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Sliding mode with adaptive estimation force control of robot manipulators interacting with an unknown passive environment

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SUMMARY

In this paper, a force control algorithm for robot manipulators is introduced, where the dynamics of non-rigid environment interacting with the robot is assumed unknown. The controller design is based on the combination of sliding mode control techniques and the adaptive estimation theory, so the introduced controller compensates the structured or unstructured uncertainty of the environment. The main source of feedback information is received from a wrist force sensor. The designed controller includes additional absorption terms in order to minimise end-point velocity error and to suppress the impact effects at the beginning of the force application.

KEYWORDS: Sliding mode; Robot control; Unknown environment; Adaptive estimation force.

1. INTRODUCTION

In assembly or grinding tasks, the robot applies a set of desired forces to the environment therefore a force controller is required to regulate the applied forces.

A challenging task of the robotics research community is to enhance the manufacturing flexibility of the robots. In the case of tasks requiring contact with the environment, the force controller has to deal with a great variety of contact situations. Therefore, the controller has to encounter passive mechanisms or environments with unknown dynamic characteristics.

A great deal of work[1–4] has been devoted in the quantitative investigation of the robot-environment dynamics, since the understanding of the dynamic behaviour is crucial for the design of force controllers. It has been noticed that when the environment is stiff then the force feedback branch presents high gains. Additionally, the system is marginally stable due to the low damping of the stiff environments. On the other hand, force applied to soft environments or mechanisms may cause oscillatory response. Finally, it has been concluded that the nonlinear dynamics and kinematics of the robot affect the stability of the system.

Eppinger and Seering[1] noticed two basic factors that influence the dynamics of the system robot-environment: the high eigenvalues of the environment and the compliance of the robot arm. The authors proved that a controller designed under the assumption that the environment dynamics is negligible could lead to an unstable system. Otherwise, the system is marginally stable with light damping.

In a second paper, Eppinger and Seering[2] investigated the structure of a force controller and its performance with respect to the dynamics of the environment and the actuator. They concluded that low pass filtering and PI controlling add destabilising poles, while a PD control and a lead compensation adds zeros, which provide considerably high phase lead at low frequencies and the system nonlinearities influence the performance of the controller.

An and Hollerbach[3] studied the effects of the robot kinematics on the stability of the controller. They proved that a kinematics coordinate transformation in the feedback affects the dynamics of the closed loop system and may cause instabilities. The Hybrid Position/Force control exhibits such kinematically induced instabilities in the case of revolute robots, while the stiffness force control and the resolved acceleration methods, do not.

In a second paper, An and Hollerbach[4] investigated the stability conditions of force controllers. They concluded that three factors affect the stability of the system: the first factor is that force feedback signal is essentially a high-gain position feedback. The second factor is the unmodeled high frequency dynamics due to robot structural system. Finally, the third factor is the high stiffness of the environment-mechanism.

The impedance control method[5,6] is investigated to solve the problem of force control. The main idea of impedance control is to assign a prescribed dynamic behaviour for robot manipulator while its end effector is interacting with the environment. The desired performance is specified by a complete set of linear or nonlinear second order differential equations representing a mass-spring-damper system. Generally the impedance control is suitable for those tasks where contact forces must be kept small.[7]

Hogan[5] presented an impedance controller with force feedback for the regulation of the force applied by the end point of the robot. The calculation of the inverse kinematics of the manipulator is not used, in order to reduce the computational cost in the implemented controller. Hogan studied the cases where the free motion of the robot is followed by a constrained motion. In these cases, problems arise due to the difference between the compliance of the environment and the total compliance of the manipulator. For stiff environments the total compliance of the manipulator should be quite high, then the force sensors have to be

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chosen softer than the environment. The proposed impedance algorithm eliminates this problem and there is no need for limitations on the force sensor stiffness. The results from two experiments are presented in the paper. In the first experiment the controller includes force feedback, while in the second experiment the force feedback is eliminated. In both cases the controller is stable but the existence of the force feedback improves the performance of the system.

Anderson and Spong[6] introduced the Hybrid Impedance Controller (HIC) where the main characteristics of the hybrid force/position and impedance control algorithm are combined. The general scheme of this controller follows the hybrid force/position[8] architecture where the force and position tasks are split and defined in the operational space. At the end effector of the robot a force sensor is mounted. The controller has an inner loop where the inverse dynamics algorithm is implemented. Impedance terms are introduced for each degree of freedom in the operational space. The outer loop of the control system includes the compensator, which monitors the impedance of the system. In order to validate the performance of the proposed controller a peg-in-hole assembly task was simulated. The presented results show that the task is accomplished quite fast without any undesirable effect.

In the previous approaches,[5,6] the environment is assumed rigid, and only the force sensor is assumed that represents the external stiffness. The designed controllers regulate the contact forces via a pre-defined second order dynamics based on the impedance control structure.

In the following group of papers[9–14] the hybrid force/position control scheme including adaptive algorithms is used to guide the robots fine motion in contact with the environment. The adaptive algorithms either tune the gains of the control law or determine the actual values of the dynamic environment characteristics.

Koivo[9] introduced an autoregressive model with external excitation for the design of a discrete self-tuning adaptive force controller. This controller minimises the error of the velocity and of the applied force under the constraint of the ARX model. The controller operates in the joint space with a hybrid force/position architecture and time varying gains. An additional adaptive algorithm is used for the estimation of the environment characteristics in the Cartesian coordinate system.

Fukuda *et al*,[10] introduced a force control method for manipulators by considering the grasped object dynamics based on the adaptive control concept. The MRAC algorithm tunes the control system gains in order to minimise the error between the response of the manipulator and the response of a pre-defined dynamic system, which is exerted by the same input signal. The proposed controller goes further from the classical MRAC approach, it estimates also the unknown parameters of the environment. In order to validate the performance of the proposed controller the authors presented two simulations with hard and soft environment respectively. In both cases the desired force was obtained. However, in the case of the hard environment the response was quite oscillatory.

Kalaycioglu and Brown[11] implemented the hybrid position/force controller with an adaptive algorithm for the identification of the unknown environment parameters. The used estimation algorithm is based on the least squares with exponential forgetting. The integral form of the adaptation law suppresses the oscillations of the response. In addition, an internal algorithm that calculates the position for the compliant motion prevents the overshooting caused by the impact between the end-point of the robot and the environment. From the described experimental setup is observed that the force feedback-sampling rate is out of the bandwidth of the position controller. This proves that the force control introduces high frequency components in the system. The results obtained by the simulation and the experiments show fast response and very good accuracy. The estimation of the unknown parameters is fast and quite accurate.

Zhen and Goldenberg[12] presented a new control method for constraint motion of robot manipulators based on a hybrid force/position concept including an adaptive estimator. The adaptive estimator is used in order to avoid the force feedback signal derivative and the joint acceleration measurements. Except the forces normal to the constraint surface the contact friction forces were included in the formulation of the model. The simulation results show that the algorithm is robust with a quite good performance. However all the holonomic constraints can not be described by smooth functions and the simulation results are for a Cartesian manipulator where the dynamics of the robot is uncoupled. The controller performance for a revolute manipulator, where the robot dynamics is coupled, is under question.

Almerico *et al*[13] used the sliding mode techniques to build a hybrid force/position controller. In this paper major effort has been put in the application of the well-known control methods to real world industrial applications. Joint level dynamic decoupling is performed by a hardware controller using feedforward plus sliding mode terms. Linear state-feedback loops act in the task space. The sliding surface consists only by the velocities of the joints. The force controller is based on a modified impedance scheme. The authors show that the low resolution of the force sensor does not affect significantly the performance of the proposed controller. In addition, the chattering effect due to the sliding mode terms is out of the bandwidth of the actuators. The authors claim the chattering effect is filtered by the compliance of the robot joints and links.

