Imagination and Imaging in Model Building

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Modelling became one of the primary tools of mathematical economic research in the twentieth century, but when we look at examples of how nonanalogical models were first built in economics, both the process of making representations and aspects of the representing relation remain opaque. Like early astronomers, economists have to imagine how the hidden parts of their world are arranged and to make images, that is, create models, to represent how they work. The case of the Edgeworth Box, a model widely used for theoretical work in twentieth-century economics, provides a good example to explore the process of making mathematical representations of the economy. It shows how, in making these new representations, conceptual elements were developed which could not have been represented in the older verbal forms of economics.

1. Introduction. Pioneers of mathematical economics in the late nineteenth century presented their mathematizing efforts as a matter of language change, a change that would be progressive for economics as a science because economic ideas expressed in mathematics would be expressed more exactly, and reasoned about more rigorously, than when expressed in words. To make a mathematical economics, economists needed not only a mathematical language, terms, and formulae, but also to imagine a mathematically described world within which their economic ideas could be expressed. Many of the early mathematical models came via analogies, and their provenance can be understood in the terms offered by Hesse (1966; for example, see Morgan 1997). But this was not true of

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all mathematical models, and for these others, it is difficult to describe them as being the result of processes of abstraction, simplification, and idealization, such as might be given in some philosophical accounts of model building in economics (see Hamminga and De Marchi 1994). Rather, it seems, economists relied on processes of imagination and image making in modelling aspects of the world that they did not fully understand and that they could neither observe directly nor access experimentally.

2. Making the Economic World in Mathematical Models. Nelson Goodman's *Ways of World Making* (1978) stresses how scientists and artists are involved in making sense of the world in similar kinds of ways. Both groups make versions of the world. I interpret this here as both groups make representations of the world.[1] Economists took their earlier verbally made economic world as a matter of habit. But, their verbal economy—the nouns, verbs, descriptive phrases, and relations between them that economists still use—grew up over the past centuries in such a way that their theories and descriptions of the economic world could be expressed within that domain, within that representation of the world. Creating a mathematical economics was, historically, a similar process to that of creating a verbal economics. It was a process in which economists came to think about the economic world in a mathematical way and represent it to themselves in mathematical form: a process of imagining and imaging, so that over the twentieth century the elements, their meanings, how they are symbolized, and what relations are assumed all came to be taken for granted. Economists had to make their mathematical version of the economic world, just as their forebears had made their verbal one.

Making a mathematical economic world was a tall order. Model building, I suggest, flowered in economics during the late-nineteenth-century period and throughout the twentieth-century process of mathematization for two reasons. One is that exactly those kinds of qualities needed in making a new version of the world are those found in model building, namely, the abilities to be imaginative about the world and to make images

1. Goodman is careful not to use the term representation so broadly; R. I. G. Hughes (1997) understands Goodman's term in the context of model making as "denotation." See Weintraub 1991 for an account of world making in economics in the Goodman tradition.

of it in mathematical forms.[2] Second, model building provided a way of generating the vocabulary and forms of the new way of thinking, and so the "working objects" (the tag comes from Daston and Galison 1992) on which the mathematical economic description could be refined and tested. Model building, in essence, involved new conceptual elements which could not be represented in the old forms. The nature and content of the new representations and the grammar they entailed changed the way economists pictured the economy and changed the picture that economists had of the world.

3. The History of the Edgeworth Box Diagram. To illustrate and make sense of these claims, I turn to the historical process by which a small mathematical diagrammatical model—the Edgeworth Box—was formed. This diagram played a ubiquitous role in theorizing up into the mid-twentieth century and is still used today. A recent history of the model by Tom Humphrey (1996) used modernized versions of the diagram to reconstruct the history of theorizing using the diagram.[3] Humphrey's history and set of images could only have been written by someone who already knew the Box, was familiar with what it could represent, and with how it could be used. My question is rather different: How did economists *first make* the model which became so well known? I seek to recreate the unfamiliarity erased by history, to see how that model world was first imagined and imaged by the founders of the Edgeworth Box.

3.1. Edgeworth's Image. Let us first dip into Francis Edgeworth's own account of how he made his economic world in mathematics. Edgeworth's

2. I want to avoid any account in which mathematization is seen either as a process of translation or as one of transcribing. Where "translation" underestimates the cognitive problem (see Latour 1986), "transcribing" makes too much of an ontological commitment: it suggests that the laws of economics are written in mathematics and economists merely had to figure out how to decipher their own Book of Nature. (This is associated with the "perfect model" model held by some philosophers of science—see Teller (2001) for a recent attack on this view.) Recognition of the presence and importance of model building suggests not difficulties of perception and transcription but rather ones of cognition and portrayal.

