Macroeconomic Dynamics, **17**, 2013, 1467–1495. Printed in the United States of America.
doi:10.1017/S1365100512000193

CREDIT FRICTIONS AND FIRM DYNAMICS

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In this paper I develop a dynamic stochastic general equilibrium model of credit frictions in which the production technology provides a U-shaped average cost curve, enabling endogenous solutions for firm size and quantity. Firms weigh the present value of future net revenues against the opportunity cost of staying in business in their entry or exit decisions. I find that credit frictions increase variable investment costs and result in a larger firm size and a smaller number of firms in the steady state. As the economy deviates from the steady state, however, the presence of credit frictions increases fluctuation in the number of firms, raising market entry during an economic upturn and market exit during a downturn. Also, I find that allowing free entry mitigates some of the effects of credit frictions due to macroeconomic fluctuations. In addition to the homogeneous-firm model, I examine the model when firms have heterogeneous access to credit and find that different credit access gives rise to different firm sizes in the steady state. Firms with easier access to credit become larger than those with less access to credit. Heterogeneous credit access also means that these two types of firms will respond differently to a common technology shock.

Keywords: Credit Market Frictions, Collateral Constraint, Firm Size, Firm Heterogeneity, Entry and Exit

1. INTRODUCTION

Following the collapse of U.S. credit markets in the second half of 2007, growth in industrialized economies has slowed down remarkably. This slowdown, along with observations from previous financial crises, indicates that credit market conditions significantly affect real economic conditions. Credit frictions, which are typically defined as imperfect credit market conditions, cause contractions in the credit supply, raise investment costs for businesses, and spread the effects of economic shocks.

In the literature, this financial propagation mechanism has been well developed under dynamic stochastic general equilibrium (DSGE) conditions. Examples include the model of a collateral constraint [e.g., Kiyotaki and Moore (1997),

I owe my deepest gratitude to my Ph.D. advisor at Southern Methodist University, Nathan Balke, for his continuous help and invaluable advice. I thank Thomas Fomby, Tim Fuerst, Enrique Martinez-Garcia, Santanu Roy, and Pete VanderHard for their comments and encouragement. I am also grateful to an anonymous referee for insightful and helpful suggestions. Address correspondence to: Zheng Zeng, Department of Economics, Bowling Green State University, Bowling Green, OH 43403, USA; e-mail: zzeng@bgsu.edu.

 1365-1005/12

Iacoviello (2005)] and the framework of costly state verification [e.g., Townsend (1979), Bernanke and Gertler (1989), Carlstrom and Fuerst (1997), Bernanke et al. (1999)]. Even though these models differ in detail, the results of these studies support the shock propagation effects of credit frictions. Although these studies have highlighted the significance of credit frictions for aggregate economic conditions, there has been little research to date on the impact of credit frictions on firm size, number of firms, or firm heterogeneity.

This paper contributes to filling this void. To incorporate firm entry and exit, instead of imposing exogenous exit distortions [e.g., Chatterjee and Cooper (1993), Cooley and Quadrini (2001), Bilbiie et al. (2005), Bergin and Corsetti (2008)], I propose a DSGE model in which firms' production technology provides a U-shaped average cost curve. With this curve, the endogenous individual size and optimal total number of firms can be determined and solved. Firms, which are forward-looking, weigh the present value of their future revenues against the opportunity cost of staying in business, which is the value of total asset holdings, to make entry or exit decisions. A zero-profit condition holds in the steady state of the model economy.

Using this model economy as a benchmark, I introduce a model of credit friction adopted from Kiyotaki and Moore (1997). As firms seek external funds to finance capital investments, their borrowing is subject to a collateral constraint. By comparing the two models, I first find that this credit friction results in both a smaller number of firms and a larger individual firm size in the steady state. The collateral constraint increases financing and investment costs in relation to variable costs. Based on the U-shaped average cost curve, larger firms have a cost advantage as compared to the benchmark economy. As a result, the size of individual firms rises and the market selects fewer firms in the steady state as the credit constraints bind. Second, I find that the binding collateral constraint causes greater fluctuations in firm entry (or exit) when the economy deviates from the steady state. A positive technology shock increases the expected future cash flow and triggers firm entry; however, the shock also encourages firms to boost investment and increase their asset holdings. The increase in firms' asset holdings raises opportunity costs to stay in or enter the market. Whether the number of firms will be affected by this positive shock depends on the presence of the credit friction. In the model economy with friction, the collateral constraint distorts asset prices, discourages firms from adjusting their asset holdings in response to the economic shock, and therefore leaves more room for entry than the frictionless model economy.

I also find that the effect of credit frictions on aggregate economy is conditional upon entry barriers. I set up three types of entry and exit rules: "free entry," in which a zero-profit condition holds; "restricted entry," in which a barrier to entry is introduced; and "no entry," in which the number of firms is fixed. I find that aggregate economic activity in the model economy with a higher entry barrier is affected by credit frictions more than in a model with free entry. This finding is very crucial. In the literature, the well-known "credit view" concludes that credit frictions have an important impact on business cycle fluctuations [e.g., Iacoviello (2005), Carlstorm et al. (2010)]. However, there are also arguments that place much lower significance on the propagation effect of credit frictions [e.g., Kocherlakota (2000), Cordoba and Ripoll (2004)]. I do not endorse one viewpoint in all cases but instead argue that the impact of credit frictions on macroeconomic fluctuations is conditional on the market structure, i.e., the entry and exit conditions. The market with entry barriers tends to be more sensitive to the effects of credit frictions.

In addition to the homogeneous-firm model, I introduce a model economy with firm heterogeneity. Firms are initially different in their production factor intensities, which leads to heterogeneous access to credit. Investment by firms with easier access to credit is subject to a less binding collateral constraint and vice versa.

One purpose of introducing this model extension is to examine the relation between credit frictions and firm size. It is well known in the literature that small firms tend to have less access to credit or are more likely to be credit-rationed than large firms [see Calomiris and Hubbard (1990), Gertler and Gilchrist (1993, 1994), Fisher (1999)], yet whether firm size can be affected by heterogeneous access to credit remains unclear. In this model extension, I show that when credit constraints are binding, it will be less costly for firms with easier access to credit to accumulate capital and produce output than for those with less access to credit. Using the value of output and the value of total assets as two measures of firm size [see Gilchrist and Gertler (1994)], the results imply that firms with easier access to credit are larger than their counterparts; thus, credit frictions do give rise to different firm sizes in the model economy.

Finally, to examine the effects of credit friction on firm heterogeneity when the economy deviates from the steady state, I simulate the impulse responses of both types of firms to a common shock to total factor productivity (TFP) and find them fairly alike in a frictionless economy. This similarity occurs because firms share the same TFP despite their different factor intensities. Nevertheless, with a binding credit constraint, heterogeneous access to credit causes firms to make different investment and production decisions, and consequently, respond differently to the same shock.

The rest of the paper is organized as follows. In Section 2, I introduce the homogeneous-firm model and present the benchmark general equilibrium with credit frictions and firm entry and exit. I also provide the parameterization and quantitative analysis of this model economy. In Section 3, I extend the model with firm heterogeneity and discuss the results. The final section provides concluding remarks and suggests avenues for future research.

2. HOMOGENEOUS-FIRM MODEL

2.1. General Equilibrium with Entry and Exit

The benchmark model is based on a standard real business cycle (RBC) setup. The economy consists of a single representative household and N_t symmetric firms staying in business at time period t. The production technology requires labor input, capital input (which includes both physical and intangible capital), and land input. The household provides labor, but the rest of the production factors are owned by firms. The final good is the numeraire and can be used as a consumption good for the household and a capital good for the firms that decide to stay in business. The land market is cleared by the household and firms together, and the total land supply is fixed.

The firms. Firms own capital stock K_t and land stock H_t at the beginning of each time period t and hire labor L_t from the household to produce the final good Y_t, subject to the technology

$$Y_t = A_t K_t^{\alpha} H_t^{\theta} L_t^{\gamma} - \Psi,$$

where Ψ is a fixed production cost. I also assume that

$$\alpha + \theta + \gamma < 1,$$

which gives an upward-sloping marginal cost curve. This production function generates a U-shaped average cost curve. Without entry and exit distortions, the U-shaped average cost curve allows the market to determine the number of firms existing in the economy and the size of each firm in a DSGE model.

Using the funds obtained from selling the final goods Y_t, firms pay their wage bills at a common wage rate W_t and their debt B_t at an interest rate R_t, distribute their profits (dividends) d_t to the household, and issue shares S_{t+1} to the household at price P_t^k. Firms that choose to stay in business borrow new debt B_{t+1} from the household and make land investments $Q_t(H_{t+1} - H_t)$ at the unit price of land Q_t and capital investment $K_{t+1} - (1-\delta)K_t$ with depreciation rate δ. For simplicity, I assume that firms can accumulate capital directly by purchasing their own goods without producing investment goods; hence, the price of capital equals the price of final goods, P_t. Each firm staying in business maximizes the summation of current profit and share value,

$$\operatorname*{Max}_{L_{i,t}, K_{i,t+1}, H_{i,t+1}, B_{i,t+1}} \left(d_t + P_t^k\right),$$

where

$$d_t = Y_t - W_t L_t - R_t B_t + B_{t+1} - Q_t\,(H_{t+1} - H_t) - P_t\,[K_{t+1} - (1-\delta)\,K_t]. \quad \textbf{(1)}$$

Note that firms decide whether to enter or exit before deciding to invest. Figure 1 summarizes the timing of the entire production and investment process.

