Macroeconomic Dynamics, **21**, 2017, 55–105. Printed in the United States of America.
doi:10.1017/S1365100515000395

HOW LOAN MODIFICATIONS INFLUENCE THE PREVALENCE OF MORTGAGE DEFAULTS

JISEOB KIM
Korea Development Institute (KDI)

How much can government-driven mortgage modification programs reduce the mortgage default rate? I compare an economy without a modification option to one with easy modifications, and evaluate the impact of these loan modifications on the foreclosure rate. Through loan modification, mortgage servicers can mitigate their losses and households can improve their financial positions without having to walk away from their homes. When modifying loan contracts is prohibitively costly, the default rate increases 1.5 percentage points in response to a 2007-style unexpected drop in housing prices of 30%. I calibrate the cost of modification after the financial crisis to match the Home Affordable Modification Program (HAMP) modification rate of 0.68%. My quantitative exercises show that current government efforts to promote mortgage modifications reduce the mortgage default rate by 0.63 percentage points.

Keywords: Mortgage Default, Mortgage Modification, Home Affordable Modification Program (HAMP), Financial Crisis

1. INTRODUCTION

Starting in late 2007, the U.S. housing market entered a period of decline. One of the more notable changes observed was a sudden increase in the mortgage default rate. Many papers have examined the potential causes of the housing crisis and the scope for policy to improve the financial situation of affected households. In this paper, I propose a quantitative model of mortgage default that allows for modification of mortgage terms and analyze the impact of government-driven modification programs in reducing mortgage defaults.

As shown in Figure 1, commercial banks' residential mortgage charge-off rate abruptly increased after 2007. In turn, the U.S. government, along with many financial sector entities, attempted to mitigate losses incurred by creditors and to reduce foreclosure rates among debtors [Gerardi and Li (2010)]. Although the

This paper is based on a chapter in my Ph.D. dissertation at the University of Rochester. I am deeply grateful to Mark Bils for his guidance. I also thank Yan Bai, Yongsung Chang, Ryan Michaels, Ronni Pavan, Jay Hong, William Hawkins, Andrew Davis, Arpad Abraham, an associate editor, and two anonymous referees for helpful comments.
Address correspondence to: Jiseob Kim, Korea Development Institute (KDI), 15 Giljae-gil, Sejong-si 339-007, Korea; e-mail: jiseob83@gmail.com

 1365-1005/16

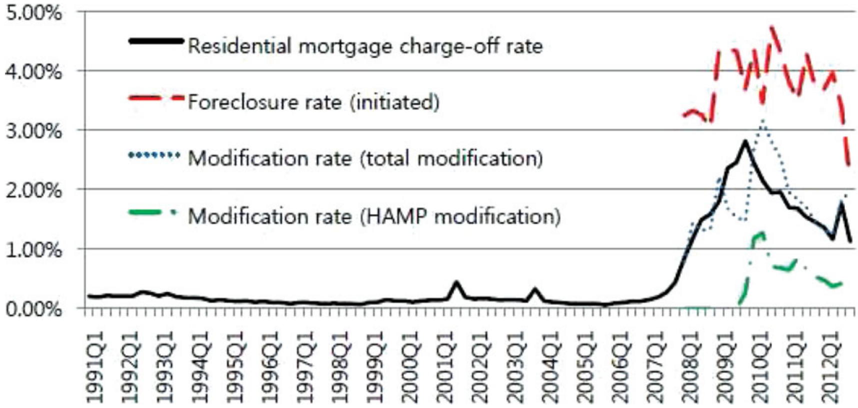

FIGURE 1. Quarterly residential mortgage charge-off rate, foreclosure rate, loan modification rate, and HAMP modification rate. All rates are annualized. *Source:* Federal Reserve, OCC Mortgage Metrics Report.

number of modifications has increased since late 2007, as shown in Figure 1, the effectiveness of loan modification programs is still unclear.

In this paper, I formulate a mortgage default model with loan modification under symmetric information between creditors and debtors. When housing price declines are correlated with negative income shocks, the budget set of mortgage holders shrinks. When drops in housing prices are large enough to produce negative housing equity, a financially constrained mortgage holder has less incentive to repay debt or sell the house to relieve budget tightness. Although default is always an option, default has further costs, including a bad credit history. When a household chooses to default, the mortgage servicer forecloses on the house and resells it in the market. However, at the same time, the financial intermediary needs to pay a foreclosure cost, estimated as 30% of the house price [Posner and Zingales (2009)]. Because household default incurs costs for both parties, there may be room for a mutually beneficial renegotiation of the loan contract. By reducing payments in order to prevent mortgage default, mortgage servicers could mitigate foreclosure costs. At the same time, mortgage holders might be better off staying in their current houses and avoiding their default-related costs.

To evaluate the benefit of modifications, I construct a dynamic model where a household makes saving decisions, housing purchase decisions, housing size decisions, down payment decisions, and selling and default decisions. For simplicity, there is only one type of mortgage contract: a 30-year fixed rate mortgage. Each household faces three types of exogenous shocks: income, housing price, and moving shocks. For example, a household might move for their children's education or for health reasons. Once a household receives a moving shock, it has to vacate its home immediately after closing housing-related contracts. Hence, the moving shock captures involuntary home exit and the indirect cost of owning a house.

In addition to this basic model, I introduce a loan modification option to examine its effect on the household default rate. Once a household initially wants to default on its mortgage debt, a mortgage servicer can reduce current mortgage debt up to the point where the value of defaulting and the value of repaying are the same. Lenders find this optimal whenever the expected present value of cash inflows under the modified loan is higher than the value of the defaulted housing net of any foreclosure costs. Under this modification scheme, the contract is efficient and all the surplus is taken by the mortgage servicer. Because there is no information asymmetry between creditors and debtors, ex ante it is a state-contingent contract, and ex post it is a loan modification contract. The structure of my modification contract is similar to that in Harris and Holmstrom (1982), where the contracted long-term wage jumps up when the outside market wage goes up.

I calibrate the model to precrisis data using the model without loan modification. This choice reflects the fact that modifying mortgage contracts before the housing crisis was likely prohibitively costly, possibly because of contract or securitization frictions [Keys et al. (2013)]. Through securitization, the ownership of mortgage bonds is transferred from loan originators to investors. Hence, mortgage servicers, which manage the mortgage schedules and coordinate the multiple investor relationships created through securitization, have less incentive to initiate any modifications. In fact, loan modification was a very rare event before the housing crisis. According to the Office of the Comptroller of the Currency, "[the] number of modifications done prior to 2008 was very small, almost negligible." Further, "As a loss mitigation tool, modifications were not typically used as assistance prior to the downturn in the economy and housing market focused on assisting homeowners with short-term credit repair."

However, after the outbreak of the housing crisis, the costs of modifying mortgage terms may have became lower because of government-driven foreclosure prevention policies. A financially constrained household that is struggling to make mortgage payments may be eligible for loan modifcation under the Home Affordable Modification Program (HAMP). Also, financial institutions that participate in the program receive monetary incentives from the government. These efforts to promote mortgage modifications reduce the effective costs of modifying loan contracts.

In my model, when a mortgage servicer modifies a loan contract, it incurs costs proportional to the debt principal. If the costs of modifying the loan contract are high enough, the mortgage servicer will let households default and recover the house value net of foreclosure costs, rather than renegotiating the loan contracts. In turn, no modifications occur in the steady state. Through calibration, I match the steady-state default rate to the precrisis mortgage default rate, which is 1.5%. In the other extreme case where there are no modification frictions, the steady-state default rate is almost zero. Depending on the modification cost, the steady-state default rate varies between these two extreme cases.

Next, I conduct an experiment mirroring recent events in the housing market. Starting in late 2007, the Case–Shiller index declined around 30% through the recession. At the same time, the mortgage foreclosure rate almost tripled (from 1.5% to 4.2%). Motivated by this observed decline in house prices, I calculate the response of the mortgage default rate from an unexpected drop in average housing prices of 30%. Because financially constrained households are more likely to default on their mortgages after an unexpected house price shock, the mortgage default rate suddenly increases. When modifying loan contracts is prohibitively costly, the default rate increases 1.5 percentage points in response to an unexpected drop in housing prices of 30%, and increases 1.6 percentage points in response to unexpected simultaneous drops in house prices of 30% and income of 10%.

The loan modification structure presented here is similar to certain aspects of the HAMP, which began in 2009. I calibrate the cost of modification after the financial crisis to match the 2011 HAMP modification rate of 0.68%. My quantitative results show that this type of mortgage modification program reduces the mortgage default rate by 0.63 percentage points. When the government doubles program spending, the mortgage default rate can be decreased by an additional 0.37 percentage points.

After the housing market crash, several foreclosure prevention policies were introduced [Robinson (2009); Gerardi and Li (2010)]. However, the potential ability of further mortgage loan modifications to complement and improve on these initiatives continued to be emphasized [White (2008); Levitin (2009)].

Then why are financial institutions hesitant to modify loan contracts? Mortgage holders' strategic behavior, especially "redefault" and "self-cure" risk, might be one reason [Foote et al. (2009); Adelino et al. (2013)]. Redefaults occur when a borrower who receives a modification still ends up in delinquency or default. Self-cure refers to delinquent borrowers who would become "current" in their repayment schedule without receiving any modification. However, empirical studies draw different conclusions regarding the potential for such strategic household behavior. Haughwout et al. (2010) finds that the redefault rate decreases as monthly payments or the debt principal are reduced. Conversely, Foote et al. (2009) and Mayer et al. (2014) find the opposite.

Contract frictions between borrowers and lenders, especially as generated by securitization, are another obstacle that hinders loan modification. Empirical research shows that securitized loans are less likely to be renegotiated than nonsecuritized loans [Piskorski et al. (2010), Agarwal et al. (2011), and Keys et al. (2013)]. Similarly, the performance of securitized loans is worse than that of nonsecuritized loans [Elul (2011)]. Again, however, the literature is split, with some arguing that mortgage loan securitization does not affect loan renegotiation [Adelino et al. (2013), Jiang (2014)].

One general agreement within this literature is that modifying loans incurs some costs. Depending on how high the cost is, a loan modification program may or may not be effective. This speaks to the need for a quantitative approach. However, to the best of my knowledge, there are no papers that analyze the effect of loan modification on reducing the foreclosure rate in a quantitative manner. In this paper, I compare an economy without a modification option to one with easy modifications and I evaluate the effectiveness of government-driven modification programs in reducing foreclosures.

