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Study of small amplitude ion-acoustic solitary wave structures and amplitude modulation in e–p–i plasma with streaming ions

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Abstract

By using reductive perturbation technique we have studied the linear and non-linear properties of ion-acoustic solitary structures in a three-component plasma containing non-thermal electrons and Boltzmann positrons and a comparatively cold ion which has got a streaming motion. The Korteweg–de Vries equation has been obtained and the dependence of small amplitude solitary structures on various plasma parameters such as streaming velocity (v_0), non-thermal parameter (β), reciprocal of electron temperature (χ), positron density (p), Mach number (M), and ion density (δ) have been studied. The possibility of formation of enveloping soliton and its characteristic features are further investigated by deriving the non-linear Schrödinger equation.

Introduction

In general, space plasmas, can be broadly modeled with Maxwellian velocity distribution. But with the advancement of satellite-based technologies and data therein it has been found in recent years, that most of these plasmas, especially those near to earth plasmas have high-energy tails and heat-flux shoulders, which may be due to the fact that these plasmas are quite inhomogeneous and semi-collisionless (Marsch *et al.* 1982; Summers *et al.*, 1994; Ma and Summers, 1998). As a result, it has been established that these plasmas (especially the electron distributions) if modeled by a generalized Lorentzian or kappa distribution gives proper result rather than by a pure Maxwellian (Summers and Thorne, 1991; 1992). Dispersion relation in various multicomponent plasma has been investigated (Bala & Kaur, 2013). Lorentzian distribution has a spectral index κ which accounts for its high-energy tail. In the limit of $\kappa \to \infty$, it leads to a Maxwellian distribution function. Astrophysical plasma shows features and phenomena that prove that they contain non-thermal distributed electrons. For instance, the solar wind plasmas are usually found to be non-thermal distributed. In close proximity to earth, the solar wind plasmas usually have a density of ∼30–100 cm^{-3} with velocity ∼500 km/s and temperature reaching up to ∼ 50 eV (Baumjohann and Treumann, 1997). With the pioneering work of Zabusky and Kruskal (1965) on the one-dimensional (1D) solitary structure of the celebrated Korteweg–de Vries (KdV) equation in plasma physics, it has also played a major role in analyzing the non-linear phenomena in physical and biological sciences. Solitons are defined as spatially localized pulsed shape stable non-linear construct which retains their shape, identity, and energy in the mutual collision and is, therefore, the exact solution of a large number of the non-linear partial differential equation. They are formed due to the balance between the non-linear effects (that causes the steeping) and dispersive effects (that causes the broadening).

The KdV equation and the non-linear Schrödinger equation (NLSE) along with their different variants having used in the study of a large number of non-linear phenomena in astrophysical and laboratory produced plasma. Nakamura and Sarma (2001) and Nakamura *et al.* (1999) have used this concept in their study of laboratory-produced plasma. The reductive perturbation technique which is used to derive the KdV equation describes the evolution of a non-modulated wave (a bare pulse without fast oscillation). The NLSE that governs the dynamics of a modulated wave packet deals with the fact that the non-linearity arising within the packet is balanced by the group dispersion resulting in the formation of a stationary envelope structured solution.

The electron–positron plasma exists in the magnetosphere of pulsars (Michel 1982, 1991) the early universe (Gibbons *et al.* 1983), the bi-polar outflow in active galactic nuclei (Miller & Witta, 1987). Though the astrophysical plasma dominantly contain electron and positron create a finite proportion of ion may be present forming effective electron–positron–ion (e–p–i) plasma. Similarly a small fraction of positron may be present in plasmas containing electrons and ions viz. tokamak plasma: in such cases the positron may be generated by the mechanism of pair production which may have possibly occurred due to collision between ran away electron (few MeV) and thermal particles (Helander & Ward, 2003). It has been also reported when ultra-intense laser pulses interact with matter as in case of internal confinement fusion, the plasma produced there may also contain positron (Liang *et al*., 1998; Gahn *et al*., 2000). The presence of ions may lead to the formation of the number of low-frequency mode which otherwise does not propagate in an electron–positron plasma. Therefore an e–p–i plasma is important in case of a laboratory plasma apart from the cosmological plasmas. The non-linear behavior in e–p–i plasma is more interesting from theoretical as well as experimental point of view.

Over the past few years the study of non-linear phenomena in electron–positron plasma has gained considerable interest. Ion-acoustic solitary structures and its dependence on various plasma parameters have been reported by Pillay and Bharuthram (1992), Ghosh and Bharuthram (2008), Mahmood and Akhtar (2008), Gill *et al.* (2003, 2004, 2007), Nejoh (1996*a*, *b*), Mushtaq and Shah (2005), and Popel *et al.* (2005). In recent times, modulation instabilities of different wave modes have been studied due to its importance in stable wave propagation. Watanabe (1977) reported about the modulation instability of monochromatic ion-acoustic wave in his experiment. Amplitude modulation in e–p–i plasma has been reported recently by Esfandyari-Kalejahi *et al.* (2006), Jehan *et al.* (2009), Kourakis *et al.* (2006), and Salahuddin *et al.* (2002). Esfandyari studied the non-linear propagation of electrostatic wave packets in e–p–i plasma with varying positron concentration and other plasma parameters. The findings of Kourakis are rather interesting. According to him the presence of background ion species modify the characteristic features of excitation and alter the stability profile of modulated wave packet. Jehan reported that with an increase in positron condition there is a decrease in the maximum growth rate of instability.

