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The propagation of linear waves in high-energy-density magnetoplasmas using a relativistic quantum magnetohydrodynamic model

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The propagation characteristics of linear waves in high-energy-density magnetoplasmas are investigated using a relativistic magnetohydrodynamic model based on the framework of relativistic quantum theory. Based on the covariant Wigner function approach, a relativistic quantum magnetohydrodynamic model is established. Starting from the relativistic quantum magnetohydrodynamic equations and the Maxwell equations, the dispersion equation for relativistic quantum magnetoplasmas is derived. The contributions of both quantum effects and relativistic effects are shown in the dispersion relations for perpendicular, parallel propagation with respect to a background magnetic field. Results show that the corrections of both quantum effects and relativistic effects are significant when choosing the plasma parameters of laser-based plasma compression schemes.

Key words: plasma dynamics, quantum plasma, plasma waves

1. Introduction

High-energy-density plasmas have been considered for a long time, mainly in the context of astrophysics, such as the interior of giant planets (Tajima & Dawson 1979; Modena *et al.* 1995), the atmospheres of neutron stars (Amiranoff *et al.* 1998) and the interior of massive white dwarfs (Schroeder, Whittum & Wurtele 1999). With the development of modern laser technology, it is possible to obtain multi-petawatt laser power in the laboratory, which has made studies of high-energy-density plasmas more popular in intense laser–solid-density plasma interaction experiments (Markowich, Ringhofer & Schmeiser 1990; Jung 2001; Opher *et al.* 2001; Bingham, Mendonca & Shukla 2004; Marklund & Shukla 2006), laser-based inertial plasma fusion (Kremp *et al.* 1999; Ali *et al.* 2007) and the next generation of laser-based plasma compression (LBPC) schemes (Brodin, Marklund & Manfredi 2008).

In high-energy-density plasmas, the number density of electrons can be as high as 10^{23}–10^{30} cm^{-3}, and the thermal de Broglie wavelength of electrons is similar to or larger than the average interparticle distance of electrons (i.e. $\lambda_B^3 n_0 \geq 1$). Under such circumstances, since the Pauli exclusion principle indicates that fermions must occupy

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different energy levels, the distribution function of electrons changes from Boltzmann to Fermi–Dirac and classical thermal pressure changes to the degeneracy pressure. When the size of the system becomes comparable to the interparticle distance, electron tunnelling effects become important due to Heisenberg's uncertainty principle.

In superdense astrophysical objects, which have electron number density of the order of 10^{28}–10^{30} cm^{-3}, the electron Fermi energy is no longer negligible compared to the electron mass energy and the speed of an electron on the Fermi surface becomes comparable to the speed of light in vacuum. Meanwhile, with the development of laser technology, the intensity of lasers is able to reach the multi-petawatt domain. In the laser–plasma interaction, the oscillation velocity of electrons is close to the speed of light when the laser intensity reaches 10^{18} W cm^{-2}. Under such circumstances, the relativistic motion effects of electrons should be considered. When the laser intensity reaches 10^{24} W cm^{-2}, ions begin to show relativistic motion and the relativistic motion effects of ions should also be considered (Tamburini *et al.* 2010).

Therefore, the physical nature of high-energy-density plasmas is actually that of relativistic quantum plasmas, and it is necessary to establish a relativistic quantum model to describe the physical processes in high-energy-density plasmas. Studies of relativistic quantum plasmas began with the pioneering theoretical works of Tsytovich (1961), Jancovici (1962) and Lindhard (1954), who derived expressions of the longitudinal and transverse response functions for non-degenerate and completely degenerate electron gases. These results were extended using the Wigner function approach (Tenreiro & Hakim 1977; Hakim & Heyvaerts 1978; Hakim & Sivak 1982; Diaz Alonso & Hakim 1984) and quantum plasmadynamics (Melrose, Weise & McOrist 2006; Melrose 2008). Based on the covariant Wigner function approach, a relativistic quantum magnetohydrodynamic model was established and the relativistic quantum correction to laser wakefield acceleration was investigated by Zhu & Ji (2010). Meanwhile, the propagation of electromagnetic waves and electron plasma waves in relativistic quantum plasmas was investigated using a relativistic quantum kinetic model (Zhu & Ji 2012).