In a related vein, there is a literature quantitatively examining what drove the observed increase in mortgage defaults. The introduction of unconventional mortgage contracts was the main reason, according to Corbae and Quintin (2015) and Campbell and Cocco (2015). Others claim that a positive housing supply shock, along with credit constraints and delays in the foreclosure process, was the driving force [Chatterjee and Eyigungor (2009, 2015)]. In this paper, the main driving forces for the huge increase in the foreclosure rate are optimistic belief in the future housing market and interest rate subsidies to low-income households, followed by an unexpected drop in house prices. Related to this mechanism, Burnside et al. (2015) explained the boom and bust of the housing market through agents' expectations about long-run fundamentals. In addition, Paul (2008), Roberts (2008), and Mian and Sufi (2009) emphasized that low-income households could easily take out mortgages with low prices before the housing market crash. I will examine the model impact of this potential unwarranted optimism and subsidies to low-income households in this paper.

The model structure presented here is similar to that in Chatterjee and Eyigungor (2009, 2015), Jeske et al. (2013), Hatchondo et al. (2014), Corbae and Quintin (2015), Arslan et al. (2015), and Guler (2015). (Davis and Van Nieuwerburgh (in press) review extensive empirical and theoretical housing market papers.) However, these papers do not have a loan modification option and thus both my steady-state and transition exercises are somewhat unusual in comparison [Davis and Van Nieuwerburgh (in press)].

The remainder of the paper is organized as follows. Section 2 introduces a basic model that does not have a loan modification option. Section 3 introduces a model with a loan modification option. Section 4 calibrates the model. Section 5 reports the steady-state results. Section 6 presents households' responses from a sudden drop in housing prices. Section 7 analyzes the effectiveness of current U.S. housing policies. Section 8 concludes the paper.

2. MODEL WITH NO MODIFICATION

Time is discrete and infinite. There are two market participants: households and mortgage servicers (or financial intermediaries). Households are either young or old. Households stochastically move from young to old with a probability of ρ_O and then die with a probability of ρ_D. The total measure of households is constant over time. The household's expected utility is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t, h_t, s_t\right),$$

where c_t is consumption, $h_t \in \{h_S, h_L\}$ is housing size with $h_S < h_L$, small and large, and $s_t \in \{0, 1\}$ is an indicator function that is 1 (or H) if a household is a homeowner, and 0 (or R) if a household is a renter. When a household is a renter, the only available housing size is h_S. However, prospective homeowners can choose to buy either $\{h_S, h_L\}$. The household's utility is defined by

$$u\left(c, h, s\right) = \left\{ \begin{array}{ll} \frac{\left[c^{1-\omega} h^{\omega}\right]^{1-\xi}}{1-\xi} & \text{if a renter, } h = h_S \\ \frac{\left[c^{1-\omega} \{h(1+\kappa)\}^{\omega}\right]^{1-\xi}}{1-\xi} & \text{if a homeowner, } h \in \{h_S, h_L\} \end{array} \right\},$$

where κ is an extra utility gain when a household lives in owner-occupied housing. With a larger house, a household receives an extra utility gain. A household can own at most one house.

A household is born with zero assets and starts life as a renter. When young, a household receives stochastic income (e) and accumulates financial assets (a). Also, the household decides whether to buy a house and assume a mortgage contract or remain a renter. If a household decides to remain a renter, it keeps accumulating financial assets. If a household decides to buy a house, it chooses the housing size and the fraction of down payment $\eta \in [0, 1]$. A transaction cost, which is a fraction χ_B of the housing value, is also paid. Only a fixed-rate mortgage (FRM) with N contract periods is available in the mortgage market.[1] A mortgage contract is used only for buying housing. There is no refinancing option or junior liens of a mortgage.

Once a household becomes a homeowner, it chooses one of three options: making periodic payments (x) while staying in the house, selling the house and becoming a renter, and defaulting on its mortgage debt. If a household defaults, it becomes a renter and is not eligible to buy a house for some period of time. With a probability of γ, the defaulted household recovers a good credit record and becomes a renter eligible to buy a house. When a household decides to sell a house, it has to pay a transaction cost, which is a fraction χ_S of the housing value. If a household repays all of the mortgage debt before becoming old, it can stay in or sell the house. Because there is no remaining debt at this point, a default option is not needed.

While owning a house, a household receives a moving shock with a probability of μ. This moving shock captures the household's involuntary move to other places. When a household with mortgage debt receives a moving shock, it is forced to sell the house or default on its mortgage debt. When a household without mortgage debt receives a moving shock, it is forced to sell the house and become a renter.

When a household becomes old, it becomes a renter, staying in an h_S housing size unit, provided by the government (or a nursing home). Hence, it is assumed that old households do not need to pay rental costs. Also, it is assumed that the old household receives a fixed amount of periodic income and dissaves financial assets before dying. Because there is no bequest motive, old age is a period for spending all of the household's remaining wealth. A fair annuity market exists during the old-age period.

In this model, the house supplier has large enough resources, and the housing supply is infinitely elastic. The house supplier prices each house with a unit housing price of $p \in \wp$. Hence, a household can choose any type of house that it can afford without affecting market house prices. However, households face a so-called housing price shock, where the unit housing price stochastically changes over time following a Markov process. Applying the law of large numbers, the measure of supplied houses with a unit price of $p \in \wp$ is constant. The house supplier also provides rental housing with a unit rental price of $\theta(p)$. For each submarket of housing with a unit price of $p \in \wp$, there should be no arbitrage opportunity from providing rental housing. Thus, for each unit housing price $p \in \wp$, the unit rental price is determined by

$$p = \theta(p) + \frac{\theta(p)}{1 + r_f} + \frac{\theta(p)}{(1 + r_f)^2} + ...,$$

where r_f is the risk-free interest rate.

Overall, the model has three types of exogenous shocks: an income shock, a unit housing price shock, and a moving shock. A moving shock affects the margin whether a homeowner exits his/her house or not. However, a unit housing price shock does not directly affect the home exit margin. Instead, it affects the margin of whether to sell a house or default on mortgage debt conditional on home exit.

2.1. Young Households

Young households can have one of four statuses: a renter who is eligible to buy a house (V_R^Y), a renter who is not eligible to buy a house because of a previous history of mortgage default (V_D^Y), a homeowner with mortgage debt (V_H^Y), or a homeowner who has repaid all of the mortgage debt (V_F^Y).

Renter who is eligible to buy a house. A renter has two options: remaining a renter (V_{RR}^Y), or becoming a homeowner with a mortgage loan contract (V_{RH}^Y). Thus, a young renter solves the following problem:

$$V_R^Y(a, e, p) = \max\left\{V_{RR}^Y(a, e, p), V_{RH}^Y(a, e, p)\right\}.$$

https://doi.org/10.1017/S1365100515000395 Published online by Cambridge University Press

If a household chooses to remain a renter, its value is given by

$$V_{\mathrm{RR}}^{\mathrm{Y}}(a, e, p) = \max_{a'} u(c, h_{\mathrm{S}}, R) + \beta\left[(1-\rho_{\mathrm{O}}) E V_{\mathrm{R}}^{\mathrm{Y}}(a', e', p') + \rho_{\mathrm{O}} V^{\mathrm{O}}(a')\right]$$

$$\text{s.t.}$$

$$c + a' + \theta(p) h_{\mathrm{S}} = (1 + r_{\mathrm{f}}) a + e,$$

$$c \geq 0,\ a' \geq 0,$$

where V^{O} is the value for old households, which will be defined later.

If a household chooses to buy a house with a mortgage contract, it has to choose a housing size (h), the down payment fraction (η), and the amount of saving (a'). Then the homeowner's state in the next period will be $(a, e, p, h, n, x, r_{\mathrm{m}})$, where (n, x, r_{m}) is the mortgage contract term, which indicates mortgage age (n), periodic payment (x), and mortgage interest rate (r_{m}). A renter can buy a house if the renter's initial assets are greater than the sum of the down payment, the transaction cost, and one periodic repayment.[2] The mortgage contract terms are endogenously determined by the household's choices and states and will be specified in the mortgage servicer's problem.

In the next period, with a probability of $(1-\rho_{\mathrm{O}})$, the household will stay young. Conditional on being young, a household receives an exogenous moving shock with a probability of μ. Then the household sells the house or defaults on its debt. If a household does not receive a moving shock, a household will remain a homeowner with mortgage debt. With a probability of ρ_{O}, the household becomes old. If a household becomes old with remaining mortgage debt, it chooses one of two options: having housing equity net of the mortgage debt or giving up net housing equity:

$$V_{\mathrm{RH}}^{\mathrm{Y}}(a, e, p) = \max_{\substack{a', h \in \{h_{\mathrm{S}}, h_{\mathrm{L}}\},\\ \eta \in [0,1]}} u(c, h, H)$$

$$+\beta \begin{bmatrix} (1-\rho_{\mathrm{O}}) \mu E \max \begin{Bmatrix} V_{HS}^{\mathrm{Y}}(a', e', p', h, 1, x, r_{\mathrm{m}}), \\ V_{\mathrm{D}}^{\mathrm{Y}}(a', e', p') \end{Bmatrix} \\ + (1-\rho_{\mathrm{O}})(1-\mu) E V_{\mathrm{H}}^{\mathrm{Y}}(a', e', p', h, 1, x, r_{\mathrm{m}}) \\ + \rho_{\mathrm{O}} E \max\left\{V^{\mathrm{O}}(a' + p'h - d'), V^{\mathrm{O}}(a')\right\} \end{bmatrix},$$

$$\text{s.t.}$$

$$c + a' + (\eta + \chi_{\mathrm{B}}) ph + x = (1 + r_{\mathrm{f}}) a + e,$$

$$(\eta + \chi_{\mathrm{B}}) ph + x \leq (1 + r_{\mathrm{f}}) a,$$

$$x = (1-\eta) ph \frac{r_{\mathrm{m}}}{1 + r_{\mathrm{m}}}\left[1 - \frac{1}{(1 + r_{\mathrm{m}})^N}\right]^{-1},$$

https://doi.org/10.1017/S1365100515000395 Published online by Cambridge University Press

$$d' = x \frac{1 + r_{\mathrm{m}}}{r_{\mathrm{m}}} \left[1 - \frac{1}{(1 + r_{\mathrm{m}})^{N-1}} \right],$$

$$x = x(a, e, p, h, \eta), \ r_{\mathrm{m}} = r_{\mathrm{m}}(a, e, p, h, \eta),$$

$$c \geq 0, \ a' \geq 0.$$

Homeowner without mortgage debt. When a homeowner repays all of the remaining mortgage debt before becoming old, it chooses either to stay in ($V_{\mathrm{FK}}^{\mathrm{Y}}$) or to sell the house ($V_{\mathrm{FS}}^{\mathrm{Y}}$). A household without mortgage debt solves the following problem:

$$V_{\mathrm{F}}^{\mathrm{Y}}(a, e, p, h) = \max \left\{ V_{\mathrm{FK}}^{\mathrm{Y}}(a, e, p, h), V_{\mathrm{FS}}^{\mathrm{Y}}(a, e, p, h) \right\}.$$

