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Numbers as a cognitive and social technology: on the nature of conventional number sequences used in economic systems

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Abstract: This paper examines the fundamental nature of numbers as they are used in economic systems. In the framework proposed, number sequences are technological objects ('tools') that are constituted by both form and function. To do their job, number sequences have to have the necessary *internal* structure – all elements (e.g. symbols) of the sequence must be distinct from one another, and the sequence must be a progression. In addition, numerical toolkits have to have the right *external* structure – they must be situated in a social network of economic agents that confers on them quantitative functions (e.g. identifying set sizes). Number sequences are the product of multilevel evolutionary processes, including psychological selection that screens sequences for their learnability by human users. Number tools are a kind of capital; they are material systems that are as real as other everyday objects. Just as changing physical tools alters the structure of productive activity, so too changing number sequences alters cognitive, behavioral, and social routines.

1. Introduction

> The body of economic theory does not address the ontological status of 'number'. Furthermore, to establish such an ontological commitment is philosophically problematic... [H]ow do numbers enter the realm of human interaction such that, when I utter 'two', my interlocutor grasps my meaning of 'two'? (Zúñiga, 1999: 312)

> It was economic calculation that assigned to measurement, number, and reckoning the role they play in our quantitative and computing civilization. (Mises, 1966: 230)

Conventional number sequences are important institutions that facilitate economic production and exchange. (Number sequences are taken to include verbal counting series, such as 'ONE , TWO , THREE ...' and also written number sequences, such as '1, 2, 3, ...'.) In the everyday operation of markets, prices,

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and quantities are expressed using numbers. Buyers and sellers rely on number sequences in order to communicate and bargain with one another and to fix mutually agreeable terms of exchange. Number sequences are part of the core connective structure of common knowledge that supports the economic system and that underpins the institutions of property, contract, price, and money. They frame agents' perceptions and constrain their behavior. They are vital for arithmetical reasoning in economic calculation and are indispensable to economic coordination in modern market economies.

Given the ubiquity and centrality of number sequences in the economic realm, it is somewhat surprising that economists have paid scant attention to the nature of numbers. They have seldom reflected on what numbers are, what characteristics they possess and how they relate to other entities. Moreover, recent edited works on ontological issues in economic philosophy and evolutionary economics are also noticeably silent on the fundamental nature of numbers (see, for example, Mäki, 2001; Klaes, 2004). Economists are not addressing fundamental philosophical questions, such as: 'Do numbers exist, and if so, in what sense?', 'Are numbers as real as material objects of everyday life?', 'Are they like technical objects, such as hammers, or more like social objects, such as $20 bills?', 'Do they exist independently of the human mind or are they in fact constructions of it?'.

The gap is all the more surprising when we consider the heroic numerical abilities that are attributed to economic agents in mainstream economic theory. All economic agents are assumed to be highly numerate beings. Numeracy of economic agents is simply taken as given. It is a datum of the economic system. Implicitly, agents are assumed to understand the true nature of real numbers. In addition, agents are able to apply that knowledge in order to exactly and exhaustively describe their environment in numerical terms. Neoclassical economists regard it as self-evident that the actions of agents can be described by a vector in 'commodity space' that has the structure of a real field R^k (where R is the set of real numbers and k is the number of dimensions – i.e. the finite number of commodities in the economy) (Debreu, 1984: 268). Thus, agents are equipped with well-defined mathematical functions for decision-making, which, in equilibrium, enables them to optimally allocate their given means amongst given ends subject to their constraints. (Indeed, neoclassical economic agents *are* these functions.) Consequently, key concepts of modern economics, such as preferences, technology, choice, and efficiency, are now defined solely with respect to the real space R^n.

Even institutional, evolutionary, and Austrian economists tend to take numbers for granted. They do not examine how number sequences come into existence in an economic sense as a result of the social division of labor, economic specialization, and economic exchange. They pay little, if any, attention to the socially recognized systems of numerical symbolization that are used to represent exchange ratios or money prices. Markets, no matter how rudimentary, are simply assumed to operate with series of well-distinguished numerical symbols that serve as inputs to computation. Thus, economists of all hues are ontologically committed to numbers for the sake of explaining economic phenomena. Numbers pervade the economic cosmos but the nature of these objects and how economic agents come to have knowledge of them are never explained.

To fill this gap, this paper explores the nature and role of these pivotal institutions in economic systems. We investigate how number sequences function in the practices of economic agents and address the most salient features of these technological objects from an economic standpoint. The next section sketches out the distinctive character of our analysis of the nature of numbers. Section 3 describes the internal structure that a system of symbols must have to serve as a numerical toolkit. Section 4 explores how number sequences have functional properties that are crucially dependent upon their external structure – their being situated in human social networks. Section 5 considers the multilevel evolutionary processes by which number sequences are selected for replication by human users, and it explains why a single number sequence rarely, if ever, achieves a monopoly position in the marketplace. Section 6 investigates the material instantiation of number sequences and how these sequences exhibit some fundamental properties of capital. Finally, Section 7 examines how number sequences are causal elements that can make a difference to the practices of economic agents. Different numerical toolkits have different effects on agents' routines and repertoires of potential behavior.

2. The approach in this article: street-level economic ontology

An obvious first place for us to turn for guidance on the nature of numbers is contemporary philosophy of mathematics. Within this branch of discourse, Platonism and fictionalism are currently regarded as the two most plausible views on the nature of mathematical objects such as numbers. According to Platonism, numbers are abstract objects that are independent of human cognition (Frege, 1959: 33–38). They do not exist in space or time, they are neither physical nor mental objects, and they do not enter into cause–effect relations with other objects. 'Numbers in particular do not emit or reflect signals, they leave no traces, their behavior causes no phenomena from which their existence may be inferred' (Giaquinto, 2001: 5). Fictionalism has garnered more and more supporters since it was first introduced in the 1980s (Field, 1989). In a nutshell, it agrees with Platonism that if there were such things as numbers, then they would be abstract objects outside of space and time. However, it rejects the Platonist ontological thesis that abstract objects exist. And if there are no such things as abstract objects, it follows that numbers do not exist. Nevertheless, fictionalism maintains, it is pragmatically useful to talk and think *as if* there are such things as numbers. Hence, numbers are fictional entities that are part of the currently accepted 'story of arithmetic'.