In this paper, we investigate the propagation characteristics of linear waves in high-energy-density magnetoplasmas using the relativistic quantum magnetohydrodynamic model that was put forward by us previously. The paper is organized as follows. In § 2, the simplified derivation of the relativistic quantum magnetohydrodynamic model based on the covariant Wigner function approach is presented. In § 3, starting from the relativistic quantum hydrodynamic equations and the Maxwell equations, the dispersion equation for relativistic quantum magnetoplasmas is deduced. In § 4, the contributions of both quantum effects and relativistic effects are shown in the dispersion relation for perpendicular, parallel propagation with respect to a background magnetic field. In § 5, the contributions of the Bohm potential, the Fermi statistics pressure and the relativistic effects are quantitatively calculated with real plasma parameters.

2. Relativistic quantum magnetohydrodynamic model

In this article, the physical system we analyse is the electron plasma. Since the mass of ions is much more than that of electrons, ions are treated as a stationary neutralizing background, and only the motion of electrons is considered. Since electrons are fermions, there will appear an electron spin current and a spin force acting on them due to the Bohr magnetization. However, for most plasmas, the spins of electrons are essentially randomly oriented, and the spin quantum effects are negligible. On the other hand, in a highly magnetized or low-temperature plasma, the spin effects can be appreciable (Marklund & Brodin 2007; Shukla & Eliasson 2011).

The covariant Wigner function approach for relativistic quantum plasmas was put forward by Hakim & Heyvaerts (1978). In the weak relativistic approximation and neglecting the spin contribution, a relativistic quantum kinetic model has been established as (Zhu & Ji 2010)

$$p^{\lambda}\partial_{\lambda}f(x,p) - \frac{e}{c}F^{\mu}{}_{\lambda}p^{\lambda}\frac{\partial}{\partial p^{\mu}}f(x,p) = 0, \tag{2.1}$$

where

$$f(x,p) = \frac{1}{(2\pi\hbar)^4}\int \mathrm{d}^4R\exp(-\mathrm{i}\pi\cdot R/\hbar)\left\langle\bar{\psi}\left(x+\frac{1}{2}R\right)\psi\left(x-\frac{1}{2}R\right)\right\rangle, \tag{2.2}$$

with $\pi^{\mu} = p^{\mu} + eA^{\mu}/c$, ψ and $\bar{\psi}$ are Dirac's fields obeying the Dirac equation and the brackets $\langle\cdots\rangle$ represent a quantum statistical average.

In order to derive a fluid model, we introduce the definitions of four-current $J^{\lambda}(x)$ and momentum-energy tensor $T^{\nu\lambda}$ as

$$J^{\lambda}(x) = -\frac{e}{m}\int \mathrm{d}^4p p^{\lambda}f(x,p) \tag{2.3}$$

and

$$T^{\nu\lambda} = \frac{1}{m}\int \mathrm{d}^4p p^{\nu}p^{\lambda}f(x,p). \tag{2.4}$$

Taking moments of (2.1) and using (2.3) and (2.4), the covariant forms of the relativistic quantum hydrodynamic equations are obtained as

$$\partial_{\lambda}J^{\lambda} = 0 \tag{2.5}$$

and

$$\partial_{\lambda}T^{\nu\lambda} = \frac{1}{c}F^{\nu}{}_{\lambda}J^{\lambda}. \tag{2.6}$$

By introducing the momentum-energy tensor $T^{\nu\lambda}$ of perfect fluids

$$T^{\nu\lambda} = -P\eta^{\nu\lambda} + \left(\frac{P}{c^2} + mn\right)U^{\nu}U^{\lambda}, \tag{2.7}$$

where P and $U^{\nu} = (\gamma c, \gamma \boldsymbol{u})$ are the pressure and four-dimensional velocity of the electron fluids, the three-dimensional vector forms of relativistic quantum magnetohydrodynamic equations are obtained from (2.5) and (2.6) as

$$\partial_t(\gamma n) + \nabla\cdot(\gamma n\boldsymbol{u}) = 0 \tag{2.8}$$

and

$$\begin{aligned}\frac{\partial\boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{u} = &-\frac{enc^2}{(P+mnc^2)\gamma}\left(\boldsymbol{E} + \frac{\boldsymbol{u}}{c}\times\boldsymbol{B} - \frac{1}{c^2}\boldsymbol{u}\boldsymbol{u}\cdot\boldsymbol{E}\right)\\ &-\frac{c^2}{(P+mnc^2)\gamma^2}\left(\nabla P + \frac{\boldsymbol{\beta}}{c}\frac{\partial P}{\partial t}\right),\end{aligned} \tag{2.9}$$

where $\gamma = 1/\sqrt{1-(u/c)^2}$ is the relativistic factor and $\boldsymbol{\beta} = \boldsymbol{u}/c$. The pressure term P in (2.9) may be decomposed into a classical part P^C and a quantum part P^Q. Under the weak