As firms decide to exit, they sell their land and capital stock to other firms in the internal market, make zero investment, issue zero debt and shares, pay dividends

https://doi.org/10.1017/S1365100512000193 Published online by Cambridge University Press

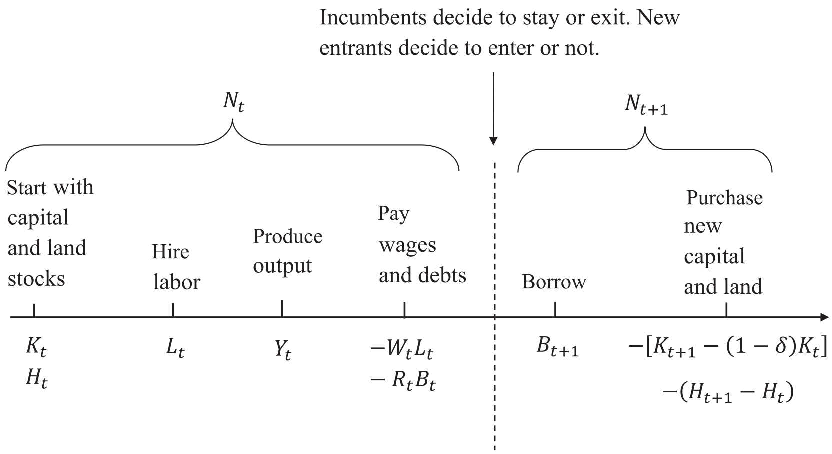

FIGURE 1. Timing of the production and investment process of firms.

to the household, represented by

$$d_t^{\mathrm{ex}} = Y_t - W_t L_t - R_t B_t + Q_t H_t + (1-\delta)\, P_t K_t, \tag{2}$$

and finally leave the market.

Because all firms are identical, there is either entry or exit in the economy but not both. When the market allows entry, each new entrant borrows funds and invests in capital and land in the same way as incumbents. To keep the model solvable, without loss of generality, I want to avoid tracking the optimal factor choices of different firms. Because both new and incumbent firms invest in the same manner, I assume that each new entrant receives a transfer Z_t equal to the sum of an incumbent's capital and land stock, $[Q_t H_t + (1-\delta) P_t K_t]$. These transfers are financed by a lump-sum tax payment T_t levied on the household. One can interpret the transfer to each new firm as an initial public offering that is not restricted by any financial constraints. With this assumption, new entrants issue shares at the same price and borrow the same amount of debt as incumbents do, and they use the resources raised by issuing shares and debt to make the same capital and land investment decisions, as well. The dividend flow provided by each of the new entrants is therefore given by

$$d_t^{\mathrm{en}} = Z_t + B_{t+1} - Q_t H_{t+1} - P_t K_{t+1}. \tag{3}$$

Though new entrants' dividend flows differ from those of the incumbents, the investment and production decisions of all existing firms in the economy are identical at any stage. Firms' optimal choice of labor, capital, and land are standard:

$$W_t = P_t \frac{\gamma\,(Y_t + \Psi)}{L_t}, \tag{4}$$

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$$1 = \beta E_t \frac{\lambda_{t+1}^c}{\lambda_t^c} \left[\frac{\alpha \left(Y_{t+1} + \Psi \right)}{K_{t+1}} + (1 - \delta) \right], \tag{5}$$

$$Q_t = \beta E_t \frac{\lambda_{t+1}^c}{\lambda_t^c} \left[\frac{\theta \left(Y_{t+1} + \Psi \right)}{H_{t+1}} + Q_{t+1} \right]. \tag{6}$$

The right-hand side of each of these three equations represents the discounted gains that firms will receive if they increase their factors of production by one unit in terms of consumption goods, and the left-hand side represents the discounted units that they must give up in order to do so.

The household. A single infinitely lived representative household derives utility from the consumption of a nondurable good C_t, leisure (labor) L_t^c, and a durable good that is measured by landholding H_t^c:[1]

$$E_0 \sum_{t=0}^{\infty} \beta^t U \left(C_t, H_t^c, L_t^c \right).$$

Assuming log utility, the household solves

$$\underset{C_t, H_{t+1}^c, B_{t+1}^c, S_{t+1}, L_t^c}{\text{Max}} E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t + \eta^h \log H_t^c + \eta^l \log \left(1 - L_t^c \right) \right)$$

subject to the following budget constraint:

$$\begin{aligned} & W_t L_t^c - Q_t \left(H_{t+1}^c - H_t^c \right) - \left(B_{t+1}^c - R_t B_t^c \right) \\ & \quad \geq C_t + T_t + P_t^k S_{t+1} - \left(P_t^k + d_t \right) S_t - X_t. \end{aligned}$$

At time period t, with land holdings H_t^c, bond holdings B_t^c coming to maturity, and share holdings S_t, the household rents labor L_t^c at a wage W_t, issues loans valued at B_{t+1}^c at the agreed gross interest rate R_{t+1} to firms, purchases C_t units of consumption good and new shares S_{t+1}, and pays the lump-sum tax payment T_t as the source of the transfers to new firms. Given that the household owns S_t shares of firms, the incumbents send $(P_t^k + d_t)S_t$ to the household at the end of the time period t. X_t denotes the funds the household receives from either new entrants or exiting firms. When firms enter, i.e., $(N_{t+1} - N_t) > 0$, they send the dividend payments and the income from selling shares to the household; thus, $X_t = (P_t^k + d_t^{\text{en}})(N_{t+1} - N_t)$. When exit occurs, the household receives $X_t = d_t^{\text{ex}}(N_t - N_{t+1})$ from the exiting firms. Although the value of X_t depends on whether entry occurs or exit does, the budget constraint of the household holds either way in model equilibrium. The proof is provided in Appendix A.

Using λ_t^c to denote the Lagrange multiplier of the budget constraint, the optimal choices of goods consumption, leisure, and land holdings are functions of the price of goods, wages, and the price of land, respectively:

$$\lambda_t^c = \frac{1}{C_t}; \tag{7}$$

$$\lambda_t^c W_t = \frac{\eta^l}{1 - L_t^c}; \tag{8}$$

$$\lambda_t^c Q_t = \beta \left(\frac{\eta^h}{H_{t+1}^c} + E_t \lambda_{t+1}^c Q_{t+1} \right). \tag{9}$$

The optimal choice of loan issuance yields the standard solution for the real interest rate:

$$R_{t+1} = \frac{\lambda_t^c}{\beta E_t \lambda_{t+1}^c}. \tag{10}$$

Also, solving for share holdings results in the pricing equation

$$P_t^k = \beta E_t \left[\frac{\lambda_{t+1}^c \left(P_{t+1}^k + d_{t+1} \right)}{\lambda_t^c} \right] \tag{11}$$

with the transversality condition

$$\lim_{l \to 0} \beta^l \lambda_{t+l}^c P_{i,t+l}^k = 0.$$

Equation (11) can be solved forward as

$$P_{i,t}^k = E_t \left[\sum_{j=1}^{\infty} \beta^j \frac{\lambda_{t+j}^c}{\lambda_t^c} d_{i,t+j} \right]. \tag{12}$$

Equation (12) indicates that the value of each share equals the discounted future profits of each firm.

Entry and exit. Firms' entry and exit decisions depend on the expected payoff and the opportunity cost of staying in business. The present value of accumulated future profits, P_t^k, plays a key role here. Higher future accumulated profits correspond to a higher expected payoff of staying in business and a higher rate of firm entry. The opportunity cost of entering or staying in the market includes the total investment $Q_t(H_{t+1} - H_t) + [K_{t+1} - (1-\delta)K_t]$ and the value of current assets $Q_t H_t + (1-\delta)K_t$ of a firm. Therefore, the zero-profit condition can be written as

$$P_t^k = Q_t H_{t+1} + K_{t+1}. \tag{13}$$

When opportunity cost can at least be covered by the present value of expected future payoff, i.e., $P_t^k \geq Q_t H_{t+1} + K_{t+1}$, incumbents will choose to stay in business and new firms will enter; otherwise, firms will exit.

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In addition to the zero-profit condition, I also consider a more general entry and exit rule characterized by

$$\xi\,(N_{t+1} - N_t) = f\,(\pi_t), \tag{14}$$

where $\pi_t \equiv P_t^k - (Q_t H_{t+1} + K_{t+1})$, $\xi \geq 0$ and $f_{\pi_t} > 0$. With $\pi = 0$ and $f(0) = 0$, the zero-profit condition holds in the steady state. Log-linearizing the entry rule in (14) implies that

$$\hat{N}_{t+1} = \hat{N}_t + \frac{f_{\pi_i}}{\xi}\left[\frac{P^k}{N}\hat{P}_t^k - \frac{QH}{N}(\hat{Q}_t + \hat{H}_{t+1}) - \frac{K}{N}\hat{K}_{t+1}\right]. \tag{15}$$

On one hand, equation (15) suggests that the higher the economic profit, the higher the rate of firm entry. On the other hand, ξ is equivalent to an adjustment cost of firm entry. A zero value of ξ suggests free entry and exit as shown in equation (13). Any strictly positive value of ξ indicates a barrier to entry or exit, with higher values of ξ corresponding with higher barriers. In an extreme case with $\xi = \infty$, firm entry is completely shut down in the model. Since I am not trying to explore the reasons for entry barriers in this paper, this adjustment cost provides a simple and clean way to incorporate different entry and exit conditions into the model.

Market clearing. To generate the resource constraint in the economy, I summarize a set of market-clearing conditions in this section.

Assuming the total land supply $\bar{H}$ is fixed over time, the household and firms together clear the land market:

$$N_t H_t + H_t^c = \bar{H}. \tag{16}$$

The additional market-clearing conditions that link the household with firms include labor, debt, and equity markets:

$$L_t^c = N_t L_t, \tag{17}$$

$$B_t^c = N_t B_t, \tag{18}$$

$$S_t = N_t. \tag{19}$$

Equation (19) indicates that each firm issues one share in equilibrium.