Although Platonism and fictionalism might have their respective merits for characterizing the primary objects of mathematics, neither view sheds much light on the status of numbers in economic systems. Given that economic agents exist entirely within space-time, how could they ever acquire knowledge of abstract objects such as numbers that purportedly exist wholly outside space-time (as Platonists would argue)? In addition, there is no evidence that when economic agents are assigning numbers to indicate the size of a set, they are intending their numerical expressions to be taken as fictional claims (as fictionalists would argue).

Consequently, the approach in this paper draws on different intellectual sources. The first is a cognitive-science account of how numbers are represented in the human mind (Wiese, 2003, 2007). Wiese's approach provides a unified concept of numbers that covers cardinal, ordinal, and nominal use-contexts for numbers. It is a functional approach – numbers are worthy of our attention because of the functions that they perform. The second source is recent philosophical study of the ontology of institutional reality in general, and of technical and social artifacts in particular (Searle, 1995, 2005; Vermaas and Houkes, 2006; Faulkner and Runde, 2009; Lawson, 2008). Thus, in the framework in this paper, numbers are technological objects ('tools' in the widest sense of the term) that are constituted by both form and function. A number sequence is a concrete system made up of elements and connections between them. Numbers have a dual nature in that they are objects that must have a particular technical *internal* structure to fulfill their functions, while at the same time these functions are also irreducibly social in nature, thereby requiring a particular *external* structure (particular connections to human users). What sets number sequences apart from other mathematical progressions is that they belong to the class of objects to which some network of economic agents has imposed the functions of a numerical toolkit.

This paper is a study in what Mäki (2001: 7) calls local (specifically, economic) ontology in that it focuses upon issues about the fundamental nature of numbers that are pertinent to their use in economic systems, irrespective of the relevance of these issues to other realms. It is also an exercise in direct descriptive ontology in that it is concerned with directly investigating institutional reality rather than the presuppositions of economic theory or the ontological commitments of economists.

Our interest is upon the status of numbers within economic systems populated by flesh-and-blood human actors. The focus is upon a street-level, economic ontology of numbers. We consider issues regarding the existence of number sequences and their use from the perspective of active human individuals rather than that of detached birds-eye observers. 'Institutional facts only exist from the point of view of the participants' (Searle, 2005: 22). This approach contrasts with the tendency within the philosophy of mathematics to de-anthropomorphize (and de-materialize) mathematical objects and strip them of their human baggage. From our perspective, number sequences are economically non-existent if economic agents have not yet learned to use them in their quantificational practices, whether or not these objects might exist in other senses. They are economically non-existent if economic agents are causally isolated from numbers and have no contact with or grasp of them, so that numbers have no causal effects on their decision-making or the operation of markets.

Because natural numbers (rather than integers and rational numbers) were the first numerical sequences to be used in the history of markets, we limit the scope of our inquiry to the ontology (fundamental nature) of natural numbers as used in economic systems. Moreover, the natural number sequences we consider are not necessarily infinite and indeed may even be highly truncated, consisting of only a few elements. In addition, we are chiefly concerned with *conventionalized* sequences of natural numbers that have actually been used by populations of economic agents in historical time rather than with idiosyncratic and/or possible, but never actualized, sequences.

3. Internal structural properties of number sequences

Before we investigate the structural properties of number sequences, we must make an important distinction between numbers and cardinality. Cardinality is a property of sets. It is the property that we try to assess when we ask how many members there are in a particular set. Loosely interpreted, cardinality is the quantity or size of a set. In contrast, numbers (as defined here) are the tools we apply in order to identify various properties of empirical objects, of which cardinality is one such property, but not the only one. A number sequence is a kind of yardstick that agents can use to measure a property of sets of empirical objects – the yardstick must not be mistaken for the property being assessed.

What then makes an object a number sequence? What criteria qualify an object to fulfill numerical purposes in the economic realm? What conditions must an object fulfill to play the role of a conventional number sequence in the lives of economic agents? Number sequences are technological objects that human beings create (oftentimes unintentionally) and that serve as an indirect means to human satisfaction. They are 'tools' in the widest sense of the term. They are devices for figuring things out and for tackling recurrent coordination problems. Like other technological objects (Faulkner and Runde, 2009), number sequences have a dualistic quality in that they are constituted by both form and function. In order for a particular sequence N to be a numerical toolkit for quantifying sets and assessing other properties of empirical objects, the form (i.e. structure) of N must satisfy two conditions:

(C1) All elements of N must be well distinguished from each other;
(C2) N must be a progression. (Wiese, 2003: 60, 304)

By saying that all elements of N must be well distinguished, we mean in practice that they can be differentiated from one another by their physical properties, such as sound (phonological shape), size, shape, and position, and that users can discriminate between them with their available cognitive and perceptual apparatus.

By saying that N must be a progression, we mean in effect that a number sequence is an object made up of elements and special connections between them (Barwise and Moss, 1991). In formal terms, a progression is a particular set N that is totally ordered by a relation R (such as the '$<$'-relation) and R is irreflexive, asymmetric, transitive, and connected in N (Quine, 1960: §54). In Wiese's criteria-based approach, '$<$' is the symbol used for the relation that sequentially orders the elements in the number sequence and not a relation (such as the 'less-than' relation) between sets of different sizes. Thus, '$x < y$' is paraphrased as 'x precedes y in the number sequence N'. Of course, agents do not have to understand these formal properties of a progression in order to use them to do the job of numbers. Their knowledge of structure is often rudimentary, fuzzy, and coarse-grained. Moreover, the cognitive division of labor means that ordinary folk can make use of the testimony of experts and teachers.