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relativistic approximation $P \ll mnc^2$, P^C and P^Q can be written as (Manfredi & Haas 2001; Haas 2011)

$$P^C = \frac{mn_0 v_{Fe}^2}{5}\left(\frac{n}{n_0}\right)^3, \tag{2.10}$$

$$P^Q = \frac{\hbar^2}{2m}\left[(\nabla\sqrt{n})^2 - \sqrt{n}\nabla^2\sqrt{n}\right]. \tag{2.11}$$

Inserting the expressions of classical pressure and quantum pressure into (2.9), we have

$$\begin{aligned}\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{u} = &-\frac{e}{m\gamma}\left(\boldsymbol{E} + \frac{\boldsymbol{u}}{c}\times\boldsymbol{B} - \frac{1}{c^2}\boldsymbol{uu}\cdot\boldsymbol{E}\right) - \frac{3nv_{Fe}^2}{5n_0^2\gamma^2}\left(\nabla n + \frac{\boldsymbol{\beta}}{cn}\frac{\partial n}{\partial t}\right)\\ &+ \frac{\hbar^2}{2m^2\gamma^2}\left\{\nabla\left(\frac{\nabla^2\sqrt{n}}{\sqrt{n}}\right) - \frac{\boldsymbol{\beta}}{cn}\partial_t[(\nabla\sqrt{n})^2 - \sqrt{n}\nabla^2\sqrt{n}]\right\}.\end{aligned} \tag{2.12}$$

The second and third terms on the right-hand side of (2.12) are the Fermi statistics pressure gradient and Bohm potential with the correction of relativistic factor. When setting $\boldsymbol{\beta}$ equal to zero, (2.12) degenerates to the non-relativistic quantum magnetohydrodynamic equation. When setting $\hbar \to 0$ and $\boldsymbol{\beta} \to 0$, the classical magnetohydrodynamic equation is recovered.

3. Dispersion equation of relativistic quantum plasmas

We assume that every quantity φ (representing $\boldsymbol{u}$, $\boldsymbol{E}$, $\boldsymbol{B}$, n) in (2.12) has the following form:

$$\varphi = \varphi_0 + \varphi_1, \tag{3.1}$$

where φ_0 is the unperturbed value and $\varphi_1 \ll \varphi_0$ is a small perturbation. Plasma equilibrium is assumed as $\boldsymbol{E}_0 = 0$, $\boldsymbol{u}_0 = 0$, $\boldsymbol{k} = (k, 0, 0)$ and $\boldsymbol{B}_0 = (B_0\cos\theta, 0, B_0\sin\theta)$, where θ is the angle between wavevector and external magnetic field. By the above assumption, we can obtain the basic linearized momentum equation

$$\frac{\partial \boldsymbol{u}_1}{\partial t} = -\frac{e}{m\gamma}\left(\boldsymbol{E}_1 + \frac{\boldsymbol{u}_1}{c}\times\boldsymbol{B}_0\right) - \frac{3v_{Fe}^2}{5n_0\gamma^2}\nabla n_1 + \frac{\hbar^2}{4m^2 n_0\gamma^2}\nabla\nabla^2 n_1 \tag{3.2}$$

and the continuity equation

$$\frac{\partial n_1}{\partial t} + n_0\nabla\cdot\boldsymbol{u}_1 = 0. \tag{3.3}$$

The electromagnetic fields $\boldsymbol{E}_1$ and $\boldsymbol{B}_1$ in (3.2) satisfy the linearized Maxwell equations:

$$\nabla\times\boldsymbol{E}_1 = -\frac{1}{c}\frac{\partial \boldsymbol{B}_1}{\partial t}, \tag{3.4}$$

$$\nabla\times\boldsymbol{B}_1 = \frac{1}{c}\frac{\partial \boldsymbol{E}_1}{\partial t} - \frac{4\pi e n_0}{c}\boldsymbol{u}_1. \tag{3.5}$$

