

Spaces of Possibility

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Abstract

We care not just how things are but how they could have been otherwise – about possibility and necessity as well as actuality. Many philosophers regard such talk as beyond the reach of respectable science, since observation tells us how things are but (allegedly) not how they could have been otherwise. I argue that, on the contrary, such criticisms are ill-founded: possibility and necessity are studied in natural science, for example through phase spaces, abstract mathematical representations of the possible states of a physical system. The possibility is objective, not merely epistemic, though it may be more restricted than pure metaphysical possibility. The elements of a phase space are very similar to Kripke's possible worlds, despite being time slices rather than total histories. The absence of explicit modal operators in the mathematical models used by scientists does not show science to be non-modal; rather, it manifests reliance on a mathematical framework for theorizing about objective possibility similar to the mathematical framework of possible worlds model theory.

1. Introduction¹

We speak not just of what *is* or *is not*, but of what *can* or *cannot* be, not just of what something does or fails to do, but of what it *can* or *cannot* do. We continually modify verb phrases with *modal* auxiliary verbs such as 'can', 'could', 'should', 'may', 'might', 'must', constructions such as 'able to', 'has to', 'needs to', and so on. We also use corresponding adjectives: instead of saying that something *can* or *cannot* or *must* happen, we also say that it is *possible* or *impossible* or *necessary*. Such a modal dimension is more or less ubiquitous in our thought and talk.

Not all modalities are alike. You spot a large insect of unknown species. You say 'Careful: it may sting'. You don't know whether it

¹ This paper descends from a lecture given at the Royal Institute of Philosophy, where it received useful feedback. For a more detailed and rigorous development of the ideas in it, see Timothy Williamson, 'Modal Science', *Canadian Journal of Philosophy* 46 (2016): 453–492. More general issues about the status of modal logic are treated in Timothy Williamson, *Modal Logic as Metaphysics* (Oxford: Oxford University Press, 2013).

can sting, but since you know that for all you know it stings, you are entitled to say that it *may* sting. Here ‘may’ concerns your state of knowledge, while ‘can’ concerns the nature of the insect, irrespective of what you or anyone else happens to know or believe about it. Linguists have studied such a distinction in many languages. Linguists call ‘may’ here an *epistemic* modal, because it concerns the relevant speakers’ states of knowledge. Many of them call ‘can’ a *circumstantial* modal, because it concerns the circumstances themselves, rather than states of knowledge about them. Other linguists call ‘can’ a *dynamic* modal, because it concerns powers to change the circumstances.²

Neither ‘circumstantial’ nor ‘dynamic’ is a terribly felicitous choice of terminology. Both words have misleadingly restrictive implications. Think of the classic metaphysical question ‘Could there have been nothing?’ It makes sense, even if it is hard to answer. It is not a question about anyone’s states of knowledge or belief. After all, we all know that there *is* something, but our knowledge does not settle the question. The metaphysician grants that there is something, but still asks whether that is *necessarily* or only *contingently* so. The question abstracts away from everything specific to our actual circumstances.

In more helpful terminology, epistemic modality is contrasted with *objective* modality. In effect, the metaphysician is asking whether it is objectively necessary in the highest degree that there is something. The distinctions between what is accidental and what is inevitable, or between what is contingent and what is necessary, concern objective modality. By contrast, the distinction between what is uncertain and what is certain concerns epistemic modality.

This talk of objective modality is meant to recall the closely connected talk of *objective probability*. Suppose that I tell you that the coin hidden in my fist is either two-headed or two-tailed, but I won’t tell you which. I am trustworthy, and you trust me. What is the probability that the coin will come up heads if tossed?

² For the large relevant literature by linguists on the semantics of modal expressions in natural language, one might start with Angelika Kratzer, ‘What “must” and “can” must and can mean’, *Linguistics and Philosophy* 1 (1977): 337–355, and *Modals and Conditionals: New and Revised Perspectives* (Oxford: Oxford University Press), and Paul Portner, *Modality* (Oxford: Oxford University Press). For an exchange on the distinction between epistemic and objective modalities see Barbara Vetter, ‘Williamsonian modal epistemology, possibility-based’, *Canadian Journal of Philosophy* 46 (2016): 766–795, and Timothy Williamson, ‘Reply to Vetter’, *Canadian Journal of Philosophy* 46 (2016): 796–802.