The upshot of these two criteria is that numbers are distinct elements in a special type of pattern that possesses a special internal relational structure. Numbers are sequential positions in a progression (such as a stable counting sequence). They are completely and solely defined by their relations to other positions in the structure in which they occur. What is significant is the pattern or structure that the elements jointly display: '"Objects" do not do the job of numbers singly; the whole system performs the job or nothing does' (Benacerraf, 1965: 69).

As a result, numbers cannot be identified or used outside of the concrete systems of which they are elements. The counting word 'FOUR ', for example, is completely and solely defined by its position as the fourth element in the conventional English counting sequence ('ONE , TWO , THREE , FOUR , FIVE . . . '). The counting word 'FOUR ' does not refer to the number 4; rather, it *is* the number 4 within the particular sequence of which it forms a part. Thus, counting words are verbal numerical tools – linguistic instances of numbers (Wiese, 2007: 769). As recited in a sequence, counting words do not denote an abstract entity or refer to anything in the outside world: 'Questions of the identification of the referents of number words should be dismissed as misguided in just the way that a question about the referents of the parts of a ruler would be seen as misguided' (Benacerraf, 1965: 71).

The question arises as to whether it is strictly necessary, in ordinary economic contexts, for number sequences to be infinite. In the everyday operation of markets, agents do not need an infinite number sequence provided they are only seeking to handle finite empirical structures, such as finite sets of economic goods. However, the finiteness of a number sequence does mean that there is an obvious upper bound to the size of empirical sets that agents can quantify numerically and it excludes agents from carrying out mathematical operations based on higher numbers or infinity.

If the elements in a system are not well-distinguished from each other and do not exhibit sequential ordering, then that system does not have the structure required to be a fully functioning number sequence in the lives of economic agents. Such a system is prone to malfunction. It could fail to quantify sets of goods consistently. Consequently, such a sequence could lead to misunderstandings between economic agents and result in a breakdown of coordination between agents' actions. More specifically, any of the following features indicates a poor fit with the structural template for fully functioning number sequences:

1. Insufficient differentiation of elements (e.g. in the Oksapmin body-counting system of Papua New Guinea, the same words 'tan besa' ('other forearm') are used for both the twenty-first and the twenty-ninth elements in the number sequence so that context is crucial for understanding a count (Saxe and Esmonde, 2004; see too Menninger, 1969: 35));
2. Truncation or limited extent (e.g. the Pirahã language in the Brazilian Amazon contains a non-recursive counting sequence that comprises just two words, 'hói' (for 'one') and 'hoí' (for 'two'), with the result that it cannot be used to quantify sets with more than two elements (Gordon, 2004));
3. Gaps – the sequence lacks elements for positions in the sequence (e.g. 'ONE , TWO , THREE , FOUR , FIVE , x, x, x, x, TEN ', where 'x' indicates gaps and not symbols in the broken sequence);
4. Redundancy and branching – the sequence contains multiple elements for the same position in the sequence (e.g. VIIII and IX for the ninth position in the Roman number sequence);
5. Loops so that the sequence starts all over again {1, 2, 3, 4, 5, 1, 2, 3, 4, 5, 1, 2, 3, 4, 5 . . .};
6. Constitutional complexity exceeding what can be learned by (typical) human users.[1]

4. External structure: properties related to human intentionality and sociality

Any set that forms a progression of well-distinguished elements can potentially be used as a number sequence. (Indeed, there are potentially infinitely many sequences that satisfy the above two conditions and could therefore play the role of numbers; none of them has intrinsic properties that make it stand out as the one and only number sequence.) For example, even though it is very truncated, the sequence of words 'Fee! Fie! Fo! Fum!' would meet the

1 Using artificial-life modeling (and the computational technique of genetic algorithms), Hurford (1999) builds in factors such as these into his fitness function for modeling the evolution of number systems.

above two criteria since its elements exhibit distinctness and sequential ordering, but no agents use this sequence for numerical purposes. In the marketplace, only very few progressions actually end up being selected as conventional numerical toolkits. Thus, being a conventional number sequence must be more than just an issue of having the right internal structure described above. Conditions (C1) and (C2) are necessary but not sufficient – they underdetermine what fundamental entities actually constitute numerical toolkits in economic systems.

For this reason, we propose an additional criterion that draws upon Searle's (1995, 2005) analysis of institutional structure. What distinguishes a conventionalized number sequence from other progressions is the following:

(C3) *N* must belong to a class of objects to which a network of at least two agents has imposed specific quantitative functions (first and foremost, the function of representing non-iconically the cardinality of empirical sets).

A network comprising two or more agents (who are connected over economic space and actually interact with each other) selects a progression to serve as a mutually recognized set of numerical tools. The functions imposed on these objects are understood to include identifying, and communicating information about, the cardinal, ordinal, and nominal relations between empirical objects. What sets numbers apart from other progressions is thus not intrinsic to numbers themselves but is dependent on the practical interests and goals of economic agents and the uses to which they put them (the functions they impose upon them). In other words, a progression must also have the right *external* structure to do the job of numbers. It is only in relation to human intentionality and social activity that progressions become conventional number sequences. Progressions do not have numerical functions intrinsically but because a network confers such functions on them (so that these functions are observer-relative rather than observer-independent) (Searle, 2005: 7). By themselves, progressions are 'representationally lifeless' (Dretske, 1988). Their power to do the job of numbers requires input from the human agents who create and use them and who assign them numerical functions. The imposition of function need not be conscious or deliberate or explicitly understood by the relevant network of agents. The uses to which numbers are put may simply be reflected in their behavior.

