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FINANCIAL GLOBALIZATION AND ECONOMIC GROWTH

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Using a two-sector endogenous growth model, the speed of convergence is determined primarily by the gap in rates of return between physical and human capital. In closed economies, for a typical situation of having relatively less physical capital than in a steady state, the return on physical capital will be significantly high, whereas the return on human capital will be relatively low. This gap in rates of return is quite large when the economy is not at its steady state. In open economies, where human capital is nontradable, the gap in rates of return is small, as is the gap between the international interest rate (which is less than the closed economies return on physical capital) and the return on human capital. Convergence in open economies will be relatively slow, and convergence in closed economies will be relatively fast, and therefore there is little gain from financial liberalization.

Keywords: Capital Mobility, Speed of Convergence, Welfare Gains, Open versus Closed Economies

1. INTRODUCTION

Using the one-sector neoclassical growth model, Gourinchas and Jeanne (2006) show the positive effects of capital mobility on convergence and growth, and point out that the welfare gains from capital mobility are small. These authors suggest that a model with human capital leads to a reduction of the speed of convergence of the closed economy and may increase welfare gains.

In this paper we want to analyze the effects of capital mobility on the speed of convergence and welfare. We use a two-sector endogenous growth model, with two capitals (physical and human) accumulable by specific production functions, in which the speed of convergence is determined primarily by the gap in rates of

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return between physical and human capital. In a closed economy, for a typical situation of having relatively less physical capital than in a steady state, the return on physical capital will be significantly high, whereas the return on human capital will be relatively low. This gap in rates of return is quite large when the economy is not in its steady state. In an open economy, where human capital is nontradable, the gap in rates of return is small, as is the gap between the international interest rate (which is less than the closed economy return on physical capital) and the return on human capital. Convergence in open economies will be relatively slow, and convergence in closed economies will be relatively fast, and therefore there is little gain from financial liberalization. The result on the speed of convergence reverses the conclusion given by the one-sector neoclassical growth model for the same experiment. The welfare gains from capital mobility are equivalent to a permanent increase in consumption by approximately 1% and in line with Gourinchas and Jeanne (2006).

With a two-sector endogenous growth model, we can analyze the transitional dynamics using a no-arbitrage condition, under which the gap in rates of return between physical and human capital gives the incentive to accumulate relatively more on one of the capitals. The no-arbitrage condition provides a simple way to analyze the transitional dynamics of the model and gives an intuition of the mechanism at work. We use this no-arbitrage condition to compare the speed of convergence of the closed and the open economies and understand the difference. We consider a small open economy with the same structure of production as the closed economy. The consumption goods and physical capital are tradable and produced with one production function, whereas human capital is nontradable and produced with another production function.

The intuition of this model for the speeds of convergence is associated with a very low level of adjustment costs, but we also analytically derive all the results for the Lucas (1988) model with adjustment costs. The presence of adjustment costs in the open economy gives the possibility of having at work the stable mechanism of relative price, which is on the basis of the simple and intuitive mechanism of our model.

Our paper is related to several important strands of theory on growth, convergence, and openness.[1] First there is the literature on two-sector endogenous growth models, such as Lucas (1988) and Rebelo (1991). The characterization of transitional dynamics and the conditions of stability in these models is discussed in Bond et al. (1996), whereas Mulligan and Sala-i-Martin (1993) provide numerical simulations for the transitional dynamics.

Using the Lucas (1988) model, Ortigueira and Santos (1997) arrive at an analytical expression for the speed of convergence of a two-sector endogenous growth model, derived in a theorem presented in the appendix of their paper. Their paper is concerned with the fact that the speed of convergence of a closed economy in the two-sector endogenous growth model does not depend on the preference parameters, as in the one-sector neoclassical growth model. Our paper presents an analytical expression not only for the speed of convergence for the same model but also for the general model and for the Lucas (1988) model with adjustment costs. We also present the expressions for the speed of convergence of the open economy. Moreover, our paper presents an intuition for the transitional dynamics based on a no-arbitrage condition, which is not developed in Bond et al. (1996) or in Ortigueira and Santos (1997). The intuition associated with the transitional dynamics and the expressions for the speed of convergence of the closed and open economies, developed in our paper, are the basis for understanding our main result.

A second strand, with a tradition in international economics, makes a distinction between tradable and nontradable goods. For one of the first formalizations in the spirit of our model see Bruno (1976). Farmer and Lahiri (2006) analyze the transitional dynamics of a two-country world economy. These authors use a two-sector endogenous growth model and make a distinction between tradable and nontradable sectors, but they do not present analytical expressions for the speed of convergence. Turnovsky (1996) develops a dependent economy using a two-sector endogenous growth model, but he does not analyze the speed of convergence. He is interested in the dynamics of the model following a shock in the parameters.

The paper is organized as follows. Section 2 presents a two-sector endogenous growth model with adjustment costs and points out the difference in the budget constraint between the closed and the open economies. Section 3 develops the closed-economy version with adjustment costs and its dynamics, and presents the intuition for the transitional dynamics through the no-arbitrage condition. Section 4 presents the small open economy with adjustment costs and its dynamics. In this section we also compare the closed and open economies, and we show the main result of the paper. In Section 5 we generalize the basic model, we present a numerical example of our main result, and we discuss the robustness of this result. We also present numerical simulations for the welfare gains from capital mobility. Finally, in Section 6, we present the concluding remarks. The Appendices contain the details of the derivation of some analytical results.

2. THE MODEL

This section presents the structure of a two-sector endogenous growth model with adjustment costs for physical capital and points out the difference in the budget constraint between a closed economy and an open economy with capital mobility.

An analysis of the general model (where the production of human capital also uses physical capital, but without adjustment costs) and its dynamics is made by Bond et al. (1996), who present an analytical and geometrical solution for the dynamics of the model.[2]

The utility function of the representative agent is given by[3]

$$U(C) = \log C, \tag{1}$$

where C represents consumption.

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There are two sectors of production, one (Y^K) for consumption goods and accumulation of physical capital, $\dot{K}$, and the other (Y^H) for accumulation of human capital, $\dot{H}$. The technologies are similar to the ones in Lucas (1988), but there are no externalities. We have

$$Y^K = AK^{\alpha}(uH)^{1-\alpha}, \tag{2}$$

$$Y^H = B[(1-u)H]. \tag{3}$$

Without loss of generality, the depreciation rate is assumed equal to zero for physical capital and human capital. α ($0 < \alpha < 1$) is a parameter with $1-\alpha$ representing the share of nontradable capital in the consumption sector and u ($0 \leq u \leq 1$) is the fraction of the human capital used in the consumption sector. A and B represent the level of technology in the consumption and the human capital sectors, respectively.