Energetic electrons are in general non-thermally distributed and are abundant in space and laboratory plasmas; it has a significant effect on non-linear collective phenomena. Such distribution is also found in the magnetosphere. In 1994, Berezhiani *et al.* (1994) gave a theory of strong-electromagnetic-wave propagation in an e–p–i plasma. Cairns *et al.* (1995) proposed a non-thermal distribution of electron during their study of ion-acoustic solitary structure observed by FREJA satellite developed by Swedish space corporation on behalf of the national Swedish space board and reported the formation of compressive and rarefactive solitons. In the later years, Singh and Lakhina (2004) reported the modification of electron-acoustic solitary structures in the presence of non-thermal electrons. In the year 2008, Verheest and Pillay studied the dust acoustic solitary structures with positively charged dust particles and non-thermal electrons. They delimited a range of parameter where both positive and negative solitary structures may coexist. Using the same non-thermal distribution given by Cairns et al. Modulation instability in envelop solitary structures in un-magnetized plasma was reported by Kourakis and Shukla (2005) and Tang and Xue (2004). Pakzad in 2009 studied ion-acoustic solitary waves and determined the parametric region for their existence. Saberian *et al.* (2011) studied the ion-acoustic solitary waves in a relativistic e–p–i plasma using Sagdeev's pseudo-potential method. The relativistic dynamical equation was initially modeled by Lee and Choi (2007); Baluku and Hellberg (2011) used the similar pseudo-potential method to study ion-acoustic solitary wave in e–p–i plasma with non-thermal electron and Boltzmann's positron.

In this paper, we studied the non-linear behavior of ion-acoustic solitary structures in an e–p–i plasma consisting non-thermal electrons and Boltzmann positrons. The findings are interesting from the theoretical and experimental point of view (astronomical and laboratory plasma). The novel finding in this paper is that we studied the small amplitude solitary structures and investigated the criteria of formation of envelope soliton. The transformation of the KdV equation into an NLSE to study amplitude modulation. The paper is organized in the following manner; section "Basic equations" provides the basic equations for the particles' dynamics. Section "Linear dispersion characteristics" carries out the linear analysis and the linear dispersion relation is derived. Here we studied the dependence of linear dispersion on different plasma parameters. Section "KdV Equation and the solitary wave structures" derives the non-linear KdV equation and analyzes the formation and properties of ion-acoustic solitary wave structures. In the section "NLSE and the envelope soliton", we investigate the properties of envelop soliton by transforming our KdV equation into an NLSE. In this section, we studied the stability criteria and also the growth rate of modulational instability. Finally, we discuss the results and conclude with some comments.

Basic equations

We consider an e–p–i plasma with non-thermal electrons and Boltzmann positrons. The dynamical equations for unidirectional propagation in such plasma are given by

$$\frac{\partial n_i}{\partial t}+\frac{\partial}{\partial x}(n_i v_i)=0, \tag{1}$$

$$\frac{\partial v_i}{\partial t}+v_i\frac{\partial}{\partial x}v_i=-\frac{e}{m}\frac{\partial\varphi}{\partial x}-\frac{1}{mn_i}\frac{\partial p}{\partial x}, \tag{2}$$

$$\frac{\partial^2\varphi}{\partial x^2}=4\pi e(n_e-n_i-n_p). \tag{3}$$

The first equation is the fluid equation of continuity for the ions, the second one is the equation of motion for the ions. Equation (3) is the Poisson's equation.

Again, the ion pressure is given by

$$p=nkT_i, \tag{4}$$

where k is the Boltzmann's constant.

Now, using normalization conditions, $\bar{t}\to\omega_{ci}t$, $\bar{x}\to((x\omega_{ci})/(V_{T_e}))$, $\overline{v_i}\to(v_i/V_{T_e})$, and $\bar{\varphi}\to((e\varphi)/(kT_{Te}))$, $\overline{n_\alpha}\to(n_\alpha/n_\alpha 0)$.

(where $\omega_{ci}=\sqrt{((4\pi n_0 e^2)/c^2)}=$ the electron plasma oscillation frequency. V_{T_e} = Fermi thermal speed of electrons), we get the set of normalized equations are given by

$$\frac{\partial n_i}{\partial t}+\frac{\partial}{\partial x}(n_i v_i)=0, \tag{5}$$

$$\frac{\partial v_i}{\partial t}+v_i\frac{\partial}{\partial x}v_i=\frac{\partial\varphi}{\partial x}-\frac{1}{n_i}\frac{\partial n_i}{\partial x}, \tag{6}$$

$$\frac{\partial^2\varphi}{\partial x^2}=n_e-\delta n_i-pn_p. \tag{7}$$

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Here $\delta = (n_{i0}/n_{e0})$ and $p = (n_{p0}/n_{e0})$ represent the equilibrium density ratios of ion-to-electron and positron-to-electron.

The non-thermal electron density is given by

$$n_e = n_{e0}\left\{\left[1 - \beta\frac{e\varphi}{kT_e}\right] + \beta\left(\frac{e\varphi}{kT_e}\right)^2\right\}\exp\left(\frac{e\varphi}{kT_e}\right)$$
$$= n_{e0}\{[1 - \beta\chi\varphi] + \beta\chi^2\varphi^2\}\exp(\chi\varphi). \quad (8)$$

Here β is the non-thermal parameter.