Consequently, a serial structure of distinct symbols has to be situated in a structure of purposeful agents to perform numerical functions. Number sequences exist within a world of objects that contains more than just physical symbols and symbolic structures. The imposition of a numerical function on a progression embeds that progression within the ends–means framework(s) of a network of sign-users. Number sequences are thus constitutively social in the sense that they stand in external relations to people and their functions depend on us. Progressions only have functions to the extent that they contribute to the achievement of agents' purposes. If there were no network of agents who need to use numerical tools in their interactions with each other, there would not be any conventional number sequences, no matter how many progressions are available in the economic system. 'No agent, no purpose, no function' (McLaughlin, 2001: 60).

The economic existence of numbers depends upon or presupposes the existence of sign-using agents that are able and want to use number tools to identify a broad range of properties of empirical objects, such as the cardinality of a set. In order for a symbol to be an element of a number sequence, it must be viewed as a particular type of sign by the right kind of minds, namely minds that have:

1. the concepts of cardinality, of sequential rank and of (non)-identity;
2. the capacity to invent and use arbitrary symbols (in Peirce's (1960) sense of non-iconic, conventional signs that bear no relation of similarity or physical or temporal contiguity with that for which they stand);
3. the capacity to position symbols in a set into a stable serial order and to grasp relations of 'comes before' or 'comes after' between those symbols;
4. the capacity to grasp empirical relations (such as 'has more elements than') between (sets of) empirical objects;
5. the capacity to make systematic (homomorphic) mappings between numerical and empirical relational structures;
6. the capacity to impose semantic interpretations upon numerical expressions that indicate properties of empirical objects in the world of things beyond the number sequence;
7. a sufficiently advanced 'mind reading' capacity for attributing beliefs and intentions to other agents who use number sequences for communicative purposes (more precisely, in Dennett's (1987) terminology, they must have at least third-order intentionality);
8. the capacity, through the intermediary of eyes, hands, and the like, to visually and manually interact with external physical symbols, media, and objects; and
9. the capacity to select and execute the relevant routines (or 'use-plans') for a number sequence in order to realize the given numerical goal.

When economic agents use number sequences to quantify sets, and wish to communicate to others the results of their calculations, they must follow very specific rules. Taken together, these rules constitute a type of 'utilization routine' or 'use-plan' for employing number tools in economic contexts, a set of instructions for doing the numerical procedure correctly. A use-plan for a number sequence is a series of actions in which users manipulate numbers in order to advance the achievement of the given goal (cf. Vermaas and Houkes, 2006: 7). The relevant set of goals includes identifying cardinal, ordinal, and nominal relations between empirical objects; that is, determining set sizes, ranks in a sequence, and the identity (or non-identity) of elements within a set.

Consider the rules for counting objects in a set (Gelman and Gallistel, 1978). First, agents must apply numbers to objects sequentially, with each number being followed by its successor in the number sequence (stable-order rule). The stable order in which agents link up elements in a number sequence with empirical objects is required to ensure that agents always end up with the same number for sets of the same cardinality. Otherwise, they could not use the last number employed in the count to indicate the cardinality of the quantified set (as required by the cardinality rule). In addition, agents must assign exactly one number to each and every object in the set being quantified (one-to-one rule). The one-to-one principle is particularly complex because it requires that the agent tag the objects in the context of an evolving partition of the quantified set. (By a partition of a set, we mean a collection of pairwise disjoint subsets of that set.) At any point in the counting process, the agent has to distinguish the subset of already counted objects from the subset of objects still to be counted, with the partition being updated continually after the assignment of each number (Nesher, 1988: 114).

These rules are conditions that have to be fulfilled by any competent user seeking to use numbers to quantify empirical objects. They are not hardwired into a number sequence, but must be acquired from other external sources. Typically, economic agents do not have to construct these routines afresh because pre-existing routines are already highly entrenched, well tested, and socially inculcated in children. Agents learn them from others in processes of cultural transmission, which include rote memorization and routine drilling in the use of counting sequences.

Rules governing use can impose constraints on who counts what and where. They can specify what kinds of objects can be counted with particular number sequences. For example, traditional Mangareva, an Austronesian language spoken in the Gambier Islands in French Polynesia, contains three distinct number sequences for counting different types of economic goods. One verbal number sequence is used to count tools, pandanus leaves, sugar cane, and breadfruit; another is used to count ripe breadfruits and octopus; and the third is used to count the first breadfruits and octopus of a season (Beller and Bender, 2008). Social codes of operation can also prohibit counting specific types of objects. Lean (1992: ch. 6) and Seidenberg (1962: 14–16) list examples of societies in which the verbal counting of people, live animals and various other objects is taboo because it brings bad luck to them. Relevant here are also social rules in North America against using the number 13 to label floors in apartment buildings and rows of seats in aircraft.

The functions of a number sequence are those effects of it that account for its ongoing replication by human carriers. Conventional number sequences have *coordination* functions as conventions (as defined by Millikan, 2003: 226–227). People use conventional number sequences in the market in order to solve recurrent coordination problems – e.g. to make their mutual wants intelligible to others. Conventional number sequences perform an important function in guiding and coordinating social actions that are dispersed over space and time. Those sequences that have survived so far have evolved in a particular way because they have been effective often enough at coordinating actions. These effects (i.e. coordination functions) are not determined solely by the purposes of the agent producing numbers from a conventional sequence. The function of a conventional number sequence is not on the same level as either the purposes of the individual agents *producing* these numbers or the purposes of the individual agents *interpreting* them (cf. Millikan, 2004: 26). Hence, number sequences possess emergent properties that are not ontologically reducible to the causal powers and purposes of individual agents. By themselves, the intentional states of individual agents on either side of a communicational exchange are not sufficient for establishing numerical functions of a progression. To be functional, number sequences require joint uptake and human cooperation at some level.[2] The function of conventional number sequences is social (interpersonal) in nature. It is socially recognized and sanctioned.

