steady state, equal to $\alpha$ (that is, $\alpha^* = \overline{\alpha} = \alpha$). Thus, the efficient level of output is equal to the natural level of output.

In this special case of the CES production function, fluctuations in output caused by oil shocks at the target level equals the fluctuations in the natural level. Then, stabilization of output around the natural level also implies stabilization around the target level. This is a special case in which the "divine coincidence" appears. Therefore, setting output equal to the target level also implies complete stabilization of inflation at zero.

In this particular case there is not a trade-off between stabilizing output and inflation. However, in a more general specification of the CES production function this trade-off appears, as it is established in the next proposition:

PROPOSITION 2. *When oil is difficult to substitute in production the efficient output responds less to oil shocks than the natural level, which generates a trade-off.*

When oil is difficult to substitute the elasticity of substitution between inputs is lower than one (that is, $\psi < 1$). In this case $\eta > 0$ and the share of oil in total costs in the steady state that replicates the target level of output is lower than that in the steady state (that is, $\alpha^* < \overline{\alpha}$), which causes the target output to fluctuate less than the natural level (that is, $|y_t^*| < |y_t^n|$). Thus, in this case it is not possible to have both inflation zero and output at the target level at all periods. In this case a "cost-push" shock arises endogenously, which generates a trade-off between stabilizing inflation and the welfare-relevant output gap. This "cost-push" shock is proportional to the difference between $y_t^*$ and $y_t^n$, as shown in equation (47).

As mentioned in the previous section, this trade-off is generated by the convexity of real marginal costs with respect to the real oil price, which produce a time-varying wedge between the marginal rate of substitution and the marginal productivity of labor. Moreover, eliminating the distortions in the steady state

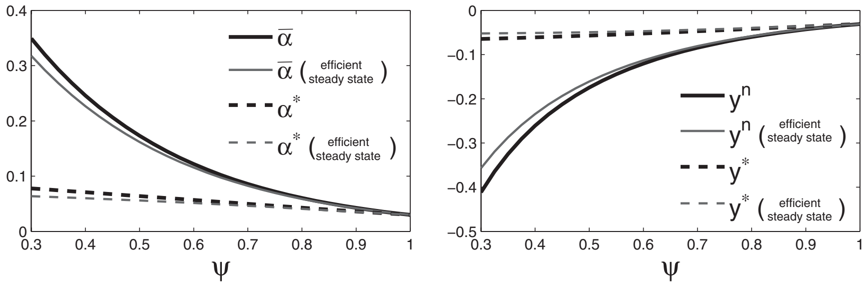


**FIGURE 1.** (a) Share of oil in total costs. (b) Natural and target level of output.

reduces the trade-off, because this wedge becomes less sensitive with respect to the oil price. However, making the steady state efficient cannot eliminate this trade-off.

Figure 1 shows the effects of the elasticity of substitution on $\alpha^*$ and $\overline{\alpha}$ and on $y^*$ and $y^n$. As mentioned in Proposition 1, when $\psi = 1$, $\alpha^* = \overline{\alpha} = \alpha$. Similarly, as in Proposition 2, lower $\psi$ increases both $\alpha^*$ and $\overline{\alpha}$, but $\alpha^*$ is always lower than $\overline{\alpha}$. Also, in this case, the efficient output fluctuates less than the natural level of output for a 1% increase in the real oil price. Because of this difference between $y^*$ and $y^n$, the endogenous "cost-push" shock also increases when the elasticity of substitution $\psi$ is lower. Moreover, this figure also shows the effects when distortions in the steady state are eliminated. In this case, both $\alpha^*$ and $\overline{\alpha}$ decrease and $y^*$ and $y^n$ become less sensitive to an oil price shock.

It is also important to analyze how the production function affects λ, the relative weight of stabilizing the welfare-relevant output gap and inflation. In the special case of a Cobb–Douglas production function, the coefficient $\gamma$ defined in the previous section equals 1 and the relative weight of the loss function between welfare-relevant output gap and inflation stabilization (λ) becomes $\frac{\kappa_y}{\varepsilon}(1 - \sigma\alpha)$. This is similar to the coefficient found by many authors for the case of a closed economy,[7] which is the ratio of the effect of output on inflation in the Phillips curve to the elasticity of substitution among goods, but multiplied by the additional term $(1 - \sigma\alpha)$.

The relative weight in the loss function between welfare-relevant output gap and inflation stabilization is decreasing in the degree of price stickiness ($\theta$) and the elasticity of substitution among goods ($\varepsilon$). When prices are more sticky (larger $\theta$), $\kappa_y$ is lower and price dispersion is higher. Similarly, a higher elasticity of substitution among goods ($\varepsilon$) amplifies the welfare losses caused by any given price dispersion. In both cases, the costs of inflation are more important and output stabilization has a lower weight than inflation stabilization.

The term $(1 - \sigma\alpha)$ captures the effects of oil shocks on inflation through costs. When the weight of oil in the production function ($\alpha$) is higher, the effects of oil shocks in marginal costs and inflation are more important. Thus, it becomes more important to stabilize inflation with respect to output.

The next proposition describes the behavior of $\lambda$ with respect to the elasticity of substitution $\psi$.

PROPOSITION 3. *The lower the elasticity of substitution between oil and labor, the lower the weight in the loss function between welfare-relevant output gap and inflation stabilization ($\lambda$).*

When the elasticity of substitution $\psi$ is lower, the effect of output fluctuations on inflation becomes smaller ($\kappa_y$). This implies a higher relative effect on inflation respect to output, and therefore lower $\lambda$. This also implies a higher sacrifice ratio, because there are necessary relatively larger changes on the interest rate in order to stabilize inflation.

Figure 2 shows the effects on $\lambda$ of the elasticity of substitution for three different values of $\alpha$. $\lambda$ takes its highest value when $\psi = 1$ and decreases exponentially for lower $\psi$. Also, higher $\alpha$ reduces $\lambda$, which means a higher weight on inflation relative to output fluctuations in the welfare function.

### 4.1. Optimal Unconstrained Response to Oil Shocks from a Timeless Perspective

When we solve for the Lagrangian (49), we obtain the following first-order conditions that characterize the solution of the optimal path of inflation and the welfare-relevant output gap in terms of the Lagrange multipliers:

PROPOSITION 4. *The optimal unconstrained response to oil shocks is given by the conditions*

$$\pi_t = \varphi_{t-1} - \varphi_t,$$
$$x_t = \frac{\kappa_y}{\lambda}\varphi_t,$$

*where $\varphi_t$ is the Lagrange multiplier of the optimization problem that has the law of motion*

$$\varphi_t = \tau_\varphi \varphi_{t-1} - \phi q_t,$$

*for $\phi \equiv \frac{\tau_\varphi}{1-\beta\tau_\varphi\rho}\varpi$, and that satisfies the initial condition*

$$\varphi_{t_o-1} = -\phi \sum_{k=0}^{\infty} \tau_\varphi^k q_{t-1-k},$$

*where $\tau_\varphi = Z - \sqrt{Z^2 - \frac{1}{\beta}} < 1$ and $Z = [(1+\beta) + \frac{\kappa_y^2}{\lambda}]/(2\beta)$.*

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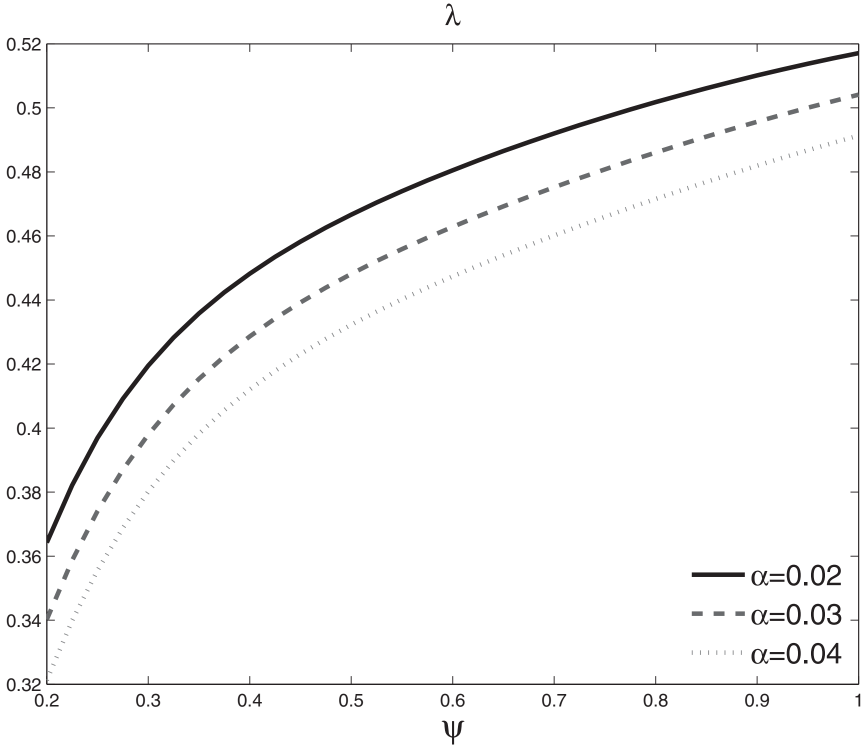


**FIGURE 2.** Relative weight between output and inflation stabilization (λ).

The proof is in Appendix C.1. From a timeless perspective the initial condition for $\varphi_{t_o-1}$ depends on the past realizations of the oil prices and it is time-consistent with the policy problem.