Using the structure of production in Lucas (1988), we are assuming that the production of goods and physical capital is more intensive in physical capital. That is, $\alpha > \beta$, where β means the share of human capital in the sector producing human capital. In the Lucas (1988) model $\beta = 0$. This condition leads to the stability of the differential equation associated with the relative price, as shown in Bond et al. (1996). As we can observe in the following sections, this condition is important to our analysis and intuition of transitional dynamics.

The planning problem for this (closed) economy is given by

$$\max_{C,u,I,K,H} \int_0^{+\infty} (\log C)\, e^{-\rho t} dt \tag{4}$$

subject to

$$AK^{\alpha}(uH)^{1-\alpha} = C + I\left[1 + m\left(\frac{I}{K}\right)\right], \tag{5a}$$

$$\dot{K} = I, \tag{5b}$$

$$\dot{H} = B[(1-u)H]. \tag{5c}$$

$K_0 > 0$ and $H_0 > 0$ are given and $C \geq 0$. The parameter ρ represents the subjective discount rate and I represents investment in physical capital.

Notice that equation (5a) is the budget constraint of the closed economy, which will be replaced by equation (18a) in the open economy. This is the only difference between the two economies. The expression for the adjustment costs, $m(\frac{I}{K})$, is represented in the following way:[4]

$$m\left(\frac{I}{K}\right) = \frac{\frac{h}{2}\left(\frac{I}{K} - a\right)^2}{\frac{I}{K}}, \tag{6}$$

$$a = g,$$

$$g = \left(r^K\right)^* - \rho > 0. \tag{7}$$

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The parameter h is the sensitivity of the adjustment costs to the ratio of investment to physical capital. We specify the adjustment costs in terms of net investment, as we assume $a = g$, and g represents the growth rate of the closed economy in the steady state.

Using this structure of production, we have the same steady state values for all variables related to production in the closed and the open economies. These economies differ in consumption and, thus, also in terms of welfare.

3. THE CLOSED ECONOMY

3.1. The Model and the Transitional Dynamics

In this subsection, we provide an intuition for the behavior of the transitional dynamics, based on a no-arbitrage condition for the accumulation of physical and human capital. This intuition will be the basis for understanding the different behavior of the speed of convergence of the closed and the open economies. We would like to point out that such an intuition is not developed in Bond et al. (1996) or Ortigueira and Santos (1997).

The Hamiltonian for the problem of the closed economy, given by equations (4) and (5a)–(5c), is

$$J = U(C)e^{-\rho t} + \eta e^{-\rho t}\left\{AK^{\alpha}(uH)^{1-\alpha} - C - I\left[1 + m\left(\frac{I}{K}\right)\right]\right\}$$
$$+\eta q e^{-\rho t} I + \mu e^{-\rho t}\{B[(1-u)H]\},$$

where η is the Lagrangian multiplier associated with equation (5a), ηq is the co-state variable in installed physical capital,[5] and μ is the costate variable in human capital.

We obtain the following first-order conditions:[6]

$$C^{-1} = \eta, \tag{8a}$$

$$r^K = \alpha AK^{\alpha-1}(uH)^{1-\alpha}, \tag{8b}$$

$$r^H = (1-\alpha)AK^{\alpha}(uH)^{-\alpha} = \frac{\mu}{\eta}B, \tag{8c}$$

$$\dot{\mu} = \mu\left(\rho - \frac{\eta}{\mu}r^H\right), \tag{8d}$$

$$q = 1 + h\left(\frac{I}{K} - a\right), \tag{8e}$$

$$\dot{q} = \left(\rho - \frac{\dot{\eta}}{\eta}\right)q - r^K - h\left(\frac{I}{K} - a\right)\frac{I}{K} + \frac{h}{2}\left(\frac{I}{K} - a\right)^2. \tag{8f}$$

The transversality conditions are

$$\lim_{t\to+\infty} \eta q e^{-\rho t} K = 0 \quad \text{and} \quad \lim_{t\to+\infty} \mu e^{-\rho t} H = 0.$$

Equation (8e) can be specified as

$$\frac{q-1}{h} + g = \frac{I}{K} = \frac{\dot{K}}{K}. \tag{9}$$

We can define $P = \mu/\eta$ as the relative price of human capital in terms of goods. The rental rates of physical and human capital are represented, respectively, by r^K and r^H. Assuming there is incomplete specialization in production, the rental rate of each capital is a function of the relative price in a two-sector and two-factor model,

$$r^K = \alpha A \phi^{\alpha-1} P^{\frac{\alpha-1}{\alpha}}, \tag{10a}$$

$$r^H = (1-\alpha) A \phi^{\alpha} P, \tag{10b}$$

where $\phi \equiv [\frac{B}{A}(1-\alpha)^{-1}]^{\frac{1}{\alpha}}$ is a constant.

Notice that $\eta = C^{-1} = U'(C)$. It follows that $\dot{P}/P = \dot{\mu}/\mu - \dot{\eta}/\eta = \dot{\mu}/\mu + \dot{C}/C$. We also define $c = C/K$, $k = K/H$, $y^K = Y^K/K$, and $y^H = Y^H/H$. Following Bond et al. (1996), the Cobb–Douglas production functions can be written as

$$y^K \equiv \frac{Y^K}{K} = \frac{1}{\alpha} r^K, \tag{11a}$$

$$y^H \equiv \frac{Y^H}{H} = B - \frac{1-\alpha}{\alpha}\frac{r^K}{P} k. \tag{11b}$$

The dynamic system is given by

$$\frac{\dot{c}}{c} = \frac{\dot{C}}{C} - \left(\frac{q-1}{h} + g\right), \tag{12a}$$

$$\frac{\dot{k}}{k} = \left(\frac{q-1}{h} + g\right) - y^H, \tag{12b}$$

$$\dot{q} = \left(\rho + \frac{\dot{C}}{C}\right) q - r^K - (q-1)\, a - \frac{h}{2}\left(\frac{q-1}{h}\right)^2, \tag{12c}$$

$$\frac{\dot{P}}{P} = \left(\rho - \frac{r^H}{P}\right) + \frac{\dot{C}}{C}, \tag{12d}$$

and it takes into account the transversality conditions for K and H.

It is enlightening to rearrange equation (12c) and substitute the expression $\frac{\dot{C}}{C}$ in equation (12d):

$$\frac{\dot{P}}{P} = \tilde{r}^K - \frac{1}{P} r^H, \tag{13}$$

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where

$$\tilde{r}^K = \frac{r^K}{q} + \frac{\dot{q}}{q} + \frac{(q-1)\,a + \frac{h}{2}\left(\frac{q-1}{h}\right)^2}{q}. \tag{14}$$

We can analyze the transitional dynamics of the system by taking into account the behavior of the relative price and the remuneration of the two capitals, following the no-arbitrage condition given by equation (13). Investing one unit of physical capital yields a net marginal product of physical capital in the consumption goods equal to $\tilde{r}^K$. On the other hand, investing $\frac{1}{P}$ units of human capital yields $\frac{\dot{P}}{P} + \frac{1}{P}r^H$.