For example, consider the case of posted-price retail markets in which sellers advertise the prices at which they will sell their goods. Sellers use the conventional Hindu-Arabic number sequence ('1, 2, 3, etc . . .') to express unit prices (e.g. $50), and buyers interpret these signs. Other forms of communication are kept to a minimum (e.g. no haggling). In this market, the function of the Hindu-Arabic number sequence is realized through cooperation at some level between both sellers (who write these numbers) and buyers (who interpret such signs) taken together. Sellers in this market are adapted to an environment in which buyers are responding, with sufficient frequency, to the numbers that sellers write in ways that reinforce sellers' using Hindu-Arabic numbers to announce prices. Buyers are adapted to an environment in which sellers, with sufficient frequency, write ask-prices in these numbers in situations such that it is in the interests of buyers to replicate expected responses to these numbers (cf. Millikan, 2004: 105). Thus, the Hindu-Arabic number sequence has been selected for performing a function that interests *both* sides of the market at once and sufficiently often to promote continued replication. It is selected for persistence by both sellers and buyers who are making use of it in expected ways, given the expected responses of their counterparts on the other side of the market. Moreover, increasing stability of those expectations and the formation of dovetailing habits on both sides of the market enhance the durability of the Hindu-Arabic number sequence (see Hodgson, 2002a: 117, 123). Consequently, the function of the Hindu-Arabic

2 A joint activity, such as economic exchange, can be cooperative down to a certain level (e.g. the level of the relevant business practices and rules, including the number sequence used) but still be competitive beyond that level – competitive in the sense that the players do not intend that the specific details of their plans dovetail harmoniously at lower levels (Bratman, 1999: 107; Searle, 1990: 413–414).

number sequence is not reducible to the intentions alone of individual sellers who use these numbers.

5. Evolutionary processes and multiple number sequences in the marketplace

Number sequences are selected for replication by the human agents who use them. Human minds and groups of human agents constitute the selection environment in which number sequences come into existence and are replicated. There are multi-level selection processes at work – first, a psychological selective process by which possible number sequences are screened by human minds for their cognizability, usefulness, and ease of imitation (and whatever other properties matter to potential users), then a higher-level selective process by which number sequences may contribute to the differential survival of individuals (and groups composed of them) who adopt particular sequences. 'If a set of beneficial social rules can survive the gauntlet of *psychological* selection, then groups of individuals who adopt those rules will be favored by *environmental* selection' (Whitman, 1998: 62, emphasis added). Structures of symbols that can actually perform numerical functions are more likely to be selected, replicated, and retained as numerical toolkits in the economic system. If we paraphrase and adapt an argument made by Christiansen (1994: 126) about language in general, we can say that number sequences are evolved and adapted to us: the selective forces acting on number sequences to fit human users are significantly stronger that the selective forces acting on humans to be able to use numbers. Thus, number sequences have to adapt themselves to their human carriers rather than vice versa. A number sequence can only survive if it can be learned and used by humans. Number systems that are excessively complex or costly for us to learn either do not come into existence or dwindle.

Evolutionary processes rarely, if ever, converge on a single number sequence. Because different number sequences appeal to different groups of users, competing number sequences (as unowned and unownable networks) can coexist with one another. Network effects from adopting a particular number sequence are limited to the net benefits of being synchronized with other users with whom one actually interacts rather than the total number of users of that sequence. Where the marginal benefits of increasing network size can be exhausted at network sizes that are small relative to the total number of users of numerical tools, multiple number sequences can co-occur in a population of agents (cf. Liebowitz and Margolis, 1994: 141).

Hence, we might observe multiple number sequences being employed in the marketplace at the same time, in complementary and competitive ways. For example, in vegetable markets, most of us are familiar with reading prices expressed using written Hindu-Arabic numbers ('$1 per pound etc') and then specifying the quantities we wish to purchase using spoken number words ('TWO pounds of carrots') . Similarly, the routine of writing a check involves

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specifying the amount of money payable in both written number words and Hindu-Arabic numerals in order to prevent fraudulent alteration. Menninger (1969: 464) provides an illustration of an old Chinese bank draft that uses three distinct written number sequences: common, official (*ta-hsieh*) and commercial (*Suzhou* or *huama*) systems. Within this document, each number sequence has its specific purpose: to specify the date, to record the amount of currency, and to identify the check number, respectively.

Although convergence to a unique sequence is not necessary, successful communication in the market does require that buyers and sellers mutually recognize the particular sequence each of them is using and its numerical relational structure. In order to correctly interpret your request 'SIX donuts, please', I must understand the relation that the counting word 'SIX' bears to the rest of the progression you are using. It should be noted, however, that successful communication does not require that the buyer and seller produce numbers from the same conventional number sequence in a particular transaction: numbers must be interpretable by the other but need not be produced by the other. While in a French market, I can point to the oranges and request 'TWO oranges' and the vendor can hand them to me and then demand 'CINQ euros' in exchange. Each of us produces a number-word construction in a different tongue but this does not impede communication as long as each hearer understands the one who speaks.

6. The 'capital character' and material instantiation of number sequences

As social technologies for economic calculation, number sequences exhibit the salient properties of capital. They are capital goods that are complementary to many economic activities and that serve to reduce the costs of economic exchange. Numerical tools are a kind of capital in the sense that they are humanly generated factors available for further production rather than nature-given or original factors of production. Numbers are always integrated economically into the economic system through their employment as numerical tools in economic calculation and time-consuming processes of making and transferring goods. Like other capital goods, numbers are dedicated by human agents to particular purposes, but, in comparison with other capital goods, numerical tools are remarkably fungible. They are 'highly flexible tools that we use ... to assess properties of objects as diverse as cardinality, weight, temperature, rank, and identity' (Wiese, 2003: 40). There also appears to be no limits on the sorts of objects to which numbers can be applied.