Also, we define the impulse response of a shock in the oil price in period $t$ ($\xi_t$) in a variable $z$ in $t+j$ as the unexpected change in its transition path. Thus the impulse is calculated by

$$I_t(z_{t+j}) = E_t(z_{t+j}) - E_{t-1}(z_{t+j}),$$

and the impulse responses for inflation, the price level, and the welfare-relevant output gap for the optimal policy are

$$I_t^{opt}(\pi_{t+j}) = \left( \frac{\rho^{j+1} - \tau_\varphi^{j+1}}{\rho - \tau_\varphi} - \frac{\rho^j - \tau_\varphi^j}{\rho - \tau_\varphi} \right) \phi \xi_t, \qquad \textbf{(50)}$$

$$I_t^{opt}(p_{t+j}) = \left( \frac{\rho^{j+1} - \tau_\varphi^{j+1}}{\rho - \tau_\varphi} \right) \phi \xi_t, \qquad \textbf{(51)}$$

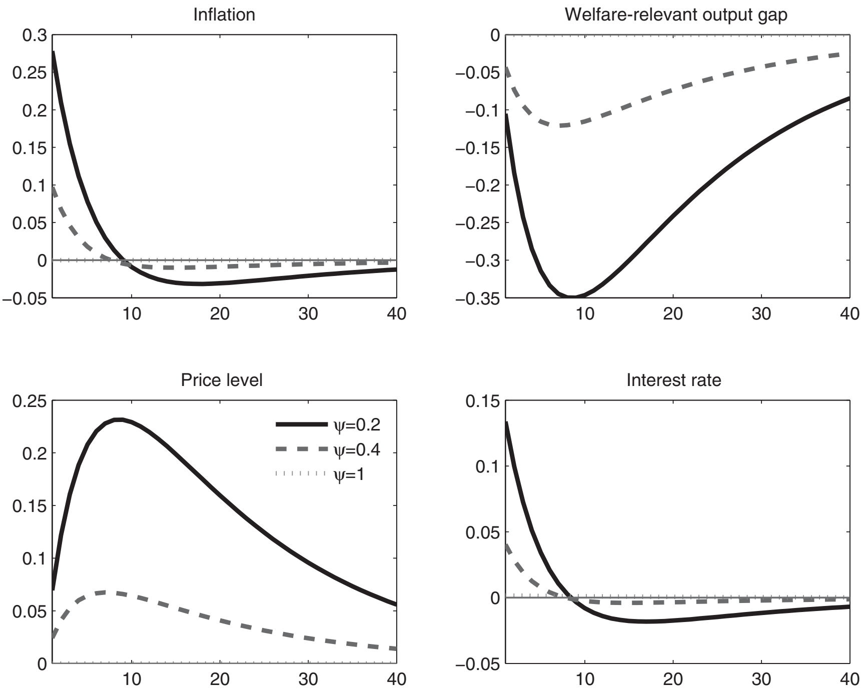


**FIGURE 3.** Impulse response to an oil shock under optimal unconstrained monetary policy.

$$I_t^{opt}(x_{t+j}) = -\frac{\kappa_y}{\lambda}\left(\frac{\rho^{j+1} - \tau_\varphi^{j+1}}{\rho - \tau_\varphi}\right)\phi\xi_t. \tag{52}$$

See Appendix C.1 for details on the derivation.

Figure 3 shows the optimal unconstrained impulse response functions of inflation, the welfare-relevant output gap, the price level, and the nominal interest rate to an oil price shock of size one for different values of the elasticity of substitution ($\psi$). Inflation and the nominal interest rate are in yearly terms. The benchmark case is a value of $\psi = 0.2$. In these graphs we can see that after an oil shock the optimal response is an increase of inflation and a reduction of the welfare-relevant output gap. The nominal interest rate also increases to partially offset the effects of the oil shock on inflation. Inflation after eight quarters become negative as the optimal unconstrained plan is associated with price stability. Thus, after some time, the price level returns to its initial level. To summarize, the optimal response to an oil shock implies an effect on impact on inflation that dies out very rapidly and a more persistent effect on output.

An increase in the elasticity of substitution from 0.2 to 0.4 reduces the size of the cost push shock and diminishes $\overline{\alpha}$ but increases $\lambda$. Thus, the impact on all the variables is reduced, inflation initially being the more affected variable. Also, the higher impact on the welfare-relevant output gap is after eight quarters. In contrast,

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when the elasticity of substitution is unity, because there is no such trade-off, both inflation and welfare-relevant output gap are zero in every period.

## 4.2. Evaluation of Suboptimal Rules: The Noninertial Plan

We can use our LQ policy problem to rank alternative suboptimal policies. One example of such policies is the optimal noninertial plan. By a noninertial policy we mean a monetary policy rule that depends only on the current state of the economy. In this case, if the policy results in a determinate equilibrium, then the endogenous variables also depend on the current state.

If the current state of the economy is given by the cost-push shock, which has the law of motion

$$u_t = \rho u_{t-1} + \varpi \xi_t,$$

where $\xi_t$ is the oil price shock and $\varpi$ is defined in the preceding section, a first-order general description of the possible equilibrium dynamics can be written in the form[8]

$$\pi_t = \overline{\pi} + f_\pi u_t, \tag{53}$$

$$x_t = \overline{x} + f_x u_t, \tag{54}$$

$$\varphi_t = \overline{\varphi} + f_\varphi u_t, \tag{55}$$

where we need to determine the coefficients $\overline{\pi}, \overline{x}, \overline{\varphi}, f_\pi, f_x$, and $f_\varphi$. To solve for the optimal noninertial plan, we need to replace (53), (54), and (55) in the Lagrangian (49) and solve for the coefficients that maximize the objective function. The results are summarized in the following proposition:

PROPOSITION 5. *The optimal noninertial plan is given by* $\pi_t = \overline{\pi} + f_\pi u_t$ *and* $x_t = \overline{x} + f_x u_t$, *where*

$$\overline{\pi} = 0,\ f_\pi = \frac{\lambda(1-\rho)}{\kappa_y^2 + \lambda(1-\beta\rho)(1-\rho)}.$$

$$\overline{x} = 0,\ f_x = \frac{\kappa_y}{\kappa_y^2 + \lambda(1-\beta\rho)(1-\rho)}.$$

Note that in the optimal noninertial plan the ratio of inflation/output gap is constant and equal to $\frac{\lambda(1-\rho)}{\kappa_y}$. The higher the weight in the loss function for output fluctuations relative to inflation fluctuations, the higher the inflation rate. Also, the more persistent the oil shocks, the lower the weight on inflation relative to the welfare-relevant output gap.

Similarly to the optimal case, the impulse response functions for inflation and output are defined by

$$I_t^{ni}(\pi_{t+j}) = f_\pi \varpi \rho^j \xi_t,$$

$$I_t^{ni}(x_{t+j}) = f_\pi \varpi \rho^j \xi_t.$$

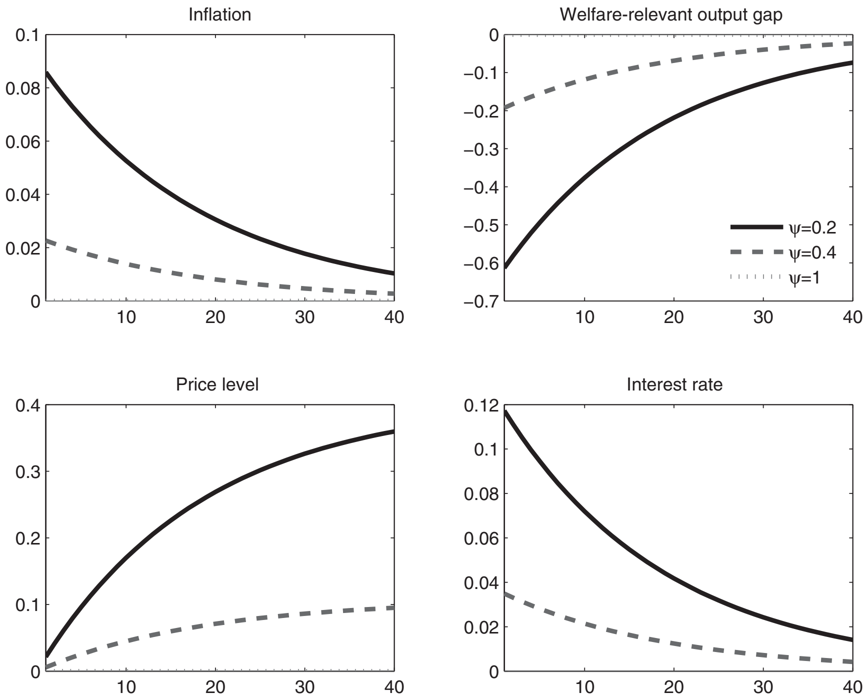


**FIGURE 4.** Impulse response to an oil shock under the optimal noninertial plan.

Figure 4 shows the responses in the optimal noninertial plan to a unitary oil price shock. As shown, the main difference with respect to the previous plan is that in the optimal noninertial plan, inflation returns to its initial level after some time, but in the optimal unconstrained plan the price level is the one that converges. This implies that inflation must be negative after some quarters in the optimal unconstrained plan. Also, the reduction in the welfare-relevant output gap is much lower on impact in the case of the optimal unconstrained plan than in that of the optimal noninertial plan. In the latter, the reduction in the welfare-relevant output is proportional to the increase in inflation.