Once a household chooses to keep the house, the value is given by

$$V_{\mathrm{FK}}^{\mathrm{Y}}(a, e, p, h) = \max_{a'} u(c, h, H) + \beta \begin{bmatrix} (1 - \rho_{\mathrm{O}}) \mu E V_{\mathrm{FS}}^{\mathrm{Y}}(a', e', p', h) \\ + (1 - \rho_{\mathrm{O}})(1 - \mu) E V_{\mathrm{F}}^{\mathrm{Y}}\left(a', e', p', h\right) \\ + \rho_{\mathrm{O}} E V^{\mathrm{O}}\left(a' + p'h\right) \end{bmatrix}$$

$$\text{s.t.}$$

$$c + a' = (1 + r_{\mathrm{f}}) a + e,$$

$$c \geq 0, \ a' \geq 0.$$

Because there is no remaining mortgage debt, a default option is not needed. When a homeowner receives a moving shock with a probability of μ, it is forced to sell the house.

If a homeowner sells the house and becomes a renter, her value is given by[3]

$$V_{\mathrm{FS}}^{\mathrm{Y}}(a, e, p, h) = \max_{a'} u(c, h_{\mathrm{S}}, R) + \beta \left[(1 - \rho_{\mathrm{O}}) E V_{\mathrm{R}}^{\mathrm{Y}}\left(a', e', p'\right) + \rho_{\mathrm{O}} V^{\mathrm{O}}\left(a'\right) \right]$$

$$\text{s.t.}$$

$$c + a' + \theta(p) h_{\mathrm{S}} = (1 + r_{\mathrm{f}}) a + e + (1 - \chi_{\mathrm{S}}) ph,$$

$$c \geq 0, \ a' \geq 0.$$

Homeowner with mortgage debt. Once a household becomes a homeowner ($V_{\mathrm{H}}^{\mathrm{Y}}$) with mortgage debt ($n \leq N-1$), it has three options: repaying the debt ($V_{\mathrm{HP}}^{\mathrm{Y}}$), selling the house ($V_{\mathrm{HS}}^{\mathrm{Y}}$), or defaulting ($V_{\mathrm{D}}^{\mathrm{Y}}$). Thus, a household with mortgage debt solves the following problem:

$$V_{\mathrm{H}}^{\mathrm{Y}}(a, e, p, h, n, x, r_{\mathrm{m}}) = \max \left\{ \begin{array}{c} V_{\mathrm{HP}}^{\mathrm{Y}}(a, e, p, h, n, x, r_{\mathrm{m}}), \\ V_{\mathrm{HS}}^{\mathrm{Y}}(a, e, p, h, n, x, r_{\mathrm{m}}), \\ V_{\mathrm{D}}^{\mathrm{Y}}(a, e, p) \end{array} \right\}.$$

If a household chooses to repay its debt, with $n < N-1$, the value is given by

$$V_{\mathrm{HP}}^{\mathrm{Y}}\left(a, e, p, h, n, x, r_{\mathrm{m}}\right)=\max_{a'} u\left(c, h, H\right)$$
$$+\beta\left[\begin{array}{c}\left(1-\rho_{0}\right) \mu E \max \left\{\begin{array}{c}V_{\mathrm{HS}}^{\mathrm{Y}}\left(a', e', p', h, n+1, x, r_{\mathrm{m}}\right), \\ V_{\mathrm{D}}^{\mathrm{Y}}\left(a', e', p'\right)\end{array}\right\} \\ +\left(1-\rho_{\mathrm{O}}\right)(1-\mu) E V_{\mathrm{H}}^{\mathrm{Y}}\left(a', e', p', h, n+1, x, r_{\mathrm{m}}\right) \\ +\rho_{\mathrm{O}} E \max \left\{V^{\mathrm{O}}\left(a'+p' h-d'\right), V^{\mathrm{O}}\left(a'\right)\right\}\end{array}\right]$$

s.t.

$$c+a'+x=\left(1+r_{\mathrm{f}}\right) a+e,$$
$$d'=x \frac{1+r_{\mathrm{m}}}{r_{\mathrm{m}}}\left[1-\frac{1}{\left(1+r_{\mathrm{m}}\right)^{N-n-1}}\right],$$
$$c \geq 0,\ a' \geq 0.$$

When a household receives a moving shock, which occurs with probability μ, it chooses to sell the house or default on its mortgage debt. Conditional on not receiving a moving shock, the household's value in the next period is given by $V_{\mathrm{H}}^{\mathrm{Y}}$.

When $n = N-1$, a household repays the last periodic payment. Then it solves the following problem:

$$V_{\mathrm{HP}}^{\mathrm{Y}}\left(a, e, p, h, N-1, x, r_{\mathrm{m}}\right)=\max_{a'} u\left(c, h, H\right)$$
$$+\beta\left[\begin{array}{c}\left(1-\rho_{\mathrm{O}}\right) \mu E V_{\mathrm{FS}}^{\mathrm{Y}}\left(a', e', p', h\right) \\ +\left(1-\rho_{\mathrm{O}}\right)(1-\mu) E V_{\mathrm{F}}^{\mathrm{Y}}\left(a', e', p', h\right) \\ +\rho_{\mathrm{O}} E V^{\mathrm{O}}\left(a'+p' h\right)\end{array}\right]$$

s.t.

$$c+a'+x=\left(1+r_{\mathrm{f}}\right) a+e,$$
$$c \geq 0,\ a' \geq 0.$$

When $n \leq N-1$, if a household with mortgage debt chooses to sell the current house and repay all of the remaining debt, the household becomes a renter eligible to buy a house:

$$V_{\mathrm{HS}}^{\mathrm{Y}}\left(a, e, p, h, n, x, r_{\mathrm{m}}\right)=\max_{a'} u\left(c, h_{\mathrm{S}}, R\right)$$
$$+\beta\left[\left(1-\rho_{\mathrm{O}}\right) E V_{\mathrm{R}}^{\mathrm{Y}}\left(a', e', p'\right)+\rho_{\mathrm{O}} V^{\mathrm{O}}\left(a'\right)\right]$$

s.t.

$$c+a'+\theta(p) h_{\mathrm{S}}+d=\left(1+r_{\mathrm{f}}\right) a+e+\left(1-\chi_{\mathrm{S}}\right) p h,$$

https://doi.org/10.1017/S1365100515000395 Published online by Cambridge University Press

$$d = x\frac{1+r_{\mathrm{m}}}{r_{\mathrm{m}}}\left[1-\frac{1}{(1+r_{\mathrm{m}})^{N-n}}\right],$$
$$c \geq 0,\ a' \geq 0.$$

If a household defaults on its mortgage debt, it is not eligible to buy a house for some period of time, as a default penalty. With a probability of γ, a defaulted household becomes eligible to make a new mortgage contract:

$$V_{\mathrm{D}}^{\mathrm{Y}}(a,e,p) = \max_{a'} u(c,h_{\mathrm{S}},R) + \beta\begin{bmatrix}(1-\rho_{\mathrm{O}})\,\gamma E V_{\mathrm{R}}^{\mathrm{Y}}(a',e',p') \\ (1-\rho_{\mathrm{O}})(1-\gamma) E V_{\mathrm{D}}^{\mathrm{Y}}(a',e',p') \\ +\rho_{\mathrm{O}} V^{\mathrm{O}}(a')\end{bmatrix}$$

s.t.

$$c + a' + \theta(p)\,h_{\mathrm{S}} = (1+r_{\mathrm{f}})\,a + e,$$
$$c \geq 0,\ a' \geq 0.$$

2.2. Old Household

When a household becomes old, it sells any housing equity or defaults and becomes a renter, staying in a small house h_{S}. For simplicity, it is assumed that there is no rental cost for old. If a household has accumulated large amounts of wealth while young, it gets utility in old age from spending on consumption goods, rather than from spending on housing services. Old people have a fixed amount of income (e_{O}). With a probability of ρ_{D}, the old household dies and there is no bequest motive. The interest rate for assets is given by $\frac{1+r_{\mathrm{f}}}{1-\rho_{\mathrm{D}}}$, which incorporates the annuity value. The model is

$$V^{\mathrm{O}}(a) = \max_{a'} u(c,h_{\mathrm{S}},R) + \beta(1-\rho_{\mathrm{D}})\,V^{\mathrm{O}}(a')$$

s.t.

$$c + a' = \frac{1+r_{\mathrm{f}}}{1-\rho_{\mathrm{D}}}a + e_{\mathrm{O}},$$
$$c \geq 0,\ a' \geq 0.$$

2.3. Mortgage Servicer's Expected Profit

Assume that the mortgage servicing market is competitive and the expected profit of mortgage servicers is zero. Mortgage servicers can freely borrow money at a risk-free interest rate of r_{f}. Such borrowing by households is not allowed. It is also assumed that there is no information asymmetry between borrowers and lenders.[4]