This approach thus has important implications for the economic ontology of numbers. Just as 'there is no such thing as an abstract or ideal capital that exists apart from concrete capital goods' (Mises, 1966: 503), so too there is no such thing as abstract numbers that exist apart from humanly generated number sequences. At the outset, numbers do not exist outside the orbit of human intentions, human production, and exchange. Numbers are cognitive and social tools that are created by us. In the beginning, they arise directly or indirectly from purposeful economic actions (understood in the broadest sense) and from the needs of everyday life. Number sequences are higher-order rule structures that humans have created and adopted as an indirect means to satisfying their ends. 'The natural numbers are the work of men, the product of human language and of human thought' (Popper, 1979: 160). They are 'our linguistic invention; our convention; our construction' (Popper, 1992: 26). Once we have created them, however, number sequences have autonomous properties which are well-determined (e.g. properties of being 'odd', 'even', 'divisible', and 'prime'), which may require great ingenuity for their discovery (Hersh, 1986: 22).

Our notion of number sequences as a structural pattern also comports well with Lachmann's (1978) conception of capital as a structure. According to this structural conception, each capital good always forms part of a whole and has to fit into a 'capital combination' so that relationships of complementarity between capital goods trump relationships of substitution. 'Complementarity' is defined as a property of means that are used for the same end – i.e. as part of the same plan (Lachmann, 1977: 200). Similarly, we have described how agents embed progressions (symbolic structures) into use-plans for number sequences. Numerical functions are ascribed to these structures relative to use-plans. The use of number sequences always involves inserting those symbolic structures into rule complexes that comprise cognitive, behavioral, social, and technical rules (cf. Dopfer and Potts, 2008: 37). In order to realize their functionality, number sequences must be supplemented with particular routines, physical media, and other devices for making computations. Only the combination of such rules yields a fully functioning number sequence. By themselves, neither a single capital good nor a single number tool can render any services. Capital goods and number tools that no longer fit into any rule complex or use-plan, lose their functionality and become obsolescent.

In addition, a number system is capital in the evolutionary-economic sense of a 'hyperstructured technology' (Potts, 2000: 117). A hyperstructure is a system of systems that exhibits a nested hierarchical structure; a hyperstructured technology thus consists of interrelated technological subsystems that in turn consist of lower-level subsystems. A number system is a technology for generating well-distinguished elements and ordering them into a progression. It too can be a hyperstructure in that it exhibits a great deal of structure (patterning) at more than one level. For example, the sequence of counting words in natural languages contains at least three levels of organization: phonemic structure, morphemic structure, and phrase structure. Consider the verbal number sequence in Adzera, an Austronesian language spoken in Morobe Province in Papua New Guinea: *bits* (one), *iru*' (two), *iru*' *da bits* (two plus one), *iru*' *da iru*' (two plus two), and *iru*' *da iru*' *da bits* (two plus two plus one) (Holzknecht, 1986). Loosely interpreted, phonemes are the smallest distinctive (and meaningless) units of sound that are combined according to specific rules into morphemes (such as *iru*'), the minimal units of semantic meaning; the latter are then in turn combined according to recursive phrase-structure rules into higher-order compound expressions (such as *iru*' *da bits*). Even after the lower-level discrete elements are combined into higher-level structures, the original elements continue to be identifiable perceptually rather than blend (see Abler, 1989).

Our approach suggests that it is a mistake to neglect the specific features of the material medium in which number sequences are made concrete. It is a mistake to purge the materiality out of numbers by quarantining them in an abstract realm outside space-time (as in the case of Platonism) or by pushing them exclusively inside the heads of agents (as in psychological constructivism). The material medium constitutes a set of rules that both enables and constrains the ongoing evolution of number sequences and methods of economic calculation. Hence, the particular physical instantiation of number sequences contributes constitutively to the creation of institutional reality. For example, the inflexibility of clay tablets as a representational medium in ancient Mesopotamia required users to economize on the range of symbols used. This may have spurred Babylonian scribes to discover the principle of place-value as a way of overcoming this constraint. In contrast, the flexibility of papyrus as a writing material may have opened up the way for Egyptian specialists to devise new symbols to substitute for groups of iterated number signs, thereby giving rise to the principle of cipherization (according to which one and only one sign is used for each power of the base represented) (Boyer, 1944: 137).

7. The interplay of number sequences and routines in the economic arena

According to the above account, materiality is as fundamental to understanding numbers as is their sociality. Like other kinds of capital, number sequences have to be instantiated in some physical medium to be used by economic agents; there can be no numerical tools that are devoid of a material substrate. What does this materiality imply for the kind of causal influence that conventional number sequences might have on economic agents? Do different number sequences have different effects on agents' repertoires of potential behavior? In this section, we sketch out possible ways in which number sequences might enter into cause–effect relations with economic agents. We offer some tentative conjectures on how number sequences can make a difference to the actions and practices of agents.

How number sequences are materially instantiated is not a minor implementational detail. The surface form of number tools can affect the perceptual and cognitive processes that are activated and the computational routines that are enacted. This point can be seen clearly if we consider two familiar number sequences: spoken number words in English (ONE , TWO , THREE . . .) and Hindu-Arabic numerals (1, 2, 3, 4 . . .). The former is a *verbal* (i.e. linguistic) sequence of

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numerical tools that is based on phonological representations, whereas the latter is a *non-verbal* sequence of numerical tools that is based on visual representations (Wiese, 2003: 260). These two sequences are not different sets of labels for referring to the same 'abstract' numbers; they are different instances of numerical toolkits with different material properties.