The transfers made to new entrants are financed by tax payments from the household:

$$Z_t = Q_t H_t + (1-\delta)\,P_t K_t \tag{20}$$

$$T_t = (N_{t+1} - N_t)\,Z_t. \tag{21}$$

Using these market-clearing conditions together with the household's budget constraint, I obtain the following resource constraint, which shows that the total output is allocated to consumption and capital investments:

$$N_t Y_t - C_t - N_{t+1} K_{t+1} + (1-\delta)\,N_t K_t = 0. \tag{22}$$

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2.2. Credit Constraints

Technically, the first task in introducing credit frictions into a DSGE model is to ensure that firms will borrow. This condition can be accomplished either by assuming a discount factor of potential borrowers lower than that of potential lenders [e.g., Kiyotaki and Moore (1997)], or by forcing potential borrowers to consume enough so that their net worth never exceeds the desired investment [e.g., Carlstrom et al. (2009)]. I introduce a fund reserve constraint mandating that firms retain profits for dividend payments; thus, firms' capital and land investment are restricted by borrowing:

$$[K_{t+1} - (1-\delta)K_t] + Q_t\,(H_{t+1} - H_t) \leq B_{t+1}. \tag{23}$$

Implicitly, this assumption works much like the model presented by Carlstrom et al. (2009), except that in my model, the household, instead of the firms (or "entrepreneurs"), is the agent that is forced to consume.

The household lends firms B_{t+1} and enforces a collateral constraint. This constraint exists because the credit contract is imperfectly enforceable, which gives a firm an incentive to finance a large investment. Once the values of new capital and land financed by the loan are high enough to exceed the discounted cash flows, the firm can sell the assets with no additional cost and shut down without paying the debt. To prevent loan default, the household never allows gross interest to exceed the value of collateral. Specifically, considering that land is the only asset desirable to the lender, the repayment of debt B_{t+1} cannot exceed the market value of their land holdings,

$$R_{t+1}B_{t+1} \leq u E_t(Q_{t+1}H_{t+1}), \tag{24}$$

where $u \in (0, 1)$ and indicates that the liquidation process costs a portion $(1-u)$ of the value of collateral.

The collateral constraint suggests that both factors of production, capital and land, have full liquidation value among the borrowers (internal market) but not necessarily between the lender and borrowers (external market). Capital has zero liquidation value and land has partial liquidation value (as $u < 1$) when they are traded outside of the production sector to the household.

Given λ_t^F and λ_t^H as the shadow prices of relaxing the fund reserve constraint [equation (23)] and collateral constraint [equation (24)], respectively, the first-order conditions for capital and land demand of firms change to

$$\left(1 + \frac{\lambda_t^F}{\lambda_t^c}\right) = E_t \frac{\beta\lambda_{t+1}^c}{\lambda_t^c}\left[\frac{\alpha\,(Y_{t+1}+\Psi)}{K_{t+1}} + \left(1 + \frac{\lambda_{t+1}^F}{\lambda_{t+1}^c}\right)(1-\delta)\right] \tag{25}$$

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and

$$\left(1+\frac{\lambda_t^F}{\lambda_t^c}\right)Q_t = E_t\frac{\beta\lambda_{t+1}^c}{\lambda_t^c}\left[\frac{\theta\,(Y_{t+1}+\Psi)}{H_{t+1}}+\left(1+\frac{\lambda_{t+1}^F}{\lambda_{t+1}^c}\right)Q_{t+1}\right]+uE_t\frac{\lambda_t^H}{\lambda_t^c}Q_{t+1}. \tag{26}$$

λ_t^F represents a premium for buying new capital or new land, and λ_t^H is the additional discount of holding land as collateral. Rewriting equation (25) yields

$$E_t R_{t+1}^k = \left(1+\frac{\lambda_t^F}{\lambda_t^c}\right)R_{t+1}^f, \tag{27}$$

where $R_{t+1}^k \equiv [\frac{\alpha(Y_{t+1}+\Psi)}{K_{t+1}} + (1+\frac{\lambda_{t+1}^F}{\lambda_{t+1}^c})(1-\delta)]$ represents the gross return on capital and $R_{t+1}^f \equiv \lambda_t^c/(\beta E_t\lambda_{t+1}^c)$ is the gross return on a risk-free asset. The factor $(1+\lambda_t^F/\lambda_t^c)$ appears as a markup on the cost of capital. When both credit constraints bind, the marginal cost to raise one unit of capital or land increases by a factor of $(1+\lambda_t^F/\lambda_t^c)$. The expected future return on land includes the payoff from relaxing the present collateral constraint, $E_t\frac{u\lambda_t^H}{\lambda_{t+1}^c}$, and the fund reserve constraint in the following time period, $E_t\frac{\lambda_{t+1}^F}{\lambda_{t+1}^c}$. Land plays two roles here: it is both a financially constrained asset and a resource that can be used to relax the collateral constraint.[2]

Equations (25) and (26) provide the basis for a credit friction, as stressed by Kiyotaki and Moore (1997), who link the movements in land price to the value of collateral, and consequently to the demand for capital. In particular, the fluctuation of the price of land, Q_{t+1}, will be amplified by a markup λ_t^H, and thus will have a significant effect in making the constraints more binding.

The following loan demand equation describes the relation between λ_t^F and λ_t^H:

$$R_{t+1} = \frac{\lambda_t^c+\lambda_t^F}{\beta E_t\lambda_{t+1}^c+\lambda_t^H}. \tag{28}$$

At market equilibrium, loan demand balances out with loan supply [equation (10)] and yields

$$\lambda_t^H = \lambda_t^F\frac{\beta E_t\lambda_{t+1}^c}{\lambda_t^c}. \tag{29}$$

Equation (29) shows that because firms and the household share the same discount factor, firms do not find borrowing attractive in the frictionless economy. The presence of the credit friction requires both the fund reserve and collateral constraints to be binding.

2.3. The Steady State and Parameterization

To compare the benchmark model with the model incorporating credit frictions, constraints (23) and (24) are set to be binding in the steady state. The solution for the markup λ^F/λ^c is a function of the relative factor intensity:

$$\frac{\lambda^F}{\lambda^c} = \left[\frac{\beta}{\delta}\left(\frac{1}{\beta} - 1 + \delta\right)\frac{\theta}{\alpha} - \left(\frac{1-\beta}{\beta u} - 1\right)\right]^{-1} - 1. \tag{30}$$

Equation (30) suggests that the following condition is required to have the credit constraints binding in the steady state (i.e., $\lambda^F > 0$):

$$\frac{\theta}{\alpha} < \frac{\delta}{\beta u}\frac{1-\beta}{1-\beta+\beta\delta}.$$

This condition suggests that a sufficiently large output elasticity of capital is required to ensure a binding collateral constraint. Because borrowing is costly, capital needs to be desirable enough for the firms to pay its cost. A sufficiently small output elasticity of land is also necessary: Land needs to be less productive than the rest of the production factors so that firms have an incentive to hold more land than is needed for production in order to gain easier access to credit.

Five parameters (β, δ, η^h, η^l, $\bar{H}$) are chosen directly at standard values in keeping with much of the literature. The time unit is one year. This leads to a discount factor β of 0.95 and an annual capital depreciation rate δ of 0.1. In the household utility function, the weight η^h on land (housing) demand is 0.25. Labor supply elasticity η^l is chosen to be 3. The total supply of land is simply normalized to unity.

The factor intensity α and θ has a direct impact on credit frictions in the steady state. In this model, K_t, which cannot be accepted as collateral, includes both physical and intangible capital. I choose the value for intangible capital share following Corrado et al. (2005, 2006) and calculate the land income share and physical capital income share based on the capital income (CI) table of the BLS. Corrado et al. calculate the intangible capital share and tangible capital share as 15% and 25%, respectively. The tangible capital share in their model consists of physical capital and land, and is consistent with that used by the BLS in putting together its multifactor productivity estimates for the private nonfarm business sector.[3] In order to obtain the physical capital share and land share for my model, I rely on the average land share in industry capital income reported by the CI table. Based on a 0.07 land/tangible capital ratio, the land share is calibrated as 1.75% leaving the physical capital share at 23.25%. Finally, I assign a value of 0.975 to the decreasing–returns to scale parameter, following Cooley and Quadrini (2001) and obtain a 57.5% labor share. As in Corrado et al., this labor income share is constructed using raw (unskilled) labor income, which does not include human capital, and is therefore lower than traditional labor compensation.

The final parameter left to be chosen is the liquidation cost, $(1-u)$. I choose the value of u as the average maximum mortgage loan–to–value ratio, which is 75%.

Table 1 summarizes the values of all parameters in the steady state.

TABLE 1. Calibration: Homogeneous-firm model

Parameters	Description	Values
β	Discount factor	0.95
δ	Capital depreciation rate	0.1
η^h	Curvature on land	0.25
η^l	Curvature on leisure	3
A	Total factor productivity	1
θ	Land share in goods production	0.02
α	Capital share in goods production	0.38
γ	Labor share in goods production	0.58
u	Maximum "loan-to-value" ratio	0.75
H	Total supply of land	1

2.4. Main Results

The steady state. Table 2 reports the steady state values of the key variables. Comparing the frictionless economy and the economy with credit constraints, one observes that the number of firms shrinks when credit friction is present. Because friction increases capital costs in relation to variable costs, larger firms tend to have a cost advantage based on a U-shaped average cost curve, and the market naturally reduces firm entry.

Due to the entry effect, each incumbent chooses to produce more and hold more capital (and land) when the credit constraints bind. This indicates a larger firm size based on Gertler and Gilchrist (1994), who suggest that firm size is measured by either output production or capital holding.