The Boltzmann distributed positron density is given by

$$n_p = n_{p0}\exp\left(\frac{-\sigma e\varphi}{kT_e}\right) = n_{p0}\exp(-\sigma\chi\varphi) \quad (9)$$

where $\chi = (e/(kT))$ is the reciprocal of electron temperature and $\sigma = (T_e/T_p)$ is the reciprocal of positron temperature.

For charge neutrality, we have therefore

$$n_p + n_i = n_e. \quad (10)$$

Using the same normalization it can be written as

$$p + \delta = 1. \quad (11)$$

Linear dispersion characteristics

In order to investigate the linear and non-linear behavior of ion-acoustic wave in the e–p–i plasma, we make the following perturbation expansions for the field quantities n_i, n_e, n_p, v_i, and φ about their values

$$\begin{pmatrix} n_i \\ v_i \\ \varphi \end{pmatrix} = \begin{pmatrix} 1 \\ v_0 \\ \varphi_0 \end{pmatrix} + \varepsilon\begin{pmatrix} n_i^{(1)} \\ v_i^{(1)} \\ \varphi^{(1)} \end{pmatrix} + \varepsilon^2\begin{pmatrix} n_i^{(2)} \\ v_i^{(2)} \\ \varphi^{(2)} \end{pmatrix} + \cdots. \quad (12)$$

Now, assuming that all the field variables are varying as $\exp[i(kx-\omega t)]$, we get for normalized wave frequency ω and wavenumber k, we get the following dispersion relation:

$$k^2 = \left[\frac{k^2 - (\omega - kv_0)^2}{\delta - k^2 - (\omega - kv_0)^2}\right]$$
$$\times(\chi + p\sigma\chi - \beta\chi + \beta\chi^2\varphi_0 + 2\beta\chi^3\varphi_0^2). \quad (13)$$

The dispersion relation exists for a particular range of wavenumber for given values of parameters where k is given by $k > \sqrt{(\delta/(1 - \beta + p\sigma))}$ (for $v_0 = 0.2$, $\beta = 0.2$, $\delta = 0.3$, $\chi = 0.5$, $p = 0.7$ $\sigma = 0.3$; $k > 0.545$).

If we consider there is no streaming velocity of ion, that is, $v_0 = 0$ and considering quasi-linearity condition that is, $\varphi_0 = 0$, then by replacing $\chi = (1/\lambda_D^2)$ we get

$$\frac{\omega^2}{k^2} = \frac{1}{\lambda_D^2} - \frac{\delta}{k^2\lambda_D^2(1 - \beta + p\sigma)}. \quad (14)$$

This is the linear dispersion relation which has an extra term on the right-hand side when compared with Baluku and Hellberg (2011). The extra term is the result of the inclusion of ion pressure in the momentum equation. Apart from this when we compare our dispersion relation with that obtained by Baliku *et al.* everything is same. Also when we consider β = 0 this gives in per perfect correspondence with the findings of Popel *et al.* (1995). While comparing the DR with that obtained by Pakzad (2009) the interpretation may be due to the fact that the distribution is a deviation from the Maxwellian one and the deviation is measured by a term β, and similarly, there exists a minimum value of M which is discussed in detail by Baluku.

It is found that the linear dispersion characteristics become steeper with an increase in the value of χ. It implies that as the electron temperature decreases the frequency increases with increase in wavenumber (Fig. 1). However, it has been found that there is no significant change in the dispersion characteristics with change in non-thermal parameter β (Fig. 2) or positron temperature because electrons provide the neutralizing effect.

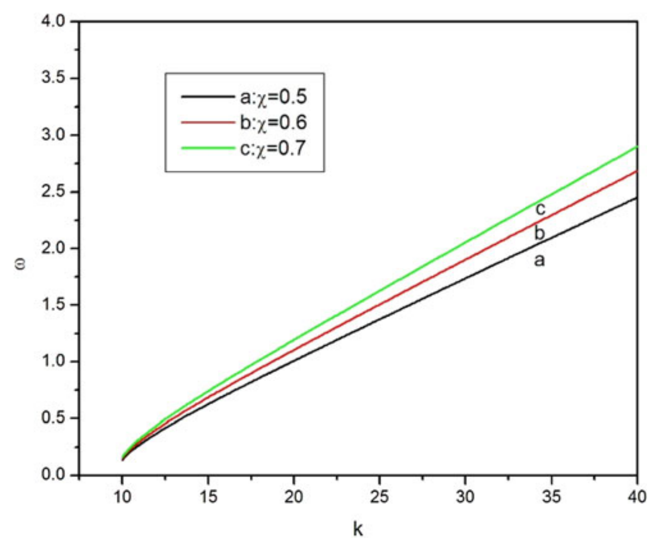

Fig. 1. Dispersion relation for different value of χ.