Both exercises, the optimal unconstrained plan and the optimal noninertial plan, show that to the extent that economies are more dependent on oil, in the sense that oil is difficult to substitute for, the impact of oil shocks on both inflation and output is greater. Also, in this case, monetary policy should react by raising the nominal interest rate more and allowing relatively more fluctuations in inflation than in output.

Furthermore, figure 4 shows the responses under the optimal noninertial plan when $\psi$ increases from 0.2 to 0.4. As shown, the impact on all the variables is reduced, because an increase of $\psi$ diminishes the size of the cost-push shock. Also, the increase of $\psi$ makes $\lambda$ larger, which makes the impact on inflation relatively

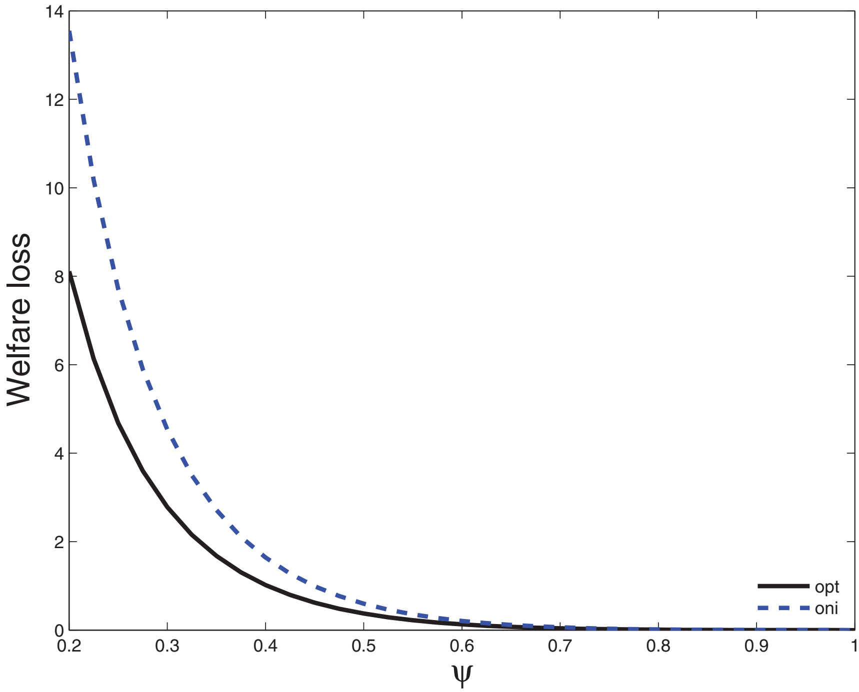


**FIGURE 5.** Welfare losses under both plans.

higher with respect to the response of the welfare-relevant output gap. As in the unconstrained case, when $\psi = 1$ the trade-off disappears. In that case, inflation is zero in every period and output equals its target level.

After analyzing the optimal plans, in the Figure 5 we plot the welfare losses for these two type of policies for different elasticities of substitution $\psi$. The welfare losses are normalized with respect to the variance of oil shocks. As shown, the welfare losses under both regimes are the same, equal to zero, when the production function is Cobb–Douglas. Moreover, when the elasticity of substitution $\psi$ decreases, the difference in the welfare losses under the two policy plans increases exponentially, which is consistent with the increase of the size of the "cost-push" shock.

## 4.3. A Simple Rule That Implements the Optimal Noninertial Plan

Optimal monetary plans can be difficult to communicate and implement, because they rely on real-time calculations of the welfare-relevant output gap and the size of the "cost-push" shock, which are unobservable variables. Because of this, in this section we estimate a simple interest rate rule that implements the optimal noninertial plan that is based only on observable variables, such as inflation and

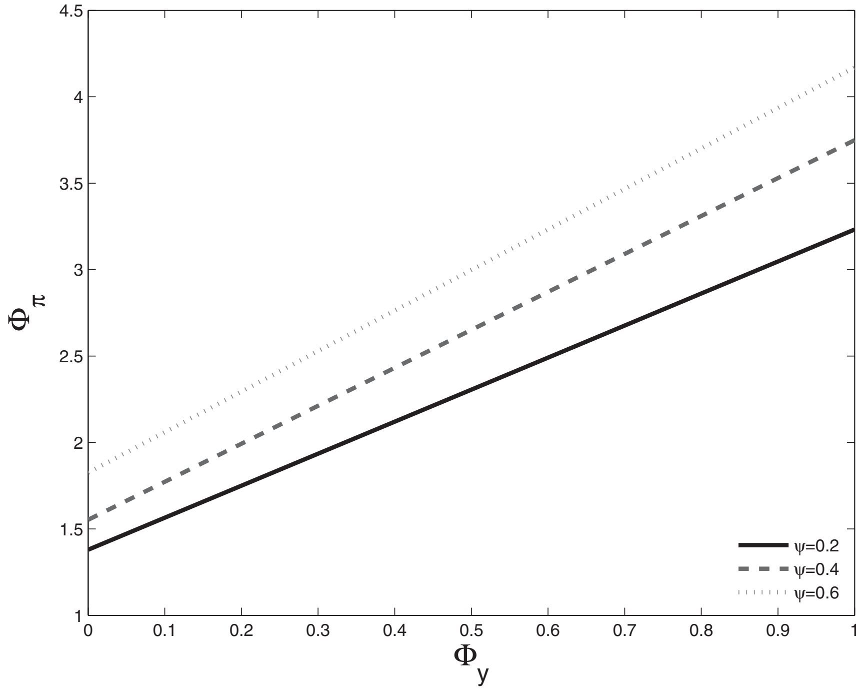


**FIGURE 6.** Simple rule coefficients that implement the optimal noninertial plan.

output. This rule has the following form:

$$r_t = \phi_\pi \pi_t + \phi_y y_t. \tag{56}$$

An advantage of using a specification such as (56) is that we can compare it with feedback rule rules that have been estimated for different economies. To estimate (56), we replace this policy rule in the dynamic IS equation (29) and use the solution from the optimal noninertial plan for inflation (53) and output gap (54) and the output targe level (43) to solve for the coefficients $\phi_\pi$ and $\phi_y$ that solve the equilibrium. The solution for these coefficients is exact because there is only one shock in the economy. Also, there is not only one set, but a continuous combination of parameters $\phi_\pi$ and $\phi_y$ that implement this optimal plan.

In Figure 6 we show the combination of parameters of the simple rule that implement the optimal noninertial plan for different values of the elasticity of substitution $\psi$. A first thing to note is that there is that there is a positive relationship between $\phi_\pi$ and $\phi_y$, which is consistent with the fact that an oil shock implies a trade-off. That is, if the response in the feedback rule to inflation is higher, then the response to output fluctuations must also be higher to compensate for the effects of oil shocks on economic activity. Moreover, when the elasticity of substitution is lower, the trade-off increases and the intercept in Figure 6 is lower. This implies that an economy with inflation targeting where oil is more difficult to substitute for should have a less aggressive response to inflation than in an economy that is less dependant to oil.

Also, consistent with a larger trade-off for lower elasticity of substitution, the response to output fluctuations must increase more for a given increase in the response to inflation fluctuations. That is, the slope in Figure 6 becomes flatter. This implies that in a flexible inflation-targeting regime, due to oil shocks considerations, a more aggressive response to inflation fluctuations must be accompanied by stronger response to output fluctuations.

## 5. CONCLUSIONS

This paper characterizes the utility-based loss function for a closed economy in which oil is used in the production process and there is staggered price setting and monopolistic competition. As in Benigno and Woodford (2005), our utility-based loss function is quadratic in inflation and the deviations of output from a target level, which is the welfare-relevant output gap.

We found that this target level differs from the natural level of output when the elasticity of substitution between labor and oil is different from one. This generates a trade-off between stabilizing inflation and output in the presence of oil shocks. Also, the cost-push shocks involved in this trade-off are proportional to oil shocks. The lower this elasticity of substitution, the greater the size of the cost-push shock. This trade-off is generated by the convexity of real marginal costs with respect to the real oil price, which produces a time-varying wedge between the marginal rate of substitution and the marginal productivity of labor. We also find that eliminating the distortions in the steady state reduces the trade-off, because this wedge becomes less sensitive with respect to the oil price. However, in contrast to Benigno and Woodford (2005), making the steady state efficient cannot eliminate this trade-off.

Furthermore, the relative weight of the welfare-relevant output gap and inflation in the utility-based loss function depends directly on this elasticity of substitution. On the contrary, the larger the share of oil in the production function, the smaller the relative weight.

These results show that to the extent that economies are more dependent on oil, in the sense that oil is difficult to substitute for in production, the impact of oil shocks on both inflation and output is higher. Also, in this case the central bank should allow less fluctuation in inflation relative to output due to oil shocks. Moreover, these results shed light on how technological improvements that reduces the dependence on oil also reduce the impact of oil shocks on the economy.

### *NOTES*

1. For example, during the 1970s and through the 1990s, most of the oil shocks seemed to be on the international supply side, either because of attempts to gain more oil revenue or because of supply interruptions, such as the Iranian Revolution and the first Gulf War. In contrast, in the 2000s the high

price of oil is more related to demand growth in the United States, China, India, and other countries. On the other hand, Kilian (2009) found that all major real oil price increases since the mid-1970s can be traced to increased global aggregate demand and/or increases in oil-specific demand.

