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ARTICLE

Coordination in price setting and the zero lower bound: a global games approach

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Abstract

This paper constructs a two-period general equilibrium model with the effective lower bound of nominal interest rates and describes price competition among monopolistically competitive firms as a coordination game. While the model has multiple equilibria with different levels of inflation (positive or zero), the equilibrium selection in line with global games implies that the economy with high expected productivity growth moves into the positive inflation equilibrium. The policy analyses indicate that monetary policy measures such as an increase in the target inflation can prevent the economy from moving into the zero inflation equilibrium even with low productivity growth.

Keywords: Inflation indeterminacy, effective lower bound, global games

JEL Classfications: D82; E31; E52

1. Introduction

In this paper, I describe price competition among firms as a coordination game in a general equilibrium model with nominal frictions and show that high productivity growth is key to realizing positive inflation. First, the effective lower bound (ELB) of nominal interest rates coupled with the Taylor-type monetary policy rule causes multiple equilibria with different levels of inflation (positive or zero) under strategic complementarity in price competition among monopolistically competitive firms. Then, I apply the equilibrium selection principle in line with global games to the model with multiple equilibria by adding a small amount of noise to private signals about productivity growth. The model with uncertainty (i.e. the model in which private signals about productivity growth contain a small amount of noise) indicates that as a result of agents' strategic behavior, equilibrium selection takes place based on the expected productivity growth rate. That is, the result implies that an economy with low economic growth tends to reach the ELB more often and consequently experience low inflation in the long run.

Equilibrium selection from among multiple equilibria with different levels of inflation is an important but underexplored issue in monetary economics. As pointed out in the seminal works of Benhabib et al. (2001, 2002), monetary models with the Taylor-type monetary policy rule have multiple steady states with different levels of inflation due to the ELB. However, most subsequent studies, particularly those based on a monetary dynamic stochastic general equilibrium (DSGE) model, intentionally and unintentionally ignore the possibility of multiple equilibria caused by the ELB and assume that long-run inflation is anchored around the target inflation rate.[1] Recently, as the ELB has become an urgent monetary policy issue in many advanced economies, some studies

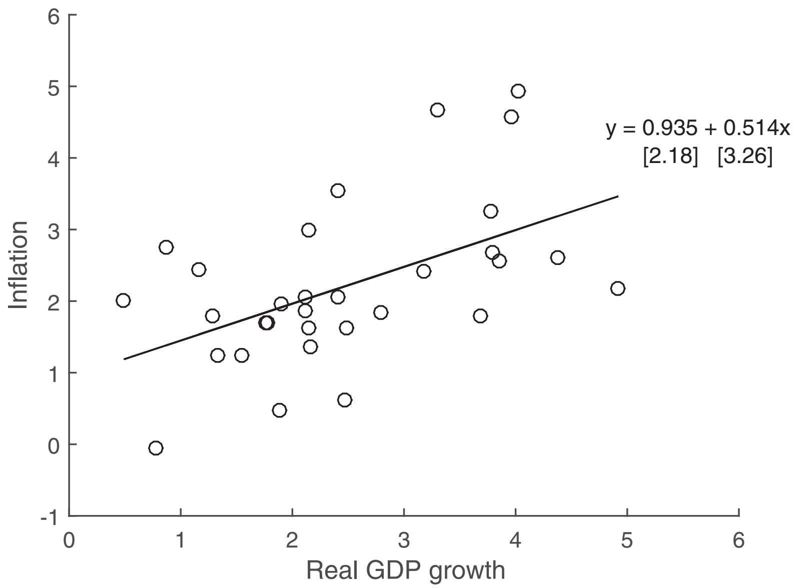

Figure 1. The long-run relationship between inflation and output growth.
Note: The figure plots the average real GDP growth rate and the average inflation rate over the last two decades (1996–2015) for OECD countries. Inflation is calculated in terms of the year-on-year change in the consumer price index for all items, excluding food and energy. In the regression equation, the values in parentheses are t-statistics. For some countries, that is, Chile, Estonia, and Slovenia, the observation period is less than 20 years because of data limitations. In addition, Mexico, Hungary, and Turkey were excluded as outliers, since these three countries experienced hyperinflation in the late 1990s.

empirically or informally examine the possibility of multiple equilibria with different levels of inflation, but there are still few theoretical investigations of equilibrium selection between them.[2]

The importance of price coordination between firms, which is investigated in this study, is intuitively appealing to policymakers as well. Most central banks in advanced economies set a positive level for target inflation and implement a monetary policy to maintain the inflation rate around the target. It is assumed that in the process of realizing positive inflation, firms raise prices together through implicit coordination via relative price adjustments. While such implicit price coordination resulting in positive inflation represents one possible equilibrium in terms of price competition among firms, another possible equilibrium is one in which firms do not dare to raise prices because other firms also do not raise prices, resulting in zero inflation. Therefore, an important question for policymakers, particularly in advanced economies that have faced prolonged low inflation, is as follows: What conditions or policies are necessary to achieve coordination among firms, together with a positive level of inflation?

While long-run inflation is assumed to be totally unrelated to economic growth in the literature on monetary economics, monetary policymakers often express the view, particularly in the context of the limited effectiveness of conventional monetary policy at the ELB, that low inflation is related to a low natural interest rate.[3] Since a low natural interest rate in the long run corresponds to low expected growth, this view essentially implies that monetary policymakers regard inflation to be closely linked to expected economic growth. This intuitive view of central bankers finds some support in empirical observations. Figure 1 plots the real gross domestic product (GDP) growth rate and the inflation rate over the last two decades (1996–2015) for Organisation for Economic Cooperation and Development (OECD) countries. The figure indicates that there is a positive and statistically significant correlation between real economic growth and inflation in the long run, which is ignored in the literature. In this paper, I explain the positive correlation using a global games approach. In that sense, the results of this study can be interpreted as providing

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one potential mechanism underlying policymakers' intuition as well as empirical evidence for the long-run relationship between economic growth and inflation.

While this study argues that equilibrium selection between the inflation and zero-inflation equilibria is essentially based on the expected growth rate, the policy analysis implies that the central bank can prevent the economy from moving into zero-inflation equilibrium through appropriate policy actions, even in the face of low economic growth. For example, the policy analysis indicates that the central bank can decrease the threshold of the growth rate for equilibrium selection by raising the target inflation rate and/or lowering the ELB through the introduction of a negative interest rate policy. This policy analysis result implies that if the prolonged low inflation recently observed in developed countries corresponds to the zero-inflation equilibrium in the model, the central bank can escape from such an equilibrium by adopting appropriate policy measures.

This study is closely related to the literature on inflation indeterminacy. Benhabib et al. (2001, 2002), the seminal studies in the literature, point out that monetary models with the ELB have multiple steady-state inflation values. Aruoba et al. (2018) assume that long-run inflation exogenously moves between two equilibrium values (one is positive and the other is close to zero) over time via a Markov switching process; they empirically investigate macroeconomic dynamics, given the exogenously selected long-run inflation. However, these previous studies do not investigate the mechanism behind equilibrium selection from among multiple equilibria with different levels of inflation. While this study adopts a backward-looking model, rather than a forward-looking model as they do, the analysis in this study is a reasonable first step for providing economic intuition to understand equilibrium selection from among multiple equilibria with different levels of inflation. Moreover, this study contributes to the literature on the application of global games to macroeconomic issues.[4] Morris and Shin (1998) discuss equilibrium selection in a currency crisis in relation to a currency's market value, while Goldstein and Pauzner (2005) discuss equilibrium selection in a bank in relation to the return on assets. More recently, some studies have applied global games to a standard business cycle model. Guimaraes et al. (2016) examine the fiscal policy confidence channel, and Taschereau-Dumouchel and Schaal (2018) analyze coordination with respect to capital utilization and points out that coordination failure can account for the long-lasting low growth after the global financial crisis. Similarly, this study applies global games to a standard monetary model such as Taylor (1999) and discusses equilibrium selection from among multiple equilibria with different levels of inflation in relation to the expected productivity growth rate. While the global games tools adopted in this paper are standard in the literature, their application to inflation indeterminacy is, to the best of my knowledge, novel. Finally, this study is related to the literature on policy analyses at the ELB (e.g. Christiano et al. (2011), Fernandez-Villaverde et al. (2014), and Iacoviello and Michelis (2016)). A distinctive difference between the present study and previous studies is that while they examine policy effects by assuming that inflation eventually returns to the target level, this study discusses policy effects on equilibrium selection from among multiple equilibria.

The remainder of this paper is organized as follows. In Section 2, I construct a simple two-period general equilibrium model and describe the price competition among firms as a coordination game. Section 3 characterizes the equilibrium for the model without uncertainty and indicates the possibility of multiple equilibria. Section 4 discusses equilibrium selection in the model with uncertainty and conducts policy analyses regarding the relationship between inflation and monetary policy. Finally, Section 5 provides the concluding remarks.

2. Model

The model is a simple two-period general equilibrium model. The private sector of the economy consists of a representative household, consumption-good firms, and a continuum of

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intermediate-good firms. The central bank implements monetary policy using nominal interest rates as a policy tool and follows the Taylor rule with an ELB on nominal interest rates. Each agent's behavior is described in turn.