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Supposing the perturbations are proportional to $\exp[\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)]$, (3.2) and (3.3) become

$$-\mathrm{i}\omega\boldsymbol{u}_1 = -\frac{e}{m\gamma}\left(\boldsymbol{E}_1+\frac{\boldsymbol{u}_1}{c}\times\boldsymbol{B}_0\right)-\frac{3\mathrm{i}v_{Fe}^2 n_1}{5n_0\gamma^2}\boldsymbol{k}-\frac{\mathrm{i}\hbar^2k^2n_1}{4m^2n_0\gamma^2}\boldsymbol{k} \tag{3.6}$$

and

$$n_1 = \frac{\boldsymbol{k}\cdot\boldsymbol{u}_1}{\omega}n_0. \tag{3.7}$$

The three components u_{1x}, u_{1y} and u_{1z} of the fluid velocity $\boldsymbol{u}_1$ can be written as

$$\left.\begin{aligned}
u_{1x} &= -\frac{\mathrm{i}e}{\omega m\gamma(1-\varDelta)}\left\{\left[1+\frac{\omega_c^2\sin\theta}{\varOmega^2(1-\varDelta)}\right]E_{1x}-\frac{i\omega\omega_c\sin\theta}{\varOmega^2}E_{1y}-\frac{\omega_c^2\sin\theta\cos\theta}{\varOmega^2}E_{1z}\right\},\\
u_{1y} &= -\frac{\mathrm{i}e}{\omega m\gamma}\left[\frac{\mathrm{i}\omega\omega_c\sin\theta}{\varOmega^2(1-\varDelta)}E_{1x}+\frac{\omega^2}{\varOmega^2}E_{1y}-\frac{\mathrm{i}\omega\omega_c\cos\theta}{\varOmega^2}E_{1z}\right],\\
u_{1z} &= -\frac{\mathrm{i}e}{\omega m\gamma}\left[-\frac{\omega_c^2\sin\theta\cos\theta}{\varOmega^2(1-\varDelta)}E_{1x}+\frac{\mathrm{i}\omega\omega_c\cos\theta}{\varOmega^2}E_{1y}+\left(1+\frac{\omega_c^2\cos\theta}{\varOmega^2}\right)E_{1z}\right],
\end{aligned}\right\} \tag{3.8}$$

where $\omega_c = eB_0/\gamma mc$ is the relativistic Larmor frequency for electrons,

$$\varDelta = \frac{3k^2v_{Fe}^2}{5\omega^2\gamma^2}+\frac{\hbar^2k^4}{4m^2\omega^2\gamma^2} \tag{3.9}$$

is the relativistic quantum correction and

$$\varOmega^2 = \omega^2-\omega_c^2\cos^2\theta-\frac{\omega_c^2}{1-\varDelta}\sin^2\theta. \tag{3.10}$$

The dispersion equation for plasmas can be derived from the linearized Maxwell equations as

$$\mathrm{Det}\left|\boldsymbol{k}\boldsymbol{k}-k^2\hat{I}+\frac{\omega^2}{c^2}\hat{\varepsilon}\right| = 0. \tag{3.11}$$

The current density and the dielectric permeability of the medium are given by

$$\boldsymbol{j} = -en_0\boldsymbol{u}_1 = \hat{\sigma}\cdot\boldsymbol{E}_1 \tag{3.12}$$

and

$$\hat{\varepsilon} = \hat{I}+\frac{4\pi\mathrm{i}}{\omega}\hat{\sigma}. \tag{3.13}$$

According to (3.11)–(3.13), the dispersion equation for relativistic quantum magnetoplasmas can be obtained as

$$\mathrm{Det}\begin{vmatrix}
\omega^2-\dfrac{\tilde{\omega}_p^2}{1-\varDelta}\left[1+\dfrac{\omega_c^2\sin^2\theta}{\varOmega^2(1-\varDelta)}\right] & \mathrm{i}\tilde{\omega}_p^2\dfrac{\omega_c\omega\sin\theta}{\varOmega^2(1-\varDelta)} & \tilde{\omega}_p^2\dfrac{\omega_c^2\sin\theta\cos\theta}{\varOmega^2(1-\varDelta)}\\
-\mathrm{i}\tilde{\omega}_p^2\dfrac{\omega_c\omega\sin\theta}{\varOmega^2(1-\varDelta)} & -k^2c^2+\omega^2\left(1-\dfrac{\tilde{\omega}_p^2}{\varOmega^2}\right) & \mathrm{i}\tilde{\omega}_p^2\dfrac{\omega_c\omega\cos\theta}{\varOmega^2}\\
\tilde{\omega}_p^2\dfrac{\omega_c^2\sin\theta\cos\theta}{\varOmega^2(1-\varDelta)} & -\mathrm{i}\tilde{\omega}_p^2\dfrac{\omega\omega_c\cos\theta}{\varOmega^2} & -k^2c^2+\omega^2\left[1-\dfrac{\tilde{\omega}_p^2}{\omega^2}\left(1+\dfrac{\omega_c^2\cos^2\theta}{\varOmega^2}\right)\right]
\end{vmatrix} = 0, \tag{3.14}$$

where $\tilde{\omega}_p^2 = 4\pi n_0e^2/m\gamma$ is the effective relativistic plasma frequency.