The fluctuation of the aggregate economy shows the costs due to the credit frictions. All firms decrease their capital holdings and increase their land holdings

TABLE 2. The steady state homogeneous-firm model

	Description	Frictionless model	Model of credit frictions
λ^F	Credit friction markup	0	0.02
Asset	Firm-level asset holdings	113.54	229.82
N	Number of firms	0.01	0
Y	Firm-level output	39	79.73
K	Firm-level capital input	100.24	201.53
H	Firm-level land input	11.16	24.31
L	Firm-level labor input	26.5	55.14
NY	Aggregate output	0.31	0.3
NK	Aggregate capital input	0.79	0.75
NH	Aggregate land input	0.09	0.09
NL	Aggregate labor input	0.21	0.21

Note: Asset $\equiv QH + K$.

because of the collateral constraint. Total land expense (NH) rises while aggregate expense in other production factors—capital and labor (NK and NL)—falls. The financing costs eventually depress the total production (NY) in the economy.

A technology shock. Appendix B provides a complete set of log-linearized equations describing the general equilibrium. The impulse-response dynamics reports the quantitative properties of the model around the steady state. An unexpected increase in total factor productivity, A_t, indicates a positive aggregate technology shock that follows a persistent AR(1) process:

$$\hat{A}_t = \rho_a \hat{A}_{t-1} + e_t^a.$$

Figure 2 illustrates the impulse responses of the model economies to an unanticipated 100–basis point increase in TFP. In the frictionless economy, the impulse responses of the factors of production are standard: A positive productivity shock results in greater investment in both land and capital. However, in the model with credit frictions, a positive shock has a weaker effect on firms' capital, output, and land demand. The binding credit constraints increase borrowing costs and consequently dampen firms' production and capital purchases. The weaker deviation of the interest rate indicates a smaller increase in loan demand because of credit frictions. The positive technology shock drives up the asset price Q_t in both models, but increases in land demand and land price are larger when the collateral constraint binds.

Figure 2 also displays the deviations of the number of firms from the steady state. Both model economies experience firm entry in response to the positive TFP shock, but there is a higher entry rate when the credit constraints bind, because these constraints moderate the responses of asset holdings (in this model, the value of capital and land) of firms. For potential new entrants, lower requirements for asset holdings ease the entry process. For incumbents, smaller asset holdings result in a lower payoff for closing their businesses. Firms tend to raise investment and accumulate more assets in response to a positive TFP shock. However, this increase in asset demand leads to a more binding collateral constraint, dampening investment and asset accumulation and eventually providing more room for entry. Alternatively, if firms experienced a negative technology shock, one would expect more firms to exit when credit frictions were introduced into the model economy. Because firms are forced to hold collateral in order to produce, they are not free to decrease their asset holdings as much as they could in a frictionless economy. These frictions eventually push more businesses to shut down. Conclusively, the credit friction restricts firms' ability to adjust their asset holdings and increase entry during an economic upturn and exit during an economic downturn.

To show the effects of credit frictions on the aggregate variables, Figure 3 shows the impulse responses of aggregate output, aggregate capital, and land holdings under the three different types of market structure: free entry, restricted entry, and no entry. Recall that setting f_π equal to zero in equation (15) shuts

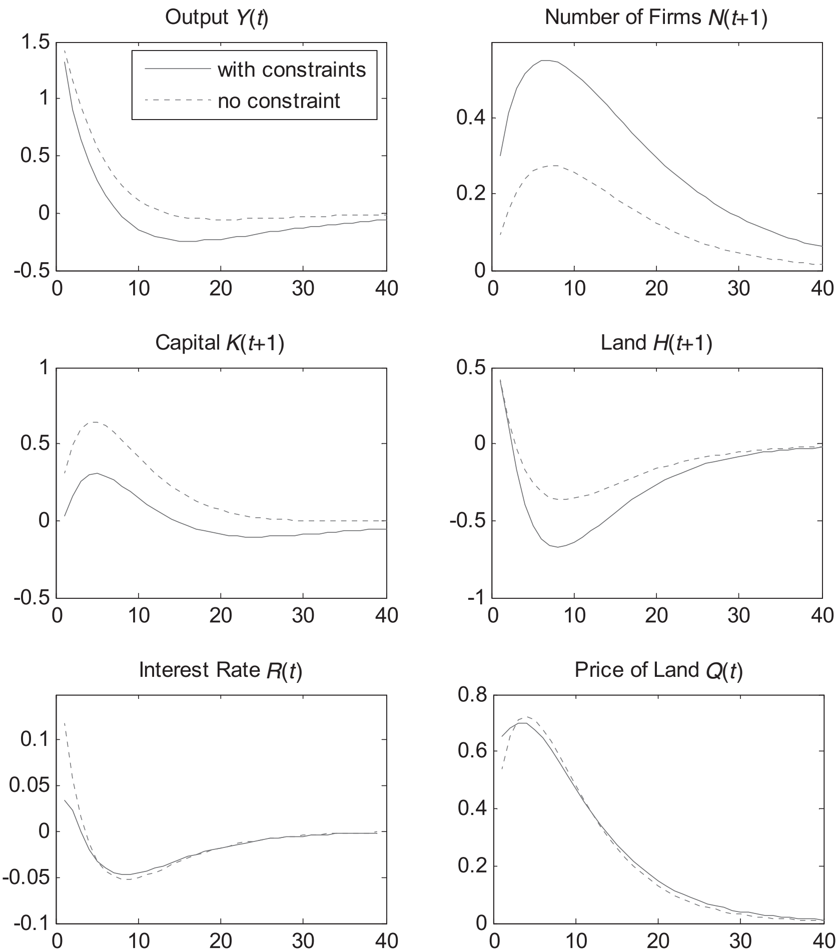

FIGURE 2. Percentage deviations of firm variables from steady state (1% positive TFP shock, homogeneous-firm model).

down entry and exit, whereas setting ξ equal to zero enables free entry and exit. According to Figure 3, stronger entry restrictions result in greater differences in how aggregate variables respond to shocks in each model. To explain this disparity, recall that credit frictions restrict deviations in the firm-level output and asset holdings from their steady state values, on one hand, whereas they make firm entry more volatile than the benchmark case, on the other. The fluctuation in firm entry moderates the dampening effect on firm-level variables. This implies that the model economy without entry and exit barriers is affected less than those with barriers.

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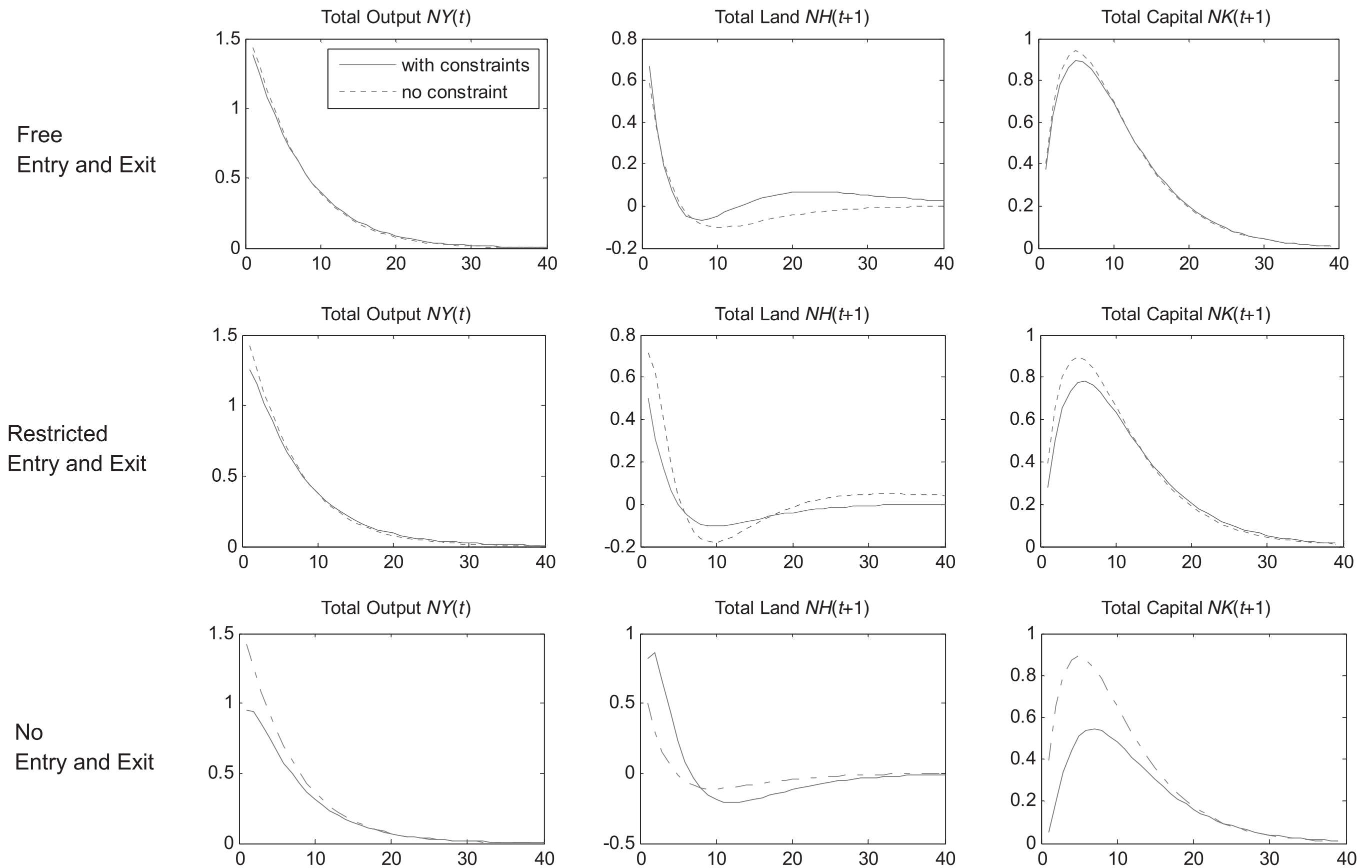

FIGURE 3. Percentage deviations of aggregate variables from steady state (1% positive TFP shock, homogeneous-firm model).