Slotine and Li[14] presented an integrated solution for the constraint motion control of a robot manipulator based on the pattern of the hybrid force/position control architecture. For the force part a common force controller based on the feedback of the force and velocity error was used, while for the motion part a sliding mode controller was developed. In designing the controller, it was assumed that uncertainty exist in the structure of both the manipulator and the environment. For this reason, an adaptive algorithm is used for the estimation of the robot and environment parameters. Except the discussion about the control design, Slotine and Li analysed some significant issues on the relation between the joint and Cartesian dynamics of robots which are used in the present paper for the design of the force control algorithm.

In the present paper a sliding force controller is developed. The environment is supposed to be an unknown passive mechanism with damping and stiffness properties while the robot dynamics is assumed known. The controller uses only the bounds of the environment dynamic characteristics. As it will be shown in the simulation results the proposed controller can compensate structured or unstructured uncertainty. The robot controller can accommodate contact forces with either linear or non-linear environments presenting coupled or uncoupled dynamics. The feedback branch is designed in order to reduce the chattering.

The stability of the system is proved via Lyapunov theory. In the case where the environment does not present damping terms and its stiffness matrix is constant, an additional adaptive estimation algorithm is implemented to reduce the uncertainty. The adaptive system estimates the unknown environment parameters and in the same time changes their bounds in order to maintain the stability of the sliding mode controller. The narrower uncertainty bounds results to the reduction of chattering effect.

The force error is formed using the signal of a high stiffness force sensor mounted on the wrist. The presented controller eliminates the need of soft force sensors because damping terms for the absorption of force oscillations are included in the control law. The main contribution of the proposed sliding force controller is that the robot can accommodate contact forces produced by linear or non-linear, coupled or uncoupled passive mechanisms.

In Section 3, the equations of robot dynamics in the Cartesian Space are formulated. In Section 4.1, the equations for the sliding controller are designed and the stability of the controller is studied. The adaptive estimation algorithm is described in Section 4.2. In Section 5, the results of the simulated system using the introduced sliding controller are presented, while in Section 6 the results using a combination of the sliding controller with an adaptive estimator are presented. Finally, in Section 7 the innovative issues of the presented method are discussed.

2. DEFINITION OF THE PROBLEM AND ROBOT DYNAMICS

In Figure 1, a simple sketch shows the basic geometry of a rigid robot interacting with a passive mechanism. The dynamic behaviour of this system in the Cartesian space is described by the following set of equations:[14]

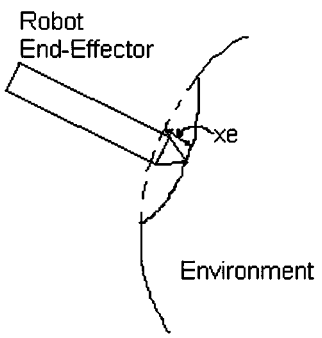

Fig. 1. Robot End-Effector with 2D environment.

$$H_r(\underline{x}_r)\ddot{\underline{x}}_r + C_r(\underline{x}_r, \dot{\underline{x}}_r)\dot{\underline{x}}_r + \underline{g}_r(\underline{x}_r) = \underline{u} - \underline{F}_s$$
$$H_e\ddot{\underline{x}}_e + C_e(\dot{\underline{x}}_e)\dot{\underline{x}}_e + K_e(\underline{x}_e)\underline{x}_e = \underline{F}_s$$
$$\underline{F}_s = K_s(\underline{x}_r - \underline{x}_e) \tag{1}$$
$$K_s = R^T K_{SO} R$$

where, $\underline{x}_r \in R^n$ represents the robot end-point displacement around the equilibrium point, $\underline{x}_e \in R^n$ represents the environment deformation around the equilibrium point, $H_r(\underline{x}_r) \in R^n \times R^n$ is the inertia matrix of the robot expressed in the Cartesian space, $C_r(\underline{x}_r, \dot{\underline{x}}_r) \in R^n \times R^n$ is the matrix of coriollis forces of the robot expressed in the Cartesian space, $\underline{g}_r(\underline{x}_r) \in R^n$ is the Gravitational forces vector, $H_e \in R^n \times R^n$ is the inertia matrix of the environment and the masses that are mounted on the robot after the force sensor (tools etc), $C_e(\dot{\underline{x}}_e) \in R^n \times R^n$ is the damping matrix of the environment, $K_e(\underline{x}_e) \in R^n \times R^n$ is the stiffness matrix of the environment, $\underline{F}_s \in R^n$ is the force vector measured by the force sensor, $K_{so} \in R^n \times R^n$ is the stiffness matrix of the force sensor expressed in its coordinate system, $R \in R^3 \times R^3$ is the rotation matrix that relates the co-ordinate system of the sensor with the co-ordinate system that is parallel to the global co-ordinate system, $K_s \in R^n \times R^n$ is the stiffness matrix of the force sensor on the global co-ordinate system, $\underline{u}$ is the control vector for the robot referred to the Cartesian space, $n \in N$ is the number of degrees of freedom of the manipulator end point.

For the description of the uncertainty of the stiffness and the damping of the environment the following formula is used:

$$B = (I + \Delta)\hat{B} \text{ and } |\delta_{ij}| \leq d_{ij}, \text{ i}=1\,..\,\text{n, j}=1\,..\,\text{n} \tag{2}$$

where $B \in R^n \times R^n$ is the stiffness matrix $K_e(\underline{x}_e)$ or the damping matrix $C_e(\dot{\underline{x}}_e)$, $\Delta \in R^n \times R^n$ is the parametric uncertainty matrix, $D \in R^n \times R^n$ is the matrix that bounds the parametric uncertainty of the stiffness matrix $\hat{K}_e(\underline{x}_e)$ or the damping $\hat{C}_e(\dot{\underline{x}}_e)$. The entries for matrices Δ and D are δ_{ij} *and* d_{ij} respectively. In the following analysis the symbol $\wedge$ is used for the estimated value of a quantity.

The matrices $K_e(\underline{x}_e)$ and $C_e(\dot{\underline{x}}_e)$ might be full and nonlinear, so environments with complete coupled dynamic systems are described. This approach is close to applications where the robot drives a passive mechanism. Although the nonlinearity of the matrices $K_e(\underline{x}_e)$ and $C_e(\dot{\underline{x}}_e)$ their estimations $\hat{K}_e(\underline{x}_e)$ and $\hat{C}_e(\dot{\underline{x}}_e)$ can be nonlinear or constants.

The matrices $H_r(\underline{x}_r)$, $C_r(\underline{x}_r, \dot{\underline{x}}_r)$, and vector $\underline{g}_r(\underline{x}_r)$ are given by the following relations [14]:

$$H_r = J^{-T}H^*(\underline{q})J^{-1}$$
$$C_r = J^{-T}C^*(\underline{q}, \dot{\underline{q}})J^{-1} - J^{-T}H^*(\underline{q})\mathrm{J}^{-1}\dot{\mathrm{J}}\mathrm{J}^{-1} \tag{3}$$
$$\underline{g}_r = \frac{\partial U}{\partial x_r}$$
$$\underline{u} = J^{-T}\underline{\tau}$$

where, $H^*(\underline{q}) \in R^m \times R^m$ is the inertia matrix of the robot in joint space, $C^*(\underline{q}, \dot{\underline{q}}) \in R^m \times R^m$ is the coriollis matrix of the robot in joint space, $J \in R^n \times R^m$ is the Jacobian matrix of the robot, U is the potential of the gravitation field, $\underline{\tau} \in R^m$ is

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the torque vector in the joint space, $\underline{q}$ is the vector of the joint coordinates and m is the number of degrees of freedom of the robot manipulator.

From the above equations, it is clear that the dynamics of the robot expressed in the Cartesian space is strictly related to the joint displacements and velocities.