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4. Dispersion relation of linear waves

In this section, the dispersion relations are discussed in the following cases: propagating without external magnetic field, propagating parallel to the background magnetic field and propagating perpendicular to the background magnetic field.

4.1. *Without external magnetic field ($\boldsymbol{B}_0 = 0$)*

When there is no external magnetic field, the dispersion equation (3.14) reduces to

$$\text{Det}\begin{vmatrix} \omega^2 - \dfrac{\tilde{\omega}_p^2}{1-\varDelta} & 0 & 0 \\ 0 & -k^2c^2 + \omega^2 - \tilde{\omega}_p^2 & 0 \\ 0 & 0 & -k^2c^2 + \omega^2 - \tilde{\omega}_p^2 \end{vmatrix} = 0. \tag{4.1}$$

By solving (4.1), we can obtain the dispersion relation of electron plasma waves in relativistic quantum plasmas as

$$\omega^2 = \tilde{\omega}_p^2 + \frac{3k^2 v_{Fe}^2}{5\gamma^2} + \frac{\hbar^2 k^4}{4m^2\gamma^2}. \tag{4.2}$$

When setting $\gamma \to 1$, (4.2) is degenerated to the dispersion relation for electron plasma waves in non-relativistic quantum plasmas, and the well-known dispersion relation for Langmuir oscillation in classical cold plasmas will be derived from (4.2) by setting $\gamma \to 1$ and $\hbar \to 0$. Equation (4.2) also indicates that Langmuir oscillations can propagate in cold plasmas due to quantum effects.

The second solution of (4.1) is

$$\omega^2 = \tilde{\omega}_p^2 + k^2c^2, \tag{4.3}$$

which is the dispersion relation of electromagnetic waves in relativistic quantum plasmas. When setting $\gamma \to 1$, (4.3) is reduced to the well-known dispersion relation for electromagnetic waves in classical plasmas. Equation (4.3) indicates that Fermi statistics pressure and Bohm potential do not affect the dispersion relation of electromagnetic waves.

4.2. *Parallel propagation ($\theta = 0$)*

When the wavevector $\boldsymbol{k}$ is parallel to the external magnetic field $\boldsymbol{B}_0$, the dispersion equation (3.14) becomes

$$\text{Det}\begin{vmatrix} \omega^2 - \dfrac{\tilde{\omega}_p^2}{1-\varDelta} & 0 & 0 \\ 0 & -k^2c^2 + \omega^2\left(1 - \dfrac{\tilde{\omega}_p^2}{\omega^2 - \omega_c^2}\right) & \mathrm{i}\tilde{\omega}_p^2 \dfrac{\omega_c \omega}{\omega^2 - \omega_c^2} \\ 0 & -\mathrm{i}\tilde{\omega}_p^2 \dfrac{\omega_c \omega}{\omega^2 - \omega_c^2} & -k^2c^2 + \omega^2\left(1 - \dfrac{\tilde{\omega}_p^2}{\omega^2 - \omega_c^2}\right) \end{vmatrix} = 0. \tag{4.4}$$

By solving (4.4), we can obtain the dispersion relation for electron plasma waves propagating parallel to the external magnetic field in relativistic quantum plasmas as

$$\omega^2 = \tilde{\omega}_p^2 + \frac{3k^2 v_{Fe}^2}{5\gamma^2} + \frac{\hbar^2 k^4}{4m^2\gamma^2}. \tag{4.5}$$

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Comparing (4.2) and (4.5), it is found that the electron plasma waves propagating parallel to the external magnetic field have the same dispersion relation as those propagating without an external magnetic field. Therefore, we can conclude that the dispersion relation of electron plasma waves is not affected by the parallel external magnetic field.