2.1 Household

The representative household supplies labor to obtain wage income W_tL_t, where W_t denotes the nominal wage, and L_t denotes the hours worked. In addition, because the household, as a stockholder, owns all firms in the economy, it also receives dividends D_t as another source of income. The household allocates its income to consumption, c_t, and savings, the latter of which takes the form of a nominal one-period bond, B_t. The household's budget constraint for $t = 1$ and 2 is

$$P_1c_1 + B_1 = W_1L_1 + D_1, \tag{1}$$
$$P_2c_2 = R_1B_1 + W_2L_2 + D_2, \tag{2}$$

where P_t is the price level, and R_1 is the nominal interest rate from Periods 1 to 2. The household's budget constraint is rewritten in real terms by dividing it by P_t:

$$c_1 + b_1 = w_1L_1 + d_1, \tag{3}$$
$$c_2 = \frac{R_1}{\pi_2}b_1 + w_2L_2 + d_2, \tag{4}$$

where $\pi_t \equiv P_t/P_{t-1}$ is the gross inflation rate, and the real variables are denoted by lower-case letters. Given that the initial price level is normalized to one ($P_0 = 1$), the inflation rate in Period 1 is equal to the price level ($\pi_1 = P_1$).

The household's inflation expectation is assumed to be formed in an adaptive and backward-looking manner:

$$E_1\left[\frac{1}{\pi_2}\right] = \frac{1}{\pi_1}. \tag{5}$$

While forward-looking inflation expectations are much more common in recent monetary economics literature, adaptive and backward-looking inflation expectations, as in Taylor (1999), are assumed here for the following reasons. First, assuming forward-looking inflation expectations anchored around the target inflation rate may induce counterintuitive policy implications in a low-inflation economy. For example, consider an economy in which the central bank's inflation target is fully credible and coincides with the long-run inflation expectations, as is always assumed in a standard forward-looking new Keynesian model. In the two-period model used in this study, it is equivalent to assuming that long-run inflation expectations are completely anchored at the target inflation rate, $E_1[1/\pi_2] = 1/\pi^*$, as Goodfriend (2002) assumes in their two-period new Keynesian model. If this is the case, the central bank can perfectly control inflation expectations by adjusting the target inflation, and consequently, given inflation expectations' strong influence on current inflation, the central bank would never struggle with the low inflation problem, which is counterintuitive for policymakers, particularly those suffering from that problem.[5] Hence, choosing a particular path for inflation expectations is reasonable in, for example, a quantitative policy analysis of an economy with a well-anchored inflation target, but it may not be appropriate in a theoretical investigation of inflation indeterminacy in a low-inflation economy. The second reason for using adaptive backward-looking inflation expectations is that they have recently been justified by a number of empirical studies (e.g. Fuhrer (2012), Madeira and Zafar (2015), and Malmendier and Nagel (2015)), particularly for households. Fully-adaptive inflation expectations are, of course, a simplified way of modeling inflation expectations, but given that the mechanism underlying inflation expectations in a forward-looking model is still controversial, I believe that fully-adaptive inflation expectations constitute one of the tractable and conservative ways of modeling inflation expectations to simplify the analysis.[6]

The household maximizes the sum of utility for $t = 1$ and 2 by choosing its consumption and labor supply:

$$\max_{c_t, L_t} E_1 \sum_{t=1}^{2} \left[\log c_t - \psi L_t\right],$$

subject to the budget constraint (4) and inflation expectation (5).[7] The first-order conditions for c_t, B_t and L_t yield the Euler equation

$$\frac{1}{c_1} = E_1 \left[\frac{R_1}{\pi_2} \frac{1}{c_2}\right], \tag{6}$$

as well as the labor supply function

$$\frac{w_t}{c_t} = \psi. \tag{7}$$

2.2 Firm

The representative consumption-good firm produces the final good, y_t, by aggregating intermediate goods $y_{i,t}$, using the constant elasticity of substitution (CES) aggregator $y_t = \left(\int_0^1 y_{i,t}^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}}$, where $\theta > 1$ is the elasticity of substitution. Let $p_{i,t}$ be the price of each intermediate good. The price index, P_t, is then defined as

$$P_t = \left(\int_0^1 p_{i,t}{}^{1-\theta} di\right)^{\frac{1}{1-\theta}}, \tag{8}$$

and the demand for each intermediate good is derived as a result of the representative consumption-good firm's profit maximization:

$$y_{i,t} = \left(\frac{p_{i,t}}{P_t}\right)^{-\theta} y_t. \tag{9}$$

A continuum of intermediate-good firms produces differentiated intermediate goods using labor $l_{i,t}$, subject to the following linear technology:

$$y_{i,t} = A_t l_{i,t}, \tag{10}$$

where A_t is aggregate productivity in period t. Here, A_1 is normalized to one (i.e. $A_1 = 1$), and A is then defined as $A_2 = A_2/A_1 \equiv A$. That is, A is the expected productivity growth from Periods 1 to 2. For the time being, it is assumed that A is perfectly forecastable and common knowledge among firms. Subsequently, a small amount of noise is added to A, which plays a crucial role in equilibrium selection.

Under monopolistic competition, the intermediate-good firm i maximizes its profits by setting the price of its differentiated products subject to the menu cost for price changes, ξ. Hence, intermediate-good firm i chooses its prices, $p_{i,t}$, to maximize the profit function $\Pi\left(p_{i,t}; c_t, w_t, P_t, A\right)$, which is defined as

$$\Pi\left(p_{i,t}; c_t, w_t, P_t, A\right) \equiv \left[\frac{p_{i,t}}{P_t} y_{i,t} - w_t l_{i,t} - \mathbb{I}_{\left\{t=1 \& \frac{p_{i,t}}{p_{i,t-1}} \neq 1\right\}} \xi\right], \tag{11}$$

subject to (9), (10), and the market clearing condition, $y_t = c_t$. Here, the initial price level for each firm i is assumed to be one ($p_{i,0} = 1$) for all firms.[8] Note that menu costs ξ apply only when the firm changes its price in Period 1. In other words, all prices are completely flexible in Period 2. In addition, note that the expected productivity growth A is the relevant variable for the optimization problem in Period 1, as well as that in Period 2, because it influences c_2, and as a result, c_1, via the Euler equation (6). In particular, a high A leads to a high c_2 and c_1, thus increasing the incentive to raise prices.[9] As a result of the optimization of intermediate-good firms, the optimal pricing strategy is defined as

$$p^*_{i,t}(c_t, w_t, P_t, A) = \arg\max_{p_{i,t}} \Pi\left(p_{i,t}; c_t, w_t, P_t, A\right). \tag{12}$$

Given that the initial price level for each firm i is equal to one ($p_{i,0} = 1$), the price change and the price level in Period 1 are equal, $\pi_{i,1} = p_{i,1}$, and can be used interchangeably in Period 1. Therefore, when analyzing price competition in Period 1, the optimal strategy for price change $\pi^*_{i,1}(c_1, w_1, P_1, A) = \arg\max_{\pi_{i,1}} \Pi\left(\pi_{i,1}; c_1, w_1, P_1, A\right)$ instead of price level $p^*_{i,1}(c_1, w_1, P_1, A)$ is used for the analysis hereafter.

2.3 Central Bank

The central bank sets the nominal interest rate R_1, according to the following Taylor rule, with an ELB (κ),

$$R_1 = \max\left[A\bar{\pi}\left(\frac{\pi_1}{\bar{\pi}}\right)^{\phi}, \kappa\right], \tag{13}$$

where $\bar{\pi}$ is the target inflation rate, and ϕ is the responsiveness to inflation. The Taylor rule indicates that the nominal interest rate is equal to the neutral interest rate $A\bar{\pi}$ times the response to the inflation gap. Responsiveness to inflation is assumed to be more than one (i.e. $\phi > 1$), meaning that the Taylor rule satisfies the Taylor principle. In addition, because this study focuses on equilibrium selection between the equilibrium around the target inflation rate and that around the zero-inflation rate, the target inflation rate is assumed to be positive (i.e. $\bar{\pi} > 1$).

The max function indicates that the central bank sets the nominal interest rate to κ if the nominal interest rate based on the Taylor rule is lower than κ. In the literature, the ELB is usually set to one; however, since some central banks have introduced a negative interest rate policy, it may be necessary to set the ELB lower than one. Section 4 considers the ELB as a policy variable and examines the effects of lowering the ELB.

3. Equilibrium Without Uncertainty

This section characterizes the equilibrium of the model described in the previous section. In particular, I focus on how the inflation rate in Period 1, π_1, is determined in equilibrium as a result of firms' strategic pricing decisions. Note that since there is no uncertainty regarding expected productivity growth A, it is perfectly forecastable and common knowledge across firms. The case without this assumption is discussed in Section 4.

3.1 Best Response Function and Nash Equilibrium

To characterize the equilibrium, the model is solved by backward induction. The model is solved for Period 2 first and then for Period 1, given the equilibrium values for Period 2. Given that prices are completely flexible in Period 2, firms' optimization yields the real wage in Period 2, $w_2 = A(\theta - 1)/\theta$, and the labor supply function (7). Then, the production function (10) yields consumption, $c_2 = A(\theta - 1)/(\theta\psi)$, and the labor supply, $l_2 = (\theta - 1)/(\theta\psi)$, in equilibrium. Note that because all real variables are determined irrespective of the price level P_2 in the flexible price model, the price level and inflation in Period 2, P_2 and π_2, are undetermined in the model. Consequently, in Period 1, firms face a static optimization problem.

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Given the equilibrium values in Period 2, we can show that all macroeconomic state variables in Period 1 can be expressed as a function of inflation, π_1. The macroeconomic state variables relevant to firms' pricing decisions in Period 1 are (c_1, w_1, π_1), as shown in (11). Given that (5) and (13) indicate that both $E_1[1/\pi_2]$ and R_1 are functions of π_1, the Euler equation (6) implies that c_1 can be expressed as a function of π_1, given the equilibrium value of c_2. Finally, with the equilibrium value of c_1 as a function of π_1, the labor supply function (7) gives the real wage w_1 as a function of π_1.