The mortgage servicer's expected profit from contracting with an (a, e, p)-type household that chooses a housing size of h and a down payment of η at the time

https://doi.org/10.1017/S1365100515000395 Published online by Cambridge University Press

of the loan contract is

$$\Pi^0(a,e,p,h,\eta) = -(1-\eta)\,ph + x(a,e,p,h,\eta) + \frac{E\Pi\left(a',e',p',h,1,x,r_{\mathrm{m}}\right)}{1+r_{\mathrm{f}}},$$

where the first term shows the total outstanding loans and the second term is the periodic repayment after the loan contract is made. The last term is the expected cash inflow from the second period of the contract. For notational simplicity, let $\Delta \equiv (a,e,p,h,n,x,r_{\mathrm{m}})$ and $\Delta' \equiv (a',e',p',h,n+1,x,r_{\mathrm{m}})$. After the first period of the contract, the mortgage servicer's expected cash inflow is

$$\begin{aligned}
\Pi(\Delta) &= (1-\rho_{\mathrm{O}})\,\mu I_{\mathrm{MS}}(\Delta)\left\{x\frac{1+r_{\mathrm{m}}}{r_{\mathrm{m}}}\left[1-\frac{1}{(1+r_{\mathrm{m}})^{N-n}}\right]\right\}\\
&+ (1-\rho_{\mathrm{O}})\,\mu I_{\mathrm{MD}}(\Delta)\left\{(1-\chi_{\mathrm{D}})\,ph\right\}\\
&+ (1-\rho_{\mathrm{O}})(1-\mu)\,I_{\mathrm{P}}(\Delta)\left\{x+\frac{E\Pi\left(\Delta'\right)}{1+r_{\mathrm{f}}}\right\}\\
&+ (1-\rho_{\mathrm{O}})(1-\mu)\,I_{\mathrm{S}}(\Delta)\left\{x\frac{1+r_{\mathrm{m}}}{r_{\mathrm{m}}}\left[1-\frac{1}{(1+r_{\mathrm{m}})^{N-n}}\right]\right\}\\
&+ (1-\rho_{\mathrm{O}})(1-\mu)\,I_{\mathrm{D}}(\Delta)\left\{(1-\chi_{\mathrm{D}})\,ph\right\}\\
&+ \rho_{\mathrm{O}} I_{\mathrm{OP}}(\Delta)\left\{x\frac{1+r_{\mathrm{m}}}{r_{\mathrm{m}}}\left[1-\frac{1}{(1+r_{\mathrm{m}})^{N-n}}\right]\right\}\\
&+ \rho_{\mathrm{O}} I_{\mathrm{OD}}(\Delta)\left\{(1-\chi_{\mathrm{D}})\,ph\right\},
\end{aligned}$$

where χ_{D} is the foreclosure cost incurred by a mortgage servicer and $\Pi(\Delta)$ is the expected cash inflow after realizing the household's income and housing price but before realizing the household's age and moving shock status. With a probability of $1-\rho_{\mathrm{O}}$, a household stays young. With a probability of μ, a household receives a moving shock. Conditional on being young with a moving shock, a household chooses either to sell or to default. $I_{\mathrm{MS}}(\Delta)$ and $I_{\mathrm{MD}}(\Delta)$ are indicator functions that are 1 if a household chooses to sell or default, respectively, conditional on a moving shock, and 0 otherwise. If a household decides to sell the house, the mortgage servicer recovers the entire loan, $x\frac{1+r_{\mathrm{m}}}{r_{\mathrm{m}}}[1-\frac{1}{(1+r_{\mathrm{m}})^{N-n}}]$. If a household defaults following the moving shock, the mortgage servicer recovers the collateral value (housing value) net of the foreclosure cost, $(1-\chi_{\mathrm{D}})ph$.

Conditional on being young and not receiving a moving shock, a household chooses either repayment, selling, or default. $I_{\mathrm{P}}(\Delta)$, $I_{\mathrm{S}}(\Delta)$, and $I_{\mathrm{D}}(\Delta)$ are indicator functions that are 1 if a household chooses to repay, sell, or default, respectively, and 0 otherwise. If a household repays its debt, the expected cash inflow is $x+\frac{E\Pi(\Delta')}{1+r_{\mathrm{f}}}$. The first term is the household's periodic payment, and the second term is the expected cash inflow when the mortgage contract persists. If a household sells the house, the expected cash inflow is $x\frac{1+r_{\mathrm{m}}}{r_{\mathrm{m}}}[1-\frac{1}{(1+r_{\mathrm{m}})^{N-n}}]$. If a household defaults on its debt, the cash inflow is $(1-\chi_{\mathrm{D}})ph$.

https://doi.org/10.1017/S1365100515000395 Published online by Cambridge University Press

A household becomes old with a probability of ρ_{O}. $I_{\mathrm{OP}}(\Delta)$ and $I_{\mathrm{OD}}(\Delta)$ are indicator functions that are 1 if a household repays or defaults on its debt just after becoming old, respectively, and 0 otherwise.[5] Thus, the mortgage servicer can recover the entire mortgage loan, $x\frac{1+r_{\mathrm{m}}}{r_{\mathrm{m}}}[1-\frac{1}{(1+r_{\mathrm{m}})^{N-n}}]$, conditional on the household's repayment. If the household chooses not to repay its debt, because of having negative equity in its house, the mortgage servicer's cash inflow is given by $(1-\chi_{\mathrm{D}})ph$.

If a household repays all of the remaining debt, $n \geq N$, the expected cash inflow is zero. That is,

$$\Pi\left(\Delta\right) = 0 \text{ if } n \geq N.$$

Because the mortgage market is competitive, the mortgage servicer's expected profit is zero for every feasible state:

$$\Pi^{0}\left(a, e, p, h, \eta\right) = 0.$$

In addition to the zero-profit condition, the periodic payment and interest rate are pinned down by the following fixed-rate mortgage condition:[6]

$$(1-\eta)\,ph = x\left(a,e,p,h,\eta\right) + \frac{x\left(a,e,p,h,\eta\right)}{1+r_{\mathrm{m}}\left(a,e,p,h,\eta\right)} + \frac{x\left(a,e,p,h,\eta\right)}{\left[1+r_{\mathrm{m}}\left(a,e,p,h,\eta\right)\right]^{2}} + \ldots + \frac{x\left(a,e,p,h,\eta\right)}{\left[1+r_{\mathrm{m}}\left(a,e,p,h,\eta\right)\right]^{N-1}}.$$

2.4. Definition of a Steady-State Equilibrium

A steady-state equilibrium consists of value functions, household policy functions, mortgage contract schedules, and an invariant distribution Ψ such that

1. Household policies are optimal given the mortgage contract schedule.
2. The mortgage servicer's zero-profit condition and the fixed-rate mortgage condition hold for every state (a, e, p) and every feasible mortgage contract term $\{r_{\mathrm{m}}(a,e,p,h(a,e,p),\eta(a,e,p)),\, x(a,e,p,h(a,e,p),\eta(a,e,p))\}$.
3. The cross-sectional distribution Ψ is invariant given optimal policies and mortgage contract schedules.

3. LOAN MODIFICATION MODEL

In this section, I add a loan modification option to the basic model. The main aim of introducing the possibility of loan modification is to evaluate the effectiveness of modifications in reducing mortgage defaults when the average housing price suddenly drops, as in the recent recession.

When a mortgage holder wants to default on her mortgage debt, a mortgage servicer has the option of reducing the mortgage debt principal to the point where the values of defaulting and not defaulting are the same. However, the amount of written-off debt must be small enough so that the expected present value of

https://doi.org/10.1017/S1365100515000395 Published online by Cambridge University Press

cash inflows with a modified loan is higher than the housing value (or collateral value) net of foreclosure costs. This type of modification prevents the marginal defaulters from walking away from their homes. At the same time, mortgage servicers mitigate their losses. Because there is no information asymmetry between borrowers and lenders, this is a state-contingent contract at the time of the loan contract, ex ante. When a household receives a loan modification after experiencing any type of bad shock, it is a contract with a loan modification, ex post.

Conditional on the household choosing default without receiving a moving shock, mortgage servicers reduce the current debt burden x to $\tilde{x}_1$, which satisfies the following:

$$V_{\mathrm{D}}^{\mathrm{Y}}(a, e, p) = V_{\mathrm{HP}}^{\mathrm{Y}}(a, e, p, h, n, \tilde{x}_1, r_{\mathrm{m}}). \quad \mathbf{(1)}$$

Because the values of defaulting and repaying are now the same, households can stay in their homes by repaying a modified amount $\tilde{x}_1$. However, mortgage servicers have an incentive to agree to modification only when they expect a larger return after the modification. That is, the modified amount $\tilde{x}_1$ has a lower bound:

$$\underbrace{(1-\chi_{\mathrm{D}})\, ph}_{\text{Expected cash inflow with default}} \leq \underbrace{\tilde{x}_1 + \frac{E\Pi\left(a', e', p', h, n+1, \tilde{x}_1, r_{\mathrm{m}}\right)}{1+r_{\mathrm{f}}}}_{\text{Expected cash inflow with modification}} - \alpha A. \quad \mathbf{(2)}$$

When a mortgage servicer modifies a loan contract, it incurs costs proportional to the debt principal. Let $A (= x\frac{1+r_{\mathrm{m}}}{r_{\mathrm{m}}}[1 - \frac{1}{(1+r_{\mathrm{m}})^{N-n}}])$ be the current debt principal. Then the modification cost is given by αA. Note that a' on the right-hand side is the household's saving after the contract is modified.

Conditional on the household's choice of defaulting after the moving shock, mortgage servicers reduce the current debt burden x to $\tilde{x}_2$, which is determined by

$$V_{\mathrm{D}}^{\mathrm{Y}}(a, e, p) = V_{\mathrm{HS}}^{\mathrm{Y}}(a, e, p, h, n, \tilde{x}_2, r_{\mathrm{m}}). \quad \mathbf{(3)}$$

The modified loan must again provide a larger cash inflow to the mortgage servicers than the original contract. That is,

$$\underbrace{(1-\chi_{\mathrm{D}})\, ph}_{\text{Expected cash inflow with default}} \leq \underbrace{\tilde{x}_2 \frac{1+r_{\mathrm{m}}}{r_{\mathrm{m}}}\left[1 - \frac{1}{(1+r_{\mathrm{m}})^{N-n}}\right]}_{\text{Expected cash inflow with modification}} - \alpha A. \quad \mathbf{(4)}$$

Given the household's state, a household chooses either to repay, sell, or default. Suppose a household initially chooses default. If condition (2) or (4) does not hold, such households will not receive a modification and end up defaulting. If condition (2) or (4) holds, the loan terms are modified and households get the value of the modified contract, which is equal to the value of default, as shown in (1) and (3). Therefore, regardless of whether a household's loan is modified, ex post, its optimal policy functions are the same as in the base model, ex ante, conditional on the loan rate schedule. For simplicity, I assume that a loan is modified only when a household is young.