3. Humphrey's history focuses on the *usage* of the Box diagram, especially its tremendous versatility to deal with theoretical questions in various domains. It is a superb history for anyone who ever thought the Box diagram was for merely for illustration. His history effectively modernized along one dimension (the diagrams) in order to tell the history of another dimension (the theoretical results). We might both be interpreted as following Lakatos' (1976) example in *Proofs and Refutations*, with actual history below the line and reconstructed history above. We chose to reconstruct along different lines. (For more general accounts of how models are used in economics, see Morgan 2001, 2002, and 2003.)

version of the diagram was introduced in his now famous *Mathematical Psychics* ([1881] 2003), a book of almost impenetrable erudition from this Irish economist. For Edgeworth, mathematics was a form of expression, a language, and because of its special qualities it was a tool or instrument both for expression of economic ideas and for reasoning about them. But in Edgeworth's mind it was also an instrument of imagination.

Edgeworth's imagination about economic behavior is funded by a general analogy with mathematical physics though the analogical content gets less fundamental and more illustrative as he argues about two individuals and two goods to exchange, where parties are free to contract only by *mutual* consent and without competition from other traders.[4] He defines the locus of points at which exchange might be contracted as those where, whichever direction a move is made away from that set of points, one trader gets more and the other less utility. This set of points is termed the "contract curve."

He then set about demonstrating the qualities of his defined contract curve by a series of mixed mathematical and verbal reasoning, in terms both spatial and algebraic, partly written down and partly in the imagination, to assure himself that the characteristics of the "contract curve" are sensibly proved by several different approaches (see 20–28).[5] At a certain point in his mathematical discourse, Edgeworth moved into one of his imagined worlds, and his original version of the "Box" appeared (see Figure 1), encased in the following text:

> It is not necessary for the purpose of the present study to carry the analysis further. To gather up and fix our thoughts, let us <u>imagine a simple case</u>—Robinson Crusoe contracting with Friday. The *articles* of contract: wages to be given by the white, labour to be given by the black. Let Robinson Crusoe = X. <u>Represent</u> y, the labour given by Friday, by a horizontal [*sic*] line measured northward from an assumed point, and measure x, the remuneration given by Crusoe, from the same point along an *eastward* line (See accompanying figure 1). Then any point between these lines <u>represents</u> a contract. It will very generally be in the interest of both parties to vary the articles of any contract taken as random. But there is a class of contracts to the variation of which the consent of *both* parties cannot be obtained, of *settlements*. These settlements are <u>represented</u> by an *indefinite num-*

4. It should be stressed that the theory of exchange points in markets with many competing traders (the "perfect competition" solution) was already worked out by 1881. Edgeworth was attacking the more difficult cases of few traders and without competition.

5. Page numbers refer to the 1881 original.

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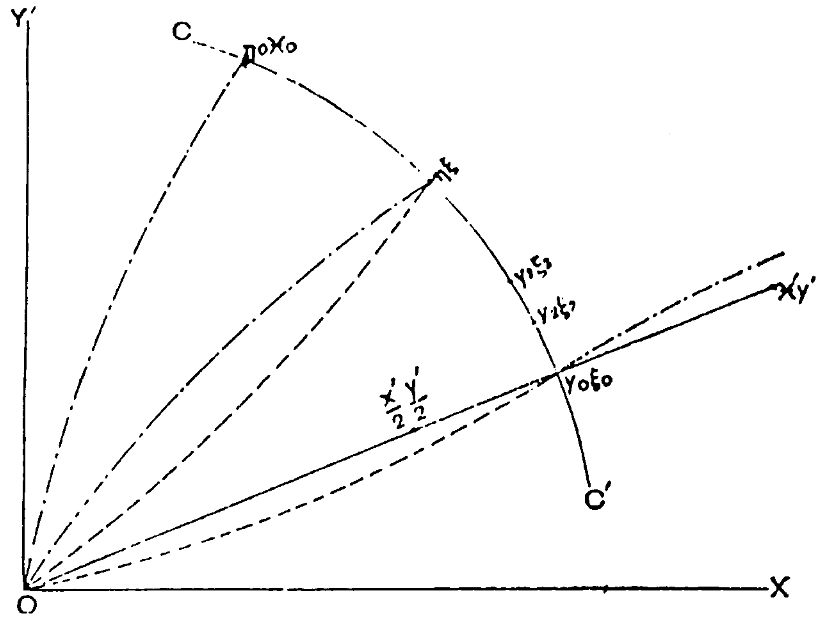

Figure 1. F. Y. Edgeworth's original diagram of 1881 (28).