2. There are a few exceptions. For instance, Natal (2009) showed that extending our work, including oil in the consumption goods bundle in a CES form, amplifies the trade-off between stabilizing inflation and the welfare output gap. In a different approach, Nakov and Pescaroti (2010) also find a trade-off when modeling explicitly the oil production in the global economy, which is generated by a dynamic distortion due to imperfect competition in the oil market.

3. In contrast, Blanchard and Galí (2007) use a Cobb–Douglas production function.

4. In particular, using the demand for oil in steady state, we have $\overline{\alpha} \equiv \mu. \overline{QM}/\overline{Y}$ is estimated as the ratio of (Oil and other fuels used for production)/(value added), from the National Income Product accounts (www.bea.gov). The average value of $\overline{QM}/\overline{Y}$ is 2.5% for the period 1972–2006 and $\mu = 1.15$ in our calibration; thus $\overline{\alpha} = 1.15 \times 2.5\% = 2.895\%$.

5. Maximizing equation (41) implies minimizing (45) subject to a predeterminated value of $v_{t_o}$. Moreover, because the objective function is purely quadratic, a linear approximation of $v_{t_o}$ suffices to describe the initial commitments, given by $v_{t_o} = \pi_{t_o}$.

6. More precisely, we are interested in studying the model when $0 < \psi \leq 1$ and $\sigma$ is not too high, because $\lambda$ is positive for $\psi \leq 1$ and $\sigma < (\overline{\alpha}\psi)^{-1}$, which is a very high value for the threshold because $\overline{\alpha}$ is lower than one and small.

7. See for example Woodford (2003) and Benigno and Woodford (2005).

8. Note that in this section we focus on the simplest case of the noninertial plan, in which all endogenous variables depend only the current state of the economy. In contrast, Benigno and Woodford (2005) work with a different noninertial plan, in which the Lagrange multipliers satisfy the first-order conditions of the unconstrained problem.

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# APPENDIX A: THE DETERMINISTIC STEADY STATE

The nonstochastic steady state of the endogenous variables for $\overline{\Pi} = 1$ is given by Table A.1, where $\overline{\alpha} \equiv \alpha^{\psi}(\overline{Q}/\overline{MC})^{1-\psi}$ is the share of oil in total costs in the steady state. Notice that the steady state values of real wages, output, and labor depend on the steady state ratio of oil prices to the marginal cost. This implies that permanent changes in oil prices would generate changes in the steady state of these variables. Also, as in the standard New Keynesian models, the marginal cost in the steady state is equal to the inverse of the mark-up.

Because monopolistic competition affects the steady state of the model, output in the steady state is below the efficient level. We call this feature a distorted steady state and $\Phi \equiv 1 - \overline{MC}$ accounts for effects of the monopolistic distortions in steady state.

Because the technology has constant returns to scale, we have that

$$\begin{aligned}\frac{\overline{V}_L}{\overline{U}_C}\frac{\overline{L}}{\overline{Y}} &= \left(\frac{\overline{W/P}}{\overline{MC}}\frac{\overline{L}}{\overline{Y}}\right)\overline{MC}\\ &= (1-\overline{\alpha})(1-\Phi).\end{aligned}$$

The ratio of the marginal rate of substitution multiplied by the ratio labor/output is a proportion $(1-\overline{\alpha})$ of the marginal costs. This expression helps us to obtain the wedge between the consumption and labor in the utility function in the steady state:

$$\begin{aligned}\frac{\overline{V}_L}{\overline{U}_C}\frac{d\overline{L}}{d\overline{Y}} &= \left(\frac{\overline{V}_L}{\overline{U}_C}\frac{\overline{L}}{\overline{Y}}\right)\left(\frac{d\overline{L/L}}{d\overline{Y/Y}}\right)\\ &= (1-\overline{\alpha})(1-\Phi)\left[1-\delta(\sigma+v)\right]\\ &\equiv 1-\Phi_L.\end{aligned}$$

**TABLE A.1.** The deterministic steady state

| | |
|---|---|
| Interest rate | $\overline{R} = \beta^{-1}$. |
| Marginal costs | $\overline{MC} = \left(\frac{\varepsilon-1}{\varepsilon}\right)(1-\tau)$. |
| Real wages | $\overline{W/P} = \frac{1-\overline{\alpha}}{\overline{MC}}\left(\frac{1-\overline{\alpha}}{1-\alpha}\right)^{\frac{1}{1-\psi}}$. |
| Output | $\overline{Y} = \left(\frac{1-\overline{\alpha}}{\overline{MC}}\right)^{\frac{1}{\sigma+v}}\left(\frac{1-\overline{\alpha}}{1-\alpha}\right)^{\frac{1+\psi v}{\sigma+v}\frac{1}{1-\psi}}$. |
| Labor | $\overline{L} = \left(\frac{1-\overline{\alpha}}{\overline{MC}}\right)^{\frac{1}{\sigma+v}}\left(\frac{1-\overline{\alpha}}{1-\alpha}\right)^{\frac{1-\sigma\psi}{\sigma+v}\frac{1}{1-\psi}}$. |

# APPENDIX B: THE SECOND-ORDER SOLUTION OF THE MODEL

## B.1. THE RECURSIVE AS EQUATION

We divide the equation for the aggregate price level (17) by $P_t^{1-\varepsilon}$ and make $P_t/P_{t-1} = \Pi_t$:

$$1 = \theta\,(\Pi_t)^{-(1-\varepsilon)} + (1-\theta)\left[\frac{P_t^*(z)}{P_t}\right]^{1-\varepsilon}. \qquad \textbf{(B.1)}$$

Aggregate inflation is a function of the optimal price level of firm $z$. Also, from equation (15) the optimal price of a typical firm can be written as

$$\frac{P_t^*(z)}{P_t} = \frac{N_t}{D_t},$$

where after using the definition for the stochastic discount factor, $\zeta_{t,t+k} = \beta^k(\frac{C_{t+k}}{C_t})^{-\sigma}\frac{P_t}{P_{t+k}}$, we define $N_t$ and $D_t$ as follows:

$$N_t = E_t\left[\sum_{k=0}^{\infty}\mu\,(\theta\beta)^k\,F_{t,t+k}^{\varepsilon}Y_{t+k}C_{t+k}^{-\sigma}\mathrm{MC}_{t+k}\right], \qquad \textbf{(B.2)}$$

$$D_t = E_t\left[\sum_{k=0}^{\infty}(\theta\beta)^k\,F_{t,t+k}^{\varepsilon-1}Y_{t+k}C_{t+k}^{-\sigma}\right]. \qquad \textbf{(B.3)}$$

$N_t$ and $D_t$ can be expanded as

$$N_t = \mu Y_t C_t^{-\sigma}\mathrm{MC}_t + E_t\left[\Pi_{t+1}^{\varepsilon}\sum_{k=0}^{\infty}\mu\,(\theta\beta)^{k+1}\,F_{t+1,t+1+k}^{\varepsilon}Y_{t+1+k}C_{t+1+k}^{-\sigma}\mathrm{MC}_{t+1+k}\right], \qquad \textbf{(B.4)}$$

$$D_t = Y_t C_t^{-\sigma} + E_t\left[\Pi_{t+1}^{\varepsilon-1}\sum_{k=0}^{\infty}(\theta\beta)^{k+1}\,F_{t+1,t+1+k}^{\varepsilon-1}C_{t+1+k}^{-\sigma}Y_{t+1+k}\right], \qquad \textbf{(B.5)}$$

where we have used the definition for $F_{t,t+k} = P_{t+k}/P_t$.

The Phillips curve with oil prices is given by the three equations

$$\theta\,(\Pi_t)^{\varepsilon-1} = 1 - (1-\theta)\left[\frac{P_t^*(z)}{P_t}\right]^{1-\varepsilon}, \qquad \textbf{(B.6)}$$

$$N_t = \mu Y_t^{1-\sigma}\mathrm{MC}_t + \theta\beta E_t\,(\Pi_{t+1})^{\varepsilon}\,N_{t+1}, \qquad \textbf{(B.7)}$$

$$D_t = Y_t^{1-\sigma} + \theta\beta E_t\,(\Pi_{t+1})^{\varepsilon-1}\,D_{t+1}, \qquad \textbf{(B.8)}$$

where we have reordered equation (B.1) and we have used equations (B.2) and (B.3), evaluated one period forward, to replace $N_{t+1}$ and $D_{t+1}$ in equations (B.4) and (B.5), and used the law of iterated expectations.

## B.2. THE SECOND-ORDER APPROXIMATION OF THE MODEL

In this section we present a log-quadratic (Taylor series) approximation of the fundamental equations of the model around the steady state. A detailed derivation is provided in the next section of this Appendix. The second-order Taylor series expansion serves to compute the equilibrium fluctuations of the endogenous variables of the model up to a residual of order $O(\|\xi\|^2)$, where $\|\xi_t\|$ is a bound on the size of the oil price shock. Up to second order, equations (26) to (29) are replaced by the set of log-quadratic equations Table A.2.