Once the state variables (c_1, w_1, π_1) are expressed as functions of π_1, the profit function for each firm (11) in Period 1 can be rewritten as

$$\tilde{\Pi}\left(\pi_{i,1};\pi_1, A\right) \equiv \Pi\left(p_{i,1};c_1, w_1, P_1, A\right). \tag{14}$$

Note that $p_{i,1}$ is replaced by $\pi_{i,1}$ because of the normalization $p_{i,0} = 1$ for all i. Given that the inflation rate in Period 1 is defined as $\pi_1 = \left(\int_{i\in[0,1]} \pi_{i,1}^{1-\theta} di\right)^{\frac{1}{1-\theta}}$, by (8) and is not influenced by each firm's pricing decision $\pi_{i,1}$ because of the assumption that each firm has the measure zero, the profile of other firms' pricing strategy, that is, $\left(\pi_{j,1}\right)_{j\neq i}$, can be replaced by aggregate inflation π_1. Therefore, the best response function for firm i in Period 1 is expressed as a function of π_1,

$$\pi_{i,1}^{*}\left(\pi_1;A\right) = \arg\max_{\pi_{i,1}} \tilde{\Pi}\left(\pi_{i,1};\pi_1, A\right),$$

and the symmetric Nash equilibrium for price competition in Period 1 is defined as strategy $\pi_1^E(A)$, satisfying $\pi_1^E(A) = \pi_{i,1}^{*}\left(\pi_1^E(A);A\right)$.

3.2 Multiple Equilibria

In this subsection, I first discuss the possibility of multiple equilibria with different levels of inflation in a general case where a firm's action space in Period 1 is a continuum of prices. Then, given the analysis of the general case, I limit the action space to a binary set to discuss equilibrium selection from among multiple equilibria with different levels of inflation.

3.2.1 The Case With a Continuum of Prices

Figure 2 shows the best response functions for firm i and the Nash equilibria for price competition in Period 1 for the case of $A = 1.002$ and $\bar{\pi} = 1.02$.[10] In the figure, the horizontal axis represents the inflation rate in Period 1, π_1, and the vertical axis represents the optimal pricing strategy of firm i $\pi_{i,1}^{*}(\pi_1;A)$ in response to inflation π_1. The figure shows the best response functions for the following three cases: (a) an economy without the ELB but with menu costs ($\kappa = -\infty$ and $\xi > 0$, represented by the thin dashed line), (b) an economy with the ELB but without menu costs ($\kappa = 1$ and $\xi = 0$, represented by the thick dashed line), and (c) an economy with the ELB and menu costs ($\kappa = 1$ and $\xi > 0$, represented by the thick bold line). Since the focus here is on symmetric Nash equilibria only, the Nash equilibria for each case are indicated by the intersections of the best response function and the 45° line.

Let us start with the economy without the ELB ($\kappa = -\infty$ and $\xi > 0$, represented by the thin dashed line). The figure indicates that the best response function in this economy is always above (below) the 45° line for all $\pi_1 > \bar{\pi}$ ($\pi_1 < \bar{\pi}$), and consequently, there is only one Nash equilibrium at $\pi_1^E = \bar{\pi}$. To understand this result intuitively, we consider the case of $\pi_1 < \bar{\pi}$. Given that the inflation rate is lower than the target level, the central bank decreases the nominal interest rate below its neutral level ($R_1 < A\bar{\pi}$), according to the Taylor rule in (13). Since this central bank policy reaction lowers the real interest rate, $E_1[R_1/\pi_2]$, and increases aggregate consumption c_1 via the Euler equation (6), the incentive for each firm to decrease its price, $\pi_{i,1}$, is mitigated. As a result, each firm accepts an increase in its relative price, which implies that in this case, the firm's optimal pricing strategy is to always set their price higher than π_1. That is,

$$\pi_{i,1}^{*}\left(\pi_1;A\right) > \pi_1 \text{ for all } \pi_1 < \bar{\pi}.$$

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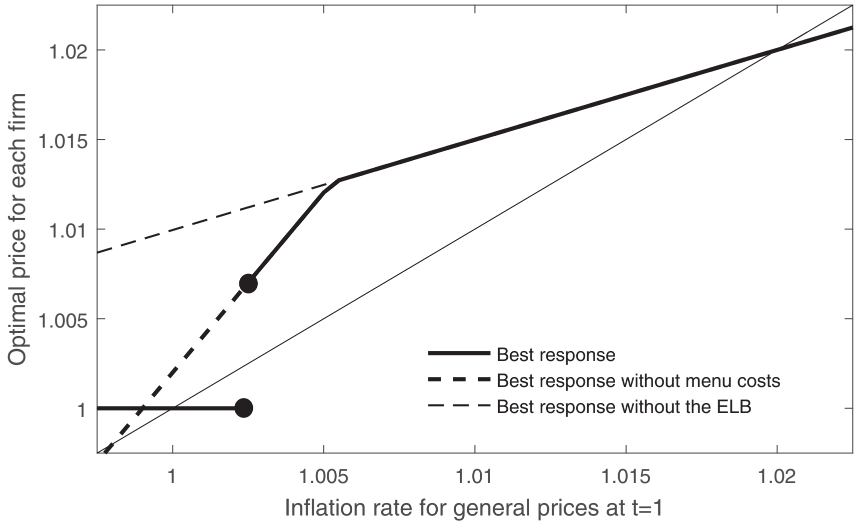

Figure 2. Best response functions for $\bar{\pi} = 1.02$.

Similarly, we have $\pi^*_{i,1}(\pi_1;A) < \pi_1$ for all $\pi_1 > \bar{\pi}$, thus leading to the sole Nash equilibrium at $\pi_t = \bar{\pi}$. Menu costs do not affect the best response function unless they are very large, because the optimal price is sufficiently high to allow the benefits of price changes to outweigh menu costs. This result for the economy without the ELB implies that if the economy is never constrained by the ELB, the central bank can maintain the inflation rate at its target level by appropriately adjusting nominal interest rates and controlling aggregate demand. Therefore, when applying the model to an economy that is far from the ELB, we do not have to be careful about strategic complementarity in price competition among firms, as in a standard new Keynesian model.

Next, we examine the economy with the ELB (i.e. $\kappa = 1$). The thick dashed line represents an economy with the ELB but without menu costs ($\kappa = 1$ and $\xi = 0$), and the thick bold line represents an economy with both the ELB and menu costs ($\kappa = 1$ and $\xi > 0$). Figure 2 shows that the best response function in these economies has a kink at $\pi_1 < \bar{\pi}$, and its slope becomes steeper when π_1 is below the kink. Consequently, there is another intersection with the 45° line at $\pi_1 = 1/A$ for the economy without menu costs (represented by the thick dashed line) and at $\pi_1 = 1$ for the economy with menu costs (represented by the thick bold line), implying that in an economy with the ELB, there are two symmetric Nash equilibria, an inflation equilibrium ($\pi_1 = \bar{\pi}$) and a negative or zero-inflation equilibrium ($\pi_1 = 1/A$ or $\pi_1 = 1$). The best response function in the economy with menu costs (represented by the thick bold line) jumps to $\pi_{i,1} = 1$ at some point in a low-inflation environment and becomes flat for inflation below this point because firms lose the incentive to make small price changes, given the menu costs. The kink in the best response functions is caused by the ELB, and its location corresponds to the point at which the nominal interest rates reach the ELB. The best response function is steeper than the 45° line when the economy is constrained by the ELB because the central bank cannot lower nominal interest rates to deal with a decline in inflation, and consequently, cannot increase c_1 as in the economy without the ELB. That is, when the economy is constrained by the ELB, a decline in π_1 leads to a vicious cycle, where

$$\pi_1 \downarrow \Rightarrow E_1\pi_2 \downarrow \Rightarrow R_1/\pi_2 \uparrow \Rightarrow c_1 \downarrow \Rightarrow \pi^*_{i,1} \downarrow .$$

Consequently, each firm has an incentive to decrease its price more than the initial decline in π_1, which implies that the best response function is steeper than the 45° line when nominal interest rates are constrained by the ELB.

This result is in line with the argument by Benhabib et al. (2002) that any models with the ELB have multiple equilibria with different values of inflation. While the model developed here is backward looking rather than forward looking like their model, the logic underlying the multiplicity of equilibria is very similar; in addition to an inflation equilibrium, the economy has a zero-inflation equilibrium because the central bank cannot lower nominal interest rates at the ELB to deal with deflationary pressure. Given that both equilibria satisfy the equilibrium condition $\pi_1^E = \pi_{i,1}^* \left(\pi_1^E; A\right)$, it is not possible to predict which of the two equilibria will prevail.

3.2.2 The Case With a Binary Set of Prices

To discuss equilibrium selection from among multiple equilibria with different levels of inflation, multiple equilibria are characterized in a more stylized model. Specifically, we assume that a firm's action space in Period 1 is not a continuum of prices, but rather, is limited to a binary set, $\{\bar{\pi}, 1\}$. That is, each firm i in Period 1 faces the option of increasing the price at the target inflation rate, $\pi_1 = \bar{\pi}$, or leaving the price unchanged, $\pi_1 = 1$. This modification may look drastic at first glance, but as the previous section has shown, firms have no incentive to set prices other than these two values, $\{\bar{\pi}, 1\}$, in the symmetric Nash equilibria with perfect information. Therefore, as long as our focus is on equilibrium selection between symmetric Nash equilibria, limiting firms' action space to the binary set does not lead to a significant loss of generality; rather, it works to avoid unnecessary complications.[11] Hereafter, because the action space is limited to a binary set, menu costs are not needed and are thus set to zero, $\xi = 0$, for simplicity.