3. DESIGN OF THE FORCE CONTROLLER

In the present study two controllers are proposed based on the sliding control method. The first controller can be used in the case where the environment has linear or non-linear damping and stiffness terms, which can be bounded. The second controller can be used when the damping effect of the environment is small or zero and the stiffness matrix is constant. In this case, an adaptive algorithm is combined with the sliding controller. The adaptive law is used to reduce the uncertainty bounds that are selected for the sliding controller. The reduction of the uncertainty bounds decreases considerably the chattering in the response, due to the action of the non-linear part of the controller. The adaptive estimator is based on the gradient method.[14]

The design of the force controller is based on the following assumptions:

- The environment is a passive mechanism described by its stiffness, damping and inertia characteristics.
- The inertia of the devices -gripper or tool- mounted after the force sensor, are included at the inertia of the environment, and all of them are considered known.
- The stiffness and the damping parameters of the environment are not known, however their bounds are known.
- The dynamic characteristics of the robot are known.

The most common force applications include environments where stiffness characteristics are dominant. However, some force applications are more complicated and the environment is characterised by damping and inertia terms.

The second assumption is related to the structure of the environment. The inertia of the environment or the passive mechanism can be determined because in most of the cases the inertia of the environment is actually the inertia of the gripper and of the grasped objects, and it is quite smaller than the inertia of the robot manipulator. In most of the cases, the stiffness and the damping of the environment are not known or are poorly known. The controller designed under this assumption increases the production flexibility of the robot, because it can manipulate a variety of environment/mechanisms where the knowledge of their stiffness or damping characteristics is quite poor.

It is assumed that the robot dynamics is known since the robot dynamics could be determined once or could be provided by the robot manufacturer. In cases of force application, the variation of the positions of the robot joints around the equilibrium point is relatively small, therefore the robot end-point velocities and the relevant coriollis forces are quite low.

3.1 Sliding mode force controller

The sliding surface based on the force error is given by the following formula:

$$\underline{s} = (\dot{\underline{F}}_s - \dot{\underline{F}}_d) + \Lambda(\underline{F}_s - \underline{F}_d) \tag{4}$$

Where $\underline{F}_d$: is the desired force.
Λ: is a positive definite diagonal matrix

According to Filippov's construction,[15] the condition $\dot{\underline{s}} = 0$ results to the convergence of the sliding error $\lim_{t \to \infty} \|\underline{s}\|_2 = 0$.

Using the condition $\dot{\underline{s}} = 0$ the optimum control law $\hat{\underline{u}}$ for each time step is obtained. The sliding error includes a differential component of the force error and not an integral one. The integration of the force error reduces the effect of the noise in the force signal. However, the differential component makes the system to respond faster. In addition, the noise in the force signal corresponds to tiny position error due to the high stiffness of the force sensor. Almerico *et al*[13] showed that the low resolution on the force signal and the existed noise do not affect so much to performance of their controller. Finally, the system has enough time to process the measured force signal in order to reject noise and to produce a smooth force derivative due to the difference between the force feedback sampling rate and the actuation bandwidth.[13]

Applying the Fillipov's construction to achieve $\dot{\underline{s}} = 0$, the best approximation law of a continuous control law is obtained:

$$\begin{aligned} \hat{\underline{u}} = {} & \underline{F}_s + C_r \dot{\underline{x}}_r + \underline{g}_r + H_r K_s^{-1}[\ddot{\underline{F}}_d - \Lambda(\dot{\underline{\mathrm{F}}}_\mathrm{s} - \dot{\underline{\mathrm{F}}}_\mathrm{d})] \\ & + \mathrm{H_r H_e^{-1}}(\underline{\mathrm{F}}_\mathrm{s} - \hat{\mathrm{C}}_\mathrm{e}(\dot{\underline{\mathrm{x}}}_\mathrm{e})\dot{\underline{\mathrm{x}}}_\mathrm{e} - \hat{\mathrm{K}}_\mathrm{e}(\underline{\mathrm{x}}_\mathrm{e})\underline{\mathrm{x}}_\mathrm{e}) \end{aligned} \tag{5}$$

where, the environment deformation x_e and its derivatives can be estimated by:

$$\begin{aligned} \underline{x}_e &= \underline{x}_r - K_s^{-1}\underline{F}_s \\ \dot{\underline{x}}_e &= \dot{\underline{x}}_r - K_s^{-1}\dot{\underline{F}}_s \end{aligned} \tag{6}$$

The equivalent control law given by equation (5) is the continuous part of the control law that would maintain $\dot{\underline{s}} = 0$, if the dynamics were exactly known. In order to obtain the stability of the system despite the environment uncertainty an additional discontinuous term across the surface $\underline{s} = 0$ is added to $\hat{\underline{u}}$:[14]

$$\underline{u} = \hat{\underline{u}} - Q\frac{\underline{s}}{\|\underline{s}\|_2 + \delta} \tag{7}$$

where Q is a positive definite matrix, and δ is a very small positive number.

The condition $\lim_{t \to \infty} \underline{s} = 0$ minimises the force error and its derivative, because the sliding surface can be interpreted as a set of first order linear filter in series for the state space errors where the matrix Λ provides the gains for these filters. However, the condition $\lim_{t \to \infty} \underline{s} = 0$, leads to invariant sets for the state space of the robot manipulator in the case of constant desired forces. For these invariant sets, the velocity of the robot end point is equal to the velocity of the environment but not zero. The condition $\lim_{t \to \infty} \underline{s} = 0$ leads to the following equations:

$$\left.\begin{aligned} \underline{F}_d &= \underline{F}_s \\ \dot{\underline{F}}_s &= \underline{0} \end{aligned}\right\} \Rightarrow \dot{\underline{x}}_r = \dot{\underline{x}}_e \tag{8}$$

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This means that for a constant desired force the control law completes the task but maybe the end point of the robot still moves with a velocity equal to the velocity of the environment. This leads to a non-desirable oscillatory response of the system.

In addition, the proposed control law cannot absorb sufficiently the robot end-point oscillations. In order to suppress the undesirable invariant sets, oscillations or impacts an absorption term is added to the control law:

$$\underline{u} = \underline{\hat{u}} - Q\frac{\underline{s}}{\|\underline{s}\|_2 + \delta} - \mathrm{K_d}\underline{\dot{\mathrm{x}}}_{\mathrm{r}} \qquad (9)$$

The matrix $\mathrm{K_d}$ is determined using Lyapunov stability theory. The following Lyapunov function candidate is formulated:

$$V(t) = \frac{1}{2}\underline{s}^T K_s^{-1}\underline{s} + \frac{1}{2}\underline{\dot{x}}_r^T H_r \underline{\dot{x}}_r \qquad (10)$$

K_s^{-1}: is a positive definite matrix as it is proved in Appendix I.

The selected Lyapunov function candidate represents the strain energy of the error that is stored in the sensor and the kinetic energy of the robot end-point expressed in the Cartesian space. The strain and the kinetic energy are positive by definition.

Differentiation of equation (10) with respect to time yields:

$$\dot{V}(t) = \underline{s}^T K_s^{-1}\underline{\dot{s}} + \underline{\dot{x}}_r^T H_r \underline{\ddot{x}}_r + \frac{1}{2}\underline{\dot{x}}_r^T \dot{H}_r \underline{\dot{x}}_r \leq 0 \qquad (11)$$

Since the matrix $(\dot{H}_r - 2C_r)$ is skew-symmetric,[9] the following relation is true:

$$\frac{1}{2}\underline{\dot{x}}_r^T(\dot{H}_r - 2C_r)\underline{\dot{x}}_r = 0 \qquad (12)$$

Substituting equation (12) in the equation (11) gives:

$$\dot{V}(t) = \underline{s}^T K_5^{-1}\underline{\dot{s}} + \underline{\dot{x}}_r^T H_r \underline{\ddot{x}}_r + \underline{\dot{x}}_r^T C_r \underline{\dot{x}}_r \qquad (13)$$

Lets assume that the non-linear term of the proposed control law is given by the following equation:

$$Q\frac{\underline{s}}{\|\underline{s}\|_2 + \delta} = H_r\left(N\frac{\underline{s}}{\|\underline{s}\|_2 + \delta} + \underline{m}\right) \qquad (14)$$

where $N \in R^m \times R^m$ is a positive definite diagonal matrix and the entries m_i of the vector m are given by the following equation:

$$m_i = p_i \cdot sign(s_i),\ i = 1, \ldots, 3 \qquad (15)$$

$$p_i = |a_{1_i}| + |a_{2_i}| \qquad (16)$$

where

$$\underline{a}_1 = H_e^{-1} D_c \hat{C}_e(\underline{\dot{x}}_e)\underline{\dot{x}}_e \text{ and } \underline{a}_2 = H_e^{-1} D_k \hat{K}_e(\underline{x}_e)\underline{x}_e \qquad (17)$$

The matrices D_c and $\mathrm{D_k}$ are the bounds of the corresponding uncertainty matrices Δ_c and Δ_k. From the above formulas, it is obvious that vector $\underline{m}$ depends on the bounded dynamics of the environment and compensates the uncertainty.