Objectively, the probability is either 100% (if it is two-headed – we can ignore complications such as its landing on its edge) or 0% (if it is two-tailed). But if you are betting on the outcome, with your limited knowledge, it is wiser to assign a probability of 50% to heads. That is the best candidate for the *epistemic probability* of heads, the probability on your evidence. By contrast, physical chance and actual frequency are forms of objective probability. Also contrasted with objective probability is *subjective* or *doxastic probability*, your credence or degree of belief. Roughly, subjective or doxastic probability stands to belief as epistemic probability stands to knowledge; both concern the cognitive states of agents, as opposed to what obtains independently. For instance, if a fervent monarchist is certain that the coin will come up heads, because it is *the queen's* head, his subjective probability for heads is 100%. If the coin is two-headed, he is lucky, because his subjective probability happens to match the objective probability. If the coin is two-tailed, he is unlucky, because his subjective probability happens to maximally mismatch the objective probability.

Just as physical chance and actual frequency are different kinds of objective probability, so there are different kinds of objective modality. More specifically, there are different kinds of objective *possibility*, and for each of those there is a dual kind of *necessity*. This duality is a special sort of interdefinability. Given a sense of 'possible', one can define the dual sense of 'necessary' by stipulating that, for any state of affairs S, it is necessary for S to obtain if and only if it is not possible for S not to obtain. Conversely, given a sense of 'necessary', one can define the dual sense of 'possible' by stipulating that it is possible for S to obtain if and only if it is not necessary for S not to obtain.

The auxiliary 'can' often expresses some kind practical possibility, a kind of objective possibility highly restricted by circumstances. For instance, as I write, I can touch my computer screen, but I cannot touch the books on my shelves, because they are a metre too far away. In the corresponding very restrictive sense, right now it is necessary that I am not touching those books. Obviously, there is also a less restrictive sense in which I can touch the books but cannot touch the moon, and a still less restrictive sense in which I can touch the moon (the world contains the technological resources for that).

Some kinds of objective possibility hold the past history fixed, to give a sense in which it is impossible for you never to have been born. Other kinds of objective possibility allow the past history to vary, to give a sense in which it is possible for you never to have been born. Some kinds of objective possibility hold the laws of physics fixed, to give a sense in which the actual laws could not

have failed. Other kinds of objective possibility allow the laws of physics to vary, to give a sense in which the actual laws could have failed.

In a given situation, something is *nomic* possible if and only if it is compatible with the laws of nature in that situation; dually, it is *nomic* necessary if and only if those laws entail it. Nomic possibility and necessity are objective modalities. But some kinds of objective possibility may be nomic impossible: perhaps, in some objective way, the actual laws of nature could have failed.

Arguably, there is a most general objective kind of modality, *metaphysical modality*. If something has metaphysical necessity, it has *every* objective kind of necessity. In particular, metaphysical necessity entails both practical necessity and nomic necessity. Dually, if something has *any* objective kind of possibility, it has metaphysical possibility. In particular, both practical possibility and nomic possibility entail metaphysical possibility. Thus one may regard any objective modality as a restriction of a metaphysical modality.

It was Saul Kripke who made the distinction between metaphysical and epistemic modality prominent.³ He contrasted the objective distinction between (metaphysically) necessary and contingent truths with the epistemic distinction between truths we can know *a priori* and truths we can only know *a posteriori*, giving examples of both necessary truths knowable only *a posteriori* and contingent truths knowable *a priori*; the details do not matter for present purposes. Since Kripke, nomic and metaphysical modalities have been normal parts of a metaphysician's stock-in-trade. Humbler objective modalities have long been normal parts of any human's cognitive stock-in-trade. One need not be a metaphysician to care whether an insect can sting, or whether one can reach an apple.