Equations (B.i) and (B.ii) are obtained by taking a second-order Taylor series expansion of the aggregate labour and the real marginal cost equation, after using the labor market equilibrium to eliminate real wages. $\widehat{\Delta}_t$ is the log-deviation of the price dispersion measure $\Delta_t$, which is a second-order function of inflation, and its dynamic is represented with equation (B.iii).

*The marginal cost equation and the labor market equilibrium.* The real marginal cost (12) and the labor market equations (4 and 23) have the second-order expansion

$$mc_t = (1-\overline{\alpha})\,w_t + \overline{\alpha}q_t + \frac{1}{2}\overline{\alpha}\,(1-\overline{\alpha})\,(1-\psi)\,(w_t - q_t)^2 + O\left(\|\xi_t\|^3\right), \qquad \textbf{(B.9)}$$

$$w_t = vl_t + \sigma y_t, \qquad \textbf{(B.10)}$$

$$l_t = y_t - \psi\,(w_t - mc_t) + \widehat{\Delta}_t, \qquad \textbf{(B.11)}$$

where $w_t$ and $\widehat{\Delta}_t$ are, respectively, the log of the deviation of the real wage and the price dispersion measure from their respective steady state. Notice that equations (B.10) and (B.11) are not approximations, but exact expressions. Solving equations (B.10) and (B.11) for the equilibrium real wage gives

$$w_t = \frac{1}{1+v\psi}\left[(v+\sigma)\,y_t + v\psi mc_t + v\widehat{\Delta}_t\right]. \qquad \textbf{(B.12)}$$

Plugging the real wage in equation (B.9) and simplifying,

$$\begin{aligned} mc_t &= \chi\,(\sigma+v)\,y_t + (1-\chi)\,(q_t) + \chi v\widehat{\Delta}_t \\ &+ \frac{1}{2}\frac{1-\psi}{1-\overline{\alpha}}\chi^2\,(1-\chi)\,[(\sigma+v)\,y_t - q_t]^2 + O\left(\|\xi_t\|^3\right), \end{aligned} \qquad \textbf{(B.13)}$$

where $\chi \equiv (1-\overline{\alpha})/(1+v\psi\overline{\alpha})$. This is the equation (B.ii) in the previous section. This expression is the second-order expansion of the real marginal cost as a function of output and the oil prices. Similarly, we can express labor in equilibrium as a function of output and oil prices,

$$l_t = y_t - \delta\,[(v+\sigma)\,y_t - q_t] + \frac{\chi}{1-\overline{\alpha}}\widehat{\Delta}_t + \frac{1}{2}\frac{1-\psi}{1-\overline{\alpha}}\delta\chi^2\,[(v+\sigma)\,y_t - q_t]^2 + O\left(\|\xi_t\|^3\right), \qquad \textbf{(B.14)}$$

for

$$\delta \equiv \psi\chi\frac{\overline{\alpha}}{1-\overline{\alpha}},$$

where $\delta$ measures the effects of oil shocks on labor.

https://doi.org/10.1017/S1365100510000106 Published online by Cambridge University Press

**TABLE A.2.** Second-order Taylor expansion of the equations of the model

| | |
|---|---|
| **Labor market** | |
| $l_t = y_t - \delta\left[(\upsilon+\sigma)\, y_t - q_t\right] + \frac{\chi}{1-\alpha}\widehat{\Delta}_t + \frac{1}{2}\frac{1-\psi}{1-\alpha}\delta\chi^2\left[(\upsilon+\sigma)\, y_t - q_t\right]^2 + O\left(\lVert\xi\rVert^3\right).$ | B.i |
| **Aggregate supply** | |
| Marginal Costs | |
| $mc_t = \chi\,(\upsilon+\sigma)\, y_t + (1-\chi)\, q_t + \frac{1}{2}\frac{1-\psi}{1-\alpha}(1-\chi)\,\chi^2\left[(\upsilon+\sigma)\, y_t - q_t\right]^2 + \chi\upsilon\widehat{\Delta}_t + O\left(\lVert\xi\rVert^3\right).$ | B.ii |
| Price dispersion | |
| $\widehat{\Delta}_t = \theta\widehat{\Delta}_t + \frac{1}{2}\varepsilon\frac{\theta}{1-\theta}\pi_t^2 + O\left(\lVert\xi\rVert^3\right).$ | B.iii |
| Phillips Curve | |
| $v_t = \kappa mc_t + \frac{1}{2}\kappa mc_t\left[2\,(1-\sigma)\, y_t + mc_t\right] + \frac{1}{2}\varepsilon\pi_t^2 + \beta E_t v_{t+1} + O\left(\lVert\xi\rVert^3\right),$ | B.iv |
| where we have defined the auxiliary variables: | |
| $v_t \equiv \pi_t + \left(\frac{\varepsilon-1}{1-\theta} + \varepsilon\right)\pi_t^2 + \frac{1}{2}(1-\theta\beta)\,\pi_t z_t.$ | B.v |
| $z_t \equiv 2\,(1-\sigma)\, y_t + mc_t + \theta\beta E_t\left(\frac{2\varepsilon-1}{1-\theta\beta}\pi_{t+1} + z_{t+1}\right) + O\left(\lVert\xi_t\rVert^2\right).$ | B.vi |
| **Aggregate demand** | |
| $y_t = E_t y_{t+1} - \frac{1}{\sigma}(r_t - E_t\pi_{t+1}) - \frac{1}{2}\sigma E_t\left[(y_t - y_{t+1}) - \frac{1}{\sigma}(r_t - \pi_{t+1})\right]^2 + O\left(\lVert\xi\rVert^3\right).$ | B.vii |

*The price dispersion equation.* The price dispersion measure is given by

$$\Delta_t = \int_0^1 \left[\frac{P_t(z)}{P_t}\right]^{-\varepsilon} dz.$$

Because a proportion $1-\theta$ of intermediate firms set prices optimally, whereas the other $\theta$ set the price last period, this price dispersion measure can be written as

$$\Delta_t = (1-\theta)\left[\frac{P_t^*(z)}{P_t}\right]^{-\varepsilon} + \theta \int_0^1 \left[\frac{P_{t-1}(z)}{P_t}\right]^{-\varepsilon} dz.$$

Dividing and multiplying by $(P_{t-1})^{-\varepsilon}$ the last term of the RHS,

$$\Delta_t = (1-\theta)\left[\frac{P_t^*(z)}{P_t}\right]^{-\varepsilon} + \theta \int_0^1 \left[\frac{P_{t-1}(z)}{P_{t-1}}\right]^{-\varepsilon} \left(\frac{P_{t-1}}{P_t}\right)^{-\varepsilon} dz.$$

Because $P_t^*(z)/P_t = N_t/D_t$ and $P_t/P_{t-1} = \Pi_t$, using equation (18) in the text and the definition for the dispersion measure lagged on period, this can be expressed as

$$\Delta_t = (1-\theta)\left[\frac{1-\theta\,(\Pi_t)^{\varepsilon-1}}{1-\theta}\right]^{\varepsilon/(\varepsilon-1)} + \theta \Delta_{t-1}\,(\Pi_t)^{\varepsilon}, \qquad \textbf{(B.15)}$$

which is a recursive representation of $\Delta_t$ as a function of $\Delta_{t-1}$ and $\Pi_t$.

Benigno and Woodford (2005) showed that a second-order approximation of the price dispersion depends solely on second-order terms on inflation. Thus, the second-order approximation of equation (B.15) is

$$\widehat{\Delta}_t = \theta \widehat{\Delta}_{t-1} + \frac{1}{2}\varepsilon \frac{\theta}{1-\theta}\pi_t^2 + O\left(\|\xi_t\|^3\right), \qquad \textbf{(B.16)}$$

which is equation (B.iii) in the previous section. Moreover, we can use equation (B.16) to write the infinite sum

$$\sum_{t=t_o}^{\infty} \beta^{t-t_o}\widehat{\Delta}_t = \theta \sum_{t=t_o}^{\infty} \beta^{t-t_o}\widehat{\Delta}_{t-1} + \frac{1}{2}\varepsilon\frac{\theta}{1-\theta}\sum_{t=t_o}^{\infty}\beta^{t-t_o}\pi_t^2 + O\left(\|\xi_t\|^3\right),$$

$$(1-\beta\theta)\sum_{t=t_o}^{\infty} \beta^{t-t_o}\widehat{\Delta}_t = \theta \widehat{\Delta}_{t_o-1} + \frac{1}{2}\varepsilon\frac{\theta}{1-\theta}\sum_{t=t_o}^{\infty}\beta^{t-t_o}\pi_t^2 + O\left(\|\xi_t\|^3\right).$$

Dividing by $(1-\beta\theta)$ and using the definition of $\kappa$,

$$\sum_{t=t_o}^{\infty} \beta^{t-t_o}\widehat{\Delta}_t = \frac{\theta}{1-\beta\theta}\widehat{\Delta}_{t_o-1} + \frac{1}{2}\frac{\varepsilon}{\kappa}\sum_{t=t_o}^{\infty}\beta^{t-t_o}\pi_t^2 + O\left(\|\xi_t\|^3\right). \qquad \textbf{(B.17)}$$

The discounted infinite sum of $\widehat{\Delta}_t$ is equal to the sum of two terms, the initial price dispersion and the discounted infinite sum of $\pi_t^2$.