> *ber* of points, a locus, the *contract-curve* CC′, or rather, a certain portion of it which may be supposed to be wholly in the space between our perpendicular lines in a direction trending from south-east to north-west. This available portion of the contract-curve lies between two points, say $\eta_0 x_0$ north-west, and $y_0 \xi_0$ south-east; which are respectively the intersections with the contract-curve of the *curves of indifference* for each party drawn through the origin. Thus the utility of the contract represented by $\eta_0 x_0$ is for Friday zero, or rather, the same as if there was no contract. As that point he would as soon be off with the bargain—work by himself perhaps. (Edgeworth [1881] 2003, 28–29; Edgeworth's italics, my underlining)

Thus Edgeworth imagines his Robinson Crusoe and Friday lining up at right angles to each other in the same plane, shoulder to shoulder as befits those who must mutually agree before exchange can take place. Edgeworth's (x,y) space is a plane, and the indifference curves are projections from three dimensional utility surfaces; thus he imagines and makes his image accordingly (and so he correctly writes that in Figure 1 we draw the Y axis horizontally northwards). The individuals (X and Y) are not fully and separately distinguished on the diagram from those things which they have to exchange (*x* and *y*).

It seems so natural to economists nowadays to represent the two goods along these two axes, but it was not so in the late-nineteenth century when economic diagrams were still in their infancy. Edgeworth probably began with Alfred Marshall's 1879 trade diagrams, which used this convention to show the trading relations between two countries, in which goods were represented on the two axes and the whole of the space between was open for trade. Edgeworth's diagram refers to individual traders alongside their goods, and provides an indifference curve for each individual and their contract curve. And while it seems initially that the whole space is open for trade as in Marshall, the argument defining the contract curve in conjunction with the indifference curves through the origin (i.e., points at which utility is equivalent to that obtained from zero exchange) rules out some areas of the ninety-degree total space. Edgeworth is so impressed by his own diagram and the way that it allows him to work out some results which had previously failed to yield to general analysis, that he writes that his figure "is proved to be a <u>correct representation</u>" and that the diagram provides "<u>an abstract typical representation</u>" of a process (Edgeworth [1881] 2003, 36; my underlining).

3.2. Turning Edgeworth's Diagram into a "Box." The further historical development of the Edgeworth Box enables us to explore some detailed questions about this process of model building and to open up questions about the representation. We start by asking: What does the Box represent nowadays, as seen in Humphrey's modernized version of Edgeworth's diagram (our Figure 2)? Two adjacent sides of the Box denote a fixed amount of the two goods or services or resources available, so that the Box represents a world with given and fixed resources. The two antagonistically placed origin points mark the direction of stance of the two traders (here A and B), each with their own two axes (the adjacent ones), on which their own shares of resources by endowment and by exchange can be marked. This is how the Box diagram looked by the 1950s, but this was not how Edgeworth imagined the economic world in 1881. The most striking thing is that his diagram is not a box at all. In his diagram, the world is represented differently—there each trader measures off his resources for exchange along one axis only; one trader X is trying to make a contract by trading his own *x* for some *y* offered by Y. Note also that with Edgeworth's original uncapped axes imported from Marshall's trading diagrams, there is no total amount or limit on the amount of resource that can be exchanged. What might now be taken as the irreducible shape of the Box—namely, a closed set of two amounts of exchangeable items represented by the sides of the box, and two traders at opposite corners,