Then, under low expected productivity growth, there are multiple equilibria with different levels of inflation.

Theorem 1. *Assume that the expected productivity growth is low enough to satisfy*

$$A < \min\left\{\frac{\kappa}{\bar{\pi}^{1-\phi}}, \frac{1-\bar{\pi}^{1-\theta}}{1-\bar{\pi}^{-\theta}}\frac{\theta\kappa}{\theta-1}\right\}.$$

Then, there are two symmetric Nash equilibria with different levels of inflation, characterized by $\pi_{i,1}^* (\bar{\pi}; A) = \bar{\pi}$ *and* $\pi_{i,1}^* (1; A) = 1$.

It is easy to show the existence of the first Nash equilibrium because it is a Nash equilibrium in the general case as well. To obtain the second one, it is necessary to satisfy the condition for the upper bound of expected productivity growth. On the one hand, the first upper bound is necessary for the ELB to bind at $\pi_1 = 1$. In contrast, the second upper bound is necessary to have $\tilde{\Pi}(1; 1, A) > \tilde{\Pi}(\bar{\pi}; 1, A)$, where $\tilde{\Pi}\left(\pi_{i,1}; \pi_1, A\right)$ is the profit function defined in (14). This condition implies that if the expected productivity growth is higher than the second upper bound, keeping the interest rate at the value of the ELB (i.e. $R_1 = \kappa$) is sufficiently accommodative for firms to raise the price of their products and set $\pi_{i,1} = \bar{\pi}$, even if others do not, that is, $\pi_1 = 1$. When $\bar{\pi} = 1.02$, $\kappa = 1.0$, and $\phi = 1.5$, the second condition is slightly tighter than the first one and requires $A < 1.0098$. This means that expected productivity growth should be less than 0.98%, implying that multiple equilibria exist under low but positive productivity growth, as in Japan.

4. Equilibrium Selection and Policy Analysis

This section discusses equilibrium selection between the inflation equilibrium ($\pi_1 = \bar{\pi}$) and the zero-inflation equilibrium ($\pi_1 = 1$) characterized in the previous section and then investigates the policy implications. While the previous section showed that there are possibly two symmetric Nash equilibria with different levels of inflation, it is well known that the multiplicity of equilibria depends on the model's information structure. In particular, the global games literature suggests that a tiny amount of noise in economic fundamentals leads to equilibrium selection in a model with strategic complementarity. If we can choose one equilibrium out of multiple equilibria, this

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allows us to use the model for policy analyses by considering the policy effects on the threshold of equilibrium selection. The following considers how a small amount of noise in expected productivity growth A leads to equilibrium selection in the price competition described in the previous section and conducts some policy analyses regarding the relationship between inflation and monetary policy.[12]

4.1 Symmetric Bayesian Nash Equilibrium

To discuss equilibrium selection in line with global games, expected productivity growth A is assumed to be imperfectly forecastable and common knowledge among firms. Specifically, the information structure is modified such that (i) the expected productivity growth A is drawn from the uniform distribution $U\left[\underline{B}, \bar{B}\right]$ and (ii) each firm i in Period 1 receives a noisy signal a_i, following the uniform distribution $U[A-\varepsilon, A+\varepsilon]$.[13] Note that the posterior belief with signal a_i is given by

$$\begin{cases} U\left[\underline{B}, a_i+\varepsilon\right] & \text{if } a_i<\underline{B}+\varepsilon \\ U\left[a_i-\varepsilon, a_i+\varepsilon\right] & \text{if } \underline{B}+\varepsilon \leq a_i \leq \bar{B}-\varepsilon \\ U\left[a_i-\varepsilon, \bar{B}\right] & \text{if } a_i>\bar{B}-\varepsilon \end{cases}.$$

As a result of this modification, the strategy for each firm i in Period 1, $\pi_{1,i}$, should be formulated as mapping from the signal to the binary action space:

$$\pi_{1,i}:[A-\varepsilon, A+\varepsilon] \rightarrow \{\bar{\pi}, 1\}.$$

It is easily shown that if the variance of ε is zero, and thus the value of A is common knowledge among firms, the same multiple equilibria as in the previous subsection are obtained. Therefore, the question is which strategy $\pi_{1,i}(\cdot)$ forms a symmetric Bayesian Nash equilibrium in the model with a noisy signal.

Let $0 \leq n \leq 1$ be the fraction of firms that choose $\pi_{i,1}=\bar{\pi}$. Then, inflation in Period 1, π_1, can be formulated as a monotonically increasing function of n,

$$\pi_1(n) \equiv \left[n\bar{\pi}^{1-\theta}+(1-n)\right]^{\frac{1}{1-\theta}},$$

for all $\bar{\pi}>1$. In addition, the difference between firms' profits when they increase their price, $\pi_1=\bar{\pi}$, and when they leave their price unchanged, $\pi_1=1$, is defined as a function of n and A,

$$u(A, n) \equiv \tilde{\Pi}\left(\bar{\pi}, \pi_1(n); A\right)-\tilde{\Pi}\left(1, \pi_1(n); A\right),$$

where $\tilde{\Pi}(\cdot)$ is the profit function given in (14). In other words, given the values of n and A, increasing the price, $\pi_{i,1}=\bar{\pi}$ (keeping the price unchanged, $\pi_{i,1}=1$), is the optimal choice for a firm, if and only if $u(A, n)>0$ ($u(A, n)<0$).

To ensure that the model has reasonable properties, the following assumption is made.

Assumption 1. *The parameter values satisfy the following two conditions:*

$$\left(1+\frac{2-\phi}{\theta-\phi}\right)\left(\frac{\pi_1}{\bar{\pi}}\right)^{2-\phi}>\frac{\bar{\pi}^{\theta-1}-1}{\bar{\pi}^{\theta}-1} \frac{\theta}{\theta-1} \quad \textit{for all } \pi_1 \in[1, \bar{\pi}], \tag{15}$$

$$A>\frac{\kappa \bar{\pi}^{\phi-1}}{2}. \tag{16}$$

Condition (15) implies that the reaction of nominal interest rates to inflation, ϕ, under the Taylor rule (13) should not be too strong. For example, when $\bar{\pi}=1.02$ and $\theta=6$, the reaction parameter in the Taylor rule should satisfy $\phi<2.0$, which is consistent with the conventional values in the literature. Further, condition (16) implies that the expected growth rate should be higher

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than a certain value, which depends on the target inflation rate. This condition is trivially satisfied in the real economy because when $\bar{\pi} = 1.02$, $\kappa = 1.0$, and $\phi = 1.5$, this condition requires $A > 0.505$, which means that the expected productivity growth rate should be above −49.5%. When these conditions are satisfied, the following two lemmas are obtained:

Lemma 1. *Assume that condition (15) is satisfied. Then,* $u(A, n)$ *increases with respect to* A *and* n.

The proof is provided in the Appendix. Intuitively, $u(A, n)$ increases with respect to A for the following two reasons. First, a higher A raises the neutral interest rate $A\bar{\pi}$ in (13) and consequently lowers the probability of reaching the ELB. Second, a higher A mitigates the decline in consumption c_1 at the ELB because the Euler equation (6) indicates that c_1 is almost proportional to c_2 when R_1 is fixed at κ. In other words, a higher A mitigates the expansion of the real interest rate gap, $R_1/\pi_2 - A$ at the ELB. On the other hand, $u(A, n)$ increases with respect to n because of strategic complementarity among firms in price competition: If more firms choose to raise prices and, consequently, inflation π_1 increases, firms have a greater incentive to raise their prices in order to avoid a change in relative prices. In monopolistic competition, since the firm's optimal pricing entails setting its price relative to others to the real marginal cost (i.e. real wages) plus a fixed mark-up, firms try to avoid undesirable demand changes due to a change in relative prices caused by aggregate inflation. Note, however, that in a general equilibrium setting, the marginal cost also changes in response to aggregate inflation because of the central bank's policy response. For instance, when the aggregate inflation rate declines, the central bank lowers the interest rate to boost consumption, thus leading to higher real wages and disincentivizing firms to lower the price of their products following the decline in aggregate inflation. This implies that strategic complementarity exists only if the central bank's response to inflation is moderate, which condition (15) requires.[14]

Lemma 2. *There are* $\bar{A}$ *and* $\underline{A}$ *satisfying*

$$u(A, n) > 0 \text{ for all } n \in [0, 1], \text{ and } A \geq \bar{A},$$
$$u(A, n) < 0 \text{ for all } n \in [0, 1], \text{ and } A \leq \underline{A}.$$

The proof is provided in the Appendix. This lemma implies that if expected productivity growth A is common knowledge and higher (lower) than a certain value, choosing $\pi_{i,1} = \bar{\pi}$ ($\pi_{i,1} = 1$) is the dominant strategy for firm i. Economic intuition for this lemma is as follows. For the upper bound $\bar{A}$, it can be shown that the ELB does not bind for all $n \in [0, 1]$ when A is higher than a certain value. Therefore, as long as condition (15) ensures that the reaction of nominal interest rates to inflation, ϕ, is not too small, the central bank can appropriately adjust R_1. Thus, firms have an incentive to choose $\pi_{i,1} = \bar{\pi}$ rather than $\pi_{i,1} = 1$, even when n is small or π_1 is low. For the lower bound $\underline{A}$, on the other hand, it can be shown that the ELB always binds to $n \in [0, 1]$ when A is lower than a certain value. Since the Euler equation (6) implies that consumption c_1 at the ELB is proportional to A, firms eventually lose the incentive to raise prices as A becomes increasingly lower.