*The Philips Curve.* The second-order expansions for equations (B.6), (B.7), and (B.8) are

$$\pi_t = \frac{(1-\theta)}{\theta}(n_t - d_t) - \frac{1}{2}\frac{(\varepsilon-1)}{1-\theta}(\pi_t)^2 + O\left(\|\xi_t\|^3\right), \qquad \textbf{(B.18)}$$

$$n_t = (1-\theta\beta)\left(a_t + \frac{1}{2}a_t^2\right) + \theta\beta\left(E_t b_{t+1} + \frac{1}{2}E_t b_{t+1}^2\right) - \frac{1}{2}n_t^2 + O\left(\|\xi_t\|^3\right), \quad \textbf{(B.19)}$$

$$d_t = (1-\theta\beta)\left(c_t + \frac{1}{2}c_t^2\right) + \theta\beta\left(E_t e_{t+1} + \frac{1}{2}E_t e_{t+1}^2\right) - \frac{1}{2}d_t^2 + O\left(\|\xi_t\|^3\right), \quad \textbf{(B.20)}$$

where we have defined the auxiliary variables $a_t$, $b_{t+1}$, $c_t$ and $e_{t+1}$ as

$$a_t \equiv (1-\sigma)\,y_t + mc_t, \qquad b_{t+1} \equiv \varepsilon\pi_{t+1} + n_{t+1},$$
$$c_t \equiv (1-\sigma)\,y_t, \qquad e_{t+1} \equiv (\varepsilon-1)\,\pi_{t+1} + d_{t+1}.$$

Subtract equations (B.19) and (B.20), and using the fact that $X^2 - Y^2 = (X-Y)(X+Y)$, for any two variables $X$ and $Y$,

$$\begin{aligned} n_t - d_t &= (1-\theta\beta)\,(a_t - c_t) + \frac{1}{2}\,(1-\theta\beta)\,(a_t - c_t)\,(a_t + c_t) \\ &+ \theta\beta E_t\,(b_{t+1} - e_{t+1}) + \frac{1}{2}\theta\beta E_t\,(b_{t+1} - e_{t+1})\,(b_{t+1} + e_{t+1}) \\ &- \frac{1}{2}\,(n_t - d_t)\,(n_t + d_t) + O\left(\|\xi_t\|^3\right). \end{aligned} \quad \textbf{(B.21)}$$

Plugging in the values of $a_t$, $b_{t+1}$, $c_t$, and $e_{t+1}$ into equation (B.21), we obtain (B.22)

$$\begin{aligned} n_t - d_t &= (1-\theta\beta)\,mc_t + \frac{1}{2}\,(1-\theta\beta)\,mc_t\,[2\,(1-\sigma)\,y_t + mc_t] \\ &+ \theta\beta E_t\,(\pi_{t+1} + n_{t+1} - d_{t+1}) \\ &+ \frac{1}{2}\theta\beta E_t\,(\pi_{t+1} + n_{t+1} - d_{t+1})\,[(2\varepsilon-1)\,\pi_{t+1} + n_{t+1} + d_{t+1}] \\ &- \frac{1}{2}\,(n_t - d_t)\,(n_t + d_t) + O\left(\|\xi_t\|^3\right). \end{aligned} \quad \textbf{(B.22)}$$

Taking equation (B.18) forward one period, we can solve for $n_{t+1} - d_{t+1}$:

$$n_{t+1} - d_{t+1} = \frac{\theta}{1-\theta}\pi_{t+1} + \frac{1}{2}\frac{\theta}{1-\theta}\frac{(\varepsilon-1)}{1-\theta}\,(\pi_{t+1})^2 + O\left(\|\xi_t\|^3\right). \quad \textbf{(B.23)}$$

Replace equation (B.23) in (B.22) and make use of the auxiliary variable $z_t = (n_t + d_t)/(1-\theta\beta)$ :

$$\begin{aligned} n_t - d_t &= (1-\theta\beta)\,mc_t + \frac{1}{2}\,(1-\theta\beta)\,mc_t\,[2\,(1-\sigma)\,y_t + mc_t] \\ &+ \frac{\theta}{1-\theta}\beta\left[E_t\pi_{t+1} + \left(\frac{\varepsilon-1}{1-\theta} + \varepsilon\right)E_t\pi_{t+1}^2 + (1-\theta\beta)\,E_t\pi_{t+1}z_{t+1}\right] \\ &- \frac{1}{2}\frac{\theta}{1-\theta}\,(1-\theta\beta)\,\pi_t z_t + O\left(\|\xi_t\|^3\right). \end{aligned} \quad \textbf{(B.24)}$$

Notice that we use only the linear part of equation (B.23) when we replace $n_{t+1} - d_{t+1}$ in the quadratic terms because we are interested in capturing terms only up to the second order of accuracy. Similarly, we make use of the linear part of equation (B.18) to replace

$(n_t - d_t) = \frac{\theta}{1-\theta}\pi_t$ on the right-hand side of equation (B.24). Replace equation (B.22) in (B.18),

$$\pi_t = \kappa mc_t + \frac{1}{2}\kappa mc_t\,(2\,(1-\sigma)\,y_t + mc_t)$$
$$+\beta\left[E_t\pi_{t+1} + \left(\frac{\varepsilon-1}{1-\theta}+\varepsilon\right)E_t\pi_{t+1}^2 + (1-\theta\beta)\,E_t\pi_{t+1}z_{t+1}\right]$$
$$-\frac{1}{2}\,(1-\theta\beta)\,\pi_t z_t - \frac{1}{2}\frac{(\varepsilon-1)}{1-\theta}\,(\pi_t)^2 + O\left(\|\xi_t\|^3\right), \quad \textbf{(B.25)}$$

for

$$\kappa \equiv \frac{(1-\theta)}{\theta}\,(1-\theta\beta)\,,$$

where $z_t$ has the following linear expansion:

$$z_t = 2\,(1-\sigma)\,y_t + mc_t + \theta\beta E_t\left(\frac{2\varepsilon-1}{1-\theta\beta}\pi_{t+1} + z_{t+1}\right) + O\left(\|\xi_t\|^3\right). \quad \textbf{(B.26)}$$

Define the following auxiliary variable:

$$v_t = \pi_t + \frac{1}{2}\left(\frac{\varepsilon-1}{1-\theta}+\varepsilon\right)\pi_t^2 + \frac{1}{2}\,(1-\theta\beta)\,\pi_t z_t. \quad \textbf{(B.27)}$$

Using the definition for $v_t$, equation (B.25) can be expressed as

$$v_t = \kappa mc_t + \frac{1}{2}\kappa mc_t\,[2\,(1-\sigma)\,y_t + mc_t] + \frac{1}{2}\varepsilon\pi_t^2 + \beta E_t v_{t+1} + O\left(\|\xi_t\|^3\right), \quad \textbf{(B.28)}$$

which is equation $(B - iv)$.

Moreover, the linear part of equation (B.28) is

$$\pi_t = \kappa mc_t + \beta E_t\,(\pi_{t+1}) + O\left(\|\xi_t\|^2\right),$$

which is the standard New Keynesian Phillips curve, in which inflation depends linearly on the real marginal costs and expected inflation.

Replace the equation for the marginal costs (B.13) in the second-order expansion of the Phillips curve (B.28),

$$v_t = \kappa_y y_t + \kappa_q q_t + \kappa\chi v\widehat{\Delta}_t + \frac{1}{2}\varepsilon\pi_t^2$$
$$+\frac{1}{2}\kappa\left(c_{yy}y_t^2 + 2c_{yq}y_t q_t + c_{qq}q_t^2\right) + \beta E_t v_{t+1} + O\left(\|\xi_t\|^3\right), \quad \textbf{(B.29)}$$

where the coefficients of the linear part are given by

$$\kappa_y \equiv \kappa\chi\,(\sigma+v)\,,$$
$$\kappa_q \equiv \kappa\,(1-\chi)\,,$$

and those of the quadratic part are

$$c_{yy} = \chi\,(\sigma+v)\,[2\,(1-\sigma) + \chi\,(\sigma+v)] + (1-\psi)\,\frac{\chi^2\,(1-\chi)\,(\sigma+v)^2}{1-\overline{\alpha}},$$

$$c_{yq} = (1-\chi)\left[2(1-\sigma)+\chi(\sigma+v)\right]-(1-\psi)\frac{\chi^2(1-\chi)(\sigma+v)}{1-\overline{\alpha}},$$

$$c_{qq} = (1-\chi)^2+(1-\psi)\frac{\chi^2(1-\chi)}{1-\overline{\alpha}}.$$

Equation (B.29) is a recursive second-order representation of the Phillips curve. However, we need to express the price dispersion in terms of inflation in order to have the Phillips curve as a function only of output, inflation, and the oil shock. Equation (B.29) can also be expressed as the discounted infinite sum

$$v_{t_o} = \sum_{t=t_o}^{\infty}\beta^{t-t_o}\left[\kappa_y y_t+\kappa_q q_t+\kappa\chi v\widehat{\Delta}_t+\frac{1}{2}\varepsilon\pi_t^2 + \frac{1}{2}\kappa\left(c_{yy}y_t^2+2c_{yq}y_tq_t+c_{qq}q_t^2\right)\right]+\left(\|\xi_t\|^3\right);$$

after making use of equation (B.17), the discounted infinite sum of $\widehat{\Delta}_t$, $v_{t_o}$, becomes

$$v_{t_o} = \sum_{t=t_o}^{\infty}\beta^{t-t_o}\left[\kappa_y y_t+\kappa_q q_t+\frac{1}{2}\varepsilon(1+\chi v)\pi_t^2+\frac{1}{2}\kappa\left(c_{yy}y_t^2+2c_{yq}y_tq_t+c_{qq}q_t^2\right)\right] + \frac{\chi v\theta}{1-\beta\theta}\widehat{\Delta}_{t_o-1}+\left(\|\xi_t\|^3\right). \quad \textbf{(B.30)}$$

This is the Phillips curve expressed as a infinite sum of output, inflation, and oil shock.

