

# Low velocity ion slowing down in a strongly magnetized plasma target

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## Abstract

Ion projectile stopping at velocity smaller than target electron thermal velocity in a strong magnetic field is investigated within a novel diffusion formulation, based on Green-Kubo integrands evaluated in magnetized one component plasma models, respectively, framed on target ions and electron. Analytic expressions are reported for slowing down orthogonal and parallel to an arbitrary large magnetic field, which are free from the usual uncertainties plaguing the standard perturbative derivations.

**Keywords:** Ion stopping; Low projectile velocity; Magnetized plasmas

Ion beam stopping in dense plasma submitted to an arbitrary large and steady magnetic field  $\mathbf{B}$  is a recurrent topic encompassing a huge range of practical situations of very high interest. This range includes ultracold plasmas (Killian, 2007), cold electron setups used for ion beam cooling (Nersisyan *et al.*, 2007), as well as many very dense systems involved in magnetized target fusions (Cereceda *et al.*, 2000, 2005), or inertial confinement fusion. This latter thermonuclear scheme presently advocates a highly regarded fast ignition scenario (Tabak *et al.*, 1994; Deutsch *et al.*, 1996), based on fem to laser produced proton or heavier ion beams impinging a precompressed capsule containing a thermonuclear fuel (Roth *et al.*, 2001; Deutsch, 2003) in it. Then,  $\mathbf{B}$  values up to  $10^{10}$  G may be reached in the laboratory (Krushelnik *et al.*, 1997). Such a topic is also of intense astrophysical concern (Winske & Gary, 2007).

These interaction geometries highlight low velocity ion slowing down (LVISD) as playing a fundamental role in asserting the confining capabilities and thermonuclear burn efficiency in dense and strongly magnetized media.

Our present goal is to demonstrate that transverse and parallel LVISD to  $\mathbf{B}$  may be given analytic expressions through a derivation free from ambiguities usually plaguing the most sophisticated combination of binary collision approximation and dielectric response (Nersisyan *et al.*, 2007). We thus

implement a radically novel approach (Dufty & Berkovsky, 1995; Dufty *et al.*, 2004) to LVISD when projectile velocity  $V_b$  remains smaller than target electron thermal velocity  $V_{\text{the}}$ . We thus consider ion stopping

$$S(V_b) = dEb(V_b)/dx, \quad (1)$$

near  $V_b = 0$ . The ratio  $S(V_b)/V_b$  usually monitors a linear stopping profile, up to 100 keV/a.m.u (Paul & Schinner, 2005) in cold matter. Similar trends are also reported in highly ionized plasma with  $\mathbf{B} = 0$  (Deutsch, 1986; Deutsch & Maynard, 2000) or  $\mathbf{B} \neq 0$  (Nersisyan *et al.*, 2007).

From now on, we intend to make use of a very powerful connection between very low velocity ion stopping and particle diffusion through Einstein characterization of ion mobility associated to thermal electron fluctuations in target, around the slow ion projectile visualized as an impurity immersed in a dense and homogeneous electron fluid.

Technically, we are then led to use the recently proposed and exact relationship (Dufty *et al.*, 2004)

$$\lim_{V_b \rightarrow 0} \frac{S(V_b)}{V_b} = k_B T_e D^{-1}, \quad (2)$$

connecting the ratio of stopping to  $V_b$  in the zero velocity limit with the ion diffusion coefficient in target.

In magnetized plasma  $D$  can be readily expressed in terms of Green-Kubo integrands involving field fluctuations in the

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target electron fluid, under the form

$$D = \frac{c^2}{B^2} \int_0^\infty d\tau \langle \vec{E}(\tau) \vec{E}(0) \rangle, \quad (3)$$

in terms of an equilibrium canonical average of the two-point autocorrelation function for fluctuating electric fields (Marchetti *et al.*, 1987, 1984; Suttorp & Cohen, 1984).

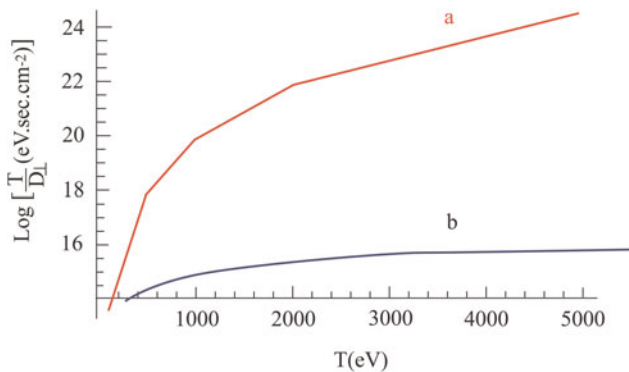
At this juncture we need to frame the Green-Kubo integrands in suitable magnetized one component-plasma (OCP) models (Marchetti *et al.*, 1987; Cohen & Suttorp, 1984) for the transverse and parallel geometry, respectively. This procedure implies that the slowly incoming ions are evolving against a background of faster fluctuating target electrons ( $V_b < V_{the}$ ) providing the OCP rigid neutralizing background thus validating the OCP assumption.