As stated in equations (6) and (7), we have $a = g$. Following equation (9), in the steady state we also have $I/K = \dot{K}/K = g$ and, thus, $q^* = 1$. It follows that in the steady state, $(\tilde{r}^K)^* = (r^K)^* = (\frac{1}{P}r^H)^* = r^*$, by equations (14) and (13). $r^*$is the steady state real interest rate, equal for this closed economy and for the small open economy of Section 4.[7]

The transitional path applies to any situation around the steady state, when $k_0 \neq k^*$. For example, by assuming that $k_0 < k^*$, physical capital is relatively less abundant and its remuneration will be relatively higher than that of human capital: $\tilde{r}^K > \frac{1}{P}r^H$. Notice that there will be an incentive to invest relatively more in physical capital. Assuming that the adjustment costs go to zero, the remuneration of the two capitals will depend only on the relative price.[8] Taking into account equations (10b) and (10a), $\frac{1}{P}r^H$ will be increasing and $\tilde{r}^K$ will be decreasing during the transition to the steady state. As human capital becomes relatively less abundant, its relative price, P, will be increasing as well as its marginal productivity, r^H.

Equation (14) illustrates the main effect of the adjustment costs on the dynamic system of the closed economy, given by equations (12a)–(12d). With $k_0 < k^*$ the adjustment costs ($h > 0$) leads to $q > 1$ and we expect that $\tilde{r}^K < r^K$, as suggested by the two first terms in equation (14). Thus, by equation (13), there will be a reduction in the incentive to invest relatively more in physical capital, as $\tilde{r}^K - \frac{1}{P}r^H < r^K - \frac{1}{P}r^H$. This leads to a reduction of the speed of convergence of the closed economy as the adjustment costs increases. The result derived in equation (16) supports this intuition.

We defined earlier $P = \frac{\mu}{\eta}$, or the ratio of the co-state variables of the Hamiltonian. That is, the relative price P is the relative shadow price of human capital over physical capital. The transitional dynamics of the relative price can also be presented taking these shadow prices into account. In the case without adjustment costs, we have by equations (12c) and (8a) $-\dot{C}/C = -(r^K - \rho) = \dot{\eta}/\eta$. It follows that $\dot{P}/P = \dot{\mu}/\mu - \dot{\eta}/\eta = [-(\frac{r^H}{P} - \rho)] - [-(r^K - \rho)]$. Assuming $k_0 < k^*$ and thus $P_0 < P^*$, we have $r^K > r^*$ and $\frac{1}{P}r^H < r^*$. Then $\dot{P}/P = r^K - r^H/P > 0$, because the decrease of the shadow price of human capital is smaller than the decrease of the shadow price of physical capital.

The intuition developed here for the closed economy can also be applied to the open economy. It follows that $\dot{P}/P = r^* - r^H/P > 0$, and again, the decrease of the shadow price of human capital is smaller than that of external debt (which is associated with the value of physical capital). But with $r^K > r^*$ during the transition, the decrease associated with the value of physical capital is higher in the closed economy and, thus, the increase of the relative price is higher in the closed economy. This idea illustrates why the speed of convergence of the closed economy is higher than that of the open economy.

We will return to these expressions of the transitional dynamics when we compare the closed and open economies in Subsections 4.2 and 5.1.

3.2. The Speed of Convergence

Analyzing the case when human capital is produced only with human capital, with $\alpha > \beta = 0$ as in Lucas (1988), it is possible to characterize the transitional dynamics of the model through a system of the two differential equations on q and P.

In this case, the ratio of consumption to physical capital c derived from equation (5a) depends only on q and P, as the ratio of production of tradable goods to physical capital y^K in equation (11a) is a function only of P. Thus, we can analyze the dynamics of the system with the differential equations of Tobin's q and of the relative price P.[9] With $r^H = PB$, $(r^K)^* = B = (\frac{r^H}{P})^*$.

The two differential equations to be considered for the transitional dynamics are

$$\dot{q} = \left(\rho + \frac{\dot{C}}{C}\right) q - r^K - (q-1)\, a - \frac{h}{2}\left(\frac{q-1}{h}\right)^2, \tag{12c}$$

$$\frac{\dot{P}}{P} = \left(\rho - \frac{r^H}{P}\right) + \frac{\dot{C}}{C}, \tag{12d}$$

where

$$\frac{\dot{C}}{C} = \left(\frac{q-1}{h} + g\right) + \frac{1}{c}\left(\frac{1}{\alpha}\frac{\alpha-1}{\alpha} r^K \frac{\dot{P}}{P} - \frac{q}{h}\dot{q}\right). \tag{15}$$

We obtain equation (15) after replacing the expression $\dot{c}/c$ of equation (12a), by following two steps. First, we divide equation (5a) by K, noting that $c = C/K$. Second, we obtain a new expression for $\frac{\dot{c}}{c}$ using this transformation of equation (5a) and the equations (10a) and (11a).

We derive an analytical solution for the speed of convergence with a linearized version of this model and observe its behavior by changing the parameter of adjustment costs. The derivation of the negative eigenvalue of the linearized system is in Appendix A. We would like to point out that c does not appear in the linearized version of equations (12c) and (12d). Notice that the constant associated with changes in c in equation (15) is equal to zero, as in the steady state $\dot{P} = \dot{q} = 0$.

Let $\lambda_{C(h)}$ be the speed of convergence of this closed economy with adjustment costs.[10] It follows that

$$2\lambda_{C(h)} = -\rho + \left[\rho^2 + 4\frac{1-\alpha}{\alpha}B\left(\frac{1-\alpha}{\alpha}B + \rho\right)\Delta\right]^{\frac{1}{2}}, \qquad \textbf{(16)}$$

where

$$\Delta = \frac{1}{1 + h\left(\frac{1+\alpha}{\alpha}\frac{1-\alpha}{\alpha}B + \rho\right)}.$$

With $h = 0$, we have $\Delta = 1$ and from equation (16) $\lambda_{C(h=0)} = \frac{1-\alpha}{\alpha}B$. This expression corresponds to the speed of convergence of the closed economy without adjustment costs, or $\lambda_C = \lambda_{C(h=0)}$.[11] This expression can also be derived directly from equation (13) with $\widetilde{r}^K = r^K$, as $q \to 1$ when $h \to 0$. It follows that $\dot{P}/P = r^K - \frac{1}{P}r^H$. Notice that we can check the stable mechanism of the relative price by taking into account equations (10a) and (10b): $r_P^K < 0$ and $r_P^H > 0$. The linearization of the differential equation on $\dot{P}/P$ leads to an expression equal to $\lambda_{C(h=0)}$.