Both of these sequences serve as 'routine prompts' (Sfard, 2008: 209) – situational elements that evoke, usually non-deterministically, specific routine courses of action. They function as 'cognitive framers' – factors that shape how an agent processes complex cognitive tasks. Number sequences activate particular arithmetic fact-retrieval processes and computational routines, while inhibiting others. In this sense, number sequences exert causal influence on agents by constraining and channeling their actions while leaving their purposes and preferences unaltered. In some contexts, verbal number tools are more likely to activate 'street mathematics' than school-prescribed routines (Nunes *et al.*, 1993). For example, in response to a spoken request: 'What is THREE HUNDRED AND FOURTEEN dollars take away TWO HUNDRED AND EIGHTY-SEVEN dollars?', a person might use the 'counting-up' routine for doing subtraction. 'Well, it's THIRTEEN up to THREE HUNDRED, and FOURTEEN makes TWENTY-SEVEN dollars'. This method involves thinking about subtraction problems as finding a distance between two numbers on a mental number line (itself a framing effect) and finding salient benchmark numbers that are easy to manipulate (typically, integral multiples of a power of base ten). This algorithm was used by shopkeepers to figure out change in cash transactions prior to the introduction of cash registers (Gibilisco and Crowhurst, 2008: 23).

In contrast, for an equivalent written computation exercise using Hindu-Arabic numbers ('$314 – $287 = ?'), an agent is likely to use the customary decomposition algorithm (using 'borrowing') for multi-digit subtraction.[3] The spatial, columnar configuration of the written digits provides external constraints that focus the agent's attention on executing the subroutine in one place-value column at a time, thereby decomposing the complex subtraction problem into a sequence of simpler pattern-completing stages. In addition, the convention of the 'crutch', first championed by Brownell (1939), alleviates the demands on the agent's working memory by using external symbolic storage: it involves striking through the digit in the next-left column from which an amount is borrowed, in order to keep track of the borrowing process.

In oral arithmetic, agents use spoken number words 'transparently' and referentially in the sense that they link them to specific cardinalities (numerical quantities) while manipulating them in calculations (Nunes, 1999: 42). In

3 The standard algorithm has been taught in US schools since the 1940s. For a brief review of the historical development of subtraction algorithms used in the US since colonial times, see Ross and Pratt-Cotter (2000). See Smith (1925: 97–101) for a brief history of subtraction algorithms in Europe since Fibonacci (1202).

contrast, with written arithmetic algorithms, the agents treat Hindu-Arabic number symbols as 'opaque' and non-referential, as decoupled from specific numerical quantities. Hence, agents can stop thinking about the cardinalities involved during intermediate stages of the calculation process (Bruner, 1973; Kaput and Shaffer, 2002). As they 'borrow' and 'carry' digits, agents do not have to pay attention to whether they are handling units, tens or hundreds. Their actions are syntactically guided rather than semantically controlled.

The standard written algorithms for basic arithmetic are not rarefied mathematical procedures; they are concrete business routines that are an integral part of the social technology of computation and decision-making in markets. They have been used in setting prices, registering price differences, calculating and dividing profits, computing interest, determining exchange rates, and keeping financial accounts. Moreover, these algorithms (especially for multiplication and division) were specifically developed by *commercial* arithmetic teachers for use with pen and paper in fourteenth-century Italy (Van Egmond, 1976: 343). They were adapted to the specific use-contexts of merchants and tradesmen and were primarily regarded as techniques of commerce in early modern England, after the Hindu-Arabic figures had begun to work their way into business accounts in the sixteenth century (Thomas, 1987: 111; Jenkinson, 1926: 264). They started life as the arithmetic of the marketplace and were only transferred to the main educational system centuries later. The 'art of calculation was no more considered a part of a liberal education than was the art of shoe-making' (Cajori 1896: 207). Furthermore, these computational routines proved to be remarkably durable in all branches of commerce: during the eighteenth and nineteenth centuries, British accountants and businessmen relied on these pen-and-paper reckoning methods and 'made virtually no use of mechanical aids to computation' (Edwards, 1989: 48).

The standard written algorithms were selected by lead users after a long process of experimentation. These procedures yield the correct solution relatively efficiently, without requiring the agent to understand their mathematical underpinnings.[4] They are user-friendly enough to be carried out reliably by people of average ability. They economize in the use of scarce cognitive resources. Standardized written algorithms increase the proportion of human computational activities that are governed by habits. 'One is unlikely to exercise any choice over method and while the calculation is being carried out one does not think much about why one does it in that way' (Plunkett, 1979: 2). In truth, Plunkett actually castigates such 'cognitive passivity', recommending instead

4 There is extensive empirical evidence that the surface format of number tools and their associated algorithms have large effects on the type and magnitude of arithmetic errors that agents commit. For a review, see Nunes (1999) and Campbell and Epp (2004). Simple arithmetic with written number words has response times and commission error rates that are approximately 30% greater than those for arithmetic with Hindu-Arabic numerals (Campbell, 1994: 31).

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'active' methods, definite choices of technique and thorough understanding. On the other hand, Whitehead (1911: 61) would regard it is as a 'profoundly erroneous truism', to which Plunkett himself and many others seem to fall victim, 'that we should cultivate the habit of thinking of what we are doing. The precise opposite is the case. Civilization advances [and we would add, the emergent computational power of markets is enhanced] by extending the number of operations which we can perform without thinking about them.' Standardized written algorithms enabled ever-wider circles of economic actors to engage in complex and extended forms of economic calculation and profit-and-loss accounting.