Remarkably, there has also been a long tradition of philosophical scepticism about objective modality, often associated with the idea that objective modal distinctions are somehow *pre-scientific*. Sometimes they are dismissed as simply illusions; sometimes they are grudgingly allowed but treated as superficial or gerrymandered. Section 2 sketches the sceptical concerns. Section 3 is a case study of one general way in which contemporary natural science explores spaces of objective possibility. It is the sceptics who are suffering from an illusion: that, with respect to the objective modalities, science is a modality-free zone. Although much remains to be understood as to exactly what role objective modality plays in a natural

³ See Saul Kripke, *Naming and Necessity* (Oxford: Blackwell, 1980).

science, the answer ‘none’ is no good. If that should be obvious, many clever philosophers have not found it so.

2. Varieties of scepticism about objective modality

The patriarch of scepticism about objective modality is, of course, David Hume. For him, as standardly – and, as far as I can tell, correctly – interpreted, we have no idea of power or necessary connection as observer-independent features of objects. We see the glass dropping to the floor, and think ‘It must smash’, but that is just our way of expressing our psychological expectation that it *will* smash. In saying ‘must’ rather than ‘will’, we make no further conjecture, true or false, about the glass itself. On Hume’s view, talk of ‘objective necessity’ makes no real sense.

Surprisingly, Hume’s argument for scepticism about objective modality continues to be influential, despite its reliance on an eighteenth-century empiricist picture of perception. He assumes that our perceptual impressions are objectively amodal – they concern purely what things are, not what they can or must be – so that, however we patch them together into ideas, the results are always objectively amodal too. But why assume that all perceptual impressions are objectively amodal? Why be so sure that we never see that some object *can be reached*? Of course, someone else with much shorter arms may see the same object from the same angle in the same conditions yet not see that it can be reached, but so what? The sort of perception most relevant to science is not just perceiving objects; it is perceiving *that* something is so. We need to perceive-that in order to gather observational data against which to test scientific theories, or indeed to test simple everyday hypotheses about our environment.⁴

Of course, a theorist may somehow distinguish a narrow class of ‘purely perceptual’ that-contents from a wider class of linguistically articulable that-contents, inferred from the purely perceptual that-contents together with background beliefs. But even if one accepts such a controversial distinction, why assume that the purely perceptual that-contents will all be amodal? To guide our actions, we often need modal information about what we and other creatures and things in our environment can do, to identify the genuine options, opportunities, and dangers. Moreover, since we often act under time pressure in a fast-changing environment, we often need such

⁴ For a strong case for modal perception, see Margot Strohminger ‘Perceptual knowledge of nonactual possibilities’, *Philosophical Perspectives* 29 (2015): 363–375.

modal information without delay. Since inference from background beliefs is a time-consuming and cognitively costly operation, there are obvious evolutionary pressures for perception to deliver the required modal information directly. Thus we might expect at least some of the supposed purely perceptual that-contents to be modal. Some major psychological theories of perception postulate exactly that – for instance, Gibson’s theory of affordances.⁵ Such theories cannot be dismissed on the basis of a picture of perception good enough for 1750.

Moreover, if for some reason all the supposed purely perceptual that-contents *are* amodal, why assume that every that-content we can entertain is definable in terms of purely perceptual that-contents? Such an empiricist dogma is a speculative hypothesis, arguably less plausible than the hypothesis that the capacity for applying objective modal distinctions is innate in humans. These are matters for scientific investigation, not blind faith. In short, the Humean critique fails to establish even a presumption of guilt in our thinking about objective modal matters.

The leading twentieth-century critic of modality was Willard Quine.⁶ However, his prime target was Rudolf Carnap’s account of *logical* modality. Since Carnap conceived logical necessity semantically, as a kind of analyticity, he did not really have *objective* modality in mind. Much of Quine’s specific critique is irrelevant to objective modality. Nevertheless, a more general Quinean critique is relevant. For Quine takes our best theory of the world to be fundamental physics, and with it mathematics, both of which he conceives as amodal. Thus our best understanding of the world has no place for objective modality. Quine’s scornful remarks about Aristotelian essentialism as an implicit commitment of quantified modal logic assume that essentialism is quite alien to contemporary science. Thus if quantified modal logic turns out to be implicit in much contemporary science, there are serious internal tensions in Quine’s position.