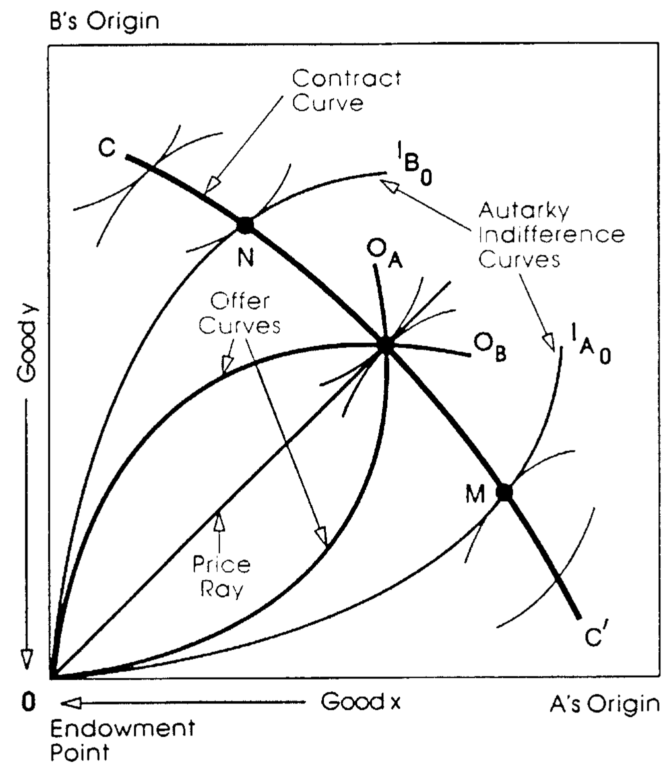

Figure 2. A modernized version of Edgeworth's diagram made into a Box by T. Humphrey in 1996 (41).

each with two axes of potential commodities to trade with—are not there from the beginning. How did Edgeworth's diagram become a box? How did the individuals arrive in an antagonistic stance?

Vilfredo Pareto, an Italian mathematical economist of great note, without comment places the individuals at the SW and NE corners in his 1906 book, thus lining them up opposite each other. He represents a fixed quantity of both goods, but, by extending the axes beyond the rectangle, invites the possibility of extension in some later diagrammatic treatment. In 1924, Arthur Bowley (in the first textbook in mathematical economics) presents almost a box, which follows Edgeworth's in orientation (i.e., potentially NW-SE), but his Box remains open, or rather unclosed, and his axes appear flexible in length.[6] He moves what we would now call

6. Historians of economics have argued over whether Edgeworth's original diagram

"the endowment point" (the initial amount of each good held by each individual) into the middle of the Box, but names it as the "origin" point, the point from which trading would commence according to the indifference map, as in Edgeworth, not the zero both goods point for each trader as it later becomes. Whereas Bowley's two axes continue but do not meet, Tibor Scitovsky (1941), like Pareto, extends his axes beyond the Box, as does Abba Lerner ([1933] 1952), who makes his Box represent production not exchange.[7]

Paying attention to the contents of the Box shows how the new conceptual elements associated with the model are developed into an analytical apparatus. Edgeworth's 1881 substantial developments of Marshall's 1879 initial trade diagrams consist in mapping utility concepts into the commodity space by adding his contract curve and indifference curves to represent the preferences of the two individual traders. These are the critical conceptual innovations that Edgeworth developed in this field, and they are discussed and diagrammed for the first time here. Pareto provides indifference maps, and shows the trading range in which welfare improvements can be negotiated in relation to (some) price rays. Bowley's innovation is to represent the possibility of initial endowments and he shows Marshall's offer curves clearly on the same map. Scitovsky develops an analysis of what happens to the utility maps when the size of the commodity space changes. Finally, in 1946, Wassily Leontief puts together all the conceptual elements of indifference maps, offer curves, contract curve, and price rays in a representation which uses the Box to show individual preference maps, their optimum exchange points, and their trading responses to different price ratios (see Figure 3). Although some of the ideas associated with these elements have a longer history, the conceptual apparatus is not something which existed before and outside of the diagram, rather they are new with the representation and develop alongside the model.

As we have seen with this case, model building is a creative, building, process. The historical sequence shows how very different Edgeworth's beginning point in Figure 1 is both from the final development of the model in Figure 3 and from the modernized version of his diagram in Figure 2. When economists *first* make an image of the economy, it is not that they know the world and subtract elements from it to isolate certain parts. Rather, like the early astronomers, the early mathematical econo-

can properly be called a box and over the relative contributions of Edgeworth, Bowley, and Pareto to its genesis and development (see particularly Creedy 1986 for arguments and sources).

7. Lerner's 1933 student paper was known by circulation, but was only finally published in 1952; see Humphrey 1996, 61–62 for further information.