It is assumed that the damping term in the control law is given by:

$$K_d\underline{\dot{x}}_r = \frac{H_r^{-1}\underline{s} + \underline{\dot{x}}_r}{\|H_r^{-1}\underline{s} + \underline{\dot{x}}_r\|_2^2}\underline{\dot{x}}_r^T\left[\left(\underline{\hat{u}} - Q\frac{\underline{s}}{\|\underline{s}\|_2 + \delta} - \underline{F}_s\right) + G\underline{\dot{x}}_r\right] \qquad (18)$$

where $G \in R^n \times R^n$ a positive definite matrix.

Using the equations (13) to (18) the following inequality for the time derivative of the Lyapunov function candidate is obtained:

$$\dot{V}(t) \leq -\underline{s}^T N\frac{\underline{s}^T}{\|\underline{s}\|_2 + \delta} - \underline{\dot{x}}_r^T G \underline{\dot{x}}_r \qquad (19)$$

From the above analysis, it is concluded that:

$V(t)$ is a positive definite function,

$\dot{V}(t)$ is a negative definite function, and

$V(t) \to \infty$ as $\|\underline{s}\|_2 \to \infty$ and $\|\underline{x}_r\|_2 \to \infty$

Therefore, the equilibrium point $\|\underline{s}\|_2 = 0$ and $\|\underline{x}_r\|_2 = 0$ is globally asymptotic stable. Moreover if the matrices N and G are selected to be equal to $a \cdot K_s^{-1}$ and $a \cdot H_r$ respectively with $a \geq 0$, then the equilibrium point is exponentially stable.

From the above analysis it is concluded that the presented control algorithm has some differences from the classical approach given by Asada and Slotine.[15] The first is that matrix N is multiplied with vector $\frac{\underline{s}}{\|\underline{s}\|_2 + \delta}$. In the classical approach matrix N is multiplied with vector $sign(\underline{s})$. The use[16] of $\frac{\underline{s}}{\|\underline{s}\|_2 + \delta}$ vector in the feedback of the presented sliding controller reduces the chattering due to the following inequality:

$$\frac{s_i}{\|\underline{s}\|_2 + \delta} \leq 1,\ \forall\, s_i \in R \qquad (20)$$

where s_i is the i-th element of vector $\underline{s}$

According to this result higher convergence rates of the Lyapunov function can be achieved for the same amplitude of the chattering effect.

The second difference lies upon the selection of the elements of the feedback matrix Q. Slotine and Asada[15] are using the Frobenius-Perron theorem in order to determine the elements of the matrix Q, which is a difficult task. In the proposed method the feedback vector is calculating quite easily through the equation (14).

Finally in the control law a damping term for the end point of the robot is included. As it is shown in equation (18)

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this term takes into consideration both the sliding error and the robot end point velocity. Therefore, the desired force is applied without any undesirable overshooting in the response of the robot end point.

3.2 *Adaptive estimation of the unknown dynamics of the environment-mechanism*

The sliding mode control law causes considerable chattering in cases where a wide range of uncertainty is considered. An adaptive estimation algorithm is introduced to estimate some dynamic properties of the environment in order to reduce the bounds of the uncertainty, while the stability and the basic structure of the control system is not affected. The proposed algorithm is valid when the damping of the environment is zero or very small and the stiffness matrix is constant. These assumptions are very close to most force control cases.

The following error function is defined by:

$$\underline{e}=(\hat{K}_e-K_e)\underline{x}_e \tag{21}$$

A matrix $Y(\underline{x}_e)$ can be determined by $K_e\cdot\underline{x}_e=Y(\underline{x}_e)\cdot\underline{p}$, where vector $\underline{p}$ is the vector of the constant parameters of the matrix K_e. In the same way, it is true that $\hat{K}_e\cdot\underline{x}_e=Y(\underline{x}_e)\cdot\hat{\underline{p}}$. Using these relations, the estimation error $\underline{e}$ can be written in the following form:

$$\underline{e}=Y(\underline{x}_e)\cdot\tilde{\underline{p}} \tag{22}$$

where

$$\tilde{\underline{p}}=\hat{\underline{p}}-\underline{p} \tag{23}$$

The equation (21) is equivalent to the following equation:

$$\underline{e}=\hat{\underline{F}}_s-\underline{F}_s \tag{24}$$

where

$$\hat{\underline{F}}_s=H_e\ddot{\underline{x}}_2+\hat{K}_e\underline{x}_e \tag{25}$$

The error $\underline{e}$ is calculated using equation (24), because the vector $\hat{\underline{F}}_s$ is determined from equation (25) and the $\underline{F}_s$ vector is measured by the force sensor.

Applying the gradient adaptation law[15] to the error $\underline{e}$, the following update equation for the $\hat{K}_e$ is obtained:

$$\dot{\hat{\underline{p}}}=-\gamma\cdot Y^T(\underline{x}_e)\cdot\underline{e} \tag{26}$$

where γ is a positive constant.

The stability of the adaptation algorithm is investigated using the following Lyapunov function candidate:

$$V(t)=\frac{1}{2}\tilde{\underline{p}}^T\cdot\tilde{\underline{p}} \tag{27}$$

This Lyapunov function candidate expresses the magnitude of the error of the estimated variables. Since $V(t)$ expresses a magnitude-distance in a metric space- it is positive. Convergence of Lyapunov function to zero leads to the reduction of the estimated variable's error. The time derivative of this Lyapunov function is the following:

$$\dot{V}(t)=-2\cdot\gamma\cdot\underline{e}^T\cdot\underline{e}\leq 0 \tag{28}$$

According to the above inequality the adaptive algorithm is stable. Although the stability is achieved the convergence of the estimated parameter error $\tilde{\underline{p}}$ to $\underline{0}$ is not always guaranteed. A brief study of the conditions that should be satisfied in order to achieve the convergence is presented in Appendix II.

The estimated stiffness matrix by the update law is used for the reduction of the uncertainty. In the case of unknown environment stiffness and zero damping the only assumption that is needed it hat stiffness matrix of the environment have to be constant. Using equation (2) for constant K_e the following relation is obtained:

$$K_e=(I+\Delta_k)\hat{K}_e \tag{29}$$

The time derivative of the stiffness uncertainty matrix is obtained by differentiating equation (29).

$$\dot{\Delta}_k=-(I+\Delta_k)\dot{\hat{K}}_e\hat{K}_e^{-1} \tag{30}$$

By applying equation (2) for $\Delta=\Delta_k$, can be assumed that a positive constant q_{ij} exists satisfying the following equation:

$$|\delta_{k\,ij}|+q_{ij}=d_{k\,ij},\ \mathrm{i}=1,\ldots,3\ \text{and}\ \mathrm{j}=1,\ldots,3 \tag{31}$$

After the differentiation of equation (31) the following equation is obtained:

$$\dot{d}_{k\,ij}=sign(\delta_{k\,ij})\dot{\delta}_{k\,ij} \tag{32}$$

The combination of equation (30) and equation (32), relates the estimated matrix $\hat{K}_e$ with the matrix of the bounds D_k. In order to estimate the matrix Δ_k, from equation (2) and equation (20) the following is obtained.

$$\underline{e}=-\Delta_k\cdot\hat{K}_e\cdot\underline{x}_e \tag{33}$$

The system in equation (33) is indeterminate but one solution is given by the following relation:

$$\Delta_k=-\underline{e}\cdot\frac{\underline{x}_e}{\|\underline{x}_e\|_2^2}\hat{K}_e^{-1} \tag{34}$$

The algorithm drives the variables $d_{k\,ij}$ to be equal to the positive constant q_{ij} and $|\delta_{k\,ij}|=0$.