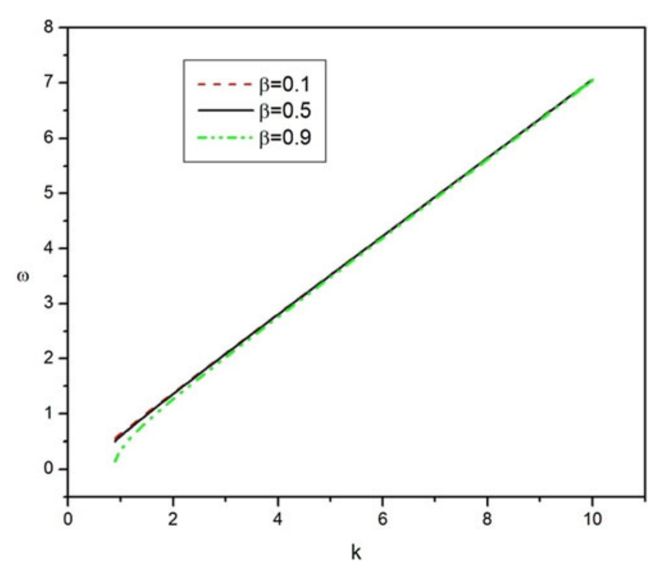

Fig. 2. Dispersion curve for different value of non-thermal parameter β.

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KdV Equation and the solitary wave structures

To investigate the behavior of small amplitude ion-acoustic wave, in the e–p–i plasma we need to study the KdV equation. The KdV equation describing the non-linear behavior of e–p–i plasma waves is derived by using the standard reductive perturbation technique. We introduce the usual stretching of space and time variables

$$\xi = \varepsilon^{1/2}(x - Mt) \text{ and } \tau = \varepsilon^{3/2}t, \tag{15}$$

where ε is a small dimensionless expansion parameter as a measure of the weakness of non-linearity, and M is a constant representing the phase velocity of the waves.

Now Eqs (5)–(7) are written in terms of the stretched coordinates ξ and τ, and the perturbation expansion given in Equation (12) is substituted. Solving for the equations lowest order in ε with the boundary condition that all the variables, that is, $n_i^{(1)}$, $v_i^{(1)}$, and $\varphi^{(1)}$ tends to zero as $|\xi| \to \infty$

$$v_i^{(1)} = \frac{(M - v_0)}{\{1 - (M - v_0)^2\}}\varphi^{(1)} \text{ and } n_i^{(1)} = \frac{\varphi^{(1)}}{\{1 - (M - v_0)^2\}} \tag{16}$$

Going to the next higher order terms in ε and after a few algebraic operations we get the KdV equation as

$$\frac{\partial\varphi}{\partial\tau} + A\frac{\partial\varphi}{\partial\xi} + B\frac{\partial^3\varphi}{\partial\xi^3} = 0 \tag{17}$$

where

$$A = \frac{1}{2(M - v_0)} - \frac{\delta}{Cn_{e0}(M - v_0)\{1 - (M - v_0)^2\}}$$

and

$$B = \frac{\{1 - (M - v_0)^2\}}{2(M - v_0)C}.$$

Here C is given by $C = n_{e0}\ [(1 - \beta)\chi + 3\beta\chi^3\varphi_0 - p^2\sigma\chi]$.

The second and third terms of Eq. (17) are respectively the non-linear and dispersive terms. The coefficients A and B corresponding to non-linear and dispersive effect both depend on v_0. The solitary structure is the result of a balance between the non-linear and dispersive effects. While the non-linear effect steepens the solitary wave profile, the dispersive effect tries to broaden the solitary wave.

The solitary profiles depend heavily on the streaming velocity. With the increase in streaming velocity the width and amplitude of the solitary structures increase (Fig. 3). Similarly, with an increase in the positron density p (equivalent to a decrease in ion density) the solitary profiles become steeper (Fig. 4). This may be accounted for the fact that the positrons being lighter move faster than ions and they lead the solitary profiles of ion-acoustic waves. In either case, the ion density and positron density are complementary to each other and considering the mass of positrons or positive ions is immaterial as suggested by Verheest and Pillay (2008) and Baluku and Hellberg (2011).

It is clear from Figure 5 that with a slight change in Mach number M both the amplitude and width of the solitary profile increases manifold. The reason behind this is simple. Because of the wave velocity the bulk of the plasma moves in unison thereby reflecting the high potential profiles. The width is also more due to the fact that when more particles form the wave structure there is more possibility that the particles disperse from it. Figure 6 shows that as the electron temperature decreases (which is equivalent to increase in χ) the solitary profiles become steeper. However, there is no significant change in the width. This is the result of the fact that as the temperature of the electrons is less, the randomness is also low which positively affects the ion concentration in the solitary profile thereby increasing its amplitude.

But in contrast to other parameters, it is found in Figure 7 that with the increase in non-thermal parameter the amplitude of the solitary wave decreases but its width does not change. As the non-thermal parameter directly relates to the deviation from equilibrium it is evident that as we move from equilibrium conditions the particle density falls and the dispersive forces act more effectively compared with the non-linear effects. Figure 8 further shows that the positron temperature enhances the peak of the solitary waves. This is due to the fact that as the temperature of the