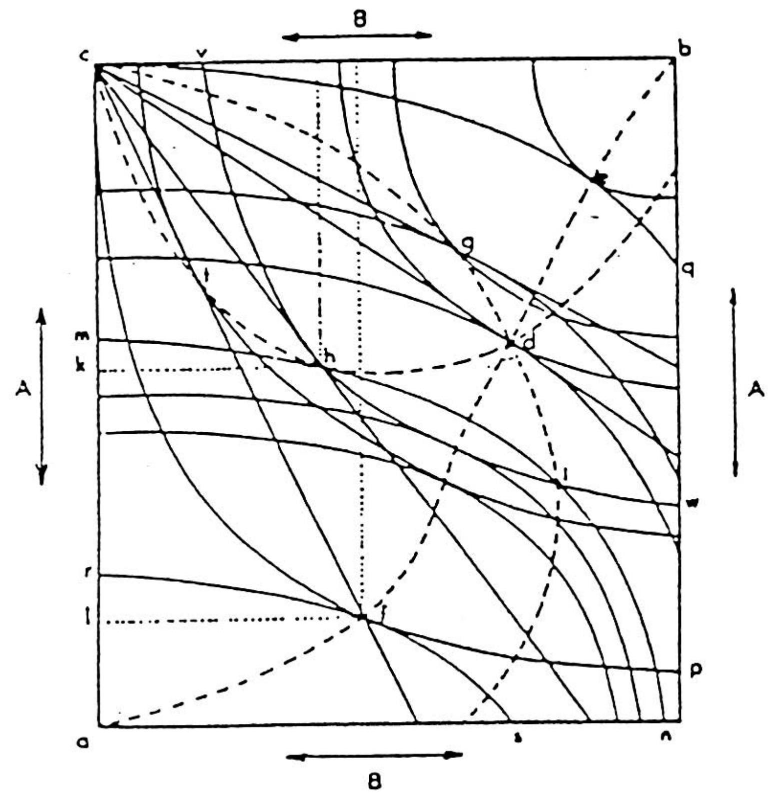

Figure 3. W. W. Leontief's Box diagram of 1946 (77).

mists used their imagination about the workings of the world to make representations of those workings in models. Gradually, later economists developed those first images, adding further elements to the representation. And, with the development of such mathematical representations, economists did not just find a new way of portraying the economic world, but developed new concepts as well.

4. Questions of Representation. The important thing about these new representations of the economic world is that they enabled economists to argue in conceptual spaces, beyond or behind perceptual space. The world of people and goods might be illustrated by an artist's depiction of the Edgeworth Box in terms of two people and their goods, but the economic

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concepts have to be visualized—imagined and imaged—into that same space by the economist.[8]

This difference between the perceptual space of illustration and the conceptual space of visualization is discussed by Michael Mahoney (1985) in relation to perspective drawing and the new mechanics of the Scientific Revolution. He sets out to destroy the Edgerton Thesis, namely that there was a direct causal link between the Renaissance improvements in drawing of machines and the development of the science of mechanics. Mahoney argues that although the engineer-artists of those days drew in new ways (they learned to provide accurate representations of physical objects in three-dimensional space) they did not draw new things. These new ways of drawing did not reveal the workings of the machines. Rather, the science of mechanics at that time treated the machine as

> an abstract, general system of quantitative parameters linked by mathematical relations . . . [so] it is difficult to see how more accurate depiction of the basic phenomena as physical objects could have conduced to their abstraction into general systems. For the defining terms of the systems lay in conceptual realms ever farther removed from the physical space the artists had become so adept at depicting. Those terms could not be drawn; at best, they could be diagramed. (Mahoney 1985, 200)

It was already the case that reasoning about mechanics was conducted in the language of mathematics; geometric reasoning and mathematical diagrams remained the main form of representing these relations and were used for reasoning through the Renaissance. But, as Mahoney remarks with great insight, "It is the mind's eye that is looking here, and it is peering into the structural relations among quantities belonging to many different conceptual (rather than perceptual) spaces" (1985, 209). Mathematics marks out the difference in the source of imagination from the mind's eye compared to the body's eye.

For the economists in my case, learning to represent the economy in new ways was drawing new things. The mathematically expressed economic elements inside the Edgeworth Box—the indifference curves, the contract curve, the points of tangency and equilibrium, etc.—are new, mind's eye, conceptual elements, not old, body's eye, perceptual elements. Scitovsky's 1941 use of the diagram provides an excellent example of this

8. In a much extended version of this paper (first given at the ECHE conference in Montreal in March 2002, and to be reported in my forthcoming book on models), I also analyze an artist's depiction of the Box diagram and use this both to explore the idealization account of model building and to demonstrate more effectively these claims about the difference between conceptual and perceptual space.

point. The critical point of his article is the difference between allocative efficiency in which the total resources in the economy are fixed (denoted by a fixed size box) and those in which the resources change (denoted by a change in box size). The representation of the effect of this change proves to be quite difficult to understand for the modern user of such boxes. It is tempting for the reader of the diagram to suppose that, by expanding the box, there are just longer axes, more goods (for example, cheese and wine) to be exchanged for given indifference maps (representing tastes, which have no reason to alter). But of course these indifference lines represent contours in conceptual space, and increasing the total resources effectively expands the box from the middle. As the axes are lengthened, perceptual space expands, but so does the conceptual space, so that the original contract curve opens out to provide a region in the middle through which the new contract curve runs.