The second solution of (4.4) is

$$\frac{k^2c^2}{\omega^2} = 1 - \frac{\tilde{\omega}_p^2}{\omega(\omega \pm \omega_c)}, \tag{4.6}$$

which is the dispersion relation of the left-handed wave (L wave) and the right-handed wave (R wave) in relativistic quantum plasmas, respectively. When setting $\gamma \to 1$, (4.6) reduces to the well-known dispersion relation for L wave and R wave in non-relativistic magnetoplasmas. Equation (4.6) also indicates that Fermi statistics pressure and Bohm potential do not affect the propagation of L waves and R waves.

4.3. *Perpendicular propagation ($\theta = \pi/2$)*

When the wavevector $\boldsymbol{k}$ is perpendicular to the external magnetic field $\boldsymbol{B}_0$, the dispersion equation (3.14) becomes

$$\text{Det}\begin{vmatrix} \omega^2\left[1 - \dfrac{\tilde{\omega}_p^2}{\omega^2(1-\varDelta) - \omega_c^2}\right] & \mathrm{i}\tilde{\omega}_p^2\dfrac{\omega_c\omega}{\omega^2(1-\varDelta)-\omega_c^2} & 0 \\ -\mathrm{i}\tilde{\omega}_p^2\dfrac{\omega_c\omega}{\omega^2(1-\varDelta)-\omega_c^2} & -k^2c^2 + \omega^2\left[1 - \dfrac{\tilde{\omega}_p^2(1-\varDelta)}{\omega^2(1-\varDelta)-\omega_c^2}\right] & 0 \\ 0 & 0 & -k^2c^2+\omega^2-\tilde{\omega}_p^2 \end{vmatrix} = 0. \tag{4.7}$$

The first solution of (4.7) is

$$\omega^2 = \tilde{\omega}_p^2 + k^2c^2, \tag{4.8}$$

which is the dispersion relation of the ordinary wave (O wave) in relativistic quantum magnetoplasmas. Comparing (4.3) and (4.8), we can conclude that the dispersion relation of electromagnetic waves is not affected by the perpendicular external magnetic field, since the electromagnetic waves propagating perpendicular to the external magnetic field have the same dispersion relation as those propagating without an external magnetic field. Equation (4.8) also indicates that quantum effects do not affect the propagation of O waves.

The second solution of (4.7) is

$$\frac{k^2c^2}{\omega^2} = 1 - \frac{\tilde{\omega}_p^2}{\omega^2}\frac{\omega^2(1-\varDelta) - \tilde{\omega}_p^2}{\omega^2 - \tilde{\omega}_h^2}, \tag{4.9}$$

which is the dispersion relation of the extraordinary wave (X wave) in relativistic quantum magnetoplasmas, and

$$\tilde{\omega}_h^2 = \tilde{\omega}_p^2 + \omega_c^2 + \frac{3k^2v_{Fe}^2}{5\gamma^2} + \frac{\hbar^2k^4}{4m^2\gamma^2} \tag{4.10}$$

is the dispersion relation of relativistic upper hybrid waves. By setting $\gamma \to 1$ and $\hbar \to 0$, (4.9) and (4.10) are degenerated to the well-known dispersion relations of X wave and the upper hybrid oscillation in classical magnetoplasmas, respectively.

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5. Discussion and conclusion

High-energy-density plasmas widely exist in the interior of white dwarf stars, intense laser–solid-density plasma interaction experiments as well as the next generation of LBPC schemes. It is expected that the electron number density can reach 10^{28} cm^{-3} and the temperature of electrons approximately 10 keV in LBPC schemes. In this section, we adopt the parameters of LBPC schemes for quantitative calculation, and the parameters are chosen as $n_0 = 3 \times 10^{28}$ cm^{-3}, $T = 10$ keV and $B_0 = 10^5$ Gs.

In quantum plasmas, since the de Broglie wavelength λ_B of electrons becomes comparable to, or even larger than, the average interparticle distance of electrons (i.e. $\lambda_B^3 n_0 \geq 1$), quantum effects are expected to play a crucial role in plasma dynamics. From the expression $\lambda_B^3 n_0 \geq 1$, we have

$$\frac{n_0}{T^{3/2}} \geq 10^{16}\ \mathrm{cm}^{-3}\,\mathrm{K}^{-3/2}. \tag{5.1}$$

Obviously, the parameters of LBPC schemes satisfy the quantum condition $\lambda_B^3 n_0 \geq 1$ and the weak relativistic condition $v_F^2/c^2 \ll 1$.