From the above analysis it is concluded that the proposed adaptive algorithm obtains two targets. The first target is the determination of the values of the environment stiffness matrix elements. The second target is the change of the uncertainty bounds according to the updated values of the estimated stiffness matrix of the environment.

To achieve the first target a simple gradient adaptive algorithm is used. Although the stability of the algorithm is achieved the convergence of the estimated values to the real ones is not guaranteed but it depends on the values of the

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vector $\underline{x}_e$ in order to maintain the stability of the sliding controller.

The achievement of second target is based on the assumptions of constant K_e matrix and the constant distance between the elements of matrices D and Δ (see equation (31)). The second assumption is done in order to maintain the stability of the proposed sliding controller.

The presented adaptive algorithm uses the estimation of the environment stiffness matrix for the reduction of the stiffness uncertainty bounds. According to this mechanism dynamics is 'transferred' from the non-linear part of the control law to the equivalent control law without any reduction of the robustness of the sliding controller. The interconnection between the sliding controller and the adaptive algorithm is shown in Figure 2.

4. SIMULATION RESULTS FOR THE DEVELOPED CONTROLLERS

In order to validate the performance of the developed controllers two different simulations were performed. In the first simulation the environment damping is neglected while the stiffness matrix is assumed constant and diagonal. The adaptive law estimates the stiffness parameters of the environment and reduces the initially selected bounds of the stiffness. In the second simulation the environment stiffness is coupled and the used elastic model is nonlinear. In this case the adaptive estimator is not combined with the force sliding controller without the adaptive estimator, because the proposed adaptive estimator is valid for constant stiffness matrix only.

The robot manipulator used in the simulation is a two D.O.F. planar rigid manipulator with a force sensor mounted at the robot end point. The dynamic parameters of the robot manipulator are the following:

$$l_1 = 1m,\ l_2 = 1m$$

$$m_1 = 10kg,\ m_2 = 10kg$$

where l_1 and l_2 are the lengths of the robot links, m_1 and m_2 are the masses of the robot links.

The stiffness of the force sensor is the following:

$$K_{SO} = \begin{bmatrix} 10^4 & 0 \\ 0 & 10^4 \end{bmatrix} (N/m)$$

5.1 *Example with Constant Stiffness Matrix*

In the first example the inertia and stiffness matrices are the following:

$$H_e = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \left(\frac{N\ \sec^2}{m} \right), \qquad K_e = \begin{bmatrix} 10^3 & 0 \\ 0 & 10^3 \end{bmatrix} (N/m)$$

The control parameters are the following:

$$\hat{K}_e = \begin{bmatrix} 10^4 & 0 \\ 0 & 10^4 \end{bmatrix} (N/m), \qquad D_k = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$N = \begin{bmatrix} 50 & 0 \\ 0 & 50 \end{bmatrix}, \quad \Lambda = \begin{bmatrix} 1.5 & 0 \\ 0 & 1.5 \end{bmatrix}, \quad G = \begin{bmatrix} 35 & 0 \\ 0 & 35 \end{bmatrix}, \quad \gamma = 100$$

In Figures 3–4 the desired and achieved forces applied to the environment in x and y directions are shown. The desired force waveform is drawn with dashed lines while the response is drawn with solid line. As it can be seen the steady state error is tiny, less than 0.2% and the settling time is about 2 seconds. The transient part of the response presents quickly damped oscillations due to the additional damping term provided by the control system. Significantly tiny is the size of the chattering effect in the force response. In the steady state the chattering amplitude is near to zero due to the action of the adaptive estimator, which reduces the bounds of the environment stiffness uncertainty.

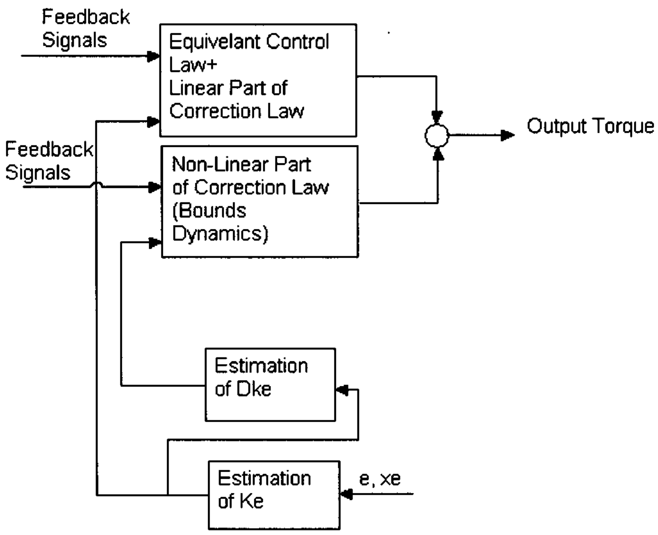

Fig. 2. Relation between the adaptive law and the control system.

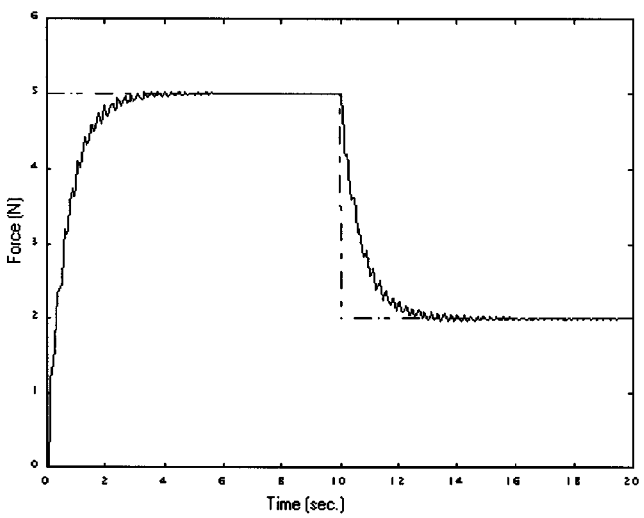

Fig. 3. Desired and achieved force on the environment along x-axis.

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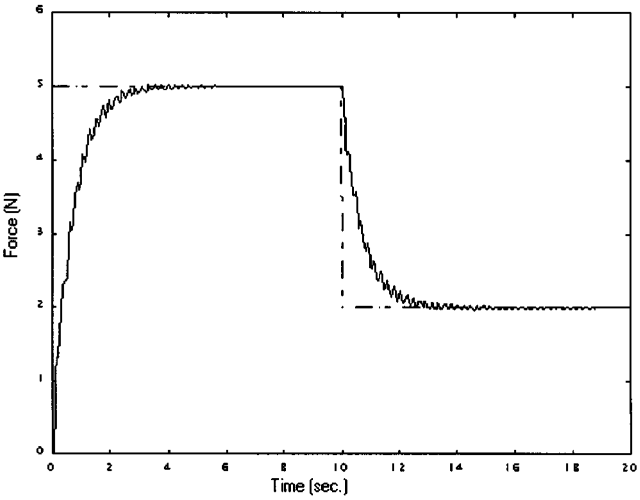

Fig. 4. Desired and achieved force on the environment along y-axis.