Given these properties of $u(A, n)$ in Lemmas 1 and 2, the following theorem is obtained in line with the global games literature.

Theorem 2. *Let* A^* *be expected growth* A *solving the equation* $\int_0^1 u(A, n), dn = 0$. *Assume that (i) conditions (15) and (16) are satisfied, and that (ii)* $\underline{B} < \underline{A} - 2\varepsilon$ *and* $\bar{B} > \bar{A} + 2\varepsilon$, *are satisfied. Then, as* $\varepsilon \to 0$, *the threshold strategy,* $\pi_{i,1}(a_i) = \bar{\pi}$ *for all* $a_i > A^*$ *and* $\pi_{i,1}(a_i) = 1$ *for all* $a_i < A^*$, *forms a unique symmetric Bayesian Nash equilibrium.*

The proof is provided in the Appendix. Theorem 1 argues that equilibrium selection takes place based on productivity growth A, and the threshold is determined as the productivity growth rate at which the expected net profit from raising prices is zero with a uniform prior for n. Therefore,

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Table 1. Calibration values

Parameter	Value or target
Monetary policy rule, ϕ	1.5
Elasticity of substitution, θ	6.0
Target inflation, $\bar{\pi}$	1.02
Effective lower bound, κ	1.0
Labor disutility, ψ	$l_2 = 0.33$

this theorem implies that low economic growth leads to low inflation in the long run. While there is some empirical evidence that supports this theorem relating long-run inflation to the economic growth rate, and although monetary policymakers, as mentioned in the Introduction, often make this link, it is not commonly encountered in the monetary economics literature. For instance, in standard new Keynesian economics, long-run inflation has nothing to do with the expected growth rate and is determined only by the target inflation rate set by the central bank. Against this background, the theorem presented here can be interpreted as providing one potential mechanism underlying policymakers' intuition, as well as empirical evidence of the long-run relationship between economic growth and inflation.

4.2 Quantitative Analysis

Given equilibrium selection according to the threshold of expected productivity growth A^* established in Theorem 1, this subsection presents some quantitative analyses and policy experiments regarding the relationship between inflation and monetary policy. In the following, the model parameters are calibrated, and then the threshold A^* is computed by solving the model numerically.

The calibration values are presented in Table 1. The responsiveness of nominal interest rates to inflation, ϕ, and the elasticity of substitution, θ, are set to $\phi = 1.5$ and $\theta = 6.0$, which satisfy condition (15). The target inflation rate $\bar{\pi}$ and the value of ELB κ are set to $\bar{\pi} = 1.02$ and $\kappa = 1.0$. These policy parameters will be changed later in a policy experiment to investigate the effect of policy changes on the threshold A^*. Finally, the parameter for labor disutility, ψ, is chosen so that the labor supply satisfies $l_2 = 0.33$. Under these calibrated parameter values, the threshold value of expected productivity growth A^* is computed by numerically solving $\int_0^1 u(A, n)\, dn = 0$ for A.

Figure 3 shows $u(A, n)$ (the difference in firms' profits when they raise their prices versus when they leave prices unchanged) with respect to n (the fraction of firms that choose $\pi_{i,1} = \bar{\pi}$) under three different values of A ($A = 1.00, 0.993$, and 0.985). There are several notable features in the figure. First, there is a kink in $u(A, n)$, and the slope becomes considerably steeper when n is smaller than the kink point. As in the economy without uncertainty, shown in Figure 2, the kink is caused by the existence of the ELB; because the central bank cannot address low inflation by lowering the nominal interest rate at the ELB, firms lose the incentive to choose $\pi_{i,1} = \bar{\pi}$ when fewer firms raise their price (i.e. when n decreases). Second, as suggested by Lemma 1, Figure 3 shows that $u(A, n)$ increases with respect to both A and n. However, note that $u(A, n)$ is constant with respect to A if the economy is not constrained by the ELB (i.e. if n is larger than the point of kink). Therefore, the value of A does not influence firms' pricing strategies if the economy is not constrained by the ELB. Given that $u(A, n) > 0$ for all $A > \bar{A}$, as suggested by Lemma 3, the figure indicates that the economy will always be in inflationary equilibrium, $\pi_1 = \bar{\pi}$, if A is so high that the economy will never be constrained by the ELB. In other words, the possibility of reaching the ELB induces the problem of equilibrium selection between the inflation and zero-inflation equilibria.

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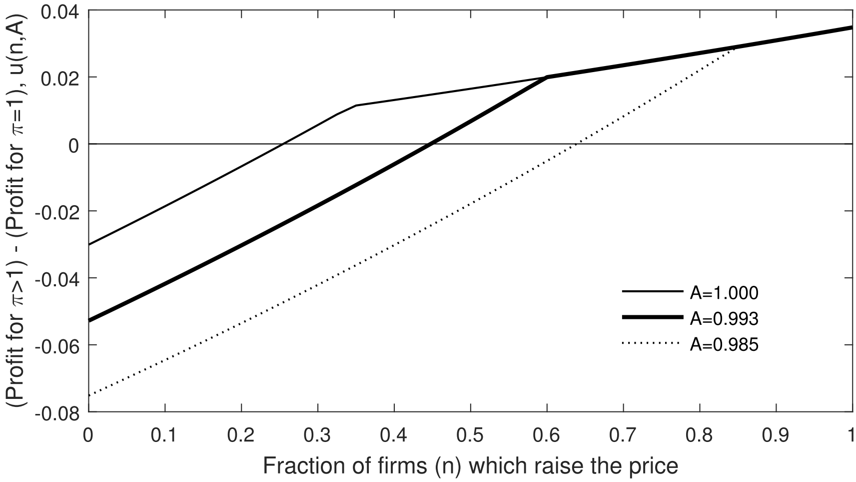

Figure 3. Net profit when firms raise their prices.

Figure 3 graphically shows how $\int_0^1 u(A, n)\, dn = 0$ can be solved for A. In the figure, $\int_0^1 u(A, n)\,, dn = 0$ indicates that the size of the area above zero is equal to that below zero. Therefore, when $A = 0.993$ (represented by the thick bold line), then $u(A, n)$ satisfies $\int_0^1 u(A, n)\, dn = 0$, but when $A > 0.993$, $(A < 0.993)$, then $\int_0^1 u(A, n)\, dn > 0$ $\left(\int_0^1 u(A, n)\, dn < 0\right)$, meaning that $A = 0.993$ is the threshold for equilibrium selection between the inflation and zero-inflation equilibria. That is, based on the baseline calibration values, if the expected productivity growth is higher (lower) than -0.7%, the economy is in inflation (zero-inflation)equilibrium.

4.3 Policy Experiment

This subsection investigates the effects of policy changes on the productivity growth threshold for equilibrium selection between inflation equilibrium and zero-inflation equilibrium. For this purpose, several values for (i) the target inflation rate, $\bar{\pi}$, (ii) the ELB, κ, and (iii) the response to inflation under the Taylor rule, ϕ, are chosen, and the effects of changes in the values on the threshold of equilibrium selection, A^*, are then examined using comparative statics.

First, the left panel of Figure 4 shows the results of the policy experiments for the target inflation rate, $\bar{\pi}$. In the figure, the horizontal axis represents the value of $\bar{\pi}$, while the vertical axis represents the productivity growth threshold, A^*. The figure indicates that the threshold productivity A^* is a decreasing function of target inflation $\bar{\pi}$, which implies that the central bank can prevent the economy from moving into zero-inflation equilibrium, even in the face of low expected productivity growth, by raising the target inflation rate. For instance, the figure indicates that the central bank can reduce the productivity growth threshold from -0.7% to -1.6% by raising the target inflation rate from 2% to 4%. Since inflation expectations in this model are assumed to be formed in a purely backward-looking manner, the rise in target inflation provides a stimulus to the economy, not by directly lowering the real interest rate, but rather by giving rise to the expectation of an accommodative monetary policy stance. For instance, because the increase in $\bar{\pi}$ from 1% to 2% implies that the central bank will continue monetary easing even when $\pi_1 > 1\%$, the increase in $\bar{\pi}$ raises c_1 and π_1 by changing private agents' expectations of monetary policy behavior. On the other hand, in any forward-looking model, raising the target inflation rate would have much larger effects to prevent the economy from moving into zero-inflation equilibrium because it directly raises

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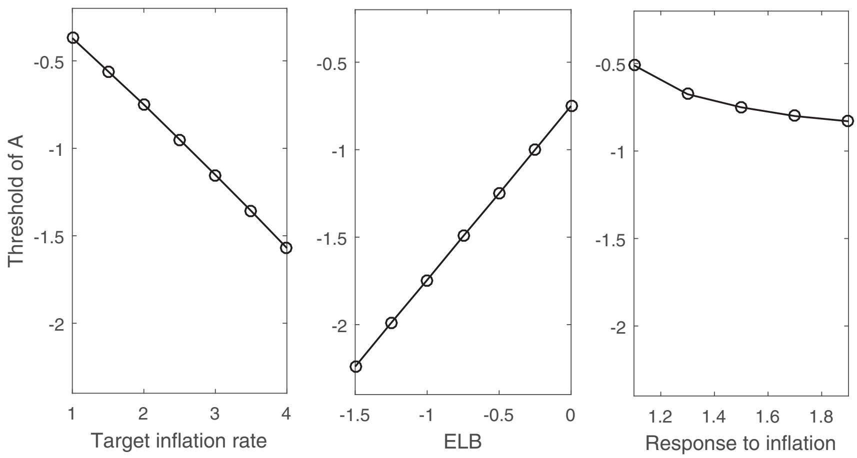

Figure 4. Results of policy experiments.
Note: The left, middle, and right panels show the results of policy experiments for the target inflation rate, $\bar{\pi}$, the value of the ELB, κ, and the response to inflation under the Taylor rule, ϕ. In the figure, the horizontal axis represents the value of $\bar{\pi}$, κ, or ϕ, while the vertical axis represents the productivity growth threshold, A^*.

inflation expectations. In this sense, assuming naive backward-looking inflation expectations is a conservative setting in terms of the policy effects of raising the target inflation rate.