Expected profit when the mortgage is originated is

$$\Pi^0(a, e, p, h, \eta) = -(1-\eta)\, ph + x(a, e, p, h, \eta) + \frac{E\Pi\left(a', e', p', h, 1, x, r_{\mathrm{m}}\right)}{1+r_{\mathrm{f}}}.$$

For notational simplicity, let $\tilde{\Delta}'(\Delta) \equiv (a', e', p', h, n+1, \tilde{x}_1(\Delta), r_{\mathrm{m}})$ be the set of state variables after a loan contract is modified. Then, after the mortgage contract is signed, the expected cash inflow is given by

$$\begin{aligned}
\Pi(\Delta) &= (1-\rho_{\mathrm{O}})\,\mu I_{\mathrm{MS}}(\Delta)\left\{x\frac{1+r_{\mathrm{m}}}{r_{\mathrm{m}}}\left[1-\frac{1}{(1+r_{\mathrm{m}})^{N-n}}\right]\right\} \\
&+ (1-\rho_{\mathrm{O}})\,\mu I_{\mathrm{MD}}(\Delta)\left[\max\left\{\begin{array}{c}(1-\chi_{\mathrm{D}})\,ph, \\ \tilde{x}_2(\Delta)\,\frac{1+r_{\mathrm{m}}}{r_{\mathrm{m}}}\left[1-\frac{1}{(1+r_{\mathrm{m}})^{N-n}}\right]-\alpha A(\Delta)\end{array}\right\}\right] \\
&+ (1-\rho_{\mathrm{O}})(1-\mu)\, I_{\mathrm{P}}(\Delta)\left\{x+\frac{E\Pi\left(\Delta'\right)}{1+r_{\mathrm{f}}}\right\} \\
&+ (1-\rho_{\mathrm{O}})(1-\mu)\, I_{\mathrm{S}}(\Delta)\left\{x\frac{1+r_{\mathrm{m}}}{r_{\mathrm{m}}}\left[1-\frac{1}{(1+r_{\mathrm{m}})^{N-n}}\right]\right\} \\
&+ (1-\rho_{\mathrm{O}})(1-\mu)\, I_{\mathrm{D}}(\Delta)\max\left\{\begin{array}{c}(1-\chi_{\mathrm{D}})\,ph, \\ \tilde{x}_1(\Delta)+\frac{E\Pi(\tilde{\Delta}'(\Delta))}{1+r_{\mathrm{f}}}-\alpha A(\Delta)\end{array}\right\} \\
&+ \rho_{\mathrm{O}} I_{\mathrm{O}}(\Delta)\left\{x\frac{1+r_{\mathrm{m}}}{r_{\mathrm{m}}}\left[1-\frac{1}{(1+r_{\mathrm{m}})^{N-n}}\right]\right\} \\
&+ \rho_{\mathrm{O}} I_{\mathrm{OD}}(\Delta)\left\{(1-\chi_{\mathrm{D}})\,ph\right\},
\end{aligned}$$

where $A(\Delta)$ is the debt principal in state Δ. $\tilde{x}_1(\Delta)$ and $\tilde{x}_2(\Delta)$ are the modified periodic repayments, which are endogenously determined by (1) and (3).[7]

When the modification cost (α) is high enough, the preceding problem converges to the no-modification case. Let $\bar{\alpha}$ be the minimum modification cost that generates zero modification in a steady state. Then, for every α with $\alpha \geq \bar{\alpha}$, loan modifications will not occur in the steady state.

When $n \geq N$, the expected cash inflow is zero:

$$\Pi(\Delta) = 0 \text{ if } n \geq N.$$

Because the mortgage servicing market is competitive, the expected profit of mortgage servicers is zero for every feasible state:

$$\Pi^0(a, e, p, h, \eta) = 0.$$

In addition to the zero-profit condition, the mortgage interest rate and the periodic payment are pinned down by the following fixed-rate mortgage

https://doi.org/10.1017/S1365100515000395 Published online by Cambridge University Press

condition:

$$(1-\eta)\,ph = x\,(a,e,p,h,\eta) + \frac{x\,(a,e,p,h,\eta)}{1+r_{\mathrm{m}}\,(a,e,p,h,\eta)} + \frac{x\,(a,e,p,h,\eta)}{[1+r_{\mathrm{m}}\,(a,e,p,h,\eta)]^2} + \dots + \frac{x\,(a,e,p,h,\eta)}{[1+r_{\mathrm{m}}\,(a,e,p,h,\eta)]^{N-1}}.$$

After the housing market crisis, government-driven mortgage modification programs were introduced, which reduced the effective cost of modification. This is captured by the reduction in the modification cost from $\bar{\alpha}$ (or equivalently ∞) to α_1, where α_1 is less than $\bar{\alpha}$. The reduction in the modification cost should be financed by outside sources, for example, by a tax, which could be modeled in several ways. For computational simplicity, I assumed that the modification cost reduction is financed through a tax paid by the old households.[8] Thus, the old households' problem under an economy with modification options is the following:

$$V^{\mathrm{O}}(a) = \max_{a'} u\,(c, h_{\mathrm{S}}, R) + \beta\,(1-\rho_{\mathrm{D}})\,V^{\mathrm{O}}\left(a'\right)$$

$$\text{s.t.}$$

$$c + a' = \frac{1+r_{\mathrm{f}}}{1-\rho_{\mathrm{D}}}a + e_{\mathrm{O}} - \tau,$$

$$c \geq 0,\ a' \geq 0,$$

where every old household pays a lump-sump tax of τ. The lump-sum tax is determined by the following market clearing condition:

$$\underbrace{\int_{\Delta\in\text{Modified households}} (\bar{\alpha}-\alpha_1)\,A\,(\Delta)\,\Psi\,(\Delta)}_{\text{Reduction in the modification cost (or government subsidy)}} = \underbrace{\int_{\Delta\in\text{Old households}} \tau\Psi\,(\Delta)}_{\text{Total modification program funding}}, \tag{5}$$

where Ψ is the stationary distribution.

Before we move to the calibration section, one more thing should be noted. A household that experiences a moving shock will always get a loan modification if $\alpha = 0$. More specifically, the following proposition holds.

PROPOSITION 1. *Suppose* $V_{\mathrm{D}}^{\mathrm{Y}}(a,e,p) > V_{\mathrm{HS}}^{\mathrm{Y}}(a,e,p,h,n,x,r_{\mathrm{m}})$, $\chi_{\mathrm{D}} > \chi_{\mathrm{S}}$, *and* $\alpha = 0$. *Then the following holds:*

$$\max\left\{(1-\chi_{\mathrm{D}})\,ph, \tilde{x}_2\,(\Delta)\,\frac{1+r_{\mathrm{m}}}{r_{\mathrm{m}}}\left[1-\frac{1}{(1+r_{\mathrm{m}})^{N-n}}\right]\right\} = \tilde{x}_2\,(\Delta)\,\frac{1+r_{\mathrm{m}}}{r_{\mathrm{m}}}\left[1-\frac{1}{(1+r_{\mathrm{m}})^{N-n}}\right].$$

Proof. See the Appendix. ■

https://doi.org/10.1017/S1365100515000395 Published online by Cambridge University Press

Hence, when the cost of modification is low enough, households can easily get modifications, especially after facing a moving shock.

3.1. Definition of a Steady-State Equilibrium

A steady-state equilibrium consists of value functions, household policy functions, mortgage contract schedules, mortgage modification schedules, tax, and an invariant distribution Ψ such that

1. Household policies are optimal given the mortgage contract schedule.
2. The mortgage servicer's zero-profit condition and the fixed-rate mortgage condition hold for every renter's state (a, e, p) and every feasible mortgage contract $\{r_m(a, e, p, h(a, e, p), \eta(a, e, p)), x(a, e, p, h(a, e, p), \eta(a, e, p))\}$.
3. The loan modification decision follows (2) and (4), and the modified amount is determined by (1) and (3).
4. The cost of modification is financed by a lump-sum tax paid by old households, as shown in (5).
5. The cross-sectional distribution Ψ is invariant given optimal policies and mortgage contract schedules.

4. CALIBRATION

I choose parameters to match the precrisis economy using the model without loan modification. As mentioned in the Introduction, loan modification was very costly and was a very rare event before the housing crisis. Hence, I choose the model without loan modification as my benchmark.

A period in this model is two years. Demographic parameters are set as $(\rho_O, \rho_D) = (\frac{1}{22}, \frac{1}{10})$. On the average, agents are young for 44 years starting from age 20 and old age lasts for 20 years after the young period.

There are three income grid points for young households, $\{e_u, e_L, e_H\}$. A household receives e_u when unemployed and receives one of $\{e_L, e_H\}$ when employed. Income for either type of employed household, $\{e_L, e_H\}$, follows an AR(1) process:

$$\log(e_{t+1}) = (1 - \rho_e)\log(\bar{e}) + \rho_e \log(e_t) + \upsilon_t,$$

where $\upsilon_t \sim$ i.i.d.$N(0, \sigma_\upsilon^2)$ and $\bar{e} = 1$ in the benchmark. $(\rho_e, \sigma_\upsilon^2) = (0.9801, 0.0318)$ are taken from Storesletten et al. (2004). Income for unemployed households is zero, $e_u = 0$.[9]

The income transition matrix captures both the probability of switching employment states and how income evolves within each state:

$$\begin{array}{c} \\ e_u \\ e_L \\ e_H \end{array} \begin{array}{c} \begin{array}{ccc} e'_u & e'_L & e'_H \end{array} \\ \begin{bmatrix} \pi_{u,u} & 1 - \pi_{u,u} & 0 \\ \pi_{l,u} & \pi_{l,l} & \pi_{l,h} \\ \pi_{h,u} & \pi_{h,l} & \pi_{h,h} \end{bmatrix} \end{array},$$

https://doi.org/10.1017/S1365100515000395 Published online by Cambridge University Press

where the following submatrix jointly follows an AR(1) process:

$$\begin{bmatrix} \pi_{\mathrm{l,l}} & \pi_{\mathrm{l,h}} \\ \pi_{\mathrm{h,l}} & \pi_{\mathrm{h,h}} \end{bmatrix}.$$

$\pi_{\mathrm{l,u}}$ is the probability that a low-income household becomes unemployed and similarly $\pi_{\mathrm{h,u}}$ is the probability that a high-income household becomes unemployed. For simplicity, I assume that $\pi_{\mathrm{l,u}} = \pi_{\mathrm{h,u}}$. I choose $\pi_{\mathrm{l,u}}$ and $\pi_{\mathrm{h,u}}$ to match the unemployment rate between 2000 and 2004, which is around 5%. $1 - \pi_{\mathrm{u,u}}$ is the probability that an unemployed household becomes employed, which I match with Shimer (2012).[10] [11] Income for old agents is around 60% of the median income for young agents (2004 SCF).