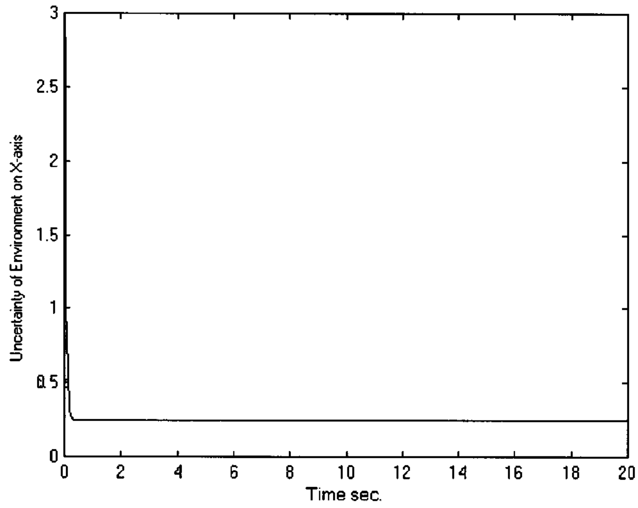

Fig. 7. Reduction of environment stiffness boundaries along x-axis.

In Figures 5–6 the velocity of the environment in x and y directions are shown. In these diagrams the action of the damping provided by the control system is more obvious.

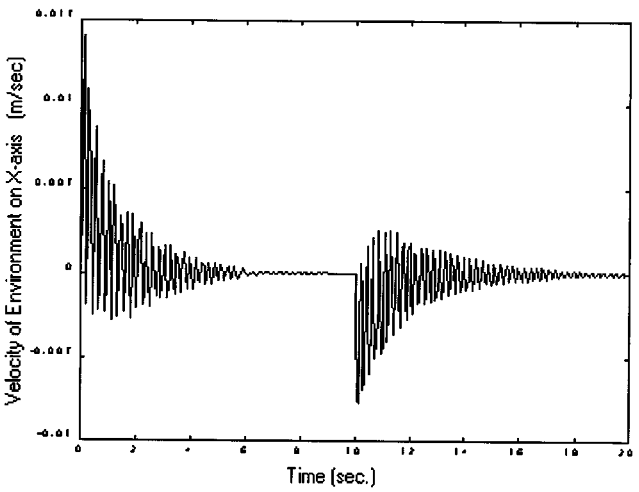

Fig. 5. Velocity of the environment along x-axis.

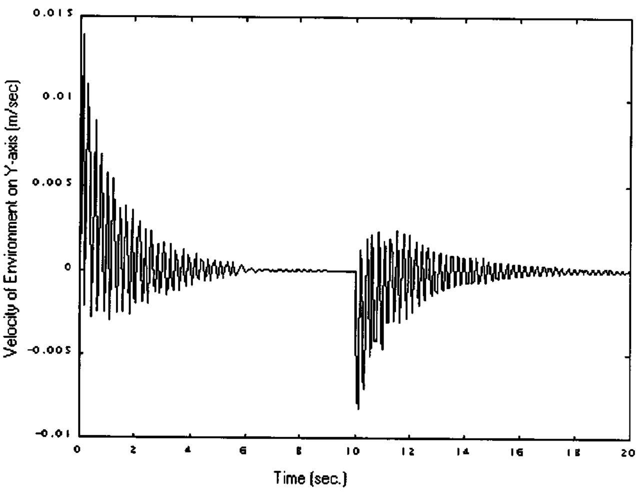

Fig. 6. Velocity of the environment along y-axis.

In Figures 7–8 the reduction of the elements of the matrix D_k are shown. The initial value of both diagonal elements of D_k is 3. At the steady state the elements of D_k are reduced to 0.25 and 1.1, respectively. In the same time the adaptive system estimates the actual values of the diagonal elements of the environment stiffness matrix. The estimation error is less than 0.01%. In the present case the elements of vector $\underline{x}_e$ have the same sign and the environment stiffness matrix is diagonal. According to the results of the Appendix II the estimated values of the environment stiffness matrix convergence exponentially to the actual ones.

This combined action of the adaptive system results to the near-smooth force response with respect to the sliding controller stability.

4.2 Example with coupled and nonlinear stiffness matrix

In the second simulation the dynamic properties of the environment are the following:

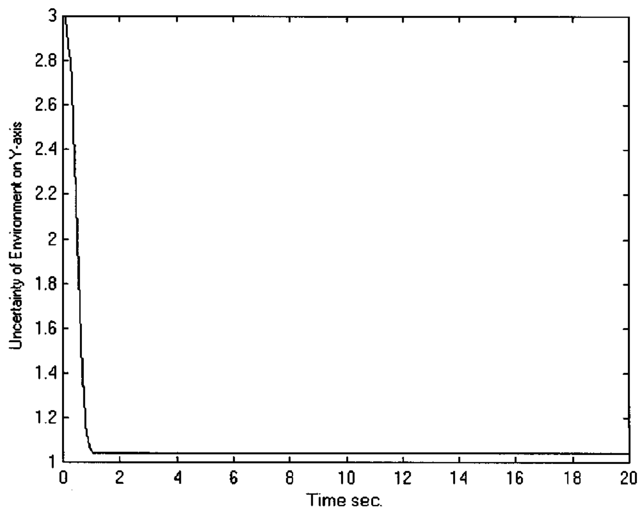

Fig. 8. Reduction of environment stiffness boundaries along y-axis.

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$$H_e=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\left(\frac{N\,\mathrm{sec}^2}{m}\right),\ K_e(\underline{x}_e)=\begin{bmatrix}f(x) & -10\\ -10 & f(x)\end{bmatrix}(N/m),$$

$$C_e=\begin{bmatrix}3 & -1\\ -1 & 3\end{bmatrix}(N\cdot\mathrm{sec}/m)$$

$$f(x)=\begin{cases}5\cdot 10^5\cdot|x|, & |x|\le 2\cdot 10^{-3}\\ 500, & |x|>2\cdot 10^{-3}\end{cases}$$

The selected control parameters are the following:

$$\hat{K}_e=\begin{bmatrix}10^4 & -15\\ -15 & 10^4\end{bmatrix}(N/m) \qquad D_k=\begin{bmatrix}25 & 2\\ 2 & 25\end{bmatrix}$$

$$\hat{C}_e=\begin{bmatrix}1.5 & -1\\ -1 & 1.5\end{bmatrix}\left(\frac{N\,\mathrm{sec}^2}{m}\right), \qquad D_c=\begin{bmatrix}5 & 5\\ 5 & 5\end{bmatrix}$$

$$N=\begin{bmatrix}50 & 0\\ 0 & 50\end{bmatrix},\quad \Lambda=\begin{bmatrix}5 & 0\\ 0 & 5\end{bmatrix},\quad G=\begin{bmatrix}35 & 0\\ 0 & 35\end{bmatrix}$$

In Figure 9 the graph of $f(x)$ is shown. It is obvious that for small environment deflections, the diagonal stiffness elements are low. Below the level of $2\cdot 10^{-3}$ (m) the diagonal stiffness elements are linear functions of the environment deflection. After this level the diagonal stiffness elements are constant and equal to 500 N/m.

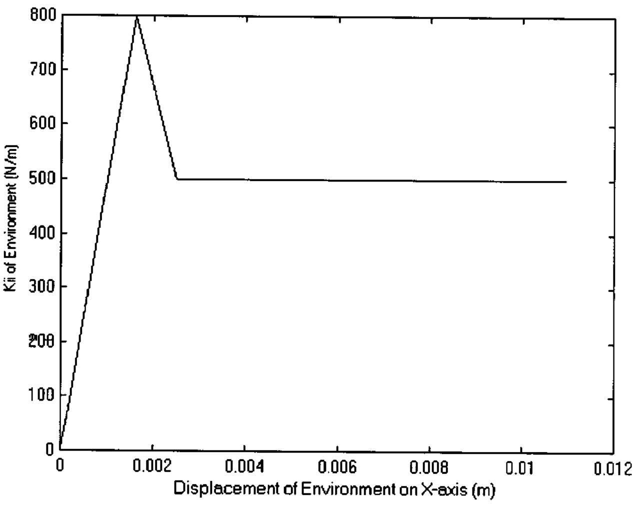

Fig. 9. Environment stiffness.