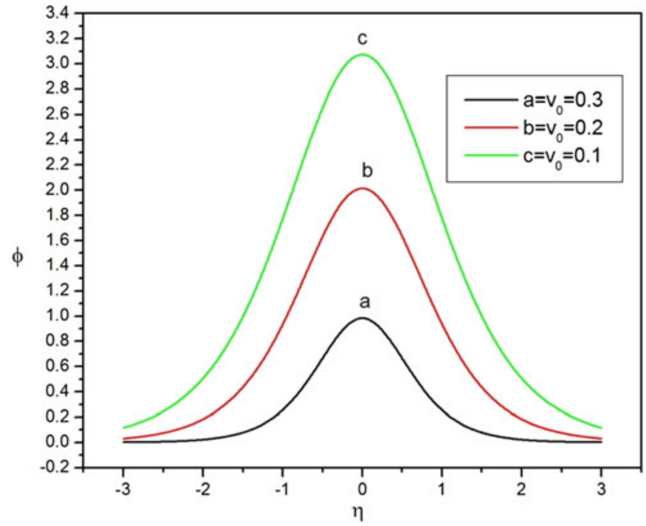

Fig. 3. Solitary profiles for different values of streaming velocity v_0.

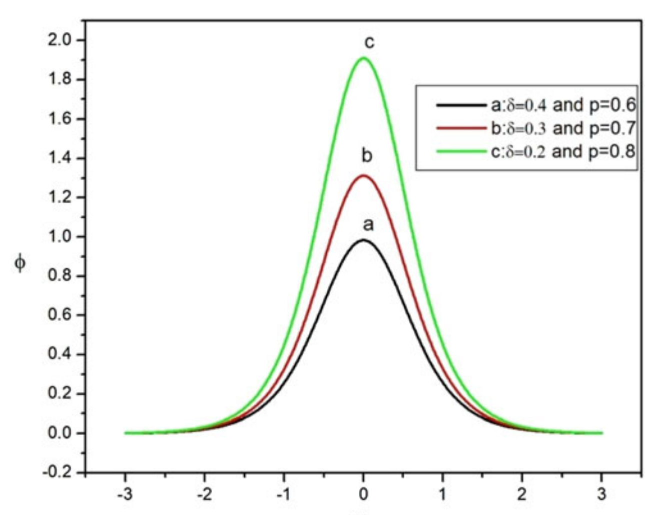

Fig. 4. Solitary profiles for different values of ion (δ) and positron (p) density.

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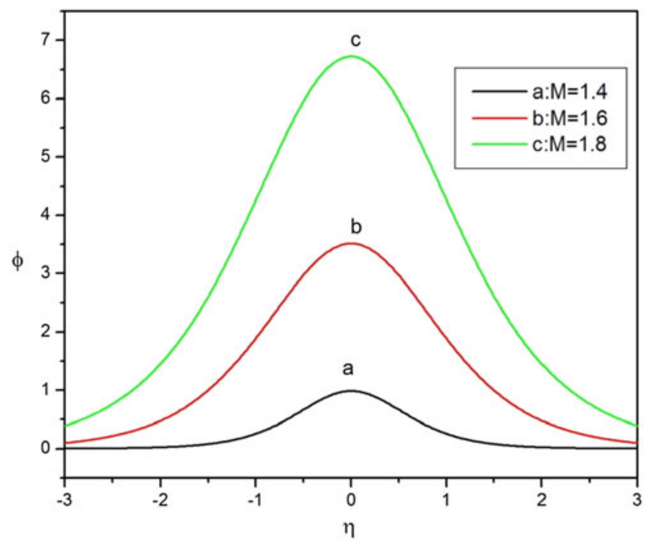

Fig. 5. Solitary profiles for different values of Mach number *M*.

positron increases, its randomness is enhanced. Positrons being lighter than positive ions offshoot the bulk and push the ions to maintain charge neutrality. The comparatively inertial ions thus stick to their positions thereby creating a rather stable ion-acoustic solitary wave profile.

NLSE and the envelope soliton

In order to study the modulational instability and the conditions of formation and properties of envelope soliton we transform our KdV equation into an NLSE by expanding Eq. (16) into a Fourier series and thereafter following regular technique:

Fourier expansion of a field quantity F is

$$F = \varepsilon^2 F_0 + \sum_{s=1}^{\infty} \varepsilon_s \{\mathrm{F}_s \exp(\mathrm{i}s\psi) + F_s^* \exp(-is\psi). \qquad (18)$$

F_0 and F_s are assumed to vary very slowly with space and time.

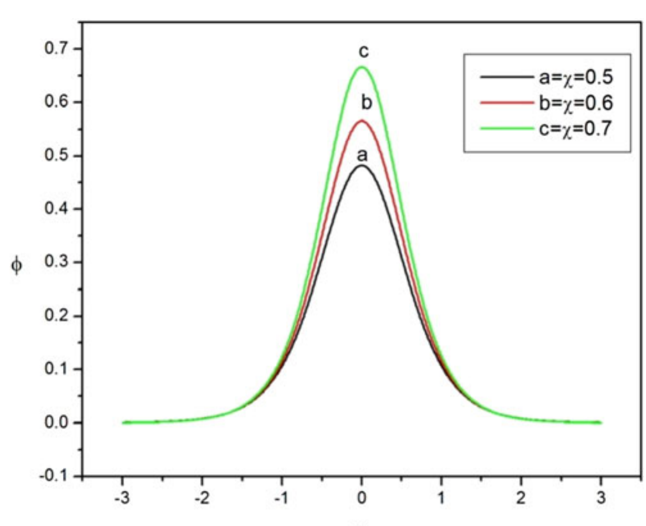

Fig. 6. Solitary profiles for different values of electron temperature reciprocal factor (χ).