Quine’s pupil David Lewis ingeniously made room for something like objective modality within a broadly Quinean worldview.⁷

⁵ See James Gibson, *The Ecological Approach to Visual Perception* (Boston, MA: Houghton Mifflin, 1979).

⁶ See especially Willard van Orman Quine, ‘Reference and modality’, in his *From a Logical Point of View* (Cambridge, MA: Harvard University Press, 1953): 139–159.

⁷ See David Lewis, ‘Counterpart theory and quantified modal logic’, *Journal of Philosophy* 65 (1968): 113–126, and *On the Plurality of Worlds* (Oxford: Blackwell, 1986).

Lewis's 'modal realism' postulates infinitely many possible worlds as large physical objects, spatiotemporally disconnected from each other, and uses them to make charitable sense of ordinary modal discourse. He does so by regimenting it in the formal language of quantified modal logic and then 'translating' the result into his preferred non-modal language of counterpart theory. Thus Lewis reduces the modal to the non-modal. On his view, when people engage in modal discourse, including objective modal discourse, they often speak truly, but what is going on can be explained in ultimately extensional, non-modal terms.

Lewis's aim of interpreting ordinary modal discourse charitably is in line with Quine's overall methodology, and the language of counterpart theory is respectable by Quinean standards. But should Quine accept Lewis's postulate of infinitely many concrete worlds? After all, without reference to physics, Lewis is making bold claims about the global structure of spatiotemporal reality: for instance, that it divides into infinitely many mutually disconnected pieces. In effect, he is proposing a grand cosmological theory. Moreover, it is inconsistent with cosmological theories in physics, which also aim to map the global structure of spatiotemporal structure. Although Lewis's postulate of disconnected pieces has some precedents in physics, the specific structures postulated are quite different. Moreover, for the sake of elegance in modal realism and of charity to claims that the laws of physics in our world are contingent, Lewis postulates worlds where those laws fail. Since those worlds are supposed to be part of overall spatiotemporal reality, he is implicitly committed to rejecting physicists' claims that their laws hold throughout spatiotemporal reality, without restriction.⁸ From a Quinean perspective, one might expect consistency with theories in physics to be given a higher priority than that. By making modal claims hostage to a deviant cosmology with little relation to developments in the natural sciences, Lewis unintentionally presents them as unscientific.

It would be misleading to call Lewis himself a modal sceptic. Indeed, with Kripke and others such as Robert Stalnaker, he was a leader of the 'modal turn', when possible worlds became part of the standard framework within which to develop philosophical theories.

Subsequently, there has been metaphysically driven pushback against the modal turn, involving a milder form of modal scepticism.

⁸ For a recent exchange on this objection to Lewis's modal realism see John Divers, 'Modal reality and (modal) logical space', *Philosophy and Phenomenological Research* 88 (2014): 726–733, and Timothy Williamson, 'Replies to Bricker, Divers, and Sullivan', *Philosophy and Phenomenological Research* 88 (2014): 744–764.

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Although modal distinctions are admitted, they are side-lined. Much of the explanatory work they were expected to do was taken away from them and reassigned elsewhere, because they were thought to be too indiscriminate to do it properly. An early instance concerned attempts in the philosophy of mind to use a modal relation of *supervenience* to explain how the physical ‘determines’ the mental: no (possible) mental difference without a (possible) physical difference. By 1980, it was already a standard complaint that supervenience is too coarse-grained to capture the sense in which the physical was supposed to be constitutively *prior* to the mental.⁹ Another key moment concerned Kripke’s use of modality to rehabilitate the old distinction between *essential* and *accidental* property possession: an object has a property essentially if and only if it is necessary that if the object exists then it has the property. In a very influential paper of 1994, Kit Fine objected that although it is impossible for either of Socrates and his singleton set {Socrates} to exist without the other, it is essential to {Socrates} to have Socrates as a member but not essential to Socrates to be a member of {Socrates}, whereas the modal account makes the two essentialist claims stand or fall together. It is too coarse-grained to capture the sense in which Socrates is supposed to be constitutively prior to {Socrates}.¹⁰ More fine-grained distinctions of essence, constitution, grounding, and the like were therefore invoked to do the explanatory work instead, and treated as metaphysically more fundamental than modality. Just as extensional distinctions had given way to intensional ones, so the latter gave way to hyperintensional distinctions.