Following Hatchondo et al. (2014), the unit housing price process follows an AR(1) process,

$$\log(p_{t+1}) = \left(1 - \rho_p\right)\log(\bar{p}) + \rho_p \log(p_t) + \varepsilon_t,$$

where $\bar{p}$ is the mean housing price and $\varepsilon_t \sim \text{i.i.d.} N(0, \sigma_\varepsilon^2)$.[12] The persistence parameter and variance of the housing price process are given by $(\rho_p, \sigma_\varepsilon^2) = (0.9409, 0.5861)$. Using the Tauchen method, I discretize the housing price process with five grid points.

Following Gruber and Martin (2003), the transaction costs of buying and selling a house are 2.5% and 7% of the housing price, respectively. From Pennington-Cross (2006), the foreclosure cost incurred by a mortgage servicer is 22% of the housing value.

There is no consensus as to how long mortgage defaulters are excluded from the mortgage market. For unsecured, or credit card, debt, a bad credit record lasts for 10 years [Chatterjee et al. (2007)]. Chatterjee and Eyigungor (2009) assume that mortgage defaulters are excluded from credit markets for 3.33 years. Chatterjee and Eyigungor (2015) assume 4 years and Guler (2015) assumes 7 years. I choose 4 years as the average exclusion period ($\gamma = 0.5$).

The risk-free interest rate is 4% per year ($r_{\mathrm{f}} = 0.08$). The preference parameters $(\omega, \xi) = (0.24, 2)$ are chosen from Chatterjee and Eyigungor (2009). The small housing size is normalized to 1. The mortgage contract lasts for 30 years (or $N = 15$).

Six free parameters remain: discount factor (β), moving shock (μ), homeowner's extra utility gain from owning (κ), average unit housing price ($\bar{p}$), big housing size (h_{L}), and modification cost (α). Because there are no modification options in the benchmark model, the modification cost parameter is not used in the benchmark calibration (or we can find the minimum value of α that makes the steady state modification rate zero, $\bar{\alpha}$).[13] I jointly match the mortgage default rate, which is 1.5% per year before the housing crisis [Corbae and Quintin(2015)], the homeownership rate, which is 68.1% (2001–2004 Census), the ratio of average annual rent to average annual income for renters, which is 0.21 (2004 SCF), the ratio of average housing value to average annual income for homeowners, which

Table 1. Calibration

Parameter	Description	Value	Target/source
	Nontarget parameters		
ρ_{O}	Probability of being old	$\frac{1}{22}$	44 years of young life
ρ_{D}	Probability of dying	$\frac{1}{10}$	20 years of old life
ρ_e	Persistence in income process	0.9801	Storesletten et al. (2004)
σ_{υ}^2	Variance of income process	0.0318	Storesletten et al. (2004)
ρ_p	Persistence in housing price	0.9409	Hatchondo et al. (2011)
σ_{ε}^2	Variance of housing price	0.5861	Hatchondo et al. (2011)
ω	Utility parameter	0.24	Chatterjee and Eyigungor (2009)
ξ	Utility parameter	2	Chatterjee and Eyigungor (2009)
χ_{B}	Transaction cost—buying	0.025	Gruber and Martin (2003)
χ_{S}	Transaction cost—selling	0.07	Gruber and Martin (2003)
χ_{D}	Foreclosure cost	0.22	Pennington-Cross (2006)
γ	Credit recovery rate	0.5	4 years of exclusion
N	Contract periods	15	30-year contract
r_{f}	Risk-free interest rate	0.08	2-year risk-free rate
e_{O}	Income for old households	0.6	2004 SCF
h_{S}	Small housing size	1	Normalized to one
$\bar{e}$	Average income	1	Normalized to one
$\pi_{l,u} = \pi_{h,u}$	Prob. of being unemployed	0.0315	Unemployment rate 5%
$1 - \pi_{u,u}$	Prob. of being employed	0.5997	Shimer (2007)
	Target parameters		
β	Discount factor	0.7	Financial asset-to-income ratio
μ	Moving shock	0.08	Mortgage default rate
κ	Homeowner's extra utility gain	4	Homeownership rate
$\bar{p}$	Average housing price	1.2	Rent-to-income ratio
h_{L}	Big housing size	1.2	House-value-to-income ratio

is 2.71 (2004 SCF), and the ratio of average financial assets to average annual income, which is 1.65 (2004 SCF). Table 1 summarizes all model parameters and targets.

5. STEADY-STATE RESULTS

In this section, I report quantitative results with and without loan modification. First, I compare steady-state economies with different modification costs. I then analyze the financial characteristics of households that decide to either default or sell their house and subsequently enter a new owner-occupied house, using model-generated data. This allows me to compare the financial characteristics of

TABLE 2. Steady state

	Data	No mod ($\alpha \geq 0.53$)	Costly mod ($\alpha = 0.265$)	Costless mod ($\alpha = 0$)
Targeted statistics with no-modification model				
Annual default rate	1.5%	**1.47%**	1.14%	0.04%
Homeownership rate	68.1%	**68.7%**	69.3%	72.03%
$\frac{\text{Average(housing price)}}{\text{Average(annual income for homeowners)}}$	2.71	**2.71**	2.71	2.80
$\frac{\text{Average(Rent)}}{\text{Average(Annual income for renters)}}$	0.21	**0.22**	0.23	0.23
$\frac{\text{Average(financial asset)}}{\text{Average(annual income)}}$	1.65	**1.66**	1.68	1.75
Nontargeted statistics				
Modification acceptance rate	0.45	0	0.41	0.98
Avg interest rate (30-year FRM real rate, 95–04)	4.87%	6.20%	6.49%	6.27%
$\frac{\text{Average(originated loan)}}{\text{Average(annual income)}}$	2.72	2.77	2.97	3.63
$\frac{\text{Average(annual periodic payment)}}{\text{Average(annual income)}}$	0.14	0.18	0.19	0.22
Coeff of variation (housing value)	0.74	0.70	0.70	0.70
Loan-to-value ratio (loan originators)	0.86	0.86	0.91	0.99
Loan-to-value ratio (defaulting households)	1.62	2.1	1.83	1.12
Home exit rate	15%	13.47%	13.40%	13.73%
Fraction of housing exit driven by selling		83.73%	87.11%	99.57%
Fraction of housing exit driven by moving shock		49.03%	49.65%	48.04%
Fraction of default driven by moving shock		81.73%	94.3%	0%
Annual modification rate		0%	0.80%	2.88%
Fraction of negative equity households		29.65%	32.08%	33.62%

households that receive loan modifications with those of similar households that cannot receive loan modifications. Finally, the effects of the moving shock and foreclosure costs on the steady state are analyzed.

5.1. Steady State

Table 2 contrasts steady-state economies with and without loan modification options. When the modification cost α is greater than or equal to 0.53 (or $\bar{\alpha} = 0.53$), the steady-state modification rate is zero, which represents the precrisis environment. When the modification cost is prohibitively high, the calibrated results show that the annual mortgage default rate is 1.47% (1.5% in the data), the homeownership rate is 68.7% (68.1% in the data), the ratio of average housing value to average income is 2.71 (2.71 in the data), the ratio of average annual rent to average annual income is 0.22 (0.21 in the data), and the ratio of average financial assets to average income is 1.66 (1.65 in the data). These are the target moments that I matched through calibration.

I now compare a steady-state economy without a modification option to one with easy modification (or $\alpha = 0$).[14] The annual mortgage default rate is reduced from 1.47% to 0.04% when a costless modification option is introduced. With costless modification, the homeownership rate increases from 68.7% to 72.03% (68.1% in the data), and the average interest rate increases from 6.2% to 6.27% (4.87% in the data). The average loan-to-value (LTV) ratio at the time of loan origination is 0.86 (0.99), and the average LTV ratio of defaulting households is 2.1 (1.12) in a "no-modification" ("costless modification") model, whereas in the data LTV at origination is around 0.86,[15] and the median LTV of defaulting households is 1.62.[16] The home exit rate among homeowners is around 13% in both models, which is close to the data.[17] The fraction of housing exit driven by selling is 99.57% in a costless modification model and 83.73% in a no-modification model (the fraction of housing exit driven by default is 0.43% in a costless modification model and 16.27% in a no-modification model). The fraction of housing exit driven by a moving shock is around 49% in both models. (The fraction of voluntary housing exit is 51%.) The fraction of negative equity households among mortgage holders is 33.62% in a costless modification model, and 29.65% in a no-modification model.

In the second column, the cost of modification is halved ($\alpha = 0.265$).[18] As the modification cost falls, the default rate decreases. At the same time, the modification rate increases. Unsurprisingly, the results for this scenario are consistent between those of the baseline and costless modification economies.

To better understand these quantitative results, we need to see the mortgage servicers' profit function. In a model with modification, the mortgage servicers' expected present value of cash inflows at the time of the loan contract is higher than that without modification, ceteris paribus, because all the modification surplus accrues to the mortgage servicers. Thus, with modification, the competitive mortgage servicers will reduce the interest rate by meeting the zero-profit condition. Because the interest rate schedule with loan modifications is lower, renters prefer to make a smaller down payment and defer more payments. Figure 2a compares the mortgage interest rate schedules with and without the loan modification option. As explained earlier, the mortgage interest rate schedule in a no-modification model is higher than that in a modification model, ceteris paribus. Therefore, with loan modification, loan originations are higher, the fraction of negative equity households is higher, the LTV ratio is higher, and the homeownership rate is higher than that with no modification scheme. Further, the number of mortgage holders under a costless modification economy is 10% higher than that under a no-modification economy (not shown in the table). Though the mortgage interest rate schedule is lower in a modification model, households take out larger amounts of mortgage debt, which increases the average interest rate.