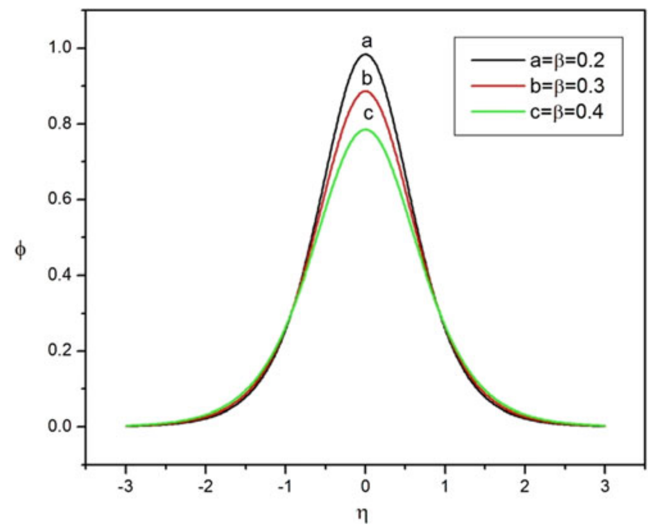

Fig. 7. Solitary profiles for different values of non-thermal parameter (β).

Expanding φ as a Fourier series we get

$$\varphi = \varepsilon^2\varphi_0 + \varepsilon\varphi_1 e^{i\psi} + \varepsilon\varphi_1^* e^{-i\psi} + \varepsilon^2\varphi_2 e^{2i\psi} + \varepsilon^2\varphi_2^* e^{-2i\psi}. \qquad (19)$$

Using perturbation expansion and using the stretched variables as

$$\rho = \varepsilon[\xi - c\tau] \text{ and } \theta = \varepsilon^2\tau. \qquad (20)$$

Therefore,

$$\frac{\partial}{\partial\tau} = -is\omega - \varepsilon c\frac{\partial}{\partial\rho} + \varepsilon^2\frac{\partial}{\partial\theta}, \qquad (21)$$

and

$$\frac{\partial}{\partial\xi} = isk + \varepsilon\frac{\partial}{\partial\rho}. \qquad (22)$$

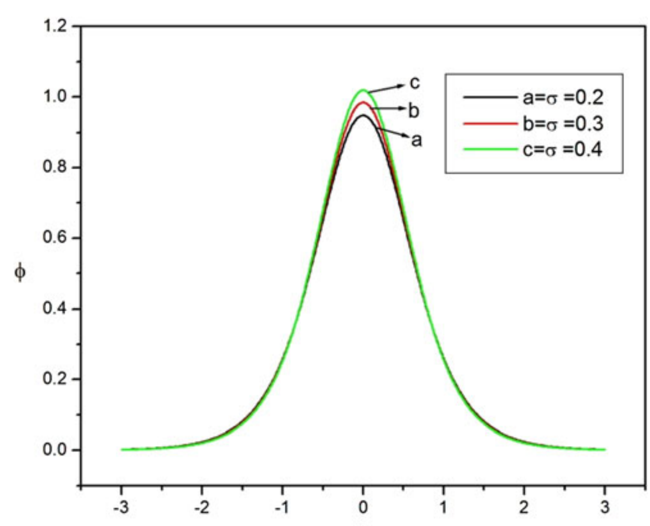

Fig. 8. Solitary profiles for different values of positron temperature reciprocal factor (σ).

Equating the coefficient of the higher order of first harmonics we get

$$i\frac{\partial\varphi}{\partial\tau} - 3kB\frac{\partial^2\varphi}{\partial\rho^2} + \frac{A^2}{6Bk}\varphi^2\varphi^* = 0. \quad (23)$$

This is the NLSE. If we consider $P = -3Bk$ and $Q = (A^2/(6Bk))$ where P represents the group dispersion coefficient and Q represents the non-linear coefficient in a regular NLSE, we obtain $PQ = -(A^2/2)$; which is always negative so that the wave is always stable thus creating dark solitons.

The growth rate of non-linear effects depends on a lot of parameters. It has been found that with the increase in streaming velocity (v_0), the non-thermal parameter (β), reciprocal of electron temperature (χ) and positron density (p) the non-linearity increases whereas it decreases with increase in the value of Mach number (M) and ion density (δ). The width of the envelop soliton is studied by the dependence of P/Q on different plasma parameters (Figs 9–12).

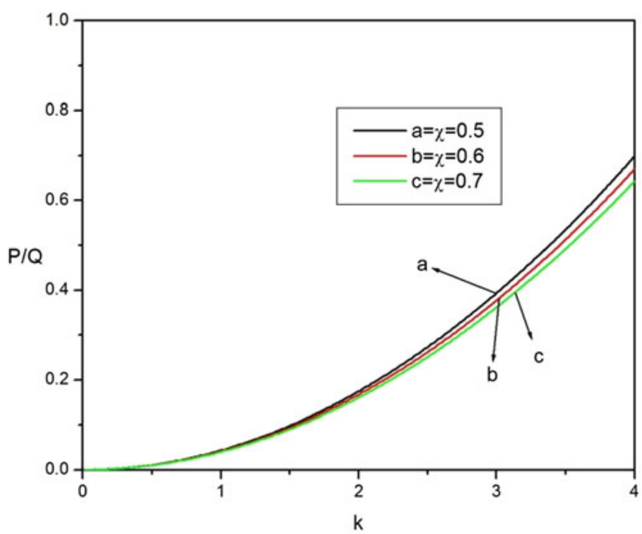

Fig. 9. *P/Q* plots for different value of reciprocal electron temperature (χ).

