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Theory of stimulated scattering of large-amplitude waves

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A nonlinear dispersion relation that governs the interaction between a high-frequency pump wave and the low-frequency modes in a plasma is derived. Previous results are generalized and discussed.

1. Introduction

A variety of waves can exist in a magnetized plasma. Their linear properties are determined by a dispersion relation $\tilde{D}(\omega, \mathbf{k}) = 0$, where ω is the wave frequency and $\mathbf{k}$ the wave vector. This relation is significantly changed, however, if a large-amplitude pump wave $(\omega_0, \mathbf{k}_0)$ propagates through the plasma. The pump wave couples to the fluctuations $(\omega, \mathbf{k})$, which means that sideband waves $(\omega_+, \mathbf{k}_+)$ and $(\omega_-, \mathbf{k}_-)$, where $\omega_\pm = \omega \pm \omega_0$ and $\mathbf{k}_\pm = \mathbf{k} \pm \mathbf{k}_0$, are excited. The pump and the sidebands interact to produce a nonlinear force on the plasma particles to drive the fluctuations $(\omega, \mathbf{k})$, which consequently now have to be described by a nonlinear dispersion relation.

The simplest case concerns a very high-frequency ($\omega_0 \gg \omega_{ce}$, where ω_{ce} is the electron gyrofrequency) pump wave that generates electrostatic sidebands by interacting with electrostatic low-frequency ($\omega \ll \omega_0$) modes $(\omega, \mathbf{k})$. Considering an electron–ion plasma, this interaction is described by the dispersion relation

$$\frac{1}{\chi_e} + \frac{1}{1+\chi_i} = -\frac{k^2}{k_+^2}\frac{|\mathbf{k}_+ \cdot \mathbf{v}_0|^2}{\omega_+^2\, \epsilon_+} + (+\leftrightarrow -), \tag{1}$$

where $\chi_e = \chi_e(\omega, \mathbf{k})$ and $\chi_i = \chi_i(\omega, \mathbf{k})$ are the susceptibilities for the electrons and ions respectively, ϵ_+ ($\approx 1 + \chi_e(\omega_+, \mathbf{k}_+)$) is the dielectric constant of the plasma governing the electrostatic sideband wave $(\omega_+, \mathbf{k}_+)$, and $\mathbf{v}_0$ is the amplitude of the induced electron velocity in the pump field. The symbol $(+\leftrightarrow -)$ on the right-hand side of (1) means an additional term where subscript $+$ has been changed to $-$. Equation (1), which is of some relevance for ionospheric heating experiments, was recently re-examined by Stenflo and Shukla (1997).

Similarly, it is straightforward to consider an electromagnetic pump wave, generating electromagnetic sidebands and electrostatic low-frequency modes, and to derive a nonlinear dispersion relation (e.g. Drake et al. 1974), where the right-hand side of (1) is replaced by

$$\frac{-k^2 |\mathbf{k}_+ \times \mathbf{v}_0|^2}{k_+^2(\omega_+^2 - \omega_{pe}^2 - k_+^2 c^2)} + (+\leftrightarrow -).$$

Here ω_{pe} is the electron plasma frequency and c the velocity of light. One can then investigate two of the most important parametric processes in plasmas, namely stimulated Raman and Brillouin scattering (see e.g. Sjölund and Stenflo 1967; Kruer 1980). Many original, but not very well known references can be found in textbooks (e.g. Tsytovich 1970; Gurevich 1978; Sitenko 1982) and review papers (e.g. Stenflo 1990), where early extensions to magnetized plasmas (e.g. Yu et al. 1974; Bujarbarua et al. 1974; Shukla and Tagare 1979) are also mentioned.

In order to treat three- and four-wave interactions between different modes, numerous recent derivations of nonlinear dispersion relations have appeared in the plasma physics literature. One comparatively general work (Stenflo 1995) showed the dispersion relation for arbitrary high-frequency waves $(\omega_0, \mathbf{k}_0)$ and $(\omega_\pm, \mathbf{k}_\pm)$. However, the low-frequency mode was assumed to be longitudinal. The purpose of the present paper is to extend that derivation so that the wave $(\omega, \mathbf{k})$ is also arbitrary. This leads to a dispersion relation that can be of use when most particular cases are to be evaluated. A related extension, limited to very high pump frequencies $(\omega_0 \gg \omega_{ce})$, has previously been presented by Stenflo (1985). Below we shall thus unify previous explicit expressions for the nonlinear dispersion relation.