This distinction between conceptual space and perceptual space also helps us to distinguish when a diagram is doing any work in the argument. If the diagram is about perceptual space but the argument about conceptual space, the reasoning will take place, as Mahoney describes it, "off the diagram" and the diagram will be, at best, an illustration, rather than a tool for experimentation and demonstration.[9] Yet, as we know from Humphrey's 1996 history, during the early-twentieth-century period, the Edgeworth Box diagram was a creative tool used to derive propositions and prove theorems in economics. It was indeed a tool for reasoning about the economic world using the conceptual resources of the diagram.

5. New Things in the New Version of the World. Although economists usually (as in the examples here) fail to express the full particulars of the individuals and their goods in their model, the Box diagram enables them to place the symbolized individuals into a different form of relationship, and to say different things about their relationship, than in the verbal economics the Box supplants. The act of representation here involves the direct visualization of the economic world into mathematical symbols and other forms of nonverbal denotation to create a new world in the model.

As a test of this proposition of newness, imagine giving a verbal exposition of Leontief's or Humphrey's Box diagrams shown here, with a sufficiently exact description that all the parts, and their relations to each other, are made clear. Such a description could be given, but only by using

9. Once every economist became familiar with the Box and its results, its status drops to being a "mere illustration" of the results derived from it, and the diagram becomes more schematic (as in Figure 2). The Box has not disappeared from modern economics because its usage continues to link with important results derived elsewhere—for example in game theory and experimental economics.

our now habitual mathematical and spatial terms expressing these economic concepts. But these same concepts and terms depended for their definition and their development on reasoning with the diagram. Thus, we can try to translate our mathematical model world into verbal terms, but it is a new world being expressed—one which we could not have expressed before we made our diagrammatic world.

I should be careful here to point out that when the Edgeworth Box is described as a mathematical model, it is not only made of mathematics. We can illustrate this best by considering the allowable movements or manipulations which can be made in the model. The notion that the two traders will be at some kind of optimum when their indifference curves meet at a tangency makes use of mathematical concepts and logic. But the apparatus of offer curves, indifference curves, and so, for example, the spaces in which trade is ruled out, depends on understanding the conceptual content of the elements in the model. Thus, Scitovsky's diagram showing the implications of increasing the resources requires manipulations of the diagram which are determined by the economic meaning of these curves, not by the logic of geometry. Both mathematical and subject-matter conceptual knowledge constrain the details of the representation and define the allowable manipulations. This is surely not particular to models in the form of diagrams, and indeed it seems likely that most if not all "mathematical" models in economics depend on economic subject information to constrain or define their rules of manipulation. From this point of view, there would be as much difficulty in "translating" the Edgeworth Box into "just mathematics" with no subject content as into "just words" with no mathematical content. The Edgeworth Box diagram carries an independent representational function:[10] it contains conceptual apparatus which could not be represented, or manipulated, in verbal form and indeed cannot be entirely expressed in purely mathematical terms.

Thus, to go back to the main arguments of the paper: it is not just that (as economists have long argued) mathematics is more exact in expression, or a more efficient workhorse, or more rigorous in argument. The point is that mathematical models represent something *different* from verbal accounts: they involve *different* concepts and use *different* kinds of arguments. The mathematically made version of the economic world is *different* from the verbally made one. Mathematical models represent

10. The phrase is from Michael Lynch (1990, 5) while the argument parallels, in a slightly different way, the claims that Morrison and I made about the autonomous functioning of models being related to a certain independence in their construction: see Morgan and Morrison 1999, chapter 2. For the relation of modelling to conceptual change in science, see Nersessian 1999.

something independently of the text; that something has conceptual content not (easily) expressible in words. It is this quality that makes models into good building blocks for a mathematically made version—a newly made version—of the economic world. But, crucially, to arrive at that newly made version of the economic world first requires imagination about how to represent the world and to make an image of that world in a model.

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