A recent manifestation of this new kind of modal scepticism is Ted Sider’s argument that metaphysical necessity is not a unified characteristic but a ragbag disjunction of miscellaneous features with no common rationale. If so, metaphysical modality is unfit for serious theoretical purposes. Like many modal sceptics, Sider specifically targets metaphysical modality, rather than objective modalities more generally. He does not consider it as a distinguished member of the broader category.¹¹

⁹ For exploration of these issues see Jaegwon Kim, *Supervenience and Mind: Selected Philosophical Issues* (Cambridge: Cambridge University Press, 1993).

¹⁰ See Kit Fine, ‘Essence and modality’, *Philosophical Perspectives*, 8 (1994): 1–16.

¹¹ See Ted Sider, *Writing the Book of the World* (Oxford: Clarendon Press, 2011), and ‘On Williamson and simplicity in modal logic’, *Canadian Journal of Philosophy* 46 (2016): 683–698, and Timothy

A common feature of modal scepticism is its presupposition that objective modalities are unscientific, therefore theoretically vulnerable. Thinking in terms of objective modalities is seen as something done by ordinary folk or metaphysicians, not as integral to natural science. Modal sceptics see themselves as fighting for science, not against it.

One might hope to refute the idea that objective modalities are alien to natural science simply by pointing to nomic modality. The distinction between nomic necessity and nomic contingency just is the distinction between what the laws of nature entail and what they do not. But that response is inadequate as it stands, for it would still need to be shown that the distinction between those true universal generalizations in the language of science that express laws of nature and those that do not is load-bearing within the science. If the phrase 'law of nature' functioned more like an honorific in science, a laurel wreath bestowed on the generalization that does all the serious work itself, then not much would have been done to vindicate the category of objective modality.

To reinforce the impression that science is fundamentally amodal, the 'harder' the science the more mathematical is the language in which its theories are formulated. There are no modal operators in the language of mathematics. From the outside one may classify mathematical theorems as metaphysically necessary, but that classification plays no role within the mathematics itself.

Nevertheless, it is an illusion that objective modalities play no role within natural science. They are integral to its subject matter. In effect, much of natural science is an exploration of objective modal space. Pure mathematics is not specifically about modal matters, but it is also not specifically about the phenomena studied in natural science. When the mathematics is applied to a physical system, it is instantiated by sets of objects relevant to that system, such as the set of its symmetries. Those objects may be objectively modal in nature, such as the set of its *possible* states, the states it *can* be in.

The point can be argued in many different ways. For instance, natural science often investigates *probabilities*, and probability is a kind of modality, since whatever has non-zero probability is possible in a corresponding sense. Of course, some probabilities in science are epistemic, concerning scientists' states of knowledge, as when they

Williamson, 'Reply to Sider', *Canadian Journal of Philosophy* 46 (2016): 699–708.

discuss which theory is most probable on the current evidence. Epistemic probabilities correspond to epistemic modalities. But sometimes scientists are interested in objective probabilities as aspects of the physical system under study, and objective probabilities correspond to objective modalities. The most famous example is quantum mechanics, but sense can also be made of objective probabilities in classical physics. Objective probabilities do not require indeterminism; they can be defined in terms of a measure over a space of initial conditions for a system.

Probabilities raise many complications, especially when defined over an infinite set of possibilities. For present purposes, it is sufficient to consider the simpler case of *phase space*. Thinking in terms of phase space is found in many areas of natural science.¹² Informally, scientists understand the phase space of a physical system as the space of its *possible* instantaneous states. For definiteness, we will consider a specific kind of phase space.