2. The dispersion relation

Starting from the general, although not sufficiently explicit, dispersion relation (Larsson and Stenflo 1976), in Appendix A we have deduced a useful dispersion relation for an electron–ion plasma. The result is

$$\frac{(1-k^2c^2/\omega^2)\,\mathbf{E}^*\cdot\mathbf{E}+c^2\mathbf{k}\cdot\mathbf{E}^*\mathbf{k}\cdot\mathbf{E}/\omega^2+\mathbf{E}^*\cdot\boldsymbol{\chi}_e\cdot\mathbf{E}+\mathbf{E}^*\cdot\boldsymbol{\chi}_i\cdot\mathbf{E}}{(\mathbf{k}\cdot\boldsymbol{\chi}_e\cdot\mathbf{E})^*(\mathbf{k}\cdot\mathbf{E}+\mathbf{k}\cdot\boldsymbol{\chi}_i\cdot\mathbf{E})}$$
$$= -\frac{c^2(k_+^2\,c^2-\omega_+^2+\omega_{pe}^2)}{\omega_0^2(1-A_+^2)\,D_+}\Bigg[\left(1+\frac{4i\nu_e}{3\tilde{\omega}_e}\right)\mathbf{K}_+$$
$$-\frac{i\omega_{ce}}{\omega_+}\frac{k_z\,\mathbf{K}_+\times(\boldsymbol{\chi}_e\cdot\mathbf{E})^*}{(\mathbf{k}\cdot\boldsymbol{\chi}_e\cdot\mathbf{E})^*}\Bigg]\cdot\mathbf{v}_{0-}\Bigg[\mathbf{K}_+^*+\frac{i\omega_{ce}}{\omega_+}\frac{k_z\,\mathbf{K}_+^*\times\boldsymbol{\chi}_e\cdot\mathbf{E}}{\mathbf{k}\cdot\boldsymbol{\chi}_e\cdot\mathbf{E}}\Bigg]\cdot\mathbf{v}_{0+}+(+\leftrightarrow-). \quad (2)$$

Here $\mathbf{E}$ is the electric field of the excited low-frequency mode, an asterisk denotes the complex conjugate, $\boldsymbol{\chi}_e$ and $\boldsymbol{\chi}_i$ are the electron and ion susceptibility tensors. The dielectric function D_+ in the denominator on the right-hand side stands for $D(\omega_+, \mathbf{k}_+)$, where D is defined as

$$D(\omega,\mathbf{k}) \equiv \left(1-\frac{\omega_{pe}^2}{\omega^2}-\frac{k^2v_t^2}{\omega^2}\right)(k^2c^2-\omega^2+\omega_{pe}^2)^2$$
$$-\frac{\omega_{ce}^2}{\omega^2}(k^2c^2-\omega^2)\;\left[(k^2c^2-w^2)\left(1-\frac{\omega_{pe}^2}{\omega^2}-\frac{k_z^2\,v_t^2}{\omega^2}\right)+\frac{k_\perp^2\,c^2\omega_{pe}^2}{\omega^2}\right]. \quad (3a)$$

The wave vector $\mathbf{k}$ has been written here as $k = k_z\,\hat{\mathbf{z}}+\mathbf{k}_\perp$, where $\hat{\mathbf{z}}$ denotes the direction of the external magnetic field, v_t^2 is the square of the electron thermal velocity times the ratio of specific heats, ν_e is the electron collision frequency, and

$$\tilde{\omega}_e = \omega+\frac{i}{\tau}+i\mathbf{k}\cdot\mathsf{H}\cdot\mathbf{k},$$

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where τ is the energy relaxation time and H is the heat-diffusion tensor. Furthermore, we have introduced the notation

$$\mathbf{v}_{0+} = \mathbf{v}_0, \quad \mathbf{v}_{0-} = \mathbf{v}_0^*,$$

$$\mathbf{K}_+ = \mathbf{k}_+ - k_{z+} A_+^2 \hat{\mathbf{z}} - iA_+ \hat{\mathbf{z}} \times \mathbf{k}_+, \tag{3b}$$

$$A_+ = \frac{\omega_{ce}}{\omega_+} \frac{k_+^2 c^2 - \omega_+^2}{(k_+^2 c^2 - \omega_+^2 + \omega_{pe}^2)}. \tag{3c}$$

Equation (2) obviously generalizes equation (28) (corrected for a misprint) in Stenflo (1995).