Moreover, restricting to proton projectiles impacting electron-proton plasma, we immediately perceive the pertinence of the diffusion-based LVISD as phrased by Eq. (2).

First, the proton beam can easily self-diffuse among its target homologues, while the same mechanism experienced by target electrons allow them to drag ambipolarly the incoming proton projectiles (Goldston & Retherford, 1995).

So, the transverse electron LIVSD can be either monitored by the well-known classical diffusion  $D_\perp \sim B^{-2}$ , or by the Bohmlike hydrodynamic one with  $D_\perp \sim B^{-1}$ . In the first case, momentum conservation at the level of the electron-ion pair implies that the ions will diffuse with the same coefficient as the electrons. On the other hand, the hydro Bohm diffusion across  $B$  is operated through clumps (Montgomery *et al.*, 1972) with a large number of particles involved in this collective process.

Transverse  $D_\perp$  and parallel  $D_{||}$  diffusion coefficient have already been discussed at length (Marchetti *et al.*, 1987; Cohen & Suttorp, 1984). Their derivation is based on the specific features of four finite frequency and propagating hydromodes in a strongly magnetized OCP with the ratio of plasma to cyclotron frequencies,  $\omega_p/\omega_b < 1$ .



**Fig. 1.** (Color online) Proton transverse LVISD in a dense plasma ( $n = 10^{21}$  e-cm<sup>-3</sup>,  $100 \leq T(\text{eV}) \leq 5000$  and  $B = 10^{10}$  G) in terms of  $T(\text{eV})$  cf Eq. (5); (a) electron stopping; (b) ion stopping.

First, two high frequency modes generalizes first Bernstein modes ( $B = 0$ ) and two finite frequency modes extend the  $B = 0$  shear modes.

So, exploring first the  $\omega_b \geq \omega_p$  domain, one can explicit the parallel and  $B$ -independent diffusion (Marchetti *et al.*, 1984, 1987)

$$D_{||}^{(0)} = \frac{3\sqrt{\pi}V_{thi}^2}{v_c} \sim 0(\omega_b^0), \quad (4a)$$

yielding back readily the unmagnetized ( $B = 0$ ) LVISD (Deutsch, 1986; Deutsch & Maynard, 2000), where  $V_{thi}^2 = \frac{k_B T}{M_p}$ , and  $v_c = \omega_p \epsilon_p \ell_n (1/\epsilon_p)$  in terms of the plasma parameter  $\epsilon_p = 1/n\lambda_D^3$ , where  $n$  denotes charged particle density, and  $\lambda_D$ , the Debye length, in a beam-plasma system taken as globally neutral with  $v_c/\omega_b \ll 1$ .

At the same level of approximation transverse diffusion reads as (Marchetti *et al.*, 1987)

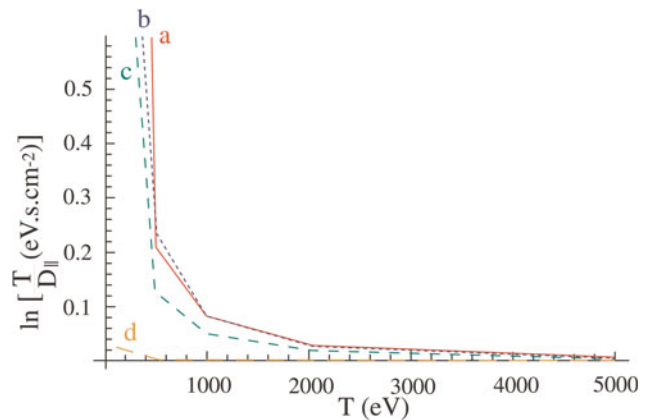
$$D_\perp^{(0)} = \frac{\Gamma^2 v_c}{3\sqrt{\pi}} \sim 0(\omega_b^{-2}), \quad (4b)$$

in terms of Larmor radius  $r_L = V_{thi}/\omega_b$ . With higher  $B$  values ( $\omega_b \gg \omega_p$ ) one reaches the transverse hydro Bohm regime featuring (Marchetti *et al.*, 1987, 1984)

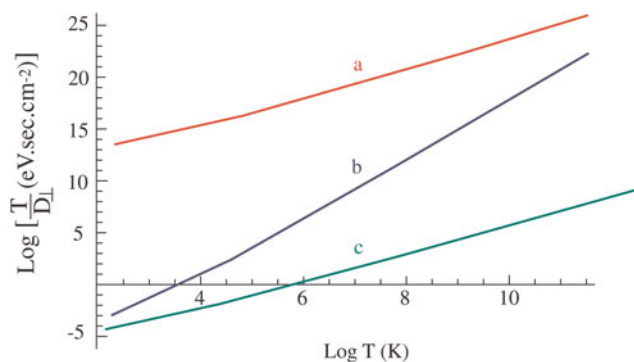
$$D_\perp^{(0)} = D_\perp^0 + \frac{0.5V_{thi}^2}{\omega_b} \epsilon_p^2 (\ln(1/\epsilon_p))^{3/2}, \quad (5)$$

while parallel diffusion retains a  $\omega_b$ -dependence through (Cohen & Suttorp, 1984)

$$D_{||}^{-1} = \frac{\Gamma^{5/2}}{\omega_p a^2} \left(\frac{3}{\pi}\right)^{1/2} \left(0.5 \text{Log}(1 + X^2) - 0.3 + \frac{0.0235}{r^2}\right), \quad (6)$$