The effect of credit frictions on firm entry is central. In a standard DSGE model with a production function that provides constant returns to scale, production costs are identical for a single-firm economy and a multifirm economy. Hence, the number of firms cannot be determined by the optimization problem without imposing exogenous entry and exit distortions. That is, the models with credit frictions developed in this setup can only provide the effect of the friction on the aggregate economy. Some have argued in the literature that the propagation effect of credit friction is negligible to a macroeconomy based on the standard DSGE model without firm entry [e.g., Cordoba and Ripoll (2004), Kocherlakota (2000)]. By allowing endogenous firm entry, I am able to separate the effects of credit frictions on individual firms and the number of firms in the economy and I find that the impact of credit friction on macroeconomic fluctuations depends on entry and exit conditions. A higher entry barrier means that credit frictions have a stronger effect on the aggregate economy.

A land demand shock. In addition to the TFP shock, I consider a preference shock to land consumption in the household's utility function,

$$U\left(C_t, H_t^c, L_t^c\right) = \log C_t + \varepsilon_t^h \eta^h \log H_t^c + \eta^l \log\left(1 - L_t^c\right),$$

where $\varepsilon^h = 1$ in the steady state and $\hat{\varepsilon}_t^h = \rho_h \hat{\varepsilon}_{t-1}^h + e_t^h$ by convention. In the model with binding credit constraints, this shock triggers a change in the price of land.

Figure 4 displays how various firm-level variables respond to such a land demand shock. The shock raises the household's demand for land consumption and drives up the land price; thus, it discourages firms from making further land investments. One can observe that credit frictions dramatically amplify the effect of this shock. In the benchmark economy, a rise in land price only triggers a slight substitution effect on the rest of the production factors: capital and labor. These trivial increases in capital and labor result in an insignificant change of output. When credit constraints bind, however, the increase in the price of land raises the value of collateral for borrowing, and temporarily relaxes the collateral constraint for the first few periods. As the price of land keeps increasing, credit frictions raise the cost of financing capital.[4] As a result, capital, labor, and output all decrease considerably. In sum, credit frictions yield large changes in firms' asset holdings and significantly affect firm entry relative to the benchmark model.

3. HETEROGENEOUS-FIRM MODEL

I will now extend the model to include two types of firms. This heterogeneity is rooted in a difference in production technologies, which leads to heterogeneous access to credit.

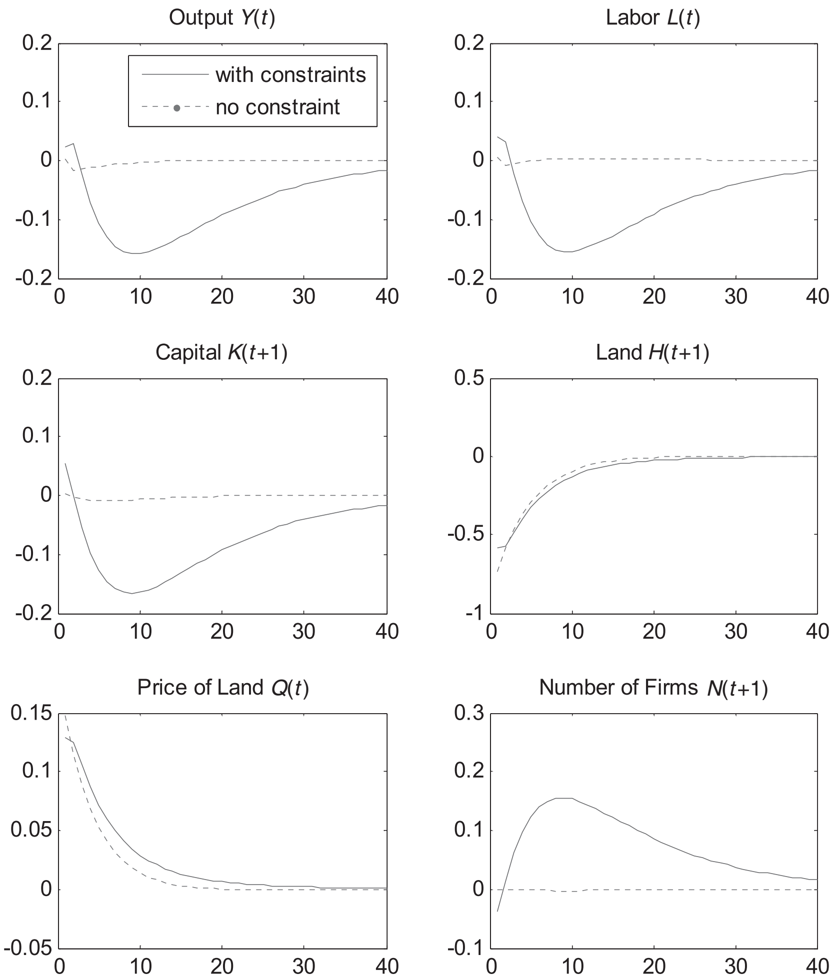

FIGURE 4. Percentage deviations of firm variables from steady state (1% land demand shock, homogeneous-firm model).

3.1. Heterogeneous Access to Credit

I carry over the assumptions of decreasing returns to scale and common fixed costs from the homogeneous firm model. The production function for each types of firms in the heterogeneous model is

$$Y_{i,t} = A_t K_{i,t}^{\alpha_i} H_{i,t}^{\theta_i} L_{i,t}^{\gamma_i} - \Psi, \quad i = 1, 2,$$

where $\alpha_1 > \alpha_2$ and $\theta_1 < \theta_2$. In other words, type I firms are capital-intensive and type II firms are land-intensive, although $\alpha_1 + \theta_1 = \alpha_2 + \theta_2$. Labor is assumed to be equally productive for both firm types ($\gamma_1 = \gamma_2 \equiv \gamma$). Within each type, firms

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are symmetric and produce a unique final good, $Y_{i,t}$, and sell their goods at price $P_{i,t}$ to the household. The household's consumption of this nondurable good, C_t, follows a CES aggregator given the elasticity of substitution $\nu/(1-\nu)$:

$$C_t = \left(\sum_i C_{i,t}^{\frac{1}{\nu}}\right)^{\nu}, \quad \nu > 1.$$

In this extension, I use the goods produced by the capital-intensive firms as the numeraire good. Therefore $P_{1,t} = 1$, and $P_{2,t}$ denotes the relative price of goods produced by the land-intensive firms in terms of the capital-intensive goods.

In order to show that the two types of firms have heterogeneous access to credit, I rewrite the fund reserve constraint and collateral constraint as follows:

$$P_{i,t}[K_{i,t+1} - (1-\delta)K_{i,t}] + Q_t(H_{i,t+1} - H_{i,t}) \leq B_{i,t+1}; \qquad \textbf{(31)}$$

$$R_{t+1}B_{i,t+1} \leq uE_t(Q_{t+1}H_{i,t+1}). \qquad \textbf{(32)}$$

Different factor intensities give rise to heterogeneous access to credit. Under binding constraints, capital has a lower liquidation value in the external market than land does. This disadvantages the capital-intensive firm more than the land-intensive firm, which has easier access to credit as a result. Heterogeneous access to credit can also be observed in the model steady state. Equation (30) can be generalized for each type i firm as follows:

$$\frac{\lambda_i^F}{\lambda^c} = \left[\frac{\beta}{\delta}\left(\frac{1}{\beta} - 1 + \delta\right)\frac{\theta_i}{\alpha_i} - \left(\frac{1-\beta}{\beta u} - 1\right)\right]^{-1} - 1.$$

Note that $\frac{\lambda_i^F}{\lambda^c}$ is larger for higher values of θ_i and smaller for higher values of α_i. This generalized form shows that credit constraints are more binding for land-intensive firms than for capital-intensive firms.

3.2. Parameterization

The parameter values $(\beta, \delta, \eta^h, \eta^l, \bar{H}, u, \gamma)$ for the heterogeneous-firm model are the same as in the homogeneous-firm model. The elasticity of substitution in consumption goods is chosen directly at a standard value of 1.5 (i.e., $\nu = 3$).

We have four parameters $(\alpha_1, \alpha_2, \theta_1, \theta_2)$ left to calibrate. To construct the data for land-intensive and capital-intensive firms, I rely on the sectoral data provided by the CI table of the BLS. Recall that I use the data of the private nonfarm business sector to calibrate the homogeneous-firm model. In this heterogeneous-firm model, I choose the private nonfarm nonmanufacturing sector as representative of the land-intensive firm and the private nonfarm manufacturing sector as representative of the capital-intensive firm. Using each sector's average land share in industry total

TABLE 3. Calibration: Heterogeneous-firm model

Parameters	Description	Values
β	Discount factor	0.95
δ	Capital depreciation rate	0.1
η^h	Curvature on land	0.25
η^l	Curvature on leisure	3
$\nu/(1-\nu)$	Elasticity of substitution in consumption goods	1.5
A	Total factor productivity	1
θ_1	Land share of capital intensive firms	0.01
θ_2	Land share of land intensive firms	0.02
α_1	Capital share of capital intensive firms	0.39
α_2	Capital share of land intensive firms	0.38
γ	Labor share in goods production	0.58
u	Maximum "loan-to-value" ratio	0.75
H	Total supply of land	1

capital income reported by the CI table, I obtain a 0.018 land income share and a 0.232 physical capital income share for the land-intensive firms, and a 0.012 land share and a 0.238 physical capital share for the capital-intensive firms.

Table 3 summarizes the values of all parameters in the steady state of the heterogeneous-firm model.

3.3. Main Results

In this section, I focus on comparing individual firm sizes in the steady state and the impact of heterogeneous access to credit on firm dynamics. The effects of credit frictions on firm entry, as well as on the aggregate economy captured by the heterogeneous-firm model, are similar to those in the homogeneous-firm model and will not be reported.