3. Dynamical systems

Dynamical systems theory is a research programme in mathematics motivated with an eye to applications in physics, chemistry, biology, engineering, and elsewhere, including the theory of chaos. It investigates a mathematically well-defined family of structured sets, dynamical systems.¹³

A given dynamical system is based on a set of objects, which in a physical application are informally understood as states of a given physical system, such as the solar system, the terrestrial weather system, or a pendulum. These states are treated as maximally specific, mutually exclusive, and jointly exhaustive. Typically, geometrical or topological structure is defined over the set of states; although the details do not matter here, much of the mathematical power of the theory derives from this structure.

The system is also equipped with a *dynamics*, which describes how it evolves over time. The dynamics is given by a family of mathematical functions, indexed by positive and negative numbers (including zero). A positive number represents a temporal distance into the

¹² See David Nolte, 'The tangled tale of phase space', *Physics Today*, 63/4 (2010): 33–38.

¹³ For an introduction to dynamical systems theory, see Steven Strogatz, *Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry, and Engineering* (Boulder, CO: Westview: 2001).

future; a negative number represents a temporal distance into the past. If you input a state of the system to the function indexed by a positive number, the function outputs the state the system will be in the associated length of time after it was in the input state. Similarly, if you input the state to the function indexed by a negative number, the function outputs the state the system was in the corresponding length of time before it was in the input state. Time may be treated either as discrete (represented by positive and negative integers) or as continuous (represented by positive and negative real numbers). Formal constraints are imposed to ensure that all the functions in the family fit together into a single coherent dynamics.

The dynamics is *deterministic* both backwards and forwards in time: the state of the system at any given moment uniquely determines its state at any later or earlier moment.

In laying down such a dynamics, we treat the system as closed, impervious to external influence, such as the intrusion of a meteorite. This of course is normally an idealization, but such idealizations are typical of natural science (for example, the treatment of a planet as a point mass), and should not disturb us.

Analysing the global structure of the dynamics is often a key step in understanding the overall behavioural patterns of the system.

When dynamical systems theory is applied to a given (type of) physical system, the states are typically assignments of numerical values to key variables associated with the system, such as the temperature at a given point. The dynamics is then usually specified by a family of equations in those variables, difference equations for discrete time or differential equations for continuous time.

It is easy to show that if some state ever repeats, in the sense that if the system is in it then it will be in it again at some later time (according to the dynamics), then the state is involved in a Nietzschean eternal recurrence: the system's history cycles infinitely. If there are only finitely many states, every state is involved in such an eternal recurrence. Indeed, Nietzsche's interest in eternal recurrence was connected with developments in physics that figured in the prehistory of dynamical systems theory.

In analysing a dynamical system, it is often convenient to partition the set of states into *orbits*. Two states are in the same orbit just in case if the system is in one of them then it was or will be in the other. This is an equivalence relation, so each state is in exactly one orbit. Thus orbits are mutually exclusive and jointly exhaustive of the set of states.

In the special case where the partition contains only one orbit, the system sooner or later traverses every state. However, that case is unusual. More normally, the states are partitioned into at least two

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orbits. This means that we must regard some states as *counterfactual*, not actually past, present, or future. The natural understanding of the states in an application is as *possible* states of the system. We cannot discard all states outside the actual orbit, because the geometrical or topological structure is defined over the set of states as a whole, not over a single orbit. Since the mathematical power of the theory depends on that structure, the utility of the application depends on retaining all the states, including the counterfactual ones. Thus the modal dimension of the system is crucial to its study.

How can we articulate the modal dimension of dynamical systems explicitly? The key is to appreciate the analogy between possible states in phase space and possible worlds in models of modal logic as developed by Saul Kripke – who was aware of the analogy when doing his pioneering work. The set of worlds in a Kripke model corresponds to the set of states in a dynamical system, or, more generally, in a phase space. Possible worlds, like possible states, are conceived as maximally specific, mutually exclusive, and jointly exhaustive. Moreover, just as we can ask what is true or false in a possible world, so we can ask what is true or false in a possible state of the dynamical system.