**Fig. 2.** (Color online) Proton parallel LVISD in a dense plasma ( $n = 10^{21}$  e-cm<sup>-3</sup>,  $100 \leq T(\text{eV}) \leq 5000$  and  $B = 10^{10}$  G) in terms of  $T(\text{eV})$ , (a) electron stopping ( $B \neq 0$ ) cf Eq. (6); (b) ion stopping ( $B \neq 0$ ) cf Eq. (6); (c) ion stopping ( $B = 0$ ) cf Eq. (4a); (d) electron stopping ( $B = 0$ ) cf Eq. (4a)



**Fig. 3.** (Color online) Proton transverse LVISD in a cold plasma ( $n = 3.5 \times 10^7 \text{ e-cm}^{-3}$ ,  $10 \leq T (\text{°K}) \leq 10^5$  and  $\mathbf{B} = 10^4 \text{ G}$ ) in terms of  $T (\text{°K})$ . (a) electron stopping (Eq. 4b); (b) electron stopping (Eq. 5); (c) ion stopping.

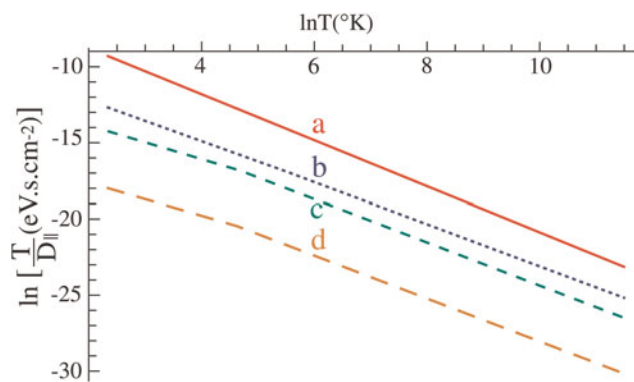
where

$$\Gamma = \frac{a^2}{3\lambda_D^2} \text{ with } a = \left(\frac{3}{4\pi n}\right)^{1/3}, \quad r = \frac{\omega_p}{\omega_b} \text{ and } X = \frac{1}{\sqrt{3}\Gamma^{3/2}}.$$

$\Gamma < 1$  encompasses, most if not all, situations of practical interest.

When electron diffusion is considered,  $V_{\text{the}}$  should be used in Eq. (5), and the above ambipolar process has to be implemented.

$D_{\perp}$  and  $D_{\parallel}$  Eqs. (5) and (6), respectively, introduced in Eq. (2) are expected to document a strong anisotropy between transverse and parallel slowing down. However, in both cases,  $\mathbf{B}$ -dependence is obviously increasing with  $\mathbf{B}^2$  (classical) or  $\mathbf{B}$  (Bohmlike). The temperature behavior is much more intriguing, as respectively displayed on Figures 1 and 2 for transverse and parallel LVISD in a highly strongly magnetized and dense target of fast ignition concern in inertial confinement fusion. One then witnesses a monotonous increase for transverse stopping (Fig. 1) contrasted to a monotonous decay for the parallel counterpart (Fig. 2).



**Fig. 4.** (Color online) Proton parallel LVISD in a cold plasma ( $n = 3.5 \times 10^7 \text{ cm}^{-3}$ ,  $\mathbf{B} = 10^4 \text{ G}$ ),  $10 \leq T (\text{°K}) \leq 10^5$ . (a) target ion slowing down ( $\mathbf{B} \neq 0$ ) cf Eq. (6); (b) target ion slowing down ( $\mathbf{B} = 0$ ) cf Eq. (4a); (c) target electron slowing down ( $\mathbf{B} \neq 0$ ) cf Eq. (6); (d) target electron slowing ( $\mathbf{B} = 0$ ) cf Eq. (4a). (b) and (c) stand in a  $\text{Log} [(M_p/M_e)^{0.5}]$  ratio.

Such a behavior is likely to be generic, because one retrieves it in the very different situation of a cold plasma used for ion beam cooling (Nersisyan *et al.*, 2007), as evidenced by the corresponding transverse (Fig. 3) and parallel (Fig. 4) behaviors.

As a summary, we implemented the very simple LVISD Eq. (2) to the *a priori* very involved ion beam-arbitrary magnetized plasma interaction. We used transverse and parallel diffusion coefficients (Marchetti *et al.*, 1987; Cohen & Suttorp, 1984) in suitably framed magnetized OCP with target electrons building up the corresponding neutralizing background. Thus, we reached analytic LVISD transverse and parallel expressions advocating contrasting temperature behavior. These quantities are of obvious significance in asserting the confinement capabilities of a very large scope of dense and strongly magnetized plasmas ranging from ultracold ones (Killian, 2007) to those featuring the highest  $\mathbf{B}$  values one can produce in the laboratory or observe in astrophysics.

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