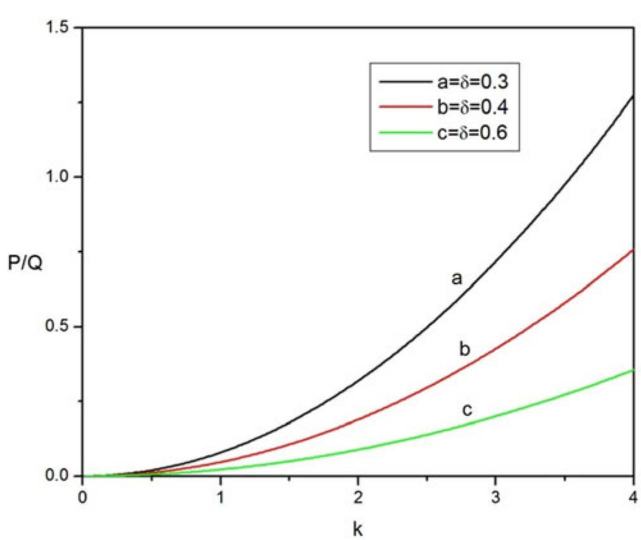

Fig. 10. *P/Q* plots for different value of ion density (δ).

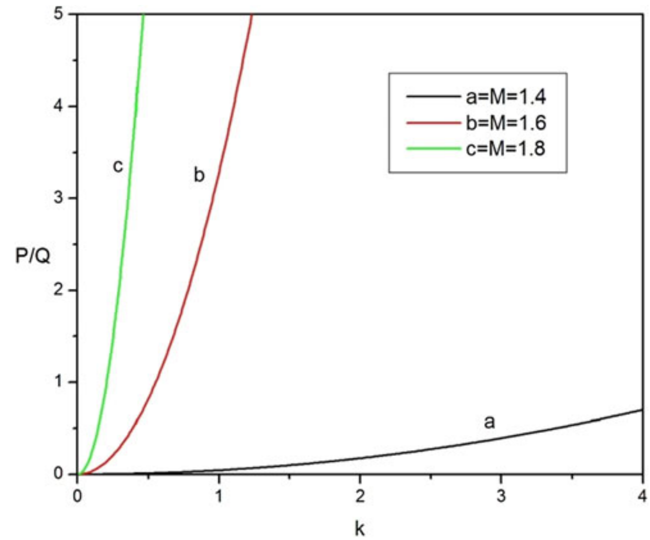

Fig. 11. *P/Q* plots for different value of Mach number (*M*).

Figure 9 shows that width of the soliton decreases with increase in the value of χ. Or in short, as the electron temperature decreases the width of the envelop soliton shrinks and vice versa. This may be due to the fact that the effect due to electron temperature is reduced due to streaming of ions which prevents localization of thermally graded electrons. Similarly, as the ion density increases the envelop soliton shrinks. It widens with the increase in positron concentration (Fig. 10).

In contrary to these, Figure 11 shows that the width of the soliton increases with increase in Mach number. This is true because of the fact that if the solitary wave is moving with a high velocity it can only propagate if the dispersive effects are less (manifested by small width).

Figure 12 is rather interesting in nature. It shows that for streaming velocity $v_0 = 0.5$, the graph suddenly becomes less sleep. It implies that the width of the soliton suddenly decreases. This may be due to the fact that there exist some kind of resonance action between the velocity of the oscillating particles and the streaming velocity. This may result in energy dissipation and the slackening of the solitary structures.

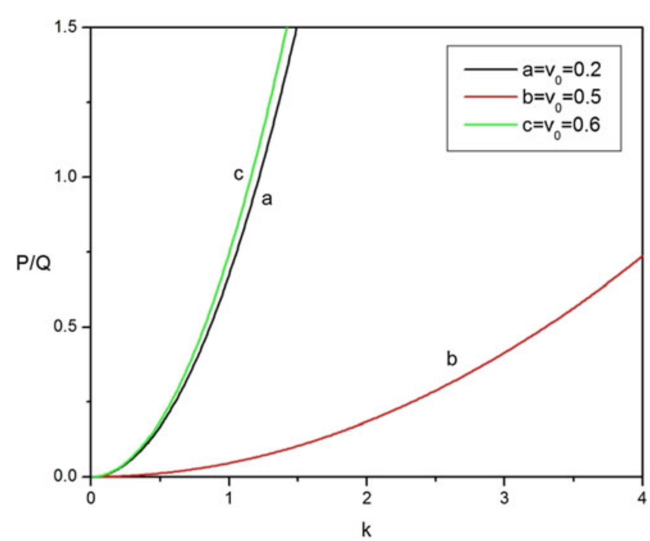

Fig. 12. *P/Q* plots for different value of streaming velocity (v_0).