Incidentally, this analogy gives no support to Lewis's modal realism; quite the opposite. Just as Kripke's theory in no way motivates regarding all possible worlds as concretely realized, so regarding all possible states of the dynamical system as concretely realized would be to impose pointless confusion onto the idea of phase space, motivated by philosophical ideology rather than by any need of the science. Both possible worlds and possible states are better conceived more abstractly. The states of the dynamical systems are specified by possible assignments of values to physical variables; whether those variables will ever actually take those values is an entirely different question.

A sentence is interpreted over a Kripke model by being assigned a proposition, modelled as a subset of the set of worlds in a model: the proposition is true at the worlds it contains and false at the other worlds, and the sentence receives exactly that truth-condition. Similarly, a sentence can be interpreted over a dynamical system by being assigned a proposition, modelled as a subset of the set of states in the dynamical system: the proposition is true at the states it contains and false at the other states, and the sentence receives exactly that truth-condition. This framework allows us to interpret modal operators over the system, just as in the simplest models of modal logic: necessity is equated with truth-in-every-state, like truth-in-every-world, and possibility with truth-in-some-state, like

truth-in-some-world. A sentence is *valid* over a dynamical system if and only if it is true in every state in the system, whatever sets of states are assigned to the atomic formulas.

The effect of this semantics is that every dynamical system validates every theorem of a strong and simple modal system, known as S5, in which purely modal matters are never contingent: what is necessary is necessarily necessary, and what is possible is necessarily possible.

Although the semantics for the modal operators over dynamical systems leaves them absolute, unrestricted by any relation of accessibility between worlds, that does not mean that the operators have generality over all *metaphysically* possible worlds. The states in a dynamical system may be much more restricted than that, and not only by laws of nature. For example, the laws of nature presumably do not require that there *be* the solar system at all, even though it is there in all states of a dynamical system of states of the solar system. Thus the states of a dynamical system need not cover all metaphysical or even all nomic possibilities. Nevertheless, the modalities defined over the dynamical system are still clearly *objective*. Their role is not to characterize what happens to be known or believed about the system.

To express distinctions more specific to dynamical systems in our language, we can define some operators in terms of the dynamical functions, since they determine which states are in the past or future of a given state. For instance, we can define tense operators such as ‘sometime in the future’, ‘always in the future’, ‘sometime in the past’, and ‘always in the past’.

The temporal operators can usefully be combined with modal operators. For example, we can say that is *possible* for a given proposition to cease to obtain: in other words, in the future of some state in which the proposition is true is a state in which the proposition is false. We cannot express that general idea using only temporal operators or only modal operators.

Thus the natural logic for dynamical systems has at least modal and temporal operators. These points help us reflect on some differences between states and worlds, despite the rough analogy between them.

Normally, a world is supposed to encompass a whole history, past and future as well as present, without distinguishing one point in it as ‘now’. Thus it is more like a whole orbit than one state in that orbit. We need not specify such a world in addition to a state because the deterministic dynamics ensures that each state belongs to only one history, one orbit. Pursuing analogies in the opposite direction, one might try comparing a state with an ordered pair of a world and a time, which fixes the state of that world at that time: such pairs are

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often used as points of evaluation in the standard semantics for a language with modal and temporal operators. However, that comparison is not quite apt, because states are repeatable, in the case of Nietzschean eternal recurrence, whereas one cannot be at the same world-time pair twice. This might suggest that states are ‘purely qualitative’ in some sense. But even that suggestion is wrong. For instance, suppose that two states are ‘mirror images’ of each other. Then there is no qualitative difference between them, yet we must not equate them as somehow the same state, since that would eliminate the distinction between an unchanging history that remains in just one of them throughout and a changing history that alternates between the two states. Dynamical systems permit such symmetries, and our methodology is, as far as we can, to avoid second-guessing the science. Thus states in dynamical systems do not quite fit into the standard philosophical categories. Nevertheless, the affinity with modal temporal logic is extremely strong.