With $h > 0$, we have $\Delta < 1$ as $\Delta_h < 0$. It follows from (16) that $\partial\lambda_{C(h)}/\partial h < 0$ and thus $\lambda_{C(h=0)} > \lambda_{C(h>0)}$. If the adjustment cost goes to infinity, $\Delta \to 0$ as well as $\lambda_{C(h)}$ in equation (16).

We would like to point out one conclusion that will be used later on when we compare the speed of convergence of the closed and open economies. Because adjustment costs have a finite value (as we will see in Section 5, the higher value referred to in the literature is $h = 32$), the speed of convergence of the closed economy is always greater than zero.

Ortigueira and Santos (1997) assume that human capital is produced only with human capital and make simulations of the speed of convergence for a closed economy with endogenous growth and adjustment costs for physical capital. They numerically find that the speed of convergence decreases as the adjustment costs increase. Our paper presents three differences from Ortigueira and Santos (1997). First, we derive analytically not only the expression for the speed of convergence without adjustment costs,[12] but also the expression with adjustment costs. Moreover, in Subsection 3.1 we present an intuition for the transitional path of the closed economy. This intuition is fundamental to comprehending the transitional path of the open economy and the difference between the speeds of convergence of the closed and the open economies. Finally, in Section 5, we also analytically derive the speed of convergence of closed and open economies for the general model without adjustment costs.[13]

4. THE ECONOMY OPEN TO CAPITAL MOBILITY

In this section, we develop a two-sector economy open to capital mobility, where physical capital and consumption will be tradable goods and human capital will be assumed nontradable. There are adjustment costs for physical capital. The speed of convergence of the open economy is smaller than the speed of convergence of the closed economy. This result is in contrast with the dynamics of the one-sector neoclassical growth model with, or without, adjustment costs.[14]

4.1. The Model and the Transitional Dynamics

The planning problem of the economy open to capital mobility is given by

$$\max_{C,u,I,K,H,D} \int_0^{+\infty} (\log C)\, e^{-\rho t} dt \tag{17}$$

subject to

$$\dot{D} = r^* D + AK^\alpha (uH)^{1-\alpha} - C - I\left[1 + m\left(\frac{I}{K}\right)\right], \tag{18a}$$

$$\dot{K} = I, \tag{18b}$$

$$\dot{H} = B\left[(1-u)H\right] \tag{18c}$$

with D_0, K_0, and H_0 given.

Let D represent net foreign bonds accumulated by the economy and let r^* be the international interest rate of this small open economy. Notice that in the open economy the budget constraint is given by equation (18a), which replaces equation (5a) for the closed economy. The expression for the adjustment costs in equation (18a) is equal to the expression in equation (6) presented in the closed economy.[15] We are assuming that r^* is equal to $(r^K)^*$, as determined in the steady state of the closed economy. This implies that the closed and the open economies have the same structure of production, as stated earlier, and we assume that the rest of the world is in the steady state. Also, notice that the closed and open economies have the same growth rate in the steady state, as $g = (r^K)^* - \rho = r^* - \rho$.

The Hamiltonian for this problem is now

$$J = U(C)e^{-\rho t} + \eta e^{-\rho t}\left\{r^* D + AK^\alpha (uH)^{1-\alpha} - C - I\left[1 + m\left(\frac{I}{K}\right)\right]\right\}$$
$$+ \eta q e^{-\rho t} I + \mu e^{-\rho t}\left\{B\left[(1-u)H\right]\right\},$$

where η, ηq, and μ are the co-state variables in net foreign bonds, installed physical capital, and human capital, respectively.

In addition to equations (8a) to (8e) for the closed economy, we obtain the following first-order conditions:

$$\dot{\eta} = \eta(\rho - r^*), \tag{19a}$$

$$\dot{q} = r^* q - r^K - h\left(\frac{I}{K} - a\right)\frac{I}{K} + \frac{h}{2}\left(\frac{I}{K} - a\right)^2. \tag{19b}$$

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The transversality conditions are now

$$\lim_{t\to+\infty} \eta q e^{-\rho t} K = 0, \quad \lim_{t\to+\infty} \mu e^{-\rho t} H = 0, \quad \text{and} \quad \lim_{t\to+\infty} \eta e^{-\rho t} D = 0.$$

As in the closed economy, we have $P = \frac{\mu}{\eta}$, giving the relative price of human capital in terms of goods. In this open economy P is also the relative price of nontradables over tradables, that is, a real exchange rate. As stated earlier, η is the value of foreign bonds. Remember that the co-state variable of installed physical capital in the open economy is also ηq, because of the adjustment costs, as in the closed economy.

The dynamic system of this open economy is given by the equations

$$\frac{\dot{c}}{c} = (r^* - \rho) - \left(\frac{q-1}{h} + g\right), \tag{20a}$$

$$\dot{d} = \left[r^* - \left(\frac{q-1}{h} + g\right)\right] d + y^K - c - \left(\frac{q-1}{h} + a\right) - \frac{h}{2}\left(\frac{q-1}{h}\right)^2, \tag{20b}$$

$$\frac{\dot{k}}{k} = \left(\frac{q-1}{h} + g\right) - y^H, \tag{20c}$$

$$\dot{q} = r^* q - r^K - (q-1)\, a - \frac{h}{2}\left(\frac{q-1}{h}\right)^2, \tag{20d}$$

$$\frac{\dot{P}}{P} = r^* - \frac{r^H}{P}, \tag{20e}$$

and it takes into account the transversality conditions for D, K, and H. We are representing $d = D/K$, implying $\dot{d}/d = \dot{D}/D - \dot{K}/K$.

The value of the relative price in the steady state P^* is given by equation (20e). As we have seen in Section 3.1, we have $q^* = 1$. With equation (20c) we have k^*. The values of d^* and c^* depend on the initial conditions.