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Results and conclusion

In this paper, we have studied the linear and non-linear characteristics of a three-component plasma consisting non-thermal electrons and Boltzmann positrons. The linear dispersion characteristics have been found to depend on electron temperature mainly and almost independent of other plasma parameters. The KdV equation that describes the small amplitude solitary waves have been derived and its dependence on plasma parameters such as streaming velocity (v_0), non-thermal parameter (β), reciprocal of electron temperature (χ), positron density (p), Mach number (M), and ion density (δ) have been studied in details. Whereas it increases in amplitude (and/or) width for some it decreases with the others. Further, we have studied the conditions of formation and characteristic features of envelop soliton by deriving the NLSE and found out that only stable but compressive envelop solitons are formed. The dependence of the width of these solitons on various plasma parameters was also analyzed. The linear dispersion relation was found to agree with those obtained previously [Baluku]. Similarly, the properties of envelop solitons were also in consonance with previous findings. This work may further be extended with variable charges on the ions and incorporating dust particles.

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APPENDIX

The normalized basic equations are

$$\frac{\partial n_{\mathrm{i}}}{\partial t}+\frac{\partial(n_{\mathrm{i}}v_{\mathrm{i}})}{\partial x}=0, \tag{A1}$$

$$\frac{\partial v_{\mathrm{i}}}{\partial t}+v_{\mathrm{i}}\frac{\partial}{\partial x}v_{\mathrm{i}}=\frac{\partial\phi}{\partial x}-\frac{1}{n_{\mathrm{i}}}\frac{\partial n_{\mathrm{i}}}{\partial x}, \tag{A2}$$

$$\frac{\partial^2\phi}{\partial x^2}=n_{\mathrm{e}}-\delta n_{\mathrm{i}}-pn_{\mathrm{p}}. \tag{A3}$$

The perturbation expansion of the field variables are given by

$$\begin{pmatrix} n_{\mathrm{i}} \\ v_{\mathrm{i}} \\ \varphi \end{pmatrix}=\begin{pmatrix} 1 \\ v_0 \\ \varphi_0 \end{pmatrix}+\varepsilon\begin{pmatrix} n_{\mathrm{i}}^{(1)} \\ v_{\mathrm{i}}^{(1)} \\ \varphi^{(1)} \end{pmatrix}+\varepsilon^2\begin{pmatrix} n_{\mathrm{i}}^{(2)} \\ v_{\mathrm{i}}^{(2)} \\ \varphi^{(2)} \end{pmatrix}+\cdots. \tag{A4}$$

The electrons have the non-thermal distributions is given by

$$\begin{aligned} n_{\mathrm{e}} &= n_{\mathrm{e}0}\left\{\left(1-\beta\frac{e\varphi}{kT_{\mathrm{e}}}\right)+\beta\left(\frac{e\varphi}{kT_{\mathrm{e}}}\right)^2\right\}\exp\left(\frac{e\varphi}{kT_{\mathrm{e}}}\right) \\ &= n_{\mathrm{e}0}\left\{\left(1-\beta\chi\varphi\right)+\beta\left(\chi\varphi\right)^2\right\}\exp\left(\chi\varphi\right) \\ &\left\{1-\left(\frac{\omega-kv_0}{k}\right)^2\right\}n_{\mathrm{i}}=\varphi^{(1)}. \end{aligned} \tag{A5}$$

The positron has the Boltzmann distribution and which is given by

$$n_{\mathrm{p}}=n_{\mathrm{p}0}\left(\frac{-\sigma e\varphi}{kT_{\mathrm{e}}}\right)=n_{\mathrm{p}0}\left(-\sigma\chi\varphi\right). \tag{A6}$$

Assuming space–time dependence of the field variables as $e^{i(kx-\omega t)}$ we get,

$$\frac{\partial}{\partial x}\equiv ik, \tag{A7}$$

$$\frac{\partial}{\partial t}\equiv -i\omega. \tag{A8}$$

From Continuity equations, we get by linearizing ε

$$-i\omega n_{\mathrm{i}}^{(1)}+ikv_{\mathrm{i}}^{(1)}-ikv_0n_{\mathrm{i}}^{(1)}=0,$$

Or,

$$v_{\mathrm{i}}^{(1)}=\left(\frac{\omega-kv_0}{k}\right)n_{\mathrm{i}}^{(1)}. \tag{A9}$$

From momentum equation of positron by linearizing ε we get

$$-i\omega v_{\mathrm{i}}^{(1)}+ikv_0v_{\mathrm{i}}^{(1)}=ik\varphi^{(1)}-ikn_{\mathrm{i}}^{(1)} \tag{A10}$$

From these two equations, we get

$$\left\{1-\left(\frac{\omega-kv_0}{k}\right)^2\right\}n_{\mathrm{i}}^{(1)}=\varphi^{(1)}.$$

From Poisson's equation

$$-k^2\varphi^{(1)}=\{\chi+(1+\chi\varphi_0)(-\beta\chi+2\beta\chi^2\varphi_0)+p\sigma\chi\}\varphi^{(1)}-\delta n_i^{(1)}. \tag{A11}$$

Replacing the value of $n_{\mathrm{i}}^{(1)}$ we get,

$$k^2=\left[\frac{k^2-(\omega-kv_0)^2}{\delta-k^2-(\omega-kv_0)^2}\right](\chi+p\sigma\chi-\beta\chi+\beta\chi^2\varphi_0+2\beta\chi^3\varphi_0^2).$$

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