Figure 2 also shows mortgage interest rate schedules faced by different household states and decisions in a model with no modification option. As household income increases, the mortgage interest rate schedule shifts down (see Figure 2b). When a household decides to buy a bigger home, the mortgage interest rate rises

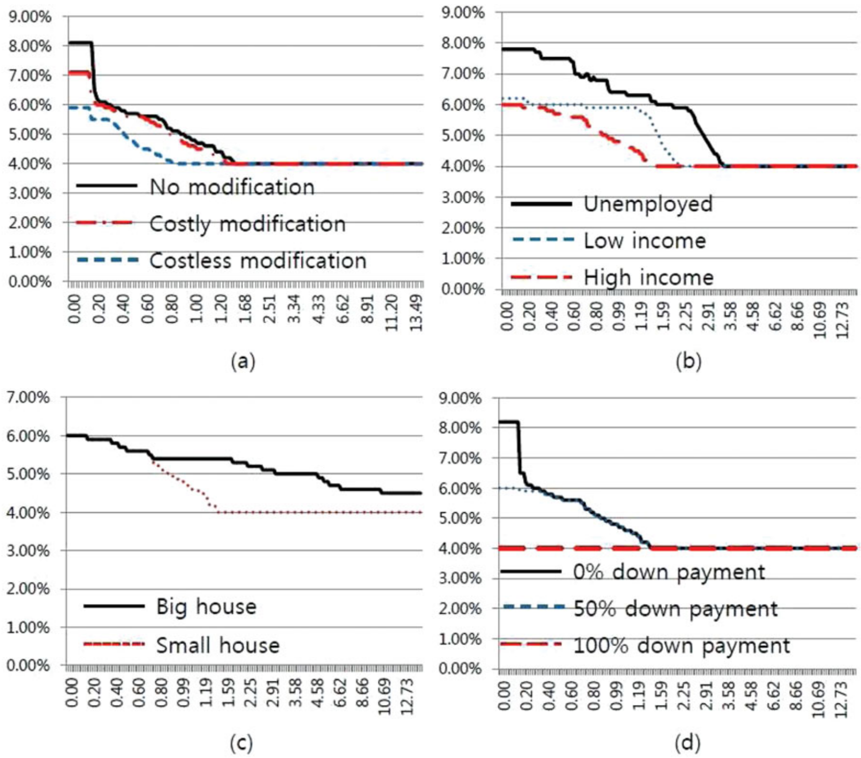

FIGURE 2. Mortgage interest rate schedules, given state variables. The horizontal axis is the asset grids.

(see Figure 2c). When a household makes a larger down payment, the mortgage interest rate schedule shifts down (see Figure 2d).

Note that these interest rate schedules are not uniquely determined. For each possible renter's state (a, e, p), a renter chooses a specific housing size and down payment among the possible set of menus (h, η), or remains as a renter. The options not chosen are off the equilibrium paths. The interest rate schedules include those off-equilibrium paths and they can be defined in diverse ways without affecting on-equilibrium paths. Hence, the interest rate schedule that I present here is one example among the multiple on- and off-equilibrium paths.

In Figure 3, I present the renter's optimal decision rules in a no-modification economy. The figure shows the renter's housing purchase decision, optimal housing size decision, and down payment decision and the equilibrium mortgage interest rates conditional on assets, income, and unit housing prices. When assets are close to zero, a renter cannot take on a mortgage contract and therefore cannot buy a house, so the mortgage loan interest rate does not exist (zero in the figure). As assets increase, a renter can buy a small house with no down payment and a

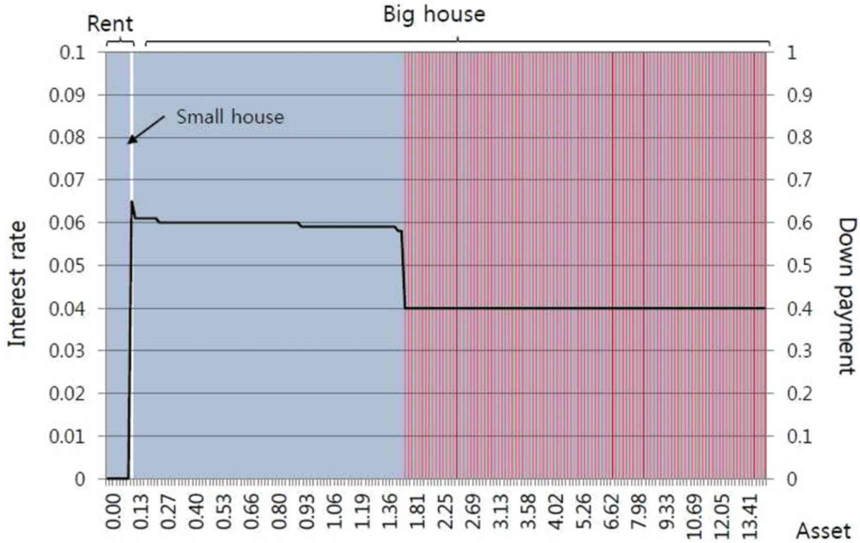

FIGURE 3. Renter's optimal housing purchase, housing size, and down payment decisions, as well as equilibrium mortgage interest rates. The horizontal axis is the asset grids. The solid line is the mortgage interest rate (left axis), the vertical lines are the down payment decision (right axis), and the shaded area is the housing purchase/housing size decision.

mortgage interest rate of 6.5% per year. With more assets, a renter chooses to buy a big house with no down payment. With yet more assets, a household makes a full down payment. Given the down payment and housing size, the equilibrium interest rate goes down as household assets increase.

5.2. Default, Selling, and Repayment Decision

In this subsection, I analyze the financial characteristics of households that choose to default, sell, or repay using model-generated data. I used the no-modification model to generate the data.[19] On receiving a moving shock, a household has two options: selling or default. Without a moving shock, a household has three options: repayment, selling, or default. First, I analyze the household's binary choice conditional on a moving shock. The dependent variable is defined by

$$I_1 = \left\{ \begin{array}{ll} 1 & \text{if a mortgage holder defaults after a moving shock} \\ 0 & \text{if a mortgage holder sells a house after a moving shock} \end{array} \right\}.$$

Table 3 shows the results. Households are more likely to default as housing value and financial assets decline, and as the remaining mortgage debt principal increases. Unemployed households are more likely to default. The signs of the coefficients are the same using the logit and probit models.

https://doi.org/10.1017/S1365100515000395 Published online by Cambridge University Press

TABLE 3. Default propensity

	No modification	
	(1) Logit	(2) Probit
Housing value	−16.45*	−9.18*
	(0.33)	(0.17)
Financial asset	−7.51*	−4.07*
	(0.22)	(0.11)
Debt principal	19.33*	10.80*
	(0.41)	(0.21)
I {Unemployed}	4.16*	2.45*
	(0.20)	(0.10)
Constant	1.97*	0.99*
	(0.12)	(0.06)
Pseudo R^2	0.8419	0.843

Notes: Dependent variable is one if a household chooses default conditional on a moving shock, and zero if a household chooses selling conditional on a moving shock. Standard errors in parentheses.
* p-value < 0.01.

When a household does not face a moving shock, it can choose one of three alternatives. Hence, I used multinomial logit and probit models to analyze the propensity for selling and default. The dependent variable is defined by

$$I_2 = \left\{ \begin{array}{ll} 1 & \text{if a mortgage holder repays} \\ 2 & \text{if a mortgage holder sells a house} \\ 3 & \text{if a mortgage holder defaults} \end{array} \right\}.$$

I take the base category in the dependent variable as repayment (or $I_2 = 1$). Table 4 shows that households are more likely to sell their houses, rather than repay, as housing value and debt principal increase, and as financial assets decrease. Households are more likely to default, rather than repay, as housing value and financial assets decrease, and as the debt principal increases. Unemployed households are more likely to sell their houses or default on their mortgages. The signs of all coefficients are the same under the multinomial logit and probit models.

In the model, negative housing equity is not a sufficient condition for default. Figure 4 shows the distribution of loan-to-value among home sellers with and without a moving shock. Conditional on a moving shock, some households decide to sell their houses, rather than default, even when they have negative housing equity (or LTV > 1). If the default penalty is large enough, a household might decide to sell its house and repay a mortgage debt that is larger than the housing value. However, a household that sells its house without receiving a moving shock always has positive housing equity (or LTV < 1).

TABLE 4. Default, selling, and repayment propensity

	No modification	
	(1) Multinomial logit	(2) Multinomial probit
	Selling	
Housing value	2.97*	2.39*
	(0.02)	(0.01)
Financial asset	−6.99*	−5.47*
	(0.06)	(0.04)
Debt principal	3.58*	2.80*
	(0.04)	(0.03)
I {Unemployed}	4.69*	3.74*
	(0.05)	(0.04)
Constant	−9.54*	−7.70*
	(0.07)	(0.05)
	Default	
Housing value	−9.62*	−6.19*
	(0.23)	(0.15)
Financial asset	−10.11*	−6.90*
	(0.20)	(0.13)
Debt principal	11.45*	7.73*
	(0.20)	(0.13)
I {Unemployed}	6.17*	4.35*
	(0.11)	(0.08)
Constant	−7.63*	−5.44*
	(0.19)	(0.13)
Pseudo R^2	0.6564	

Notes: Without a moving shock, a mortgage holder chooses repayment, selling, or default. The base category in the dependent variable, I_2, is repayment. Standard error in parentheses. * p-value < 0.01.

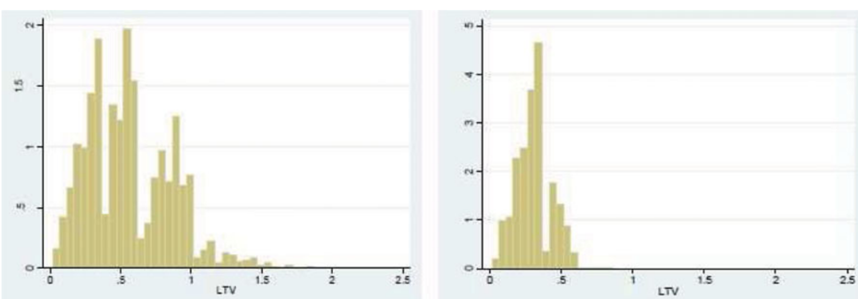

FIGURE 4. Seller's loan-to-value ratio distribution. Left, conditional on a moving shock; right, without a moving shock.

https://doi.org/10.1017/S1365100515000395 Published online by Cambridge University Press

TABLE 5. Home entry propensity

	No modification	
	(1) Logit	(2) Probit
Unit housing value	−7.69*	−3.65*
	(0.06)	(0.02)
Financial asset	4.23*	2.13*
	(0.04)	(0.02)
Income	5.70*	2.70*
	(0.06)	(0.03)
Constant	7.06*	3.09*
	(0.06)	(0.02)
Pseudo R^2	0.88	0.87

Notes: Dependent variable is one if a household chooses to enter an owner-occupied house, and zero if a household chooses to stay as a renter. Standard error in parentheses.
* p-value < 0.01.