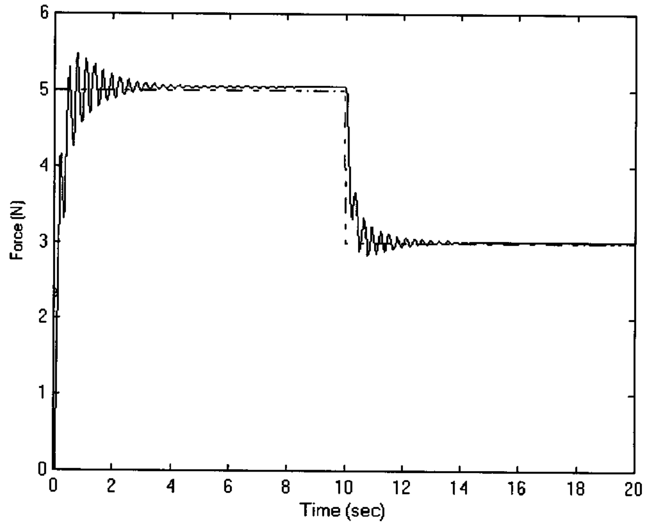

Fig. 10. Desired and achieved force on the environment along x-axis.

In Figures 10–11 the desired and achieved forces applied to the environment on x and y directions are shown. At the beginning of the force response, oscillations appear due to the low values of the diagonal elements of the stiffness matrix. These oscillations are absorbed quickly by the damping terms of the environment and the control system. The error in the steady state is tiny, less than 0.2%. The settle time for the force response in this simulation is about 2 sec. The oscillations in the force responses of the environment in this case are bigger than the corresponding oscillations appeared in the response shown in Figures 3–4. This is caused by the small values of the stiffness of the environment at the first steps of the simulation. In Figures 12–13 the velocities of the environment in x and y directions are shown. In these figures the rate of absorption of the transient response oscillations is shown better.

5 DISCUSSION ON THE METHOD

In this paper a sliding mode force controller is proposed. The sliding error is formed by the signal of the end effector

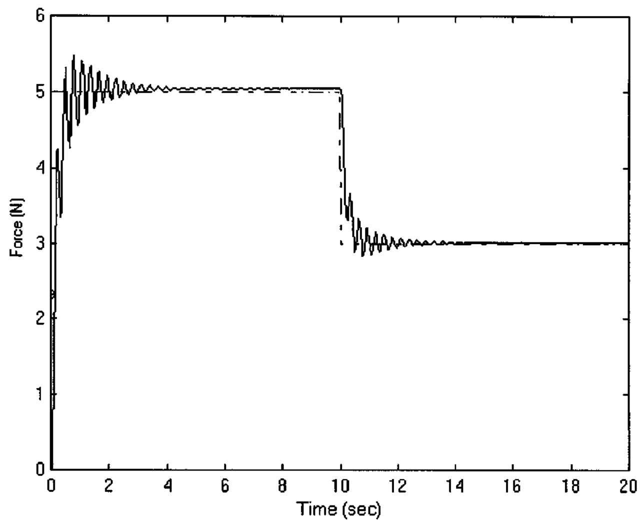

Fig. 11. Desired and achieved force on the environment along y-axis.

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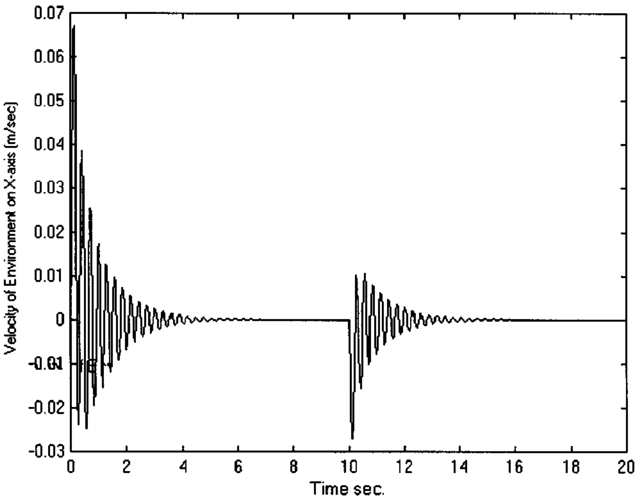

Fig. 12. Velocity of the environment along x-axis.

force sensor and the desired force. The environment acts as a passive mechanism with inertia, damping and stiffness properties. The exact models of the stiffness and damping are not known, only the bounds of them are known. Therefore, the proposed controller can encounter environments with structured or unstructured uncertainty. In the proposed sliding mode controller, the function $\underline{s}/(\|s\|_2+\delta)$ was used instead of $sign(\underline{s})$ function.[15] This results to smaller chattering amplitudes and easier determination of the nonlinear part of the control law. Also the sliding error is distributed to each axis according to the normalisation law and the chattering effect is reduced. Velocity feedback is added in order to reduce the velocities redundancy caused by the form of the sliding error in the case of static load (see equation (6)) and also to absorb the transient oscillations.

For the case where the environment stiffness is constant and the environment damping can be neglected an adaptation law is formed. The adaptation law reduces the uncertainty because it narrows the bound of the stiffness uncertainty. The small value of the uncertainty bounds results to the smaller contribution of the nonlinear part of equation (15) of the control law to the control input (see equation (7)), while the equivalent control law given by

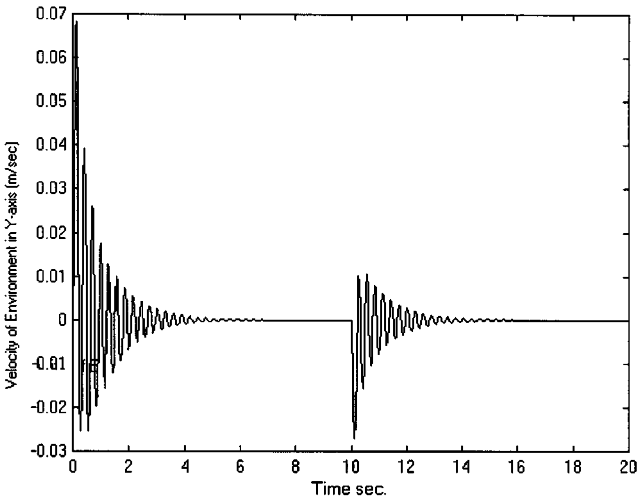

Fig. 13. Velocity of the environment along y-axis.

equation (5) is fed with the estimated values of the environment stiffness matrix.

From the simulations, it is shown that the algorithm can manipulate quite well environment with coupled, nonlinear or linear stiffness. Although the simulated environment stiffness was always lower than the force sensor stiffness, no oscillations appeared in the steady state due to the use of velocity feedback.

6. CONCLUSIONS AND FURTHER WORK

A robust force controller that can be combined with an adaptive estimation module under specific circumstances is presented. It is based on the sliding mode control. A gradient adaptive algorithm is used for the estimation of the stiffness of the environment when this is constant. The controller was implemented for revolute robot manipulator in the Cartesian space.

From the simulation results can be concluded that the proposed controller guides the robot to apply the desired forces to linear or nonlinear, coupled or uncoupled passive mechanisms quite accurately. In the simulations the steady state error is less than 0.2% in both cases. The settling time depends on the values of the selected control parameters. In the simulated cases, the control parameters are selected in order to have small settling time with reasonable amplitudes at the transient response oscillations. When the environment presents high stiffness the settling time can be further reduced while the overshooting is kept under desirable limits.