Next, the middle panel of Figure 4 shows the results of policy experiments with respect to the value of the ELB, κ. This policy experiment is motivated by the fact that some central banks have lowered the ELB by introducing a negative interest rate policy to deal with a low-inflation environment. In the figure, the horizontal axis depicts the value of κ, while the vertical axis represents the productivity growth threshold, A^*. The figure indicates that threshold productivity A^* is an increasing function with respect to the ELB value. As the value of the ELB is mainly determined by cash storage costs, it is basically an exogenous constraint for the central bank. Nevertheless, at the same time, the value of the ELB is also the central bank's choice to some extent. For instance, since the European Central Bank and the Bank of Japan have adopted a negative interest rate policy, people recognize that the value of the ELB for those central banks is negative. On the other hand, since the Federal Reserve Bank has not adopted a negative interest rate policy, people recognize that they have set their ELB to zero even though they can set negative interest rates. In that sense, this result also implies that while the central bank is not free to choose the value of the ELB, it can reduce the productivity growth threshold by lowering the value of the ELB to some extent. For instance, the figure indicates that the expected growth threshold decreases from −0.7% to −1.3% if the central bank reduces the value of the ELB from 0.0% to −0.5% by adopting a negative interest rate policy. The intuition behind this policy effect is simple: Under a lower ELB, the probability of reaching the ELB is reduced, and the increase in the real interest gap at the ELB is mitigated. The result that the negative interest rate policy may help to raise long-run inflation stands in contrast with previous studies results. For example, as mentioned earlier, Benhabib et al. (2002) constructed a model with multiple equilibria at different levels of inflation, but because their model does not incorporate any equilibrium selection mechanism, it does not provide any theoretical support for the claim that lowering the ELB through a negative interest rate policy can help rescue the economy from the equilibrium dynamics around a deflationary steady state.[15]

Finally, the right panel of Figure 4 shows the results of the policy experiments for the response to inflation under the Taylor rule, ϕ. In the figure, the horizontal axis represents the value of ϕ, while the vertical axis represents the productivity growth threshold, A^*. The figure indicates that the threshold productivity A^* is a decreasing function of the response to inflation ϕ, which implies that, while the policy effect is somewhat smaller than that of the other two policies, the central

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bank can prevent the economy from moving into zero-inflation equilibrium, even in the face of low expected productivity growth, by responding to inflation more aggressively. For instance, the figure indicates that the central bank can reduce the productivity growth threshold from −0.7% to −0.5% by raising the response to the inflation coefficient from 1.5 to 1.9. The intuition behind this policy effect is simple: When the central bank more aggressively lowers the interest rate in response to a decline in the inflation rate, it boosts consumption and disincentivizes firms to lower their prices following the decline in aggregate inflation, thus preventing the economy from moving into zero-inflation equilibrium in aggregate.

In summary, the model presented here implies that equilibrium selection between inflation equilibrium and zero-inflation equilibrium takes place depending on the expected productivity growth rate and that the central bank can decrease the productivity growth threshold by (i) raising target inflation, (ii) adopting a negative interest rate policy, and (iii) responding to deflation more aggressively, consequently preventing the economy from moving into zero-inflation equilibrium, even in the face of low productivity growth. Therefore, the results of the policy analysis in this subsection imply that if the prolonged low inflation recently observed in developed countries corresponds to zero-inflation equilibrium in the model, monetary policy measures such as increasing target inflation, introducing a negative interest rate policy, or adopting a more aggressive monetary policy rule against deflation can help end such prolonged low inflation.

5. Concluding Remarks

This study presented a simple two-period general equilibrium model in which price competition among firms is described as a coordination game. While the model has multiple Nash equilibria with different levels of inflation due to the existence of the ELB and menu costs, equilibrium selection based on global games implies that if there is a small amount of uncertainty with regard to economic fundamentals, equilibrium selection between inflation equilibrium and zero-inflation equilibrium take place depending on the expected productivity growth rate. This result contrasts with the findings of previous studies in monetary economics literature. Finally, the quantitative policy analysis indicated that monetary policy measures, such as an increase in the target inflation rate and the introduction of a negative interest rate policy, can prevent the economy from moving into zero-inflation equilibrium. This result suggests that such policies may help end the prolonged period of low inflation recently observed in developed countries.

A number of avenues for future research remain. First, while this study focused on a simple two-period model for a better understanding of the theoretical implications, it should be extended to an infinite horizon model with rational expectations in order to obtain more realistic policy implications. Second, another important step is the empirical examination of the results obtained in this study. In particular, whether changes in the target rate of inflation observed in some countries have been effective is an important empirical issue that could help to test the theoretical implications of the model developed here. These interesting and important questions remain for future research.

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Notes

1 Specifically, most studies based on a monetary DSGE model construct a model as if the ELB does not exist in the economy and focus only on equilibrium dynamics around the steady state with a positive target inflation rate. This approach has been pervasive in this literature, partly because the ELB was not an urgent monetary policy issue in most countries until the global financial crisis.

2 See, for example, Dufrenot and Khayat (2017), Aruoba et al. (2018), and Hirose (2020) for empirical investigations, and Bullard (2010) and Nakaso (2014) for policy implications without a theoretical model.
3 For instance, Kuroda (2016) argues that "[t]he [second] major challenge that made it more difficult to overcome deflation is the decline in the natural rate of interest reflecting a deceleration in Japan's growth potential. [....] Given these challenges, Japan was unable to find an appropriate cure for the chronic disease of prolonged deflation. This is how a deflationary equilibrium took hold."
4 For more details about recent applications to macroeconomic issues, see Angeletos and Lian (2016).
5 While inflation expectations in an infinite horizon model are not fixed at the target level, as in the two-period model, but will converge to it, the policy implication is almost the same. For example, Iacoviello and Michelis (2016) use a canonical forward-looking model and argue that the Bank of Japan could effectively increase inflation in Japan by raising the target inflation rate.
6 As Cochrane (2011, 2017) point out, a particular path of inflation expectations is selected in a forward-looking new Keynesian model by "trimming" other paths. Hence, the mechanism underlying inflation expectations in a forward-looking rational expectations model is still an important but challenging and controversial monetary economics issue.
7 To simplify the analysis, we assume that there is no discount factor for future utility. While this assumption influences the quantitative result in Section 4, it is not critical to the main results of this study.
8 This assumption implies that there is no price dispersion in Period 0. This is consistent with the assumption of no heterogeneity across firms in this model.
9 The mechanism that high expected productivity growth leads to an incentive for raising current prices is similar to the mechanism described in Fernandez-Villaverde et al. (2014).
10 This example means that the expected productivity growth rate is 0.2%, and the target inflation rate is 2.0%. The other parameters are set to conventional values, which are shown in Table 1.
11 Strictly speaking, restricting the action space must, of course, induce a loss of generality to some extent, but since global games with a continuum of actions are theoretically less developed for use in applied work, a binary price set is a good first step to applying global games to price competition among monopolistically competitive firms.
12 Given that forecasting expected productivity growth is critical for businesses but always involves considerable uncertainty, introducing a small amount of noise in expected productivity growth is a common approach in the literature when applying global games to a macroeconomic analysis. See, for example, Guimaraes et al. (2016) and Taschereau-Dumouchel and Schaal (2018).
13 While we assume a uniform distribution for simplicity, assuming more general distributions, including a normal distribution, does not change the main results. For more details about the distributional assumption, see Morris and Shin (2003).
14 The degree of strategic complementarity also depends on the responsiveness of real wages to aggregate demand. For instance, if labor disutility is formulated by a power function $\psi L_t^{1+\nu}/(1+\nu)$ rather than a linear function ψL_t and set to $\nu > 0$, the degree of strategic complementarity becomes weaker because real wages are more responsive to aggregate demand in the labor supply function (7). Specifically, when $\nu = 1$, Lemma 1 requires the reaction parameter in the Taylor rule to be $\phi < 1.53$ rather than $\phi < 2.0$ under $\bar{\pi} = 1.02$ and $\theta = 6$, implying that the condition is more restrictive than condition (15).
15 Note, however, that this quantitative exercise does not take into account any side effects of the negative interest rate policy, including negative effects on financial intermediaries.

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Appendix

This Appendix provides the proofs for Lemma 1, Lemma 2, and Theorem 1.