The upshot of this discussion is that conventionalized number sequences can have enduring and generalized effects on economic agents. Employing Hodgson's (2002b: 175) terminology, we might say that these institutions have the potential to *reconstitute* individuals – to change their 'fundamental properties, powers, and propensities'. As practitioners become skilled in using number tools and algorithms through frequent use, they become different individuals with new repertoires of behavioral dispositions, as Fibonacci noted: 'Once, through practice, science has turned into habit, memory and mind come to accord with hands and figures to such a degree that all act in harmony together, as if by one impulse and one breath' (1202: 1).[5] Indeed, when written arithmetic becomes habitual in childhood, 'we see little boys run, as it were, a gallop, and not make a false step... [They] add, multiply and divide as fast as a dog will trot' (Aubrey, 1972: 98–99). By changing our habits of thought, numerical institutions can transform the way people order the world and think about reality. They may also change our purposes and preferences. Murray (1978: 175) describes how in medieval Europe the Hindu-Arabic number system gave rise to a new generalized 'arithmetical mentality' – a shift towards habitual quantitative thinking and a widespread preference among the general populace for numerical precision in reporting on quantities, prices, dates, distances, and other physical phenomena. 'Growing testimony to how people actually thought and spoke . . . reveals *numerical tastes* as an increasingly salient fact of everyday life' (Murray, 1978: 186; emphasis added).[6]

Learning a number sequence for the first time amounts to much more than merely accumulating information; it involves restructuring the agent's knowledge, where 'knowledge' is understood in its expansive sense of 'the whole cognitive structure which includes *valuations* and *motivations* as well as *images* of the factual world' (Boulding, 1993: 301; emphasis added). When agents first

5 Translated by Murray (1978: 166).

6 Similarly, Crosby (1997: 49) considers the introduction of the Hindu-Arabic number system to have been a necessary (but not a sufficient) condition for the 'epochal shift' from qualitative toward *quantitative perception* in Western Europe during the late Middle Ages and Renaissance. Western Europeans were now 'thinking of reality in quantitative terms with greater consistency than any other members of their species' (p. xi).

learn to use the series of number words in their natural language, they are in effect installing an 'instant tally-making kit' inside their heads – they are loading new 'serial virtual machinery' on to the parallel hardware of their brains (Hurford, 1987: 175; Dennett, 1991: 218).[7] Thus, growing facility with number sequences can shape human agency by changing the neural connections and knowledge structures within the biological individual.

However, number sequences can do more than change the computational circuitry of cognition inside people's heads. They can also have a reconstitutive effect on the boundaries of the human cognitive system and its external structure (i.e. the connections among its components and the environment). Numerical toolkits can transform and augment the cognitive architecture for arithmetic and economic calculation so that it includes non-biological resources external to the physical boundaries of a human individual (Clark, 2003, 2006; Hutchins, 1995, 2001). Number tools are thus part of the tendency towards the 'hybridization' of cognition in general and quantitative thinking in particular. They extend the material substrate of computation beyond the agent's biological endowment for pre-numerical quantification.[8] Consequently, economic calculation is not a dematerialized phenomenon 'all in the mind'. It is partly an exosomatic process that involves tight loops of interaction among human agents and material numerical tools, other concrete objects and physical media. In economic calculation, agents make use of highly organized semiotic spaces in very physical ways – e.g. they organize material numerical symbols into tabular arrangements (e.g. spreadsheets) to support economic decision-making.

Numerical toolkits play a *constitutive* role in economic calculation that vitally depends upon their materiality. Hence, number sequences do not just exert downward causal influence by constraining and redirecting human action. They also serve as material components of hybrid computational routines (such as in the algorithms for written arithmetic). Number tools are not merely inputs to calculation mechanisms inside agents' heads but are *proper parts* of hybrid calculation processes – hybrid in the sense that they involve the participation of both human and non-human elements. Number tools become what Clark (2006: 300) calls 'fulcrums' of attention, perception, memory, and action. They provide an external scaffolding that makes certain kinds of quantitative thought possible. Numerical expressions (such as in the sentence 'The price of the iPod is TWO HUNDRED AND TEN dollars') are potential anchors by which attention can latch on to conceptual units that exceed the bounds of human perceptual

7 According to recent brain imaging studies, number words and Hindu-Arabic numbers are represented differently at the neurophysiological level (e.g., Cohen Kadosh *et al.*, 2007; Cohen Kadosh, 2008).

8 In light of current knowledge about the human brain, all economic agents are equipped with two innate biological mechanisms of pre-numerical (iconic) quantification that are located inside their heads. The first is a system for representing the exact cardinality of very small sets (i.e. of up to three elements). The second is a system that represents the approximate cardinality of large sets with a 'noisy' mental analogue magnitude (see Harper, 2008).

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discrimination (Jackendoff, 1996: 2; Hurford, 1987: 182). The very content and thinkability of the idea 'The price of the iPod is TWO HUNDRED AND TEN dollars' requires and makes use of this actual string of numerical material symbols from a conventional language.

8. Conclusion

This paper examines the nature and properties (in short, the ontology) of numbers in economic systems. This ontology depicts numbers as embedded in structures of economic agents (social networks), structures of rules (institutional networks), and structures of objects (technical networks). In particular, number sequences straddle technical and social domains. On the one hand, they are technical objects in that there is a tight connection between their function and intrinsic architecture. As with other technical objects, number sequences have to have the appropriate internal structure to be able to contribute causally to the performance of their functions. Their internal structure is not arbitrary. On the other hand, number sequences are also social objects in that their mode of existence depends crucially upon interpersonal recognition that they have the status of a mutually intelligible number sequence within the relevant network of agents. Thus, number sequences are genuinely hybrid objects in that their emergent causal properties (their 'functionality') depend crucially upon both their internal structure and the external social relations in which they stand to users. Both internal and external connections are responsible for a number sequence being the kind of thing that it is.

This ontological account is a useful step towards explaining how the emergence of numbers in an economic system can have real economic effects on the habits, preferences, and decision-making of individuals. It improves our understanding of how numerical institutions increase the scope of human action and how they enhance the computational power and coordination of markets. Further research can investigate in more detail the economic effects of different number systems on economic processes, including habitual ways of thinking and acting and patterns of entrepreneurial discovery. In addition, it is necessary to examine whether and how people's use of different number sequences affects the speed and quality of economic decision-making and the efficiency of economic calculation.

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