From the flow budget constraint, equation (18a), and by taking into account a no-Ponzi-game condition, we obtain the intertemporal budget constraint,

$$\int_0^{+\infty} C e^{-r^* t} dt = \int_0^{+\infty} \left\{ A K^\alpha (uH)^{1-\alpha} - I\left[1 + m\left(\frac{I}{K}\right)\right]\right\} e^{-r^* t} dt + D_0,$$

where $C_t = C_0 e^{(r^*-\rho)t}$. Given $D_0 = 0$, we can also derive[16]

$$\int_0^{+\infty} C e^{-r^* t} dt = q_0 K_0 + P_0 H_0.$$

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4.2. The Speed of Convergence

We now present some intuition on the transitional dynamics of the relative price in the open economy. As in Section 3 with equation (13) for the closed economy, relevant information for the open economy is provided by equation (20e). The incentive to accumulate relatively more of one of the capitals depends on their remuneration. Consider again the case $k_0 < k^*$.[17] Physical capital is relatively less abundant and its remuneration will be relatively higher than human capital as $\frac{1}{P}r^H < r^*$. Notice that now there is an alternative of investment given by the international interest rate r^*, which is fixed during the transition. However, the remuneration of physical capital changes during the transition in the closed economy, where $r^K > r^*$ and $\widetilde{r}^K > r^*$. This differentiated pattern in the closed and the open economies leads to a gap between remunerations which is smaller in the open economy. Consequently, the incentive to invest relatively more in physical capital is higher in the closed economy and this economy will converge faster to the steady state than the open economy. Using a two-sector endogenous growth model, à la Lucas (1988) and with adjustment costs, the speed of convergence is lower in the open economy than in the closed economy.

The expression for the speed of convergence of the open economy, λ_O, is determined only by the differential equation of the relative price, as the remuneration of capital only depends on the relative price. See equation (20e). Given the structure of production in Lucas (1988), where the share of physical capital in the production of human capital is zero, the speed of convergence is (or goes to) zero in this open economy, $\lambda_O \to 0$. We will see in the next section with the general model that $\lambda_O > 0$, as can be checked with equation (24).

The expression for the speed of convergence of the closed economy, given by equation (16), is positive and, thus, $\lambda_C > 0$. Consequently $\lambda_C > \lambda_O$. As we have seen in Section 3, the speed of convergence of the closed economy is greater than zero, for a positive and finite value of the adjustment costs. It follows that the speed of convergence of the closed economy is higher than the speed of convergence of the open economy.

We would like to point out again that the main result of the paper is not specific to the case where the speed of convergence of the open economy goes to zero. In the next section we will show our main result in an extended model.

5. AN EXTENSION OF THE MODEL AND NUMERICAL EXPERIMENTS

The main result of the paper was derived with the specification used in Lucas (1988). This specification for the two-sector endogenous growth model leads to a simplification and clarity of exposition of the mechanism at work in the model.

Now we present an extension of the model and numerical simulations for a positive value of β, the share of physical capital in the production of human capital. Notice that with $\beta > 0$, the system of differential equations for the speed of convergence in the closed economy with adjustment costs has three equations with three variables, implying that we do not have a clear analytical solution. But the speed of convergence of the open economy is again given by one differential equation for the relative price, as we generalize the case where the production of goods remains more intensive in physical capital ($\alpha > \beta$). See equation (20e).

Notwithstanding this, the intuitive explanation presented in Subsection 4.2 for the difference of the speed of convergence in the closed and in the open economies still applies. We rely on numerical simulations when comparing both economies with adjustment costs and with a positive share of physical capital in the production of human capital, $\beta > 0$.

With $\alpha > \beta > 0$, the equation (2) is replaced by

$$Y^H = B\,[(1-v)\,K]^{\beta}\,[(1-u)\,H]^{1-\beta}\,, \tag{21}$$

where $v(0 \leq v \leq 1)$ is the fraction of the physical capital used in the consumption sector.

The rental rates of physical and human capital are again represented, respectively, by r^K and r^H. Assuming there is incomplete specialization in production, the rental rate of each capital is also a function of the relative price:[18]

$$r^K = \alpha A \overline{\phi}^{\alpha-1} P^{\frac{\alpha-1}{\alpha-\beta}}, \tag{22a}$$

$$r^H = (1-\alpha) A \overline{\phi}^{\alpha} P^{\frac{\alpha}{\alpha-\beta}}, \tag{22b}$$

where $\overline{\phi} \equiv [\frac{B}{A}(\frac{\beta}{\alpha})^{\beta}(\frac{1-\alpha}{1-\beta})^{\beta-1}]^{\frac{1}{\alpha-\beta}}$ is a constant.

5.1. Speeds of Convergence: Closed versus Open Economies

Before presenting the numerical simulations for the speed of convergence of the closed economy with adjustment costs and a positive share of physical capital in the production of human capital, we will consider an intuitive example. As a first approximation, the speed of convergence of the closed economy for a low value of the adjustment costs can be represented by the speed of convergence without adjustment costs. Notice that with this assumption, $q = 1$ and equation (13) becomes $\dot{P}/P = r^K - r^H/P$. See also the discussion in Section 3. Linearizing this differential equation for the relative price of the closed economy around the steady state, while taking into account equations (22a) and (22b), we obtain the following analytical expression for the speed of convergence of the closed economy:

$$\lambda_C = \frac{(1-\alpha)+\beta}{\alpha-\beta}\left(r^K\right)^*. \tag{23}$$

Taking into account equations (22a) and (22b) and linearizing around the steady state equation (20e), we find that the speed of convergence of the open economy is now equal to

$$\lambda_O = \frac{\beta}{\alpha-\beta}\left(r^K\right)^*. \tag{24}$$

Comparing equations (23) and (24), it is clear that the speed of convergence is higher for the closed economy. Thus, this result is not specific to human capital being produced only with human capital ($\beta = 0$), in which case the speed of convergence goes to zero in the open economy. In fact the expression in equation (24) also makes it clear that the speed of convergence of the open economy tends to zero as β tends to zero. Notice that $\lambda_C = \frac{(1-\alpha)}{\alpha} B$ when $\beta = 0$, as in Section 3.

The difference between equations (23) and (24) is given by the expression $(1-\alpha)$ in λ_C. As long as $0 < \alpha < 1$, and given that $\alpha > \beta$, the remuneration of physical capital will be changing during the transitional path in the closed economy. In these conditions, the gap between remunerations will be higher in the closed economy than in the open economy. Only if $\alpha = 1$ would this effect disappear, as $(1-\alpha) = 0$.

With equation (20e), we have the gradual adjustment of the relative price giving also the transitional dynamics of the open economy. The marginal productivity of physical capital is not restricted to be equal to the international interest rate during the transition, $r^K \neq r^*$. This conclusion depends on the adjustments costs being positive and can be checked with equation (20d). The stable mechanism of the relative price in the open economy works for any positive value of the adjustment costs. However, this stable mechanism could not work without adjustment costs. The reason is that in the open economy without adjustment costs [$h = 0$ and $q = 1$, by equation (20d)] the remuneration of physical capital is equal to the constant international interest rate, $r^K = r^*$. As r^K is only a function of the relative price by equation (22a), this relative price will also be constant and equal to its steady state level, $P = P^*$. Thus, the remuneration of human capital over the relative price is also constant, $r^H/P = r^*$, and the stable mechanism of the relative price in equation (20e) could not work. It is important to notice that this issue does not appear in a closed economy, where there is no restriction on r^K. In conclusion, the stable mechanism of the relative price is at work in the open economy when we use adjustment costs. However, because of the comparability between the economies, we also introduce adjustment costs into the closed economy. In both economies, the adjustment costs are introduced in such a way that the steady state level is the same for the relative price and the ratio of physical to human capital. The speed of convergence of the two-sector endogenous growth model for the closed and open economies is given by one negative eigenvalue derived from the no-arbitrage condition.