Proof of Lemma 1

Firm profit $\tilde{\Pi}\left(\pi_{i,1}, \pi_1(n); A\right)$ can be written as follows:

$$\tilde{\Pi}\left(\pi_{i,1}, \pi_1; A\right) = \begin{cases} \frac{\theta-1}{\theta\psi}\pi_{i,1}^{1-\theta}\bar{\pi}^{\phi-1}\pi_1^{\theta-\phi} - \frac{1}{\psi}\left(\frac{\theta-1}{\theta}\right)^2 \pi_{i,1}^{-\theta}\bar{\pi}^{2(\phi-1)}\pi_1^{\theta+2-2\phi} & \text{if } A\bar{\pi}\left(\frac{\pi_1}{\bar{\pi}}\right)^{\phi} > \kappa \\ \frac{\theta-1}{\kappa\theta\psi}A\pi_{i,1}^{1-\theta}\pi_1^{\theta} - \frac{1}{\psi}\left(\frac{\theta-1}{\kappa\theta}A\right)^2 \pi_{i,1}^{-\theta}\pi_1^{\theta+2} & \text{otherwise} \end{cases}.$$

The first case represents firms' profits when the ELB does not bind, while the second case represents the profit when the ELB binds. Given that (i) $\pi_1(n)$ is monotonically increasing with respect to n and (ii) the profit function is continuous with respect to π_1 and A, we need to show that $u(A, \pi_1) \equiv \tilde{\Pi}(\bar{\pi}, \pi_1; A) - \tilde{\Pi}(1, \pi_1; A)$ increases with respect to π_1 and A in both the non-ELB states and at the ELB.

First, it is shown that $\partial u/\partial \pi_1 \geq 0$. When $A\bar{\pi}(\pi_1/\bar{\pi})^{\phi} > \kappa$,

$$\begin{aligned} \frac{\partial u}{\partial \pi_1} > 0 \Leftrightarrow & \left[\frac{(\theta-1)(\theta-\phi)}{\theta\psi}\bar{\pi}^{\phi-\theta}\pi_1^{\theta-\phi-1} - \frac{1}{\psi}\left(\frac{\theta-1}{\theta}\right)^2(\theta+2-2\phi)\,\bar{\pi}^{2(\phi-1)-\theta}\pi_1^{\theta+1-2\phi}\right] \\ & -\left[\frac{(\theta-1)(\theta-\phi)}{.}\theta\psi\bar{\pi}^{\phi-1}\pi_1^{\theta-\phi-1} - \frac{1}{\psi}\left(\frac{\theta-1}{\theta}\right)^2(\theta+2-2\phi)\,\bar{\pi}^{2(\phi-1)}\pi_1^{\theta+1-2\phi}\right] > 0 \\ \Leftrightarrow & \left(1 + \frac{2-\phi}{\theta-\phi}\right)\left(\frac{\pi_1}{\bar{\pi}}\right)^{2-\phi} > \frac{\bar{\pi}^{\theta-1}-1}{\bar{\pi}^{\theta}-1}\,\frac{\theta}{\theta-1}. \end{aligned}$$

The last inequality is satisfied under condition (15). Furthermore, when $A\bar{\pi}(\pi_1/\bar{\pi})^{\phi} \leq \kappa$,

$$\begin{aligned} \frac{\partial u}{\partial \pi_1} > 0 \Leftrightarrow & \left[\frac{\theta-1}{\kappa\psi}A\bar{\pi}^{1-\theta}\pi_1^{\theta-1} - \frac{1}{\psi}\left(\frac{\theta-1}{\kappa\theta}A\right)^2(\theta+2)\,\bar{\pi}^{-\theta}\pi_1^{\theta+1}\right] \\ & -\left[\frac{\theta-1}{\kappa\theta\psi}A\pi_1^{\theta-1} - \frac{1}{\psi}\left(\frac{\theta-1}{\kappa\theta}A\right)^2(\theta+2)\,\pi_1^{\theta+1}\right] > 0 \\ \Leftrightarrow & \frac{A(\theta+2)}{\theta\bar{\pi}}(\pi_1)^2 > \kappa\,\frac{\bar{\pi}^{\theta-1}-1}{\bar{\pi}^{\theta}-1}\,\frac{\theta}{\theta-1} \\ \Leftarrow & A > \frac{\theta\kappa\bar{\pi}^{\phi-1}}{\theta+2}. \end{aligned}$$

The last inequality is satisfied under condition (16).

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Next, we show that $\partial u/\partial A \geq 0$. When $A\bar{\pi}\,(\pi_1/\bar{\pi})^{\phi} > \kappa$, it is clear that $\partial u/\partial A = 0$ because $u(\cdot)$ is independent of A. Moreover, when $A\bar{\pi}\,(\pi_1/\bar{\pi})^{\phi} \leq \kappa$,

$$\frac{\partial u}{\partial A} > 0 \Leftrightarrow \left[\frac{\theta-1}{\kappa\theta\psi}\bar{\pi}^{1-\theta}\pi_1^{\theta} - \frac{2A}{\psi}\left(\frac{\theta-1}{\kappa\theta}\right)^2\bar{\pi}^{-\theta}\pi_1^{\theta+2}\right] - \left[\frac{\theta-1}{\kappa\theta\psi}\pi_1^{\theta} - \frac{2A}{\psi}\left(\frac{\theta-1}{\kappa\theta}\right)^2\pi_1^{\theta+2}\right] > 0$$
$$\Leftrightarrow \frac{2A}{\bar{\pi}}(\pi_1)^2 > \kappa\frac{\bar{\pi}^{\theta-1}-1}{\bar{\pi}^{\theta}-1}\frac{\theta}{\theta-1}$$
$$\Leftarrow A > \frac{\kappa\bar{\pi}^{\phi-1}}{2}.$$

Thus, condition (16) implies that the last inequality is satisfied.

Proof of Lemma 2

First, we show that there exists an $\underline{A}$ satisfying $u(A,\pi_1) < 0$ for all $\pi_1 \in [1,\bar{\pi}]$ and $A \leq \underline{A}$. Note that when $A < \kappa/\bar{\pi}$, ELB binds for all $\pi_1 \in [1,\bar{\pi}]$. In addition, because we know that $u(A,\pi_1)$ is increasing with respect to π_1, we need to show that there is some $\underline{A}$ satisfying $u(A,\bar{\pi}) < 0$ for all $A \leq \underline{A} \leq \kappa/\bar{\pi}$ under $R_1 = \kappa$. Inserting $R_1 = \kappa$ and deleting c_1 using the Euler equation yields:

$$u(A,\bar{\pi}) < 0 \Leftrightarrow \left[\frac{\theta-1}{\kappa\theta\psi}A\bar{\pi} - \frac{1}{\psi}\left(\frac{\theta-1}{\kappa\theta}A\right)^2\bar{\pi}^2\right] - \left[\frac{\theta-1}{\kappa\theta\psi}A\bar{\pi}^{\theta} - \frac{1}{\psi}\left(\frac{\theta-1}{\kappa\theta}A\right)^2\bar{\pi}^{\theta+2}\right] < 0$$
$$\Leftrightarrow \frac{A\bar{\pi}}{\kappa} < \frac{\bar{\pi}^{\theta-1}-1}{\bar{\pi}^{\theta}-1}\frac{\theta}{\theta-1}$$
$$\Leftarrow A < \frac{\kappa}{\bar{\pi}}.$$

The last line comes from condition (15). Thus, by setting $\underline{A} = \kappa/\bar{\pi}$, the proof for the first half of Lemma 2 is complete. Next, we show that there exists a $\bar{A}$ satisfying $u(A,\pi_1) > 0$ for all $\pi_1 \in [1,\bar{\pi}]$ and $A \geq \bar{A}$. Note that when $A > \kappa\bar{\pi}^{\phi-1}$, the ELB does not bind to all $\pi_1 \in [1,\bar{\pi}]$, and as a result, $u(A,\pi_1)$ is independent of A. In addition, because we know that $u(A,\pi_1)$ is increasing with respect to π_1, we must show that $u(A,1) > 0$ for all $A \geq \kappa\bar{\pi}^{\phi-1}$ under $R_1 = A\bar{\pi}\,(\pi_1/\bar{\pi})^{\phi}$. Inserting $R_1 = A\bar{\pi}\,(\pi_1/\bar{\pi})^{\phi}$ and deleting c_1 using the Euler equation yields:

$$u(A,1) > 0 \Leftrightarrow \left[\frac{\theta-1}{\theta\psi}\bar{\pi}^{\phi-\theta} - \frac{1}{\psi}\left(\frac{\theta-1}{\theta}\right)^2\bar{\pi}^{2(\phi-1)-\theta}\right] - \left[\frac{\theta-1}{\theta\psi}\bar{\pi}^{\phi-1} - \frac{1}{\psi}\left(\frac{\theta-1}{\theta}\right)^2\bar{\pi}^{2(\phi-1)}\right] > 0$$
$$\Leftrightarrow \bar{\pi}^{\phi-2} - \frac{\bar{\pi}^{\theta-1}-1}{\bar{\pi}^{\theta}-1}\frac{\theta}{\theta-1} > 0.$$

Thus, by setting $\bar{A} = \kappa\bar{\pi}^{\phi-1}$, the proof for the second half of Lemma 2 is complete.