## B.3. A SECOND-ORDER APPROXIMATION TO UTILITY

The expected discounted value of the utility of the representative household is

$$U_{t_o} = E_{t_o}\sum_{t=t_o}^{\infty}\beta^{t-t_o}\left[u(C_t)-v(L_t)\right]. \quad \textbf{(B.31)}$$

The first term can be approximated as

$$u(C_t) = \overline{C}\overline{u}_c\left[c_t+\frac{1}{2}(1-\sigma)c_t^2\right]+\text{t.i.p.}+O\left(\|\xi_t\|^3\right). \quad \textbf{(B.32)}$$

Similarly, the second term can be approximated as

$$v(L_t) = \overline{L}\overline{v}_L\left[l_t+\frac{1}{2}(1+v)l_t^2\right]+\text{t.i.p.}+O\left(\|\xi_t\|^3\right). \quad \textbf{(B.33)}$$

Replace the equation for labor in equilibrium in (B.33),

$$v(L_t) = \overline{L}\overline{v}_L\left(v_y y_t+\frac{1}{2}v_{yy}y_t^2+v_{yq}y_tq_t+v_{\Delta}\widehat{\Delta}_t\right)+\text{t.i.p.}+O\left(\|\xi_t\|^3\right), \quad \textbf{(B.34)}$$

where

$$v_y \equiv 1-\delta(v+\sigma),$$

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$$v_{yy} \equiv (1+v)\left[1-\delta(v+\sigma)\right]^2 + \frac{1}{2}\frac{1-\psi}{1-\overline{\alpha}}\chi^2\delta(\sigma+v)^2,$$

$$v_{yq} \equiv (1+v)\,\delta\left[1-\delta(v+\sigma)\right] - \frac{1-\psi}{1-\overline{\alpha}}\chi^2\delta(\sigma+v),$$

$$v_{\Delta} \equiv \frac{\chi}{1-\overline{\alpha}}.$$

We make use of the relation

$$\overline{L}\overline{v}_L = (1-\Phi)(1-\overline{\alpha})\,\overline{Y}\overline{u}_c, \tag{B.35}$$

where $\Phi = 1 - \frac{1-\tau}{\varepsilon/(\varepsilon-1)}$ is the steady state distortion from monopolistic competition. Replace the previous relation, equation (B.32), and equation (B.34) in (B.31), and make use of the clearing market condition $C_t = Y_t$:

$$U_{t_o} = \overline{Y}\overline{u}_c \sum_{t=t_o}^{\infty} \beta^{t-t_o}\left(u_y y_t + \frac{1}{2}u_{yy}y_t^2 + u_{yq}y_t q_t + u_{\Delta}\widehat{\Delta}_t\right) + \text{t.i.p.} + O\left(\|\xi_t\|^3\right), \tag{B.36}$$

where

$$u_y \equiv 1-(1-\Phi)(1-\overline{\alpha})\,v_y = \Phi_L,$$

$$u_{yy} \equiv 1-\sigma-(1-\Phi)(1-\overline{\alpha})\,v_{yy} = 1-\sigma-(1-\Phi_L)\,v_{yy}/\left[1-\delta(v+\sigma)\right],$$

$$u_{yq} \equiv -(1-\Phi)(1-\overline{\alpha})\,v_{yq} = -(1-\Phi_L)\,v_{yq}/\left[1-\delta(v+\sigma)\right],$$

$$u_{\Delta} \equiv -(1-\Phi)(1-\overline{\alpha})\,v_{\Delta} = -(1-\Phi)\,\chi,$$

where we make use of the change of variable

$$\Phi_L \equiv 1-(1-\Phi)(1-\overline{\alpha})\left[1-\delta(v+\sigma)\right], \tag{B.37}$$

where $\Phi_L$ is the wedge between consumption and labor in the utility function in the steady state.

Replace the present discounted value of the price distortion (B.17) in (B.36),

$$U_{t_o} = \overline{Y}\overline{u}_c E_{t_o}\sum_{t=t_o}^{\infty} \beta^{t-t_o}\left(u_y y_t + \frac{1}{2}u_{yy}y_t^2 + u_{yq}y_t q_t + \frac{1}{2}u_{\pi}\pi_t^2\right) + \text{t.i.p.} + O\left(\|q_t\|^3\right), \tag{B.38}$$

where

$$u_{\pi} \equiv \frac{\varepsilon}{\kappa}u_{\Delta} = -(1-\Phi)\,\chi\frac{\varepsilon}{\kappa}.$$

Use equation (B.30), the second-order approximation of the Phillips curve, to solve for the expected level of output:

$$\sum_{t=t_o}^{\infty}\beta^{t-t_o}y_t = -\frac{1}{\kappa_y}\sum_{t=t_o}^{\infty}\beta^{t-t_o}\Bigg[\kappa_q q_t + \frac{1}{2}\varepsilon(1+\chi v)\pi_t^2 + \frac{1}{2}\kappa\left(c_{yy}y_t^2 + 2c_{yq}y_t q_t + c_{qq}q_t^2\right)\Bigg] + \frac{1}{\kappa_y}\left[v_{t_o} - \chi v\frac{\theta}{1-\beta\theta}\widehat{\Delta}_{t_o-1}\right] + \left(\|\xi_t\|^3\right). \tag{B.39}$$

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Replace equation (B.39) in (B.38) to express it as a function of only second-order terms,

$$U_{t_o} = -\Omega \left\{ E_{t_o} \sum_{t=t_o}^{\infty} \beta^{t-t_o} \left[ \frac{1}{2}\lambda_y \left( y_t - y_t^* \right)^2 + \frac{1}{2}\lambda_\pi \pi_t^2 \right] - T_{t_o} \right\} + \text{t.i.p.} + O\left( \|q_t\|^3 \right), \quad \textbf{(B.40)}$$

where

$$\lambda_y \equiv \Phi_L \frac{\kappa}{\kappa_y} c_{yy} - u_{yy},$$

$$\lambda_\pi \equiv \Phi_L \frac{\varepsilon\left(1+\chi v\right)}{\kappa_y} - u_\pi,$$

$$y_t^* \equiv -\frac{\Phi_L \frac{\kappa}{\kappa_y} c_{yq} - u_{yq}}{\Phi_L \frac{\kappa}{\kappa_y} c_{yy} - u_{yy}} q_t;$$

additionally we have that $\Omega \equiv \overline{Y}\overline{u}_c$ and $T_{t_o} \equiv \frac{\Phi_L}{\kappa_y} v_{t_o}$.

Make use of the auxiliary variables

$$\omega_1 \equiv \left(1-\sigma\right)\Phi_L + \chi\left(\sigma + v\right),$$

$$\omega_2 \equiv \chi\left(\sigma+v\right)\left[\frac{1-\chi}{1-\overline{\alpha}} + \left(1-\Phi_L\right)\frac{\sigma\psi\overline{\alpha}}{1-\sigma\psi\overline{\alpha}}\right],$$

$$\omega_3 \equiv \Phi_L \sigma \overline{\alpha};$$

then $\lambda_y$, $\lambda_\pi$, and $y_t^*$ can be written as functions of $\omega_1$, $\omega_2$, and $\omega_3$:

$$\lambda_y \equiv \omega_1 + \left(1-\psi\right)\omega_2,$$

$$\lambda_\pi \equiv \frac{\varepsilon}{\kappa_y\left(1-\sigma\psi\overline{\alpha}\right)}\left[\omega_1 + \left(1-\psi\right)\omega_3\right],$$

$$y_t^* \equiv -\frac{1-\chi}{\chi\left(\sigma+v\right)}\left[\frac{\omega_1 - \left(1-\psi\right)\frac{\chi}{1-\chi}\omega_2}{\omega_1 + \left(1-\psi\right)\omega_2}\right] q_t.$$

Using the definitions for $\chi$, $y_t^*$ can be expressed as

$$y_t^* \equiv -\left(\frac{1+\psi v}{\sigma+v}\right)\left(\frac{\overline{\alpha}}{1-\overline{\alpha}+\eta}\right), \quad \textbf{(B.41)}$$

where

$$\eta \equiv \frac{\left(1-\psi\right)\left(1-\overline{\alpha}\right)\omega_2}{\left(1-\chi\right)\omega_1 - \left(1-\psi\right)\chi\omega_2}.$$

Denote $\alpha^*$ as

$$\alpha^* \equiv \frac{\overline{\alpha}}{1+\eta},$$

then $y_t^*$ is

$$y_t^* = -\left(\frac{1+\psi v}{\sigma+v}\right)\left(\frac{\alpha^*}{1-\alpha^*}\right) q_t. \quad \textbf{(B.42)}$$

Note from the definition of $\eta$ that when $\psi = 1$, then $\eta = 0$, $\alpha^* = \overline{\alpha} = \alpha$, and $y_t^* = y_t^n$. For a Cobb–Douglas production function, the efficient level of output equals the natural level. Also, when $\psi < 1$, then $\eta > 0$, $\alpha^* < \overline{\alpha}$, and $|y_t^*| < |y_t^n|$. For elasticity of substitution between inputs lower than one, the efficient level fluctuates less to oil shocks than the natural level. Also note that even when $\Phi_L$ is equal to zero, which summarizes the effect of monopolistic distortions on the wedge between the marginal rate of substitution and the marginal product of labor, $\eta$ is still different from zero for $\psi \neq 1$. This indicates that the efficient level of output still diverges from the natural level even when we eliminate the effects of monopolistic distortions.