In the particular case when $(\omega, \mathbf{k})$ is a longitudinal wave, we can write $\mathbf{E}$ as $-i\mathbf{k}\phi$. The left-hand side of (2) then immediately reduces to

$$\left(\frac{1}{\chi_e} + \frac{1}{1+\chi_i}\right) \Big/ k^2,$$

where $\chi \equiv \mathbf{k} \cdot \boldsymbol{\chi} \cdot \mathbf{k}/k^2$. On the right-hand side of (2) we consequently replace $\boldsymbol{\chi}_e \cdot \mathbf{E}/\mathbf{k} \cdot \boldsymbol{\chi}_e \cdot \mathbf{E}$ by $\mathbf{K}/\mathbf{k} \cdot \mathbf{K}$, where

$$\mathbf{K} = \mathbf{k} - \frac{i\omega_{ce}}{\omega} \hat{\mathbf{z}} \times \mathbf{k} - \frac{\omega_{ce}^2}{\omega^2} k_z \hat{\mathbf{z}}.$$

Equation (2) is then equivalent to equation (28) in Stenflo (1995).

In order to study waves that are not longitudinal, we have to rewrite (2) in a more explicit form, however. This will be outlined in the next section.

3. The cold electron–ion plasma

Our equations are comparatively easy to treat for a one-component (electron) plasma with immobile ions (cf. Stenflo 1994). The algebraic difficulties are, however, significantly increased for a two-component plasma with mobile ions. This is the reason why such calculations have previously been avoided. However, in order to include MHD modes, it is necessary to try to find general explicit expressions also for a plasma with two or more particle species. In this section, we limit the length of the formulae by presenting the result only for a cold collisionless electron–ion plasma. In Appendix B we show that the nominator of the left-hand side of (2) ($\mathbf{E}^* \cdot \mathsf{M} \cdot \mathbf{E}$, where $\mathsf{M} \equiv \mathsf{1} + \boldsymbol{\chi}_e + \boldsymbol{\chi}_i + (c^2/\omega^2)\mathbf{k} \times \mathbf{k} \times$) can be written as

$$\mathbf{E}^* \cdot \mathsf{M} \cdot \mathbf{E} = \frac{(k_\perp^2 c^2 - \omega^2 + \sum \omega_p^2)\, \tilde{D}(\omega, \mathbf{k})}{(k^2 c^2 - \omega^2 + \sum \omega_p^2)^2\, k_\perp^2 \theta_1} \mathbf{k} \cdot \mathbf{E}^* \mathbf{k} \cdot \mathbf{E}, \tag{4}$$

where

$$\tilde{D}(\omega, \mathbf{k}) = (\theta_1^2 - \theta_2^2)\left(1 - \frac{\sum \omega_p^2}{\omega^2}\right) - \frac{k_\perp^2 c^2}{\omega^2}\left(\theta_1 \sum \frac{\omega_p^2 \omega_c^2}{\omega^2 - \omega_c^2} - \theta_2^2\right), \tag{5a}$$

and where we have introduced the notation

$$\theta_1 = k^2 c^2 - \omega^2 + \omega^2 \sum \frac{\omega_p^2}{\omega^2 - \omega_c^2}, \tag{5b}$$

$$\theta_2 = \omega \sum \frac{\omega_p^2 \omega_c}{\omega^2 - \omega_c^2} \tag{5c}$$

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The summations in (4) and (5) are over the two particle species. We note that $\tilde{D}$ reduces to $(1-\omega_{ce}^2/\omega^2)^{-1}D$ for a cold one-component plasma. Furthermore, we have

$$\boldsymbol{\chi}\cdot\mathbf{E} = -\frac{\omega_p^2 c^2}{\omega^2}\left\{\frac{\omega^2}{(\theta_1^2-\theta_2^2)(\omega^2-\omega_c^2)}\left[\left(\theta_1-\frac{\omega_c}{\omega}\theta_2\right)\mathbf{k}_\perp+\frac{i\omega_c}{\omega}\left(\theta_1-\frac{\omega}{\omega_c}\theta_2\right)\mathbf{k}\times\hat{\mathbf{z}}\right]\right.$$
$$\left.+\frac{k_z}{k^2c^2-\omega^2+\sum\omega_p^2}\hat{\mathbf{z}}\right\}\mathbf{k}\cdot\mathbf{E}, \quad (6a)$$

and consequently

$$\mathbf{k}\cdot\boldsymbol{\chi}\cdot\mathbf{E} = -\frac{\omega_p^2 c^2}{\omega^2}\left[\frac{\omega^2(\theta_1-\omega_c\theta_2/\omega)\,k_\perp^2}{(\theta_1^2-\theta_2^2)(\omega^2-\omega_c^2)}+\frac{k_z^2}{k^2c^2-\omega^2+\sum\omega_p^2}\right]\mathbf{k}\cdot\mathbf{E}. \quad (6b)$$

Inserting (4)–(6) into (2), we have thus deduced our final formula for the interaction between the pump wave, the two sideband waves and the low-frequency mode in a cold electron–ion plasma. It is straightforward to extend these expressions to include temperature effects, but the resulting formulae will then be very lengthy.