The steady state. Table 4 compares the sizes of individual firms in the steady state. With a larger markup λ_1^F, the financing cost of capital is higher for capital-intensive firms than for land-intensive firms. Therefore, credit frictions enlarge output and asset-holding differences, as well as differences expenses for factors of production between the two types of firms. In other words, using either measure of firm size, firms with easier access to credit are larger than firms with less access to credit in the steady state. These firm-level differences are not due to entry effect but to differing levels of access to credit. Table 4 also provides a comparison between industry-level variables in the steady state. Industries that are land-intensive are able to take advantage of easier access to credit. As a result, they are larger than in the frictionless model.

TABLE 4. The steady state heterogeneous-firm model

	Description	Frictionless model	Model of frictions
λ_1	Markup of capital-intensive firms	0	0.11
λ_2	Markup of land-intensive firms	0	0.01
$Y_1 - P_2Y_2$	Difference between firm-level output	−0.95	−20.22
$K_1 - P_2K_2$	Difference between firm-level capital	−0.86	−51.9
$H_1 - H_2$	Difference between firm-level land	−4.07	−6.11
$\text{Asset}_1 - \text{Asset}_2$	Difference between firm-level asset holdings	−5.7518	−59.1806
$N_1Y_1 - P_2N_2Y_2$	Difference between industry-level output	0	0
$N_1K_1 - P_2N_2K_2$	Difference between industry-level capital	0.01	0
$N_1H_1 - N_2H_2$	Difference between industry-level land	−0.01	0
$N_1\text{Asset}_1 - N_2\text{Asset}_2$	Difference between industry-level asset holdings	−0.0042	0.0030

Note: $\text{Asset}_i \equiv QH_i + P_iK_i (i = 1, 2)$.

Impulse responses. Figure 5 demonstrates how individual firm size (measured by both output productions and asset holdings) and the number of firms respond to a positive TFP shock to all firms, whereas Figure 6 shows the effect of a TFP shock to the firms' factors of production. The differences in the responses of capital-intensive firms and land-intensive firms are fairly trivial in the frictionless economy. Even though firms differ in their factor intensities, they have the same total factor productivity. Therefore, firms' investment and production responses to the same technology shock are almost identical.

When credit constraints are present, however, firms will respond differently to the same shock. A positive TFP shock raises the investment demand of the firms and enlarges the impact of credit frictions. This deterioration is greater for capital-intensive firms. Land-intensive firms have easier access to credit and are able to increase factor investments and output more than their capital-intensive counterparts. As a result, credit frictions enlarge the difference in each type of firm's response to the same shock.

4. CONCLUDING REMARKS

In this paper, I developed a DSGE model in which the production technology is characterized by a U-shaped average cost curve. This allows endogenous firm entry and allows the market to determine the optimal number of firms. I build upon this model by introducing credit frictions, in which firms are required to

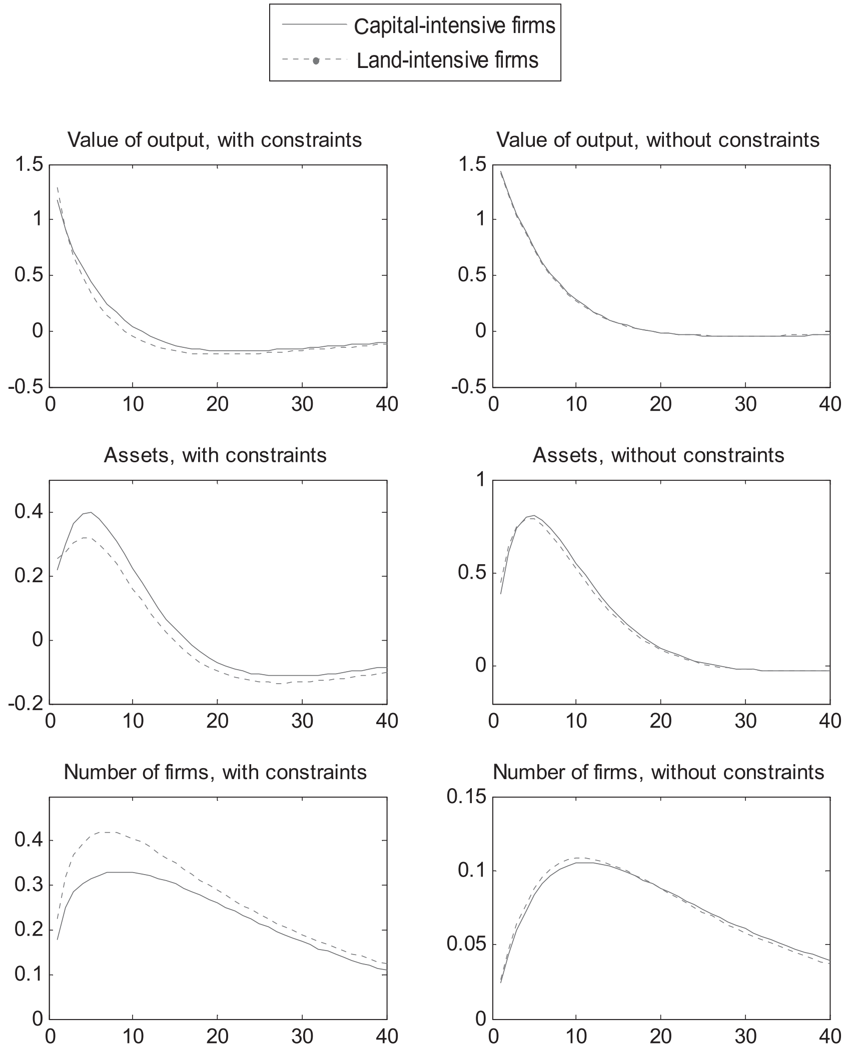

FIGURE 5. Percentage deviations of firm variables from steady state (1% positive TFP shock, heterogeneous-firm model).

borrow to finance their capital investment and their borrowings are restricted by a binding collateral constraint. These credit constraints force the existing firms in the market to hold more assets as collateral and push up asset prices. Because asset holding values represent firms' opportunity costs of entering or staying in business, credit constraints reduce the number of firms and increase the size of existing firms in the steady state. Moreover, the constraints lower firms' incentive to adjust their asset holdings in response to an economic shock and thus raise

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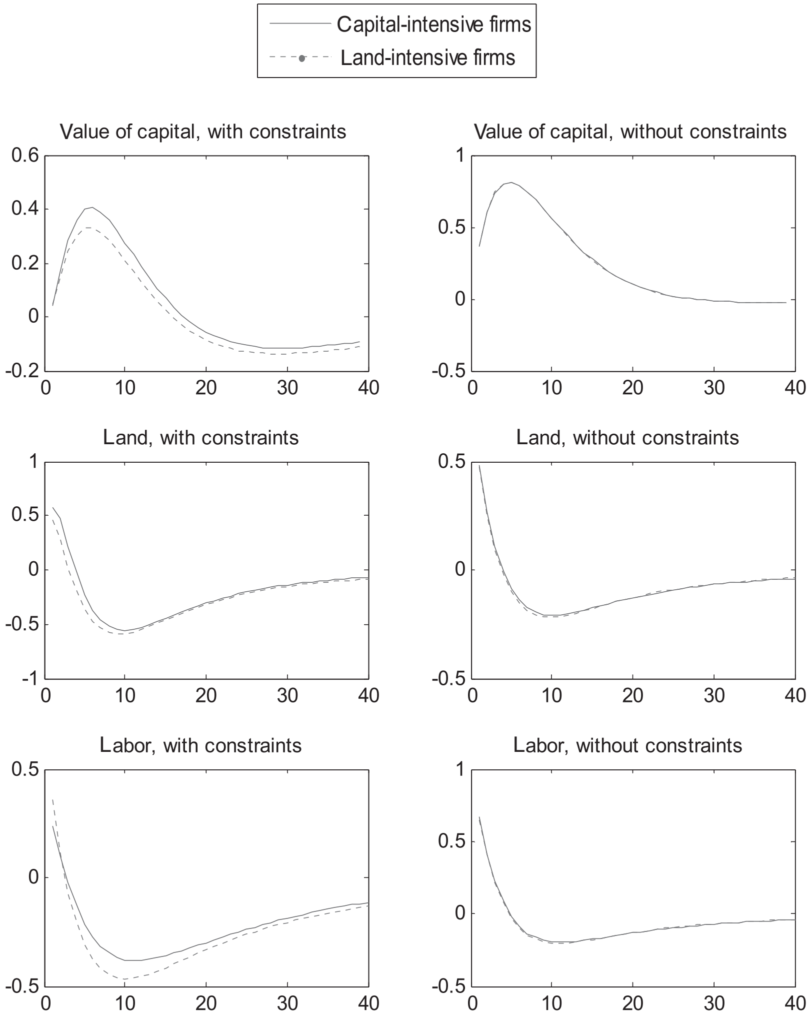

FIGURE 6. Percentage deviations of firm variables from steady state (1% positive TFP shock, heterogeneous-firm model).

the volatility of entry (or exit) when the model economy deviates from its steady state.

I also find that the effect of credit frictions on the aggregate economy depends on entry and exit conditions. In response to a TFP shock, the investment and output from individual firms deviate less from their steady state values when credit constraints are binding, whereas firm entry and exit react more sharply. As a result, unrestricted entry lessens the effect of credit frictions on the aggregate economy.

I extend the model to include firms with different production technologies and heterogeneous access to credit. In the steady state of this model, when credit frictions are present, firms with less access to credit face higher investment and production costs and accordingly are smaller than firms with easier access to credit. As the economy deviates from its steady state, heterogeneous access to credit causes different impulse responses of the two types of firms to a common technology shock.