As stated previously, this first approach to the speed of convergence of the closed economy with a low value of the adjustment costs is a specific case. Do the conclusions on the speed of convergence change with the values of the parameter of the adjustment costs used in the literature?

To find out how much of the speed of convergence of the closed economy is affected by adjustment costs, we carry out numerical simulations with a general model, as follows.

The values of the parameters of the general model are presented in Table 1.[19] The (inverse) intertemporal elasticity of substitution is given by $\sigma = 1.5$ and the

TABLE 1. Values of the parameters of the model

σ	ρ	α	β	$\delta_K = \delta_H$	A	B
1.5	0.03	0.4	0.1	0.05	1.516	0.11

Notes: σ is the (inverse) intertemporal elasticity of substitution and ρ is the subjective preference rate. α is the share of physical capital in the production of physical capital and β is the share of physical capital in the production of human capital. δ_K and δ_H are the rates of depreciation of physical capital and human capital, respectively. A and B are the parameters of technology in the production of physical and human capital, respectively. It follows that $r^* = 0.06$, $g^* = 0.02$, and $a = g^* + \delta_K = 0.07$. In the steady state $k^* = 8.9928$, $q^* = 1$, and $P^* = 25.8059$.

TABLE 2. Comparing the speed of convergence of the closed and open economies

λ_C					λ_O
$h = 0$	$h = 5$	$h = 10$	$h = 16$	$h = 32$	$h > 0$
0.25(6)	0.1003	0.071	0.055	0.0371	0.03(6)

Notes: λ_C and λ_O represent the speed of convergence of the closed and open economy, respectively. h is the parameter of adjustment costs and β is the share of physical capital in the production of human capital. See Table 1 for the values of the other parameters.

subjective preference rate by $\rho = 0.03$. Also in line with the literature, we set the share of physical capital in production of goods to be $\alpha = 0.4$, and the share of physical capital in the production of human capital is given by $\beta = 0.1$. The rate of depreciation for physical and human capital is $\delta_K = \delta_H = 0.05$. A is adjusted to maintain the values of the real interest rate and the growth rate in the steady state. Notice that using $\beta = 0$ and taking into account equation (13), we have $B = (\frac{1}{P} r^H)^* = (r^K)^* = r^* + \delta = 0.11$.[20] With these values, we obtain the following steady state values: the real interest rate is equal to $r^* = (r^K)^* - \delta = 0.06$ and the growth rate is $g = \frac{1}{\sigma}(r^* - \rho) = 0.02$. The parameter a of the adjustment costs function given in equation (6) is now equal to $a = g + \delta_K = 0.07$. We also obtain $k^* = 8.9928$, $q^* = 1$, and $P^* = 25.8059$.

The speeds of convergence for the different values of the adjustment costs are presented in Table 2. Remember that the expressions for $\lambda_C(h = 0)$ and for λ_O are given by equations (23) and (24), respectively. We simulate the values of λ_C for $h > 0$ with MATLAB and we use values of h following Ortigueira and Santos (1997). Taking into account the results presented in Table 2, we would like to make two observations. First, the speed of convergence of the closed economy is higher than that of the open economy, taking into account the values of adjustment costs used in the literature.[21] Second, the speed of convergence of the closed economy decreases with adjustment costs. We have analytically shown this result with the model presented in Section 3. We also have numerically checked equation (16) of Section 3, giving $\lambda_C(h)$ for the model as in Lucas (1988).[22]

For a two-sector endogenous growth model, the results of the simulations for the speed of convergence of closed and open economies are in contrast with the results one obtains using the one-sector neoclassical growth model in the same experiment. We have also presented at the beginning of this subsection the intuition of the main result of the paper, and in Subsection 4.2 we have analytically shown this result. Thus, we have shown our main result both analytically and with numerical simulations.

Consider again the closed economy model of Section 3. Given a negative eigenvalue of the dynamic system, all relevant variables have the same speed of convergence. Let $\overline{y} = Y/H$ be output over human capital (notice that we have labor constant and $L = 1$), $g_H = \frac{\ln H_t - \ln H_0}{t}$ be the average growth rate of human capital, H, between periods 0 and t, $g_y = \frac{\ln y_t - \ln y_0}{t}$ be the average growth rate of output per capita, y, between periods 0 and t (* stands for steady state values), and finally $\beta^C = \frac{1-e^{-\lambda^C t}}{t}$ (where λ^C is the speed of convergence of the closed economy). Thus,

$$g_y = g_H + \beta^C \ln(\overline{y}^*/\overline{y}_0). \quad \textbf{(25)}$$

Considering capital mobility as in the model of Section 4 and taking into account equation (25), there are an initial partial jump, a change in the average growth rate of human capital, and a decrease in the speed of convergence.[23] This result points to the interest in looking empirically for the effects of financial globalization on growth, not only through a direct effect, but also through an indirect effect on convergence (which may be captured by an interaction between a proxy of financial globalization and initial GDP per capita). To our knowledge, there is no systematic work dealing with this issue. We think this is an interesting issue for future research.

5.2. Welfare Gains from Capital Mobility

We would like to address the issue of welfare gains from capital mobility in the context of the two-sector endogenous growth model presented in this paper. Gourinchas and Jeanne (2006) have shown that the gains from capital mobility are small, using the one-sector neoclassical growth model. They also introduce a model with human capital, reducing the speed of convergence of the closed economy and, thus, giving the possibility of slightly increasing welfare gains. As we have seen, our two-sector endogenous growth model leads to speeds of convergence higher in the closed economy than in the open economy. It is interesting to check the welfare gains from capital mobility in the two-sector growth model, where the results for the speed of convergence for closed and open economies are in contrast with those of the one-sector neoclassical growth model.

As in Gourinchas and Jeanne (2006), we compute the equivalent variation in consumption, ω, measuring the increase in consumption for the closed economy such that the utility of the representative agent at time zero is equal in the closed and the open economy. Given $U(C) = \int_0^{+\infty} \frac{C^{1-\sigma}}{1-\sigma} e^{-\rho t} dt$ and $\sigma \neq 1$, we have

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TABLE 3. Welfare gains ω from capital mobility (in %)

ω				
$h = 0.01$	$h = 5$	$h = 10$	$h = 16$	$h = 32$
0.98	0.56	0.34	0.2	0.078

Notes: $k_0 = 0.67 * k^*$ and k_0 is the initial physical/human capital ratio. h is the parameter of adjustment costs. ω represents the welfare gains in percentage, corresponding to the equivalent variation of consumption. See Table 1 for the values of the parameters of the model.