4. Summary

Our nonlinear dispersion relation covers previous work on pump-wave-excited waves in uniform plasmas. Thus, to mention just a few different examples, it includes the situations where all the involved waves are electrostatic (see e.g. Akimoto 1994), and when an upper-hybrid pump excites upper-hybrid, electromagnetic or mixed sidebands, as well as various low-frequency modes (ion sound, magnetosonic, kinetic Alfvén waves, etc.) (see e.g. Shukla and Stenflo 1995; Yukhimuk et al. 1998), as well as when a whistler wave decays into a Langmuir wave or into a lower-hybrid sideband wave and an ultralow-frequency electromagnetic wave (see e.g. Chian 1995; Yukhimuk and Roussel-Dupre 1997). In particular, for a kinetic Alfvén wave, (5) reduces to

$$\tilde{D} = \theta_3\left[k_z^2 v_A^2 - \omega^2\left(1+\frac{k_\perp^2 c^2}{\tilde{\omega}_p^2}\right)\right],$$

where v_A is the Alfvén velocity, and

$$\theta_3 = -\frac{k_\perp^2(k^2c^2+\omega_{pe}^2)\,c^4}{k_z^2 v_A^4}, \qquad \tilde{\omega}_p^2 = \frac{\omega_{pe}^2}{1-k_z^2 v_t^2/\omega^2}.$$

Here we have generalized (5) by including the main thermal term. Obviously $\tilde{D}=0$ yields

$$\omega^2 = \frac{k_z^2 v_A^2(1+k_\perp^2\rho_s^2)}{1+k_\perp^2\lambda_e^2},$$

where ρ_s is the ion radius at the electron temperature, and λ_e is the collisionless electron skin depth (see e.g. Shukla and Stenflo 1995). Waves in very strongly magnetized ($\omega_{ce} \gg \omega_0$) plasmas (Jaiman et al. 1997; Jaiman and Tripathi 1998) can also be considered.

It should be stressed that, in the absence of collisions ($\nu_e = 0$), the coupling coefficient is a *squared* quantity. This is obvious already from an inspection of (A 1), where the coefficients $|V_{\pm}|^2$ have been written in such a form. Authors with different results may therefore be advised to reconsider their calculations.

Finally, it should be pointed out that the *resonant* interaction between three waves can be treated by slightly different methods than those discussed above (see the review paper by Stenflo 1994), and that extensions of the theory to plasmas with a slightly nonuniform background are rather straightforward.

Appendix A

Let us start with the general dispersion relation (Larsson and Stenflo 1976; Larsson et al. 1976)

$$\mathbf{E}^{*}\cdot\mathsf{M}\cdot\mathbf{E} = (4\pi)^2\left(\frac{|V_+|^2}{\mathbf{E}_+^{*}\cdot\mathsf{M}_+\cdot\mathbf{E}_+}+\frac{|V_-|^2}{\mathbf{E}_-^{*}\cdot\mathsf{M}_-\cdot\mathbf{E}_-}\right), \tag{A 1}$$

where the coupling coefficients, which have been deduced for a general (hot) uniform plasma (see e.g. Larsson and Stenflo 1976; Stenflo 1994), are presented here for simplicity for a cold plasma where the electron-to-ion mass ratio as well as ω/ω_0 are much smaller than unity. Thus

$$V_{\pm} \approx \frac{m_e n_0}{\omega}\left[\mathbf{k}\cdot\mathbf{v}^{*}\mathbf{v}_{\pm}\cdot\mathbf{v}_{0\mp}+\frac{i\omega_{ce}k_z}{\omega_{\pm}}(\mathbf{v}^{*}\times\mathbf{v}_{\pm})\cdot\mathbf{v}_{0\mp}\right], \tag{A 2}$$

where m_e is the electron mass, n_0 is the electron equilibrium density, and $\mathbf{v}$, $\mathbf{v}_{\pm}$ and $\mathbf{v}_{0\mp}$ (with $\mathbf{v}_{0+}=\mathbf{v}_0$ and $\mathbf{v}_{0-}=\mathbf{v}_0^{*}$) are the linear electron velocity perturbations of the corresponding waves (see the review by Stenflo (1994) for a detailed description, including also the terms that are neglected in (A 2)).