As further work one can propose an experimental application of the algorithm. The influence of the feedback signals noise in the performance of the controller can be investigated and the uncertainties of robot dynamics considered too into the non-linear part of the control law. Finally, the extension of the proposed method for the general problem of the Hybrid Force/Position control is a very interesting task.

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APPENDIX I

In this appendix it will be proved that the matrix K_s^{-1} is positive definite. The mathematical formulation of this proof is because the matrix K_{so}^{-1} is positive definite. For the matrix K_{so} the following relations are true:

$$k_{so_{i,i}} > 0,\ k_{so_{i,j}} = 0\ \forall\ i \neq j \tag{35}$$

The following relation gives the elements of the inverse matrix of K_{so}:

$$\{k_{so_{i,i}}\}^{-1} = 1/k_{so_{i,i}} > 0,\ \{k_{so_{i,j}}\}^{-1} = 0\ \forall\ i \neq j \tag{36}$$

Therefore the matrix K_{so}^{-1} is positive definite. The relation between K_s^{-1} and K_{so}^{-1} is the following:

$$K_s^{-1} = R^{-1} K_{so} R \tag{37}$$

The following relation should be proved:

$$\underline{x}^T K_s^{-1} \underline{x} \geq 0 \tag{38}$$

From equation (37) results to the following relation:

$$\underline{x}^T K_s^{-1} \underline{x} = (\underline{x}^T R^{-1}) K_{so}^{-1} (R\underline{x}) \tag{39}$$

If the transform matrix R is orthogonal then $R^{-1} = R^T$. This happens in our application because matrix R is the result of rotation of an orthogonal co-ordinate system. After this consideration the equation (39) results to:

$$\underline{x}^T K_s^{-1} \underline{x} = (R\underline{x})^T K_{so}^{-1} (R\underline{x}) \tag{40}$$

From equation (40) results that K_s^{-1} is positive definite because K_{so}^{-1} is positive definite.

The next step that is needed for the analysis of the stability of the control system is to prove that K_s^{-1} is symmetric. From the above analysis it results that where R is orthogonal then K_{so}^{-1} is diagonal. According to these the following relations are true.

$$K_s^{-T} = (R^{-1} K_{so}^{-1} R)^T \Rightarrow K_s^{-T} = R^{-1} K_{so}^{-1} R \Rightarrow K_s^{-T} = K_s^{-1} \tag{41}$$

APPENDIX II

In order to study the conditions under which the parameter estimation error converges to zero it will be assumed that the environment stiffness matrix satisfies the following conditions:

$$\begin{aligned} & K_e \in R^3 \times R^3 \\ & rank(K_e) = 3 \\ & \{k_e\}_{i,j} = \{k_e\}_{j,i}\ \ i = 1..3, j = 1..3 \end{aligned} \tag{42}$$

The environment stiffness matrix is symmetric $\{k_e\}_{i,j} = \{k_e\}_{j,i}$ $i = 1..3, j = 1..3$ due to the force compatibility and in the general case is coupled. The environment stiffness matrix has the following form:

$$K_e = \begin{bmatrix} k_{1,1} & k_{1,2} & k_{1,3} \\ k_{1,2} & k_{2,2} & k_{2,3} \\ k_{1,3} & k_{2,3} & k_{3,3} \end{bmatrix} \tag{43}$$

From the above results that the number of the unknown elements of the stiffness matrix is 6. Also the same from has the matrix $\tilde{K}_e$ which is equal to matrix $\hat{K}_e - K_e$. From equation (21) and equation (22) results the following system:

$$\begin{bmatrix} \tilde{k}_{1,1} & \tilde{k}_{1,2} & \tilde{k}_{1,3} \\ \tilde{k}_{1,2} & \tilde{k}_{2,2} & \tilde{k}_{2,3} \\ \tilde{k}_{1,3} & \tilde{k}_{2,3} & \tilde{k}_{3,3} \end{bmatrix} \begin{bmatrix} x_{e_1} \\ x_{e_2} \\ x_{e_3} \end{bmatrix} = \begin{bmatrix} y_{1,1} & y_{1,2} & y_{1,3} & y_{1,4} & y_{1,5} \\ y_{2,1} & y_{2,2} & y_{2,3} & y_{2,4} & y_{2,5} & y_{2,6} \\ y_{3,1} & y_{3,2} & y_{3,3} & y_{3,4} & y_{3,5} & y_{3,6} \end{bmatrix} \begin{bmatrix} \tilde{k}_{1,1} \\ \tilde{k}_{1,2} \\ \tilde{k}_{1,3} \\ \tilde{k}_{2,2} \\ \tilde{k}_{2,3} \\ \tilde{k}_{3,3} \end{bmatrix} \tag{44}$$

The solution of the above system is the following:

$$Y = \begin{bmatrix} x_{e_1} & x_{e_2} & x_{e_3} & 0 & 0 & 0 \\ 0 & x_{e_1} & 0 & x_{e_2} & x_{e_3} & 0 \\ 0 & 0 & x_{e_1} & 0 & x_{e_2} & x_{e_3} \end{bmatrix} \tag{45}$$

According to [15], in order to achieve the convergence of $\underline{\tilde{p}}$ to $\underline{0}$ the following condition has to be satisfied:

$$\int_t^{t+T} Y^T Y dt \geq aI,\ a > 0 \tag{46}$$

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After some calculations the above condition results to the following conditions:

$$\begin{array}{lll} \text{Cond.1} & \textit{Cond.2} & \textit{Cond.3} \\ \int_t^{t+T} x_{e_1}^2 dt \geq \rho, & \int_t^{t+T} (x_{e_1}^2 + x_{e_2}^2) dt \geq \rho, & \int_t^{t+T} x_{e_1} x_{e_2} dt \geq 0 \\ \int_t^{t+T} x_{e_2}^2 dt \geq \rho, & \int_t^{t+T} (x_{e_1}^2 + x_{e_3}^2) dt \geq \rho, & \int_t^{t+T} x_{e_1} x_{e_3} dt \geq 0 \;, \rho \geq 0 \\ \int_t^{t+T} x_{e_3}^2 dt \geq \rho, & \int_t^{t+T} (x_{e_2}^2 + x_{e_3}^2) dt \geq \rho, & \int_t^{t+T} x_{e_2} x_{e_3}) dt \geq 0 \end{array} \tag{47}$$

The set of Cond.1 is always real. The set of Cond.2 is satisfied always because the set of Cond.1 is satisfied. The variable ρ can be selected according to the following norm:

$$\rho = \min \left\{ \int_t^{t+T} x_{e_1}^2 dt, \int_t^{t+T} x_{e_2}^2 dt, \int_t^{t+T} x_{e_3}^2 dt \right\} \tag{48}$$

For the set of Cond.3 the following conditions have to be satisfied:

- The products $x_{e_1} x_{e_2}, x_{e_1} x_{e_3}, x_{e_2} x_{e_3}$ are positive **or**
- The components of vector $\underline{x}_e$ have at least one harmonic component each of them.

In the case where the stiffness environment matrix is decoupled it has the following form:

$$K_e = \begin{bmatrix} k_{1,1} & 0 & 0 \\ 0 & k_{2,2} & 0 \\ 0 & 0 & k_{3,3} \end{bmatrix} \tag{49}$$

From the above it follows that the number of the unknown elements of the stiffness matrix is 3. In this case the regression matrix Y is the following:

$$Y = \begin{bmatrix} x_{e_1} & 0 & 0 \\ 0 & x_{e_2} & 0 \\ 0 & 0 & x_{e_3} \end{bmatrix} \tag{50}$$

According to equation (47) and equation (50) the convergence of the parameter estimation error is achieved when the set of Con.1 is satisfied. As was mentioned before these conditions are satisfied always, so when the environment stiffness matrix is decoupled then the convergence of the parameter estimation error is always guaranteed.

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