In the same way, the natural rate of output can be expressed as

$$y_t^n = -\left(\frac{1+\psi v}{\sigma + v}\right)\left(\frac{\overline{\alpha}}{1-\overline{\alpha}}\right) q_t. \qquad \textbf{(B.43)}$$

Similarly, we can simplify $\lambda \equiv \lambda_y/\lambda_\pi$ as

$$\lambda \equiv \frac{\lambda_y}{\lambda_\pi} = \frac{\kappa_y\,(1-\sigma\psi\overline{\alpha})}{\varepsilon}\gamma,$$

where we use the auxiliary variable

$$\gamma \equiv \left[\frac{\omega_1 + (1-\psi)\,\omega_2}{\omega_1 + (1-\psi)\,\omega_3}\right].$$

Note that when $\psi = 1$, then $\gamma = 1$, and when $\psi < 1$, then $\gamma = 1$ because $\omega_2 > \omega_3$.

# APPENDIX C: OPTIMAL MONETARY POLICY

## C.1. OPTIMAL RESPONSE TO OIL SHOCKS

The policy problem consists in choosing $x_t$ and $\pi_t$ to maximize the Lagrangian

$$\mathcal{L} = -E_{t_o}\left\{\sum_{t=t_o}^{\infty}\beta^{t-t_o}\left[\frac{1}{2}\lambda x_t^2 + \frac{1}{2}\pi_t^2 - \varphi_t\left(\pi_t - \kappa_y\widehat{y}_t - \beta E_t\pi_{t+1} - u_t\right)\right]\right.$$
$$\left. + \varphi_{t_o-1}\left(\pi_{t_o} - \pi_{t_o}^*\right)\right\},$$

where $\beta^{t-t_o}\varphi_t$ is the Lagrange multiplier associated with the constraint at time $t$.

The first-order conditions with respect to $\pi_t$ and $y_t$ are respectively

$$\pi_t = \varphi_{t-1} - \varphi_t, \qquad \textbf{(C.1)}$$

$$\lambda x_t = \kappa_y\varphi_t, \qquad \textbf{(C.2)}$$

and the initial condition is

$$\pi_{t_o} = \pi_{t_o}^*,$$

where $\pi^{*}_{t_o}$ is the initial value of inflation, which is consistent with the policy problem in a timeless perspective.

Replace conditions (C.1) and (C.2) in the Phillips curve:

$$\beta E_t \varphi_{t+1} - \left[(1+\beta)\lambda + \kappa_y^2\right]\varphi_t + \lambda\varphi_{t-1} = \lambda u_t. \qquad \textbf{(C.3)}$$

This difference equation has the solution [see Woodford (2003, pp. 488–490) for details on the derivation]

$$\varphi_t = \tau_\varphi \varphi_{t-1} - \tau_\varphi \sum_{j=0}^{\infty} \beta^j \tau_\varphi^j E_t u_{t+j}, \qquad \textbf{(C.4)}$$

where $\tau_\varphi$ is the characteristic root, lower than one, of (C.3), and it is equal to

$$\tau_\varphi = Z - \sqrt{Z^2 - \frac{1}{\beta}},$$

for $Z = ((1+\beta) + \frac{\kappa_y^2}{\lambda})/(2\beta)$. Because the oil price follows an AR(1) process of the form

$$q_t = \rho q_{t-1} + \xi_t,$$

and the mark-up shock is $u_t = \varpi q_t$, $u_t$ follows the following process:

$$u_t = \rho u_{t-1} + \varpi \xi_t. \qquad \textbf{(C.5)}$$

Taking into account (C.5), equation (C.4) can be expressed as

$$\varphi_t = \tau_\varphi \varphi_{t-1} - \phi q_t, \qquad \textbf{(C.6)}$$

where

$$\phi = \frac{\tau_\varphi}{1 - \beta\tau_\varphi\rho}\varpi.$$

Iterate backward equation (C.6) and evaluate it at $t_o - 1$. This is the timeless solution to the initial condition $\varphi_{t_o-1}$,

$$\varphi_{t_o-1} = -\phi \Sigma_{k=0}^{\infty} \left(\tau_\varphi\right)^k q_{t_o-1-k}, \qquad \textbf{(C.7)}$$

which is a weighted sum of all the past realizations of oil prices.

Equations (C.1), (C.2), (C.6), and (C.7) are the conditions for the optimal unconstrained plan presented in Proposition 4.

An innovation of $\xi_t$ to the real oil price affects the current level and the expected future path of the Lagrange multiplier by an amount

$$E_t\varphi_{t+j} - E_{t-1}\varphi_{t+j} = -\frac{\rho^{j+1} - \left(\tau_\varphi\right)^{j+1}}{\rho - \tau_\varphi}\phi\xi_t,$$

for each $j \geq 0$. Given this impulse response for the multiplier, (C.1) and (C.2) can be used to derive the corresponding impulse responses for inflation and output gap,

$$E_t\pi_{t+j} - E_{t-1}\pi_{t+j} = \left[\frac{\rho^{j+1} - \left(\tau_\varphi\right)^{j+1}}{\rho - \tau_\varphi} - \frac{\rho^{j} - \left(\tau_\varphi\right)^{j}}{\rho - \tau_\varphi}\right]\phi\xi_t,$$

$$E_t x_{t+j} - E_{t-1} x_{t+j} = -\frac{\kappa_y}{\lambda}\frac{\rho^{j+1} - \left(\tau_\varphi\right)^{j+1}}{\rho - \tau_\varphi}\phi\xi_t,$$

which are expressions that appear in the main text.

## C.2. THE OPTIMAL NONINERTIAL PLAN

We want to find a solution for the paths of inflation and output gap such that the behavior of endogenous variables is a function only of the current state. That is,

$$\pi_t = \overline{\pi} + f_\pi u_t, \tag{C.8}$$

$$x_t = \overline{x} + f_x u_t, \tag{C.9}$$

$$\varphi_t = \overline{\varphi} + f_\varphi u_t, \tag{C.10}$$

where the coefficients $\overline{\pi}$, $\overline{y}$, $\overline{\varphi}$, $f_\pi$, $f_x$, and $f_\varphi$ are to be determined.

Replace (C.8), (C.9), and (C.10) in the Lagrangian and take the unconditional expected value,

$$-E\left(L_{t_o}\right) \equiv E\left\{E_{t_o}\sum_{t=t_o}^{\infty}\beta^{t-t_o}\left[\begin{array}{c}\frac{1}{2}\lambda\left(\overline{x}+f_x u_t\right)^2+\frac{1}{2}\left(\overline{\pi}+f_\pi u_t\right)^2\\ -\left(\overline{\varphi}+f_\varphi u_t\right)\left(\begin{array}{c}(1-\beta)\,\overline{\pi}-\kappa_y\overline{x}\\ +(1-\beta\rho)\,f_\pi u_t-u_t-\kappa_y f_x u_t\end{array}\right)\end{array}\right]\right\}$$
$$+E\left[\left(\overline{\varphi}+f_\varphi u_{t_o-1}\right)\left(\overline{\pi}+f_\pi u_{t_o}\right)\right]. \tag{C.11}$$

Suppressing the terms that are independent of policy and using the law of motion for $u_t$, this can be simplified as

$$-E\left(L_{t_o}\right) \equiv \frac{1}{2(1-\beta)}\left(\lambda\overline{x}^2+\overline{\pi}^2\right)-\frac{1}{2(1-\beta)}\overline{\varphi}\left[(1-\beta)\,\overline{\pi}-\kappa_y\overline{x}\right]$$
$$+\frac{1}{2}\frac{\sigma_u^2}{1-\beta}\left(\lambda {f_x}^2+{f_\pi}^2\right)-\frac{1}{2}\frac{\sigma_u^2}{1-\beta}f_\varphi\left[(1-\beta\rho)\,f_\pi-1-\kappa_y f_x\right]$$
$$+\rho\sigma_u^2 f_\varphi f_\pi .$$

The problem becomes to find $\overline{\pi}$, $\overline{y}$, $\overline{\varphi}$, $f_\pi$, $f_x$, and $f_\varphi$ that maximize the previous expression. Those coefficients are

$$\overline{\pi}=\overline{x}=\overline{\varphi}=0,$$

$$f_\pi=\frac{\lambda(1-\rho)}{\lambda\,(1-\beta\rho)\,(1-\rho)+\kappa_y^2},$$

$$f_x=-\frac{\kappa_y}{\lambda\,(1-\beta\rho)\,(1-\rho)+\kappa_y^2},$$

$$f_\varphi=\frac{\lambda}{\lambda\,(1-\beta\rho)\,(1-\rho)+\kappa_y^2},$$

which is the solution to the optimal noninertial plan given in Proposition 5.