For many purposes we need a still more expressive language. For example, we might want to say that a given proposition is true throughout one orbit and in no other states. We cannot say that in the current language. We can overcome this expressive deficiency by introducing ‘propositional quantifiers’, which permit us to generalize on variables in sentence rather than name position. Since such a variable is interpreted as taking a set of states as its value, the generality is over all sets of states, rather than over all individual states. These quantifiers can be read as ‘for every proposition’ and ‘for some proposition’. The sort of quantification over sets of states to which these propositional quantifiers correspond is frequently used in the ordinary mathematics of dynamical systems theory. For example, in the theory of chaos, it is used in defining ideas like ‘attractor’ and ‘basin of attraction’. It is of great philosophical interest, in part *because* it is not some sort of philosophical accretion on the theory of dynamical systems, but simply the natural way of articulating structure already present in that theory. Although it can be paraphrased as quantification over propositions, a perfectly faithful rendering would have to respect the fact that the variables it binds take the grammatical position of sentences, not of names for propositions, perhaps by using such quantification in the metalanguage.

Another example of the use of propositional quantification is to express the condition – which some dynamical systems satisfy but many do not – that all histories cycle, in other words that Nietzschean eternal recurrence is necessary.

Here is an example of how the modal-temporal language avoids some ‘unphysical’ distinctions that the set-theoretic framework of

the standard mathematical formulation forces on us. It requires us to distinguish between a single state and the singleton set whose only member is that state: they are distinct entities because no set is a member of itself. That distinction seems ‘unphysical’. By contrast, the modal-temporal language has no need to express such a distinction.

We can also use propositional quantification to say, truly, that there is a necessarily true proposition (the set of all states), a necessarily false proposition (the empty set), and a necessarily contingent proposition (any other set of states). These cases are of philosophical interest because they involve quantifying into the scope of a modal operator. This is exactly what Quine regarded as the third, most disgraceful stage of modal involvement.¹⁴ But here we see that such ‘essentialism’ (in the relevant coarse-grained modal sense, rather than Fine’s) is simply a by-product of articulating the modal claims implicit in a scientific theory of dynamical systems. One proposition is essentially true, one is essentially false, and others are essentially contingent. That the quantification is propositional, in other words into sentence position rather than name position, only makes it worse from Quine’s perspective, since he insisted that the only logically respectable kind of quantification is into name position (in other words first-order quantification). Quine had in mind essentialism formulated with first-order quantifiers and modal operators with a stronger metaphysical reading of the quantifiers, but his objections are so general that if they worked at all, they would work against the present version.

There is a more general moral here. Consider two strands in Quine’s philosophy. One is his methodological naturalism, roughly the idea that our philosophizing should be guided by our best overall theory of the world, which he took to boil down to fundamental physics plus mathematics. The other is his austerity about logic: no modal operators are allowed, only first-order quantifiers. Quine assumed that these two lines of thought were not in conflict. Our considerations suggest that he was wrong about that. Much of our best science has a modal dimension which can be made explicit most perspicuously in the framework of quantified modal logic.

In a further development of these themes, one can also add first-order quantifiers over individuals to the language, and determine what quantified modal logic for them is satisfied. Doing so requires looking into the internal structure of the states, which in practice are

¹⁴ See W.V.O. Quine, ‘Three grades of modal involvement’, in his *The Ways of Paradox* (New York: Random House, 1966): 156–174.

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typically assignments of values to the relevant system variables; those variables may in turn be associated with an individual. For instance, the physical system might contain three particles, with variables for the position and velocity of each particle, making six variables in all; in that sense the dynamical system would be six-dimensional. This is not the place to go into details. The results reinforce the conclusions already reached.

In general, the absence of modal operators from standard formulations of scientific theories in mathematical notation does not show the theories to be amodal, any more than the absence of modal operators in formal theories about possible worlds and their inhabitants shows those theories to be a modal. Rather, such theories are based on strong modal assumptions, without which their non-modal framework for the treatment of modal issues would embody modal fallacies.

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