$U[C^{CLOSED}(1+\omega)] = U[C^{OPEN}]$. It follows that

$$\omega = \left(U\left[C^{OPEN}\right]/U\left[C^{CLOSED}\right]\right)^{\frac{1}{1-\sigma}} - 1.$$

In Table 3, we present the welfare gains from capital mobility, for different values of the adjustment costs and the same initial capital ratio $k_0 = 0.67 * k^*$ used in Gourinchas and Jeanne (2006). We use MATLAB to solve the system of ordinary differential equations. We obtain welfare gains ω of 0.56% and 0.34%, using values for the parameter of the adjustment costs equal to 5 and 10, respectively. The value of ω is about 1% if we near eliminate the adjustment cost, with $h = 0.01$. It follows that the values we find for the welfare gains ω are small, as in Gourinchas and Jeanne (2006), and decrease with the adjustment costs. The welfare gains decrease, although the speed of convergence of the closed economy is decreasing.

6. CONCLUSION

We developed a two-sector endogenous growth model à la Lucas (1988) with adjustment costs to compare the speed of convergence of open and closed economies. As human capital was nontradable in the open economy, partial capital mobility was the only difference between these two economies. It was possible to analyze the transitional dynamics of the model with a no-arbitrage condition. The difference between the remunerations of the two capitals gives the incentive to accumulate relatively more of one of the capitals. The main implication of the two-sector endogenous growth model was a lower speed of convergence in the open economy than in the closed economy. This result reverses the conclusions of the same experiment when the one-sector neoclassical growth model is used.

We have seen that the two-sector endogenous growth model also leads to small welfare gains. Although the speed of convergence of the closed economy is decreasing, because of the increase in adjustment costs, the welfare gains also decrease.

In terms of policy implications, our study suggests the need for more reflection on the idea that capital mobility will accelerate the convergence of economies during the transition to the steady state. Given the main implication of the model, it would be of interest to analyze these issues empirically. Although the empirical literature is looking for proxies of financial openness and its effects on growth through alternative specifications, it has not analyzed the effects of capital mobility on the speed of convergence in a systematic way. Further empirical studies on this issue and its relation with growth of GDP seems to be an interesting topic for future research.

NOTES

1. Notice that the one-sector neoclassical growth model predicts a finite speed of convergence for a closed economy. Notwithstanding, a small open economy with the same characteristics would have an infinite speed of convergence. With perfect international capital mobility, the remuneration of capital in a small open economy must be equal to the international interest rate, and capital flows would eliminate any difference in remunerations instantaneously. Even allowing for adjustment costs in the small open economy, the closed economy with the same structure has a smaller speed of convergence. Abel and Blanchard (1983) characterize the dynamics of the one-sector neoclassical growth model with adjustment costs geometrically. For this model, Barro and Sala-i-Martin (2004) show numerically that the speed of convergence of the open economy is higher than that of the closed conomy. Following Barro et al. (1995), credit constraints can be introduced into an economy using physical and human capital in the production function, where only physical capital can be used as collateral for foreign borrowing. The previous conclusion does not change with the introduction of these credit constraints. For a discussion on convergence of open economies see Barro and Sala-i-Martin (2004, Ch. 3) and Obstfeld and Rogoff (1996, Ch. 7).

2. Mulligan and Sala-i-Martin (1993) study the transitional dynamics giving values to the parameters of the model.

3. With $\sigma \neq 1$, the utility function corresponds to $U(C) = \frac{C^{1-\sigma}}{1-\sigma}$. We will use this function in Section 5.

4. Ortigueira and Santos (1997) also use this specification of adjustment costs in an endogenous growth model à la Lucas (1988). See also Summers (1981) and King and Rebelo (1993).

5. Defining this co-state variable as $\eta q e^{-\lambda t}$ rather than as a single variable is a matter of convenience, as will become clear later on, when we show that q plays a key role in determining investment.

6. Some of these conditions are as in Bond et al. (1996) or Ortigueira and Santos (1997), but other are specific to the introduction of the adjustment costs in the endogenous growth model. Moreover, we can compare the conditions of the closed economy directly with the conditions of the open economy.

7. Notice that r^* is also equal to the steady state real interest rate of a closed economy without adjustment costs, given the specification of the model.

8. Notice that $h \to 0$ leads to $q \to 1$. By equation (14), whe have $\widetilde{r}^K \to r^K$, as in Bond et al. (1996).

9. In the general form where human capital is produced with both capitals, $\alpha > \beta > 0$, the linearized system has three equations depending on three variables. With such a system, it is not possible to find a clear analytical solution for the dynamics of the economy. In Section 5, we will analyze numerically the more general model.

10. Ortigueira and Santos (1997) also analyze the behavior of the speed of convergence of the nonlinearized model, and they show that the linearized model represents well the behavior of the transitional dynamics over substantial phases of the transition. Working with the linearized version of the transitional dynamics, our results are all derived analytically. In models with two negative eigenvalues, one may analyze the entire path of transition. See for example Papageorgiou and Perez-Sebastian (2007).

11. This expression is equivalent to equation (13) in Ortigueira and Santos (1997) and $\lambda_{C(h=0)} = \widehat{\lambda} = \frac{1-\beta}{\beta}(\delta - \theta + \pi + n)$. β and δ of their paper correspond to α and B in our paper. The other parameters are equal to zero, given the simplifications we made in our model.

12. Using the Lucas (1988) model, Ortigueira and Santos (1997) present an expression for the speed of convergence, only for the case without adjustment costs. The analytical solution is derived in a theorem presented in the appendix of their paper.

13. Bond et al. (1996) and Mulligan and Sala-i-Martin (1993) analyze the transitional dynamics of the general model, but they do not arrive at an expression for the speed of convergence.

14. See the discussion presented in note 1.

15. In Section 3, we explained in more detail the implications of the model for the steady state values of the variables related to production. On related expressions for the adjustment costs see Turnovsky (1996, 2002) and Fisher (2010).

16. The derivation of this equation is in Appendix B.

17. The conclusions do not change if we instead consider the case $k_0 > k^*$.

18. Notice that the expressions for r^K and r^H in equations (22a) and (22b) are equal to equations (10a) and (10b), when $\beta = 0$.

19. For a derivation of the general model, see Barro and Sala-i-Martin (2004) and Bond et al. (1996). We use the same specification of the model. These values of the parameters are in line with the literature. In the general model Barro and Sala-i-Martin (2004) use $\beta < 0.4$. Most of the literature uses $\beta = 0$, following Lucas (1988), as in Ortigueira and Santos (1997).