A question open to future research is whether credit frictions have symmetric effects on firm entry and exit. One may expect credit frictions to have a larger impact on firm entry than on firm exit, because starting a new business and accumulating assets tend to be more costly than shutting down. Campbell (1998) reports that firm exit rate varies more (i.e., has higher standard deviations over time) than firm entry rate. To develop the asymmetric effects of credit friction, economic profit must be an asymmetric function of the number of firms; that is, there exists a range in which no entry occurs, despite the possibility of positive economic profits, because of the additional entry costs imposed by credit frictions. A model with an asymmetric entry and exit rule requires an alternative setup to the one I have used in this paper.

NOTES

1. I include land holdings in the household's utility function to allow variable land supply to firms. A shock to the preference of the household in consuming land captures an exogenous change in the price of land.
2. Land investment is subject to the fund reserve constraint in order to prevent credit frictions from being erased by a type of potential unconstrained debt. Because equation (1) places no restriction on the non-negativity of the stream of dividend payments, firms could implicitly "borrow" from the household by paying negative dividends in the current time period without any penalties. Using these unconstrained funds, firms can purchase a large amount of land to undo the collateral constraint if land investment is not subject to the fund reserve constraint.
3. The aggregate capital assets reported on the CI table contain five asset types: equipment, structures, rental residential capital, inventories, and land.
4. See Appendix C for details in the timing of the effects of the shock.

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APPENDIX A: THE HOUSEHOLD'S BUDGET CONSTRAINT

The household's budget constraint when there is entry ($N_{i,t+1} > N_{i,t}$) is

$$\sum_i \left[\left(P^k_{i,t} + d_{i,t}\right) S_{i,t} + \left(P^k_{i,t} + d^{en}_{i,t}\right)(N_{i,t+1} - N_{i,t}) - T_{i,t}\right]$$
$$\geq \sum_i \left(P_{i,t}C_{i,t} + P^k_{i,t}S_{i,t+1}\right) + Q_t\left(H^c_{t+1} - H^c_t\right) + \left(B^c_{t+1} - R_tB^c_t\right) - W_tL^c_t. \tag{A.1}$$

By substituting $d^{\text{en}}_{i,t} = (Z_{i,t} + B_{i,t+1} - Q_tH_{i,t+1} - P_{i,t}K_{i,t+1})$ using equation (12), and $T_{i,t} = Z_{i,t}(N_{i,t+1} - N_{i,t})$ using the market-clearing condition, the left hand side of (A.1) can be written as

$$\left(P^k_{i,t} + d_{i,t}\right) S_{i,t} + \left(P^k_{i,t} + B_{i,t+1} - Q_tH_{i,t+1} - P_{i,t}K_{i,t+1}\right)(N_{i,t+1} - N_{i,t}). \tag{A.2}$$

Equation (A.2) suggests that when firms enter, they return taxes paid by the household in previous periods. This prevents new entrants from taking advantage of the policy of subsidy by leaving the market right after they enter and confiscating the subsidies.

When firm exit occurs ($N_{i,t+1} < N_{i,t}$), the household chooses to purchase $S_{i,t+1}$ shares from incumbent firms and at the same time owns $(N_{i,t} - N_{i,t+1})$ shares of exiting firms. The budget constraint therefore changes to

$$\sum_i \left[\left(P_{i,t}^k + d_{i,t} \right) S_{i,t+1} + d_{i,t}^{\text{ex}} (N_{i,t} - N_{i,t+1}) \right]$$
$$\geq \sum_i \left(P_{i,t} C_{i,t} + P_{i,t}^k S_{i,t+1} \right) + Q_t \left(H_{t+1}^c - H_t^c \right) + \left(B_{t+1}^c - R_t B_t^c \right) - W_t L_t^c, \qquad \textbf{(A.3)}$$

where

$$d_{i,t}^{\text{ex}} = d_{i,t} - B_{i,t+1} + Q_t H_{i,t+1} + P_{i,t} K_{i,t+1}.$$

When $d_{i,t}^{\text{ex}}$ is substituted into (A.3) using this equation, the left-hand side of (A.3) can be rewritten as

$$\left(P_{i,t}^k + d_{i,t} \right) S_{i,t+1} - d_{i,t} (N_{i,t+1} - N_{i,t}) + (B_{i,t+1} - Q_t H_{i,t+1} - P_{i,t} K_{i,t+1})(N_{i,t+1} - N_{i,t}). \qquad \textbf{(A.4)}$$

When the equity market clears, $S_{i,t} = N_{i,t}$. This indicates that (A.4) equals (A.2). Therefore, the setup of the budget constraint is consistent with the payment that the household receives regardless of entry or exit.

APPENDIX B: LOG-LINEARIZED EQUATIONS

Let the variables with circumflexes denote percentage deviations from the steady state, and let ratios of capital letters without time subscripts denote the steady state value of the respective ratios. The following set of linearized equations shows the complete framework of the model.

B.1. HOMOGENEOUS-FIRM MODEL

Technology:

$$\frac{Y}{Y+F} \hat{Y}_{.t} = \hat{A}_t + \alpha \hat{K}_t + \theta \hat{H}_t + \gamma \hat{L}_t.$$

Capital demand:

$$\beta \frac{\alpha (Y+F)}{K} \left(\frac{Y}{Y+F} E_t \hat{Y}_{t+1} - \hat{K}_{t+1} \right) = \hat{\lambda}_t^c - \beta \left[\frac{\alpha (Y+F)}{K} + (1-\delta) \right] E_t \hat{\lambda}_{t+1}^c$$
$$+ \frac{\lambda^F}{\lambda^c} \left[\hat{\lambda}_t^F - \beta (1-\delta) E_t \hat{\lambda}_{t+1}^F \right].$$

Land demand:

$$\beta \frac{\theta (Y+F)}{QH} E_t \left(\frac{Y}{Y+F} \hat{Y}_{t+1} - \hat{H}_{t+1} \right)$$
$$= \left(1 + \beta u \frac{\lambda^F}{\lambda^c} \right) \hat{\lambda}_t^c - \beta \left[\frac{\theta (Y+F)}{QH} + 1 + u \frac{\lambda^F}{\lambda^c} \right] E_t \hat{\lambda}_{t+1}^c$$

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$$+(1-\beta u)\frac{\lambda^F}{\lambda^c}\hat{\lambda}_t^F-\beta\frac{\lambda^F}{\lambda^c}E_t\hat{\lambda}_{t+1}^F$$
$$+\left(1+\frac{\lambda^F}{\lambda^c}\right)\hat{Q}_t-\beta\left[1+(1+u)\frac{\lambda^F}{\lambda^c}\right]E_t\hat{Q}_{t+1}.$$

Labor demand:

$$\frac{Y}{Y+F}\hat{Y}_{,t}-\hat{L}_{,t}=\hat{W}_t.$$

Pricing equations:

$$\hat{\lambda}_t^c+\hat{Q}_t=\beta E_t(\hat{\lambda}_{t+1}+\hat{Q}_{t+1})+(1-\beta)\frac{NH}{H^c}(\hat{H}_{t+1}+\hat{N}_{t+1}),$$
$$\hat{\lambda}_t^c+\hat{W}_t=\frac{NL}{1-L^c}(\hat{L}_t+\hat{N}_t),$$
$$\hat{\lambda}_t^c+\hat{P}_t^k=E_t\left[\hat{\lambda}_{t+1}^c+\beta\hat{P}_{t+1}^k+(1-\beta)\hat{d}_{t+1}\right].$$

Consumption Euler equations:

$$-\hat{C}_t=\hat{\lambda}_t^c,$$
$$\hat{R}_{t+1}=\hat{\lambda}_t^c-\hat{\lambda}_{t+1}^c.$$

Rules of entry and exit:

$$\xi\hat{N}_{t+1}=\xi\hat{N}_t+f_\pi\left[\frac{P^k}{N}\hat{P}_t^k-\frac{QH}{N}(\hat{Q}_t+\hat{H}_{t+1})-\frac{PK}{N}\left(\hat{P}_t+\hat{K}_{t+1}\right)\right].$$

Resource constraint and financial constraint:

$$(\hat{N}_t+\hat{Y}_t)+(1-\delta)\frac{K}{Y}(\hat{N}_t+\hat{K}_t)=\frac{C}{NY}\hat{C}_t+\frac{K}{Y}(\hat{N}_{t+1}+\hat{K}_{t+1}),$$
$$\hat{R}_{t+1}+\hat{B}_{t+1}=E_t\hat{Q}_{t+1}+\hat{H}_{t+1},$$
$$\hat{K}_{t+1}-(1-\delta)\hat{K}_{1,t}+\frac{\delta}{\beta u}(\hat{H}_{t+1}-\hat{H}_t)=\delta\hat{B}_{t+1}.$$

B.2. HETEROGENEOUS-FIRM MODEL

Technology:

$$\frac{Y_1}{Y_1+F}\hat{Y}_{1.t}=\hat{A}_t+\alpha_1\hat{K}_{1.t}+\theta_1\hat{H}_{1.t}+\gamma_1\hat{L}_{1,t},$$
$$\frac{Y_2}{Y_2+F}\hat{Y}_{2.t}=\hat{A}_t+\alpha_2\hat{K}_{2.t}+\theta_2\hat{H}_{2.t}+\gamma_2\hat{L}_{2,t}.$$

Capital demand:

$$\beta\frac{\alpha_1(Y_1+F)}{K_1}\left(\frac{Y_1}{Y_1+F}E_t\hat{Y}_{1.t+1}-\hat{K}_{1.t+1}\right)=\hat{\lambda}_t^c-\beta\left[\frac{\alpha_1(Y_1+F)}{K_1}+(1-\delta)\right]E_t\hat{\lambda}_{t+1}^c$$
$$+\frac{\lambda_1^F}{\lambda^c}\left[\hat{\lambda}_{1,t}^F-\beta(1-\delta)E_t\hat{\lambda}_{1,t+1}^F\right]$$