Equation (A 1) guided the author when the EISCAT Scientific Advisory Committee in a meeting in 1976 discussed forthcoming stimulated electromagnetic emission (SEE) experiments in Northern Scandinavia. In particular, the interaction between upper- and lower-hybrid waves and electromagnetic waves (Larsson et al. 1976) then turned out to be of interest (see also Stenflo 1990).

Using the results in Stenflo (1995), we have

$$\mathbf{E}_{\pm}^{*}\cdot\mathsf{M}_{\pm}\cdot\mathbf{E}_{\pm} = \frac{c^2 D_{\pm}}{(k_{\pm}^2 c^2-\omega_{\pm}^2+\omega_{pe}^2)^3(1-A_{\pm}^2)}\mathbf{k}_{\pm}\cdot\mathbf{E}_{\pm}^{*}\,\mathbf{k}_{\pm}\cdot\mathbf{E}_{\pm}. \tag{A 3}$$

Furthermore, noting that $\mathbf{v}_{\pm}$ is proportional to $\mathbf{K}_{\pm}$, we adopt equation (15) in Stenflo (1995) to rewrite (A 1) in the form of (2). We then also slightly generalize all formulae in this appendix by including the collisional effects in the ionospheric E region. Additional nonlinear forces will appear in the F region (Kuo et al. 1997, 1998).

Appendix B

For a cold plasma, we have

$$\mathsf{M}\cdot\mathbf{E} = \left(1-\frac{k^2c^2}{\omega^2}\right)\mathbf{E}+\frac{c^2\mathbf{k}\cdot\mathbf{E}}{\omega^2}\mathbf{k}-\sum\frac{\omega_p^2}{\omega^2-\omega_c^2}\left(\mathbf{E}-\frac{i\omega_c\,\hat{\mathbf{z}}\times\mathbf{E}}{\omega}-\frac{\omega_c^2\,\mathbf{E}_z\,\hat{\mathbf{z}}}{\omega^2}\right). \tag{B 1}$$

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Multiplying (B 1) by $\mathbf{E}^*$, we obtain

$$\mathbf{E}^*\cdot\mathsf{M}\cdot\mathbf{E} = -\frac{\theta_1\,\mathbf{E}_\perp^*\cdot\mathbf{E}_\perp}{\omega^2} + \left(1-\frac{k^2c^2}{\omega^2}-\sum\frac{\omega_p^2}{\omega^2}\right)\mathbf{E}_z^*\,\mathbf{E}_z - \frac{i\theta_2(\mathbf{E}^*\times\mathbf{E})_z}{\omega^2} + \frac{c^2\mathbf{k}\cdot\mathbf{E}^*\mathbf{k}\cdot\mathbf{E}}{\omega^2}$$

$$= -\frac{\theta_1\,|(\mathbf{k}\times\mathbf{E})_z|^2}{k_\perp^2\,\omega^2} + \frac{2i\theta_2\,\mathbf{k}\cdot\mathbf{E}_\perp(\mathbf{k}\times\mathbf{E})_z^*}{k_\perp^2\,\omega^2} + \left(1-\frac{k_z^2\,c^2}{\omega^2}-\sum\frac{\omega_p^2}{\omega^2-\omega_c^2}\right)\frac{|\mathbf{k}_\perp\cdot\mathbf{E}_\perp|^2}{k_\perp^2}$$

$$+\left(1-\frac{k_\perp^2\,c^2}{\omega^2}-\sum\frac{\omega_p^2}{\omega^2}\right)\mathbf{E}_z^*\,\mathbf{E}_z + \frac{2c^2\,\mathbf{k}_\perp\cdot\mathbf{E}_\perp\,k_z\,\mathbf{E}_z^*}{\omega^2}. \qquad \text{(B 2)}$$

Next, using the linear theory $\mathsf{M}\cdot\mathbf{E}=0$, we can replace $k_z\,\mathbf{E_z}$ in (B 2) by

$$\frac{k_z^2\,c^2}{k_\perp^2\,c^2-\omega^2+\sum\omega_p^2}\,\mathbf{k}_\perp\cdot\mathbf{E}_\perp,$$

and $(\mathbf{k}\times\mathbf{E})_z$ by $(i\theta_2/\theta_1)\mathbf{k}_\perp\cdot\mathbf{E}_\perp$. After some straightforward algebra, we then rewrite (B 2) as

$$\mathbf{E}^*\cdot\mathsf{M}\cdot\mathbf{E} = \frac{\tilde{D}\,|\mathbf{k}_\perp\cdot\mathbf{E}_\perp|^2}{(k_\perp^2\,c^2-\omega^2+\sum\omega_p^2)\,k_\perp^2\,\theta_1}, \qquad \text{(B 3)}$$

which is the same as (4).

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