20. In this last case A does not affect $(r^K)^*$. But things are different with $\beta > 0$.

21. The value of $h = 32$ is not considered plausible, and recent work points to lower values for the adjustment costs. Notice that giving higher values for the adjustment costs than those usually employed in the literature, we will have values of λ_C smaller than λ_O.

22. In the paper of Ortigueira and Santos (1997), leisure also enters the utility function and has, together with h, an effect on the speed of convergence.

23. Numerically, we can see that average growth rate of GDP per capita between periods 0 and t is higher in the open economy, as in Table 2 of Gourinchas and Jeanne (2006).

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APPENDIX A: THE NEGATIVE EIGENVALUE ASSOCIATED WITH THE CLOSED ECONOMY

Here we derive the negative eigenvalue associated with the linearization of the system given by equations (12c), (12d), and (15) in Section 3.

When human capital is produced only with human capital, the system is analyzed with the two differential equations on q and P. As presented in the text, the system is given by

$$\dot{q} = \left(\rho + \frac{\dot{C}}{C}\right) q - r^K - (q-1)\,a - \frac{h}{2}\left(\frac{q-1}{h}\right)^2,$$

$$\frac{\dot{P}}{P} = \left(\rho - \frac{r^H}{P}\right) + \frac{\dot{C}}{C},$$

where

$$\frac{\dot{C}}{C} = \left(\frac{q-1}{h} + g\right) + \frac{1}{c}\left(\frac{1}{\alpha}\frac{\alpha-1}{\alpha} r^K \frac{\dot{P}}{P} - \frac{q}{h}\dot{q}\right).$$

We would like to point out that in the linearization of this equation, the constant associated with changes in c is equal to zero, as $\dot{P}$ and $\dot{q}$ are also equal to zero in the balanced growth path.

Reorganizing terms leads to

$$\dot{q} = A_q\left[\rho q - r^K - (q-1)\,a - \frac{h}{2}\left(\frac{q-1}{h}\right)^2 + qA_P\frac{q-1}{h} + qg\right],$$

$$\frac{\dot{P}}{P} = A_P\left[(\rho - B) + \left(\frac{q-1}{h} + g\right) - \frac{1}{c}\frac{q}{h}\dot{q}\right],$$

where

$$A_P = \frac{1}{1 + \frac{1}{c}\frac{1}{\alpha}\frac{1-\alpha}{\alpha} r^K}$$

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and

$$A_q = \frac{1}{1 + qA_P\frac{1}{c}\frac{q}{h}}.$$

The linearized system can be represented by

$$\begin{bmatrix} \dot{q} \\ \dot{P} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} * \begin{bmatrix} q-1 \\ P-P^* \end{bmatrix},$$

where

$$a_{11} = \left[\rho + A_P^*\frac{1}{h}\right]A_q^*,$$

$$a_{12} = \frac{1-\alpha}{\alpha}\left(\frac{r^K}{P}\right)^* A_q^*,$$

$$a_{21} = A_P^* P^* \left[\frac{1}{h} - \frac{1}{c^*}\frac{1}{h}A_q^*\left(\rho + A_P^*\frac{1}{h}\right)\right],$$

$$a_{22} = -A_P^* A_q^* \frac{1}{c^*}\frac{1}{h}\frac{1-\alpha}{\alpha}\left(r^K\right)^*.$$

Notice that A_P^* and A_q^* correspond to the expressions for A_P and A_q, but are evaluated with the values of the variables of the model in the balanced growth path.

With $a = g = B - \rho$ and using equation (10a), one can show that $c^* = (r^K)^*/\alpha - g = \frac{1-\alpha}{\alpha}B + \rho$. It follows that

— the trace is equal to $\mathrm{Tr} = \rho$

— the determinant is equal to $\mathrm{Det} = -\frac{1-\alpha}{\alpha}B\frac{1}{h}A_P^*A_q^* = -\frac{1-\alpha}{\alpha}B(\frac{1-\alpha}{\alpha}B + \rho)\Delta$, where $\Delta = \frac{1}{1+h(\frac{1+\alpha}{\alpha}\frac{1-\alpha}{\alpha}B+\rho)}$

— and the negative eigenvalue is equal to $2\theta = \rho - \sqrt{\rho^2 + 4\frac{1-\alpha}{\alpha}B(\frac{1-\alpha}{\alpha}B + \rho)\Delta}$.

Let $\lambda_{C(h)} = -\theta$ be the speed of convergence of this closed economy with adjustment costs. By rearranging terms, we can represent the analytical solution for the speed of convergence as

$$2\lambda_{C(h)} = -\rho + \left[\rho^2 + 4\frac{1-\alpha}{\alpha}B\left(\frac{1-\alpha}{\alpha}B + \rho\right)\Delta\right]^{\frac{1}{2}}, \tag{A1}$$

where

$$\Delta = \frac{1}{1 + h\left(\frac{1+\alpha}{\alpha}\frac{1-\alpha}{\alpha}B + \rho\right)}.$$

The expression for the speed of convergence corresponds to equation (16) in the text. As $\Delta_h < 0$ it follows $\partial\lambda_{C(h)}/\partial h < 0$.

APPENDIX B: INTERTEMPORAL BUDGET CONSTRAINT

This Appendix derives the equation of the intertemporal budget constraint presented in Section 4.

From the flow budget constraint, equation (18a), and taking into account a no-Ponzi-game condition, we obtain the intertemporal budget constraint

$$\int_0^{+\infty} Ce^{-r^* t} dt = \int_0^{+\infty} \left\{ AK^{\alpha} (uH)^{1-\alpha} - I \left[1 + m \left(\frac{I}{K} \right) \right] \right\} e^{-r^* t} dt + D_0,$$

where $C_t = C_0 e^{(r^* - \rho)t}$.

The value of human and physical capital is given, respectively, by

$$P_0 H_0 = \int_0^{+\infty} \left(r^H H - PY^H \right) e^{-r^* t} dt,$$

$$q_0 K_0 = \int_0^{+\infty} \left\{ r^K K - I \left[1 + m \left(\frac{I}{K} \right) \right] \right\} e^{-r^* t} dt.$$

With Y representing total output, we have

$$Y = Y^K + PY^H = r^K K + r^H H.$$

It follows that with $Y^K = Y - PY^H$ and assuming that $D_0 = 0$, we obtain

$$q_0 K_0 + P_0 H_0 = \int_0^{+\infty} \left\{ Y^K - I \left[1 + m \left(\frac{I}{K} \right) \right] \right\}.$$

Thus

$$\int_0^{+\infty} Ce^{-r^* t} dt = q_0 K_0 + P_0 H_0.$$

This is the equation presented in the text.