Since electrons are fermions (spin-1/2 quantum particles), there will appear an electron spin current and a spin force acting on electrons due to Bohr magnetization. In a highly magnetized or low-temperature plasma (i.e. $\mu_B B_0/K_B T \geq 1$), the spin effects can be appreciable. Noting that $\mu_B B_0/K_B T \equiv \hbar\omega_{ce}/m v_{th}^2$, the spin effects are important if

$$\frac{B_0}{T} \geq 8.1 \times 10^{11}\ \mathrm{Gs\,keV}^{-1}. \tag{5.2}$$

Obviously, the parameters of LBPC schemes do not meet the above condition, and the spin effect can be ignored.

The contributions of quantum effects and relativistic effects to the dispersion relation of electron plasma waves are shown in figure 1. It is found that the Langmuir oscillations can propagate in cold plasmas due to quantum effects, and the relativistic effect reduces the frequency of plasma waves. The quantum-corrected term can reach 10^{-1} when the wavenumber of electron plasma waves is 5×10^8 cm^{-1}, and the relativistic-corrected term is 6×10^{-2} when the electron number density is $n_0 = 3 \times 10^{28}$ cm^{-3}.

Figure 2 shows the dispersion relation of R and L waves in classical and relativistic quantum magnetoplasmas. The relativistic effect reduces the frequency of R and L waves, and the relativistic-corrected term reaches 10^{-2} when the electron number density is $n_0 = 3 \times 10^{28}$ cm^{-3}. Since the Fermi degeneracy pressure and the Bohm potential are parallel to the wavevector $\boldsymbol{k}$, they have no effect on the dispersion relation of L and R waves, which are transverse waves.

Figure 3 presents the dispersion relation of X waves in classical, non-relativistic quantum and relativistic quantum plasmas. Since the X wave is an electromagnetic wave composed of partial transverse wave and longitudinal wave, both relativistic and quantum effects can modify the dispersion relation of X waves. Calculations show that the corrections of both quantum effects and relativistic effects are significant in LBPC schemes.

In summary, we present a theoretical investigation of the propagation of linear waves in relativistic quantum plasmas by applying the relativistic quantum kinetic model. The dispersion relations of plasma waves, electromagnetic waves, L waves, R waves, O waves and X waves are derived. Research shows that Langmuir oscillations can propagate in cold plasmas due to quantum tunnelling effects and Fermi statistical pressure. It is also found that quantum effects do not affect the dispersion of electromagnetic waves, L

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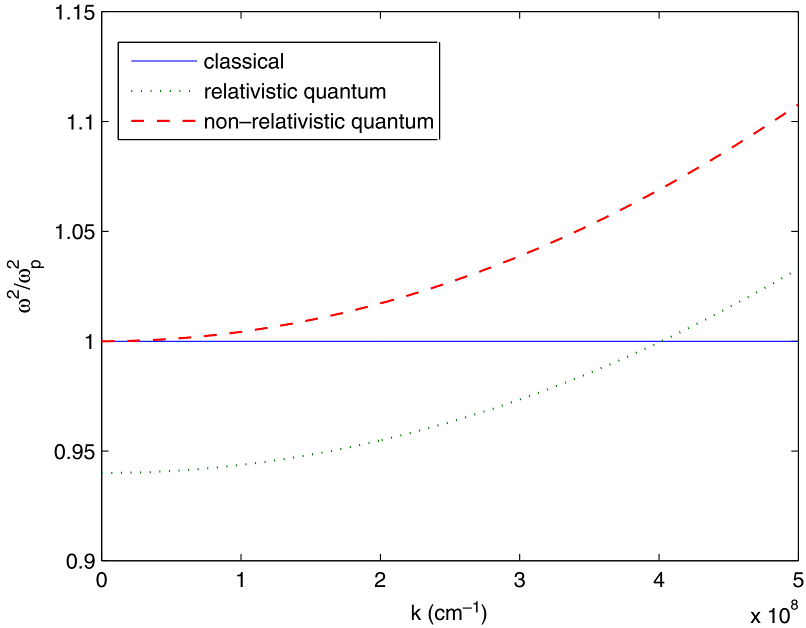

FIGURE 1. The solid line is the dispersion relation of Langmuir oscillation in classical cold plasmas, the (upper) dashed line is the dispersion relation of the electron plasma wave in non-relativistic quantum plasmas and the (lower) dashed line is the dispersion relation of the electron plasma wave in relativistic quantum plasmas. The plasma parameters are: $n_0 = 3 \times 10^{28}\ \mathrm{cm}^{-3}$, $\omega_p = 9.77 \times 10^{18}\ \mathrm{s}^{-1}$ and $v_F = 1.11 \times 10^{10}\ \mathrm{cm\,s}^{-1}$.