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$$\beta\frac{\alpha_2(Y_2+F)}{K_2}\left(\frac{Y_2}{Y_2+F}E_t\hat{Y}_{2.t+1}-\hat{K}_{2.t+1}\right)=\hat{\lambda}_t^c-\beta\left[\frac{\alpha_2(Y_2+F)}{K_2}+(1-\delta)\right]E_t\hat{\lambda}_{t+1}^c$$
$$+\frac{\lambda_2^F}{\lambda^c}\left[\hat{\lambda}_{2,t}^F-\beta(1-\delta)E_t\hat{\lambda}_{2,t+1}^F\right]+\left(1+\frac{\lambda_2^F}{\lambda^c}\right)\left(\hat{P}_{2,t}-E_t\hat{P}_{2,t+1}\right).$$

Land demand:

$$\beta\frac{\theta_1(Y_1+F)}{QH_1}E_t\left(\frac{Y_1}{Y_1+F}\hat{Y}_{1.t+1}-\hat{H}_{1.t+1}\right)$$
$$=\left(1+\beta u\frac{\lambda_1^F}{\lambda^c}\right)\hat{\lambda}_t^c-\beta\left[\frac{\theta_1(Y_1+F)}{QH_1}+1+u\frac{\lambda_1^F}{\lambda^c}\right]E_t\hat{\lambda}_{t+1}^c$$
$$+(1-\beta u)\frac{\lambda_1^F}{\lambda^c}\hat{\lambda}_{1,t}^F-\beta\frac{\lambda_1^F}{\lambda^c}E_t\hat{\lambda}_{1,t+1}^F$$
$$+\left(1+\frac{\lambda_1^F}{\lambda^c}\right)\hat{Q}_t-\beta\left(1+(1+u)\frac{\lambda_1^F}{\lambda^c}\right)E_t\hat{Q}_{t+1}$$

$$\beta\frac{\theta_2P_2(Y_2+F)}{QH_2}E_t\left(\frac{Y_2}{Y_2+F}\hat{Y}_{2.t+1}+P_{2,t+1}-\hat{H}_{2.t+1}\right)$$
$$=\left(1+\beta u\frac{\lambda_2^F}{\lambda^c}\right)\hat{\lambda}_t^c-\beta\left[\frac{\theta_2P_2(Y_2+F)}{QH_2}+1+u\frac{\lambda_2^F}{\lambda^c}\right]E_t\hat{\lambda}_{t+1}^c$$
$$+(1-\beta u)\frac{\lambda_2^F}{\lambda^c}\hat{\lambda}_{2,t}^F-\beta\frac{\lambda_2^F}{\lambda^c}E_t\hat{\lambda}_{2,t+1}^F$$
$$+\left(1+\frac{\lambda_2^F}{\lambda^c}\right)\hat{Q}_t-\beta\left(1+(1+u)\frac{\lambda_2^F}{\lambda^c}\right)E_t\hat{Q}_{t+1}.$$

Labor demand:

$$\frac{Y_1}{Y_1+F}\hat{Y}_{1,t}-\hat{L}_{1,t}=\hat{W}_t,$$
$$\frac{Y_2}{Y_2+F}\hat{Y}_{2,t}+\hat{P}_{2,t}-\hat{L}_{2,t}=\hat{W}_t.$$

Pricing equations:

$$\hat{\lambda}_t^c+\hat{Q}_t=\beta E_t(\hat{\lambda}_{t+1}+\hat{Q}_{t+1})+(1-\beta)$$
$$\times\left[\frac{N_1H_1}{H^c}(\hat{H}_{1.t+1}+\hat{N}_{1,t+1})+\frac{N_2H_2}{H^c}(\hat{H}_{2.t+1}+\hat{N}_{2,t+1})\right],$$

$$\hat{\lambda}_t^c+\hat{W}_t=\frac{N_1L_1}{1-L^c}\left(\hat{L}_{1.t}+\hat{N}_{1,t}\right)+\frac{N_2L_2}{1-L^c}(\hat{L}_{2.t}+\hat{N}_{2,t}),$$
$$\hat{\lambda}_t^c+\hat{P}_{1,t}^k=E_t\left[\hat{\lambda}_{t+1}^c+\beta\hat{P}_{1,t+1}^k+(1-\beta)\hat{d}_{1,t+1}\right],$$
$$\hat{\lambda}_t^c+\hat{P}_{2,t}^k=E_t\left[\hat{\lambda}_{t+1}^c+\beta\hat{P}_{2,t+1}^k+(1-\beta)\hat{d}_{2,t+1}\right].$$

Consumption Euler equations:

$$\frac{1-\nu}{\nu}\hat{C}_{1,t}-\left(1+\frac{1-\nu}{\nu}\right)\hat{C}_t=\hat{\lambda}_t^c,$$

$$\frac{1-\nu}{\nu}\hat{C}_{2,t}-\left(1+\frac{1-\nu}{\nu}\right)\hat{C}_t=\hat{\lambda}_t^c+\hat{P}_{2,t},$$

$$\hat{C}_t=\left(\frac{C_1}{C}\right)^{\frac{1}{\nu}}\hat{C}_{1,t}+\left(\frac{C_2}{C}\right)^{\frac{1}{\nu}}\hat{C}_{2,t},$$

$$\hat{R}_{t+1}=\hat{\lambda}_t^c-\hat{\lambda}_{t+1}^c.$$

Rules of entry and exit:

$$\xi_1\hat{N}_{1,t+1}=\xi_1\hat{N}_{1,t}+f_{\pi_1}\left[\frac{P_1^k}{N_1}\hat{P}_{1,t}^k-\frac{QH_1}{N_1}(\hat{Q}_t+\hat{H}_{1,t+1})-\frac{P_1K_1}{N_1}(\hat{P}_{1,t}+\hat{K}_{1,t+1})\right],$$

$$\xi_2\hat{N}_{2,t+1}=\xi_2\hat{N}_{2,t}+f_{\pi_2}\left[\frac{P_2^k}{N_2}\hat{P}_{2,t}^k-\frac{QH_2}{N_2}(\hat{Q}_t+\hat{H}_{2,t+1})-\frac{P_2K_2}{N_2}(\hat{P}_{2,t}+\hat{K}_{2,t+1})\right].$$

Resource constraint and financial constraint:

$$(\hat{N}_{1,t}+\hat{Y}_{1,t})+(1-\delta)\,\frac{K_1}{Y_1}(\hat{N}_{1,t}+\hat{K}_{1,t})=\frac{C_1}{N_1Y_1}\hat{C}_{1,t}+\frac{K_1}{Y_1}(\hat{N}_{1,t+1}+\hat{K}_{1,t+1}),$$

$$(\hat{N}_{2,t}+\hat{Y}_{2,t})+(1-\delta)\,\frac{K_2}{Y_2}(\hat{N}_{2,t}+\hat{K}_{2,t})=\frac{C_2}{N_2Y_2}\hat{C}_{2,t}+\frac{K_2}{Y_2}(\hat{N}_{2,t+1}+\hat{K}_{2,t+1}),$$

$$\hat{R}_{t+1}+\hat{B}_{1,t+1}=E_t\hat{Q}_{t+1}+\hat{H}_{1,t+1},$$

$$\hat{R}_{t+1}+\hat{B}_{2,t+1}=E_t\hat{Q}_{t+1}+\hat{H}_{2,t+1},$$

$$\hat{K}_{1,t+1}-(1-\delta)\,\hat{K}_{1,t}+\frac{\delta}{\beta u}(\hat{H}_{1,t+1}-\hat{H}_{1,t})=\delta\hat{B}_{1,t+1},$$

$$\delta\hat{P}_{2,t}+\hat{K}_{2,t+1}-(1-\delta)\,\hat{K}_{2,t}+\frac{\delta}{\beta u}(\hat{H}_{2,t+1}-\hat{H}_{2,t})=\delta\hat{B}_{2,t+1}.$$

APPENDIX C: THE RESPONSE OF CAPITAL TO A LAND DEMAND SHOCK

To see why the capital increases more in the model with credit frictions than in the benchmark model, we can use the two credit constraints to find out how capital and land relate to each other. Consider the following log-linearized collateral constraint and the fund reserve constraint:

$$\hat{R}_{t+1}+\hat{B}_{i,t+1}=E_t\hat{Q}_{t+1}+\hat{H}_{i,t+1}, \tag{C.1}$$

$$\delta\hat{P}_{i,t}+\hat{K}_{i,t+1}-(1-\delta)\,\hat{K}_{i,t}+\frac{\delta}{\beta u}(\hat{H}_{i,t+1}-\hat{H}_{i,t})=\delta\hat{B}_{i,t+1}. \tag{C.2}$$

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Substituting the term $\hat{B}_{i,t+1}$ into equation (C.1) using (C.2) yields

$$\hat{P}_{i,t} + \frac{1}{\delta}[\hat{K}_{i,t+1} - (1-\delta)\hat{K}_{i,t}] + \frac{1}{\beta u}(\hat{H}_{i,t+1} - \hat{H}_{i,t}) = E_t \hat{Q}_{t+1} + \hat{H}_{i,t+1} - \hat{R}_{t+1} \quad \textbf{(C.3)}$$

In the first period, when the economy gets hit by the shock, all the predetermined variables in the current period do not deviate: $\hat{K}_{i,1} = \hat{H}_{i,1} = 0$. Plugging it into the preceding equation leads to

$$\hat{P}_{i,t} + \frac{1}{\delta}\hat{K}_{i,t+1} + \left(\frac{1}{\beta u} - 1\right)\hat{H}_{i,t+1} = E_t \hat{Q}_{t+1} - \hat{R}_{t+1} \text{ for } t = 1. \quad \textbf{(C.4)}$$

As long as the right-hand side of equation (C.4) is positive, a negative $\hat{H}_{i,t+1}$ must cause a positive initial impulse in the value of capital.