Proof of Theorem 1

Some useful notations are defined before going into the proof. The threshold strategy is denoted by $\tilde{\pi}_{i,1}(a_i;A')$, where A' represents the threshold. The threshold strategy satisfies

$$\tilde{\pi}_{i,1}(a_i;A') = \begin{cases} \bar{\pi} & \text{for all } a_i \geq A' \\ 1 & \text{for all } a_i < A' \end{cases}. \tag{17}$$

We assume that all other firms adopt the threshold strategy $\tilde{\pi}_{i,1}(a_i;A')$. Then, the following two functions are defined: First, with the expected growth rate A, the fraction of firms that choose $\pi_{i,1} = \bar{\pi}$ is defined as $n(A,A') \in [0,1]$. Specifically, when $\underline{B} + \varepsilon \leq a_i \leq \bar{B} - \varepsilon$, the value of $n(A,A')$ becomes

$$n(A,A') = \begin{cases} 0 & \text{if } A < A' - \varepsilon \\ 0.5 + (A - A')/(2\varepsilon) & \text{if } A \in [A'-\varepsilon, A'+\varepsilon], \\ 1 & \text{if } A > A' + \varepsilon \end{cases} \tag{18}$$

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because the signal follows the uniform distribution $U[A-\varepsilon andA+\varepsilon]$. Second, with signal a_i, the expected net gain from raising prices is defined as $\Gamma\left(a_i, A'\right)$. When signal a_i satisfies $a_i \in \left[\underline{B}+\varepsilon, \bar{B}-\varepsilon\right]$, the firm subjectively expects that the true value of A follows a uniform distribution $U[a_i-\varepsilon, a_i+\varepsilon]$. Therefore, $\Gamma\left(a_i, A'\right)$ becomes

$$\Gamma\left(a_i, A'\right) = \frac{1}{2\varepsilon}\int_{a_i-\varepsilon}^{a_i+\varepsilon} u\left(A, n\left(A, A'\right)\right) dA,$$

where $u(\cdot)$ is given in the main text. Given these functions, the following lemma is obtained:

Lemma 3. *There is a unique A^* solving the equation $\Gamma\left(A^*, A^*\right)=0$.*

Proof. Given the signal $a_i \in \left[\underline{B}+\varepsilon, \bar{B}-\varepsilon\right]$, (18) indicates that the subjective distribution for $n\left(A, A'+\Delta A\right)$ in $A \in [a_i+\Delta A-\varepsilon, a_i+\Delta A+\varepsilon]$ is the same as that for $n\left(A, A'\right)$ in $A \in [a_i-\varepsilon, a_i+\varepsilon]$ for a small ΔA. Therefore, because $u(A, n)$ increases with respect to A, as shown in Lemma 1, $\Gamma(A, A)$ increases with respect to $A \in \left[\underline{B}+\varepsilon, \bar{B}-\varepsilon\right]$. Furthermore, we have

$$\begin{cases} \Gamma\left(a_i, A'\right) > 0 \text{ for all } a_i \geq \bar{A}+\varepsilon \text{ and } A' \\ \Gamma\left(a_i, A'\right) < 0 \text{ for all } a_i \leq \underline{A}-\varepsilon \text{ and } A', \end{cases} \tag{19}$$

because of Lemma 2 and the assumptions that $\underline{B} < \underline{A}-2\varepsilon$ and $\bar{B} > \bar{A}+2\varepsilon$, meaning that there is a unique A^* that solves the equation $\Gamma\left(A^*, A^*\right)=0$. □

The remainder of the proof consists of three steps. The first step is to show that the threshold strategy $\tilde{\pi}_{i,1}\left(a_i; A^*\right)$ forms a symmetric Bayesian Nash equilibrium. When other firms follow $\tilde{\pi}_{i,1}\left(a_i; A'\right)$, the optimal choice for firm i with signal a_i is $\pi_{i,1}\left(a_i\right)=\bar{\pi}$ if $\Gamma\left(a_i, A'\right)>0$ and $\pi_{i,1}\left(a_i\right)=1$ if $\Gamma\left(a_i, A'\right)<0$. Therefore, $\tilde{\pi}_{i,1}\left(a_i; A^*\right)$ forms a symmetric Bayesian Nash equilibrium if and only if

$$\begin{aligned} &\Gamma\left(a_i, A^*\right) < 0 \text{ for all } a_i < A^*, \\ &\Gamma\left(a_i, A^*\right) > 0 \text{ for all } a_i > A^*. \end{aligned} \tag{20}$$

Here, $\Gamma\left(a_i, A'\right)$ increases with respect to a_i because $u(A, n)$ increases with respect to A and n, and because $n\left(A, A'\right)$ increases with respect to A. Therefore, $\Gamma\left(A^*, A^*\right)=0$ immediately leads to (20), implying that $\tilde{\pi}_{i,1}\left(a_i; A^*\right)$ forms a symmetric Bayesian Nash equilibrium.

The second step of the proof shows the uniqueness of a symmetric Bayesian Nash equilibrium. In other words, it is shown that any strategy $\pi_{1,i}(\cdot)$ that forms a symmetric Bayesian Nash equilibrium must be the threshold strategy $\tilde{\pi}_{i,1}\left(a_i; A^*\right)$. To show the uniqueness, we first define $b\left(A'\right)$ as the unique a_i that solves the equation $\Gamma\left(a_i, A'\right)=0$. This is unique because (19) holds, and $\Gamma\left(a_i, A'\right)$ increases with respect to a_i. Then, we define $\tilde{b}^k\left(A'\right)$ as

$$\tilde{b}^k\left(A'\right) \equiv \begin{cases} A' & \text{if } k=0 \\ b\left(\tilde{b}^{k-1}\left(A'\right)\right) & \text{if } k \geq 1 \end{cases}.$$

Given the definition of $\tilde{b}^k\left(A'\right)$, the following lemma is posited:

Lemma 4. *Any strategy $\pi_{1,i}(\cdot)$ that forms a symmetric Bayesian Nash equilibrium satisfies*

$$\pi_{i,1}\left(a_i\right) = \begin{cases} \bar{\pi} \ \text{ if } a_i > \tilde{b}^{k-1}\left(\bar{A}+\varepsilon\right) \\ 1 \ \text{ if } a_i < \tilde{b}^{k-1}\left(\underline{A}-\varepsilon\right), \end{cases} \tag{21}$$

for all $k \geq 1$. Also, $\lim_{k\to\infty} \tilde{b}^{k-1}\left(\bar{A}+\varepsilon\right) = \lim_{k\to\infty} \tilde{b}^{k-1}\left(\underline{A}-\varepsilon\right) = A^*$.

Proof. The proof is based on the mathematical induction. For $k=1$, (21) obviously holds because of (19). Next, we assume that (21) holds for $k=m$. Thus, $\Gamma\left(a_i, \tilde{b}^{m-1}\left(\bar{A}+\varepsilon\right)\right)>0$ for all $a_i > \tilde{b}^m\left(\bar{A}+\varepsilon\right)$ because $\tilde{b}^m\left(\bar{A}+\varepsilon\right)$ solves $\Gamma\left(a_i, \tilde{b}^{m-1}\left(\bar{A}+\varepsilon\right)\right)=0$ for a_i and $\Gamma\left(a_i, \tilde{b}^{m-1}\left(\bar{A}+\varepsilon\right)\right)$ increases with respect to a_i. Therefore, any $\pi_{i,1}(\cdot)$ that forms a symmetric Bayesian Nash equilibrium must choose $\pi_{i,1}\left(a_i\right)=\bar{\pi}$ for all $a_i > \tilde{b}^m\left(\bar{A}+\varepsilon\right)$, which implies that the first line of (21) is satisfied for $k=m+1$. Similarly, it is shown that any $\pi_{i,1}\left(a_i\right)$ that forms a symmetric Bayesian Nash equilibrium must choose $\pi_{i,1}\left(a_i\right)=1$ for all $a_i < \tilde{b}^m\left(\underline{A}-\varepsilon\right)$, and the second line of (21) is satisfied for $k=m+1$. Thus, it can be shown that any strategy $\pi_{1,i}(\cdot)$ that forms a symmetric Bayesian Nash equilibrium satisfies (21) for all $k \geq 1$.

Next, we show that $\tilde{b}^{k-1}\left(\bar{A}+\varepsilon\right) \to A^*$ as $k \to \infty$. This statement is equivalent to showing that $A^* \leq b(x) < x$ for all $x > A^*$, given the definition of $\tilde{b}^k\left(A'\right)$. We have $A^* \leq b(x)$ for all $x > A^*$ because if this is not the case, $\Gamma\left(b(x), x\right)=0$ for some $b(x) < A^*$ and $x > A^*$, which contradicts Lemma 3. In addition, if $x > A^*$, we have $\Gamma(x, x) > 0$ because of Lemma 3. Thus, we have $b(x) < x$ because $b(x)$ solves $\Gamma\left(a_i, x\right)=0$ for a_i and $\Gamma\left(a_i, A'\right)$ increases with respect to a_i. Similarly, it can be shown that $\tilde{b}^{k-1}\left(\underline{A}-\varepsilon\right) \to A^*$ as $k \to \infty$. □

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At the limit, $k \to \infty$, this lemma implies that any strategy $\pi_{1,i}(\cdot)$ that forms a symmetric Bayesian Nash equilibrium must be the threshold strategy $\tilde{\pi}_{i,1}(a_i;A^*)$.

As the final step of the proof, we show that the threshold A^* can be computed by solving $\int_0^1 u(A^*, n)\, dn = 0$ when $\varepsilon \to 0$. Note that $\Gamma(A^*, A^*) = 0$ is equivalent to

$$\int_{A^*-\varepsilon}^{A^*+\varepsilon} u\left(A, n\left(A, A^*\right)\right) dA = 0.$$

By definition, $n(A, A^*) = 0.5 + (A - A^*), / (2\varepsilon)$ for $A \in [A^* - \varepsilon, A^* + \varepsilon]$. Thus, a change of variables yields

$$\int_0^1 u\left(A\left(n, A^*\right), n\right) dn = 0,$$

where $A(n, A^*) = 2\varepsilon n - \varepsilon + A^*$, which implies that when $\varepsilon \to 0$, $\Gamma(A^*, A^*) = 0$ is equivalent to $\int_0^1 u(A^*, n)\, dn = 0$.

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