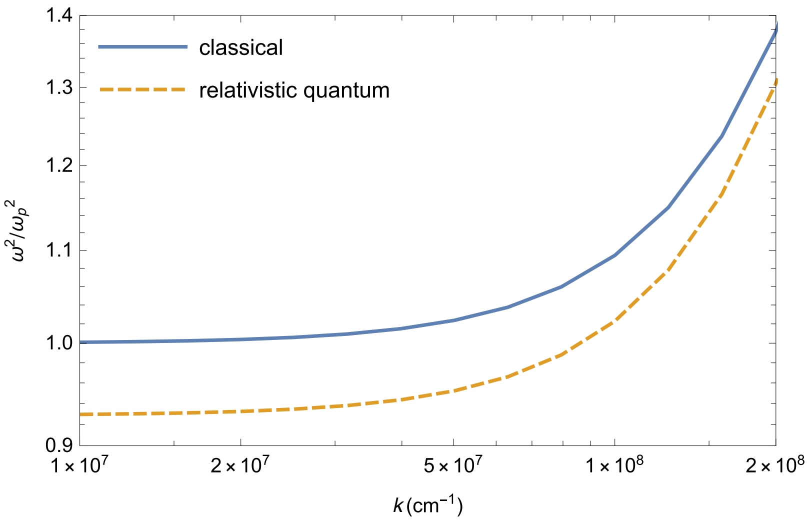

FIGURE 2. The solid line is the dispersion relation of R and L waves in classical plasmas and the dashed line is the dispersion relation of R waves in relativistic quantum plasmas. The plasma parameters are: $n_0 = 3 \times 10^{28}\ \mathrm{cm}^{-3}$ and $B_0 = 10^5$ Gs.

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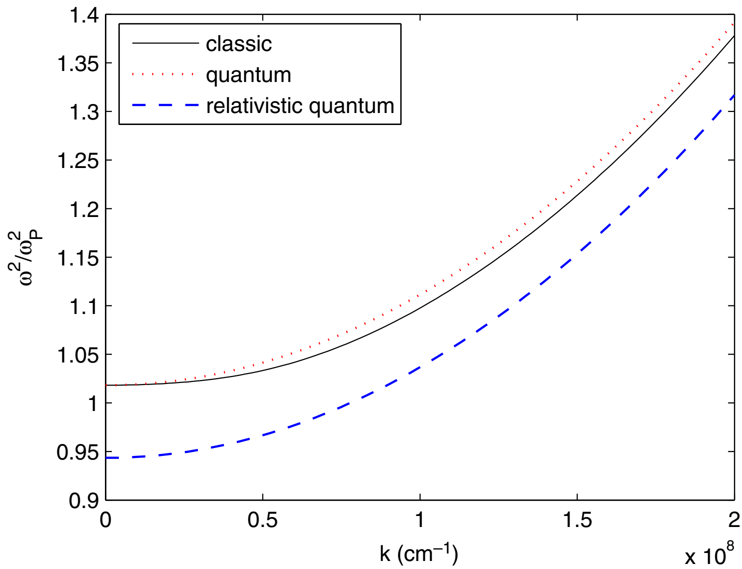

FIGURE 3. The solid line is the dispersion relation of X waves in classical plasmas, the dotted line is the dispersion relation of X waves in non-relativistic quantum plasmas and the dashed line is the dispersion relation of X waves in relativistic quantum plasmas. The plasma parameters are: $n_0 = 3 \times 10^{28}$ cm^{-3} and $B_0 = 10^5$ Gs.

waves, R waves and O waves, because they are all transverse waves. Since the X wave is an electromagnetic wave composed of partial transverse wave and longitudinal wave, the quantum effects can modify the dispersion relation of X waves. The contribution of relativistic effects is reflected in the reduction of the effective plasma frequency. Numerical evaluation indicates that the corrections produced by quantum effects and relativistic effects are significant and observable in LBPC schemes. This theoretical research may be useful for comprehending the propagation properties of the high-frequency waves in dense astrophysical objects, and also provide important reference for the experimental study of the intense laser–solid-density plasma interaction.

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Declaration of interests

The authors report no conflict of interest.

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