TABLE 6. Financial characteristics of households that decide to enter an owner-occupied house

	No mod	$\alpha = 0.265$	$\alpha = 0$
$\frac{\text{Average(home-entering households' consumption)}}{\text{Average(economywide consumption)}}$	0.97	0.99	1.00
$\frac{\text{Average(home-entering households' financial assets)}}{\text{Average(economywide financial assets)}}$	1.46	1.44	1.40
$\frac{\text{Average(home-entering households' unit house price)}}{\text{Average(economywide unit house price)}}$	0.76	0.77	0.84

5.3. Home Entry Decision

I now analyze the financial characteristics of renters who choose either to enter an owner-occupied house or not to enter. As I did in the preceding subsection, I used the no-modification model to generate the data.[20] I define an indicator function that is one if a renter decides to buy an owner-occupied house, and zero if she stays as a renter:

$$I_3 = \left\{ \begin{array}{ll} 1 & \text{if a renter enters into an owner-occupied house} \\ 0 & \text{if a renter does not enter into an owner-occupied house} \end{array} \right\}.$$

Table 5 shows the results. Households facing low house prices are more likely to buy houses. Similarly unsurprisingly, as financial assets and income increase, households are also more likely to buy houses. The signs of coefficients are the same under the logit and probit models.[21]

Table 6 shows how the cost of modification affects the financial characteristics of households that decide to enter owner-occupied houses. As the cost of

https://doi.org/10.1017/S1365100515000395 Published online by Cambridge University Press

modification decreases, households can buys house with lower mortgage loan rates. Hence, increasingly low-asset households can buy houses, even when they face a relatively high housing price shock. Because the mortgage financing cost declines as the cost of modification decreases, home buyers' average consumption increases.[22]

5.4. Financial Characteristics of Modified and Unmodified Households

I now compare the financial characteristics of households that receive a loan modification with those of households that do not receive a loan modification. Only financially troubled households default on their mortgages in a modification economy. Once a household defaults on its mortgages, a mortgage servicer decides whether to provide a loan modification or not, depending on its expected future cash inflow. Thus, some households are rescued with a modification whereas others default on their mortgages.

Table 7 shows the financial characteristics of households that either are marginal defaulters or get a loan modification under the no-modification ($\alpha = 0.53$), costly ($\alpha = 0.265$), and costless ($\alpha = 0$) modification economies. Regardless of the modification cost, households that default on their mortgages (or do not receive a loan modification) have lower income and financial assets than households that receive a loan modification. Because households with low income and savings are more likely to default even after mortgage terms are modified, because of the persistence of the income process, mortgage servicers let them default rather than provide a modification. In turn, households that default consume and save less than households that receive loan modifications.

Households that have large mortgages are less likely to receive loan modifications, conditional on financial characteristics, such as income, financial assets, and housing value.[23] Because the cost of modification is proportional to the loan principal, households with large loans are less likely to receive loan modifications. However, if we consider the unconditional average of mortgage loans, households that receive loan modifications tend to have larger mortgages than those that fail to get modifications. This result comes from the heterogeneity in households' financial status in the steady state.

As the cost of modification decreases (or modification becomes easier), only the more financially troubled households default on their mortgages. Under a costless modification economy, a large number of households are rescued from defaults. Only households that are particularly financially vulnerable default in the end. Hence, regardless of whether default is induced by a moving shock, defaulting households' income, asset position, consumption, and saving under a costless modification economy are lower than those under a costly modification economy. Furthermore, when a household receives a loan modification, its periodic mortgage burden decreases by around 5–10% of their current income (or $\frac{x-\tilde{x}}{e}$).

TABLE 7. Financial characteristics of households that receive and do not receive a loan modification

	Default after a moving shock			Modification after a moving shock		
	No mod	$\alpha = 0.265$	$\alpha = 0$	No mod	$\alpha = 0.265$	$\alpha = 0$
$\frac{\text{Average(modified/defaulted households' income)}}{\text{Average(economywide income)}}$	0.95	0.88	0.59	N/A	1.31	1.02
$\frac{\text{Average(modified/defaulted households' financial assets)}}{\text{Average(economywide financial assets)}}$	1.01	0.96	0.68	N/A	1.36	1.15
$\frac{\text{Average(modified/defaulted households' outstanding loans)}}{\text{Average(economywide financial assets)}}$	1.40	1.48	1.01	N/A	1.51	1.90
$\frac{\text{Average(amount of reduced periodic burden)}}{\text{Average(modified households' income)}}$	N/A	N/A	N/A	N/A	0.05	0.10
$\frac{\text{Average(modified/defaulted households' consumption)}}{\text{Average(economywide consumption)}}$	0.95	0.86	0.57	N/A	1.29	1.02
$\frac{\text{Average(modified/defaulted households' saving)}}{\text{Average(economywide saving)}}$	1.02	0.96	0.70	N/A	1.02	0.97
	Default without a moving shock			Modification without a moving shock		
	No mod	$\alpha = 0.265$	$\alpha = 0$	No mod	$\alpha = 0.265$	$\alpha = 0$
$\frac{\text{Average(modified/defaulted households' income)}}{\text{Average(economywide income)}}$	0.38	0.00	0.00	N/A	0.55	0.67
$\frac{\text{Average(modified/defaulted households' financial assets)}}{\text{Average(economywide financial assets)}}$	0.89	0.36	0.20	N/A	0.98	1.11
$\frac{\text{Average(modified/defaulted households' outstanding loans)}}{\text{Average(economywide financial assets)}}$	1.77	1.18	0.89	N/A	1.96	2.34
$\frac{\text{Average(amount of reduced periodic burden)}}{\text{Average(modified households' income)}}$	N/A	N/A	N/A	N/A	0.07	0.07
$\frac{\text{Average(modified/defaulted households' consumption)}}{\text{Average(economywide consumption)}}$	0.51	0.05	0.03	N/A	0.52	0.63
$\frac{\text{Average(modified/defaulted households' saving)}}{\text{Average(economywide saving)}}$	0.74	0.26	0.12	N/A	0.78	0.89

5.5. Analysis of a Moving Shock

To understand the role of the moving shock, I turn off the moving shock to see the effects on the steady-state statistics. Columns with $\mu = 0$ in Table 8 show the steady-state statistics without a moving shock. The annual default rate decreases from 1.47% to 0.18% in the no-modification model. Because many defaults are triggered by a moving shock, the default rate decreases as the moving shock effect is turned off. Under a costless modification model, the default rate is close to zero and the modification rate decreases from 2.86% to 1.5%, as the moving shock effect is eliminated.[24] The homeownership rate increases and the home exit rate decreases in both models. Because the major source of default risk is eliminated, the average mortgage interest rate goes down.[25]

5.6. Analysis of Foreclosure Costs

Columns with $\chi_D = 0$ in Table 8 present steady-state statistics when the mortgage servicer incurs zero foreclosure costs. In the data, a mortgage servicer loses 22% of the housing value when a household defaults and the mortgage servicer resells it. If the mortgage servicer instead recovers 100% of the defaulted housing value (or $\chi_D = 0$), the mortgage default rate goes up and the loan modification rate goes down in a "costless modification" model ($\alpha = 0$).[26] Because a mortgage servicer with low foreclosure costs recovers more when a household defaults, it has less incentive to modify loans. Such a mortgage servicer lets more households default and recovers the housing value without incurring any foreclosure-related costs. Because the mortgage servicer's expected cash inflow improves, the average interest rate goes down. Because the interest rate schedule shifts down, the average originated loan increases.

A similar interpretation can be made in a no-modification model. In such a model, a low-income and low-asset renter can access the mortgage market, taking advantage of the low mortgage interest rate schedule under $\chi_D = 0$. This leads to an increase in the default rate and the loan-to-value ratio and a decrease in the average interest rate.[27]

6. TRANSITION ANALYSIS WITH NO MODIFICATIONS

Starting in late 2007, average U.S. housing prices abruptly dropped. From April 2007 to April 2009, the Case–Shiller composite-20 index dropped by 30%. Motivated by this observed decline in housing prices, I calculate the transition path of the model following an unexpected drop in the average unit house price $\bar{p}$ of 30%. I also consider the case where average income $\bar{e}$ unexpectedly drops by 10% and where income and housing prices drop simultaneously by these amounts.

Figure 5 shows the transition paths in response to a permanent drop in average housing prices or income under a no-modification model. The solid line shows the response to the joint shocks, the dotted line is for the housing price shock only, and

TABLE 8. Changes in the moving shock and foreclosure cost

	Benchmark		No moving shock ($\mu = 0$)		No foreclosure cost ($\chi_D = 0$)	
	No mod	$\alpha = 0$	No mod	$\alpha = 0$	No mod	$\alpha = 0$
Annual default rate	1.47%	0.04%	0.18%	0.06%	1.52%	0.77%
Homeownership rate	68.7%	72.03%	77.10%	77.79%	68.9%	71.34%
$\frac{\text{Average(housing price)}}{\text{Average(annual income for homeowners)}}$	2.71	2.80	2.79	2.89	2.71	2.81
$\frac{\text{Average(rent)}}{\text{Average(annual income for renters)}}$	0.22	0.23	0.27	0.27	0.22	0.23
$\frac{\text{Average(financial assets)}}{\text{Average(annual income)}}$	1.66	1.75	1.48	1.69	1.66	1.75
Modification acceptance rate	N/A	0.98	N/A	0.96	N/A	0.75
Avg interest rate (30-year FRM real rate, 95–04)	6.20%	6.27%	5.08%	5.31%	5.90%	6.07%
$\frac{\text{Average(originated loan)}}{\text{Average(annual income)}}$	2.77	3.63	2.74	3.98	2.86	3.69
$\frac{\text{Average(annual periodic payment)}}{\text{Average(annual income)}}$	0.18	0.22	0.15	0.22	0.17	0.22
Coeff. of variation (housing value)	0.70	0.70	0.72	0.71	0.71	0.70
Loan-to-value ratio (loan originators)	0.86	0.99	0.87	0.99	0.90	0.99
Loan-to-value ratio (defaulting households)	2.1	1.12	2.65	1.05	2.1	1.38
Home exit rate	13.47%	13.73%	7.16%	8.34%	13.36%	13.70%
Fraction of housing exit driven by selling	83.73%	99.57%	95.91%	98.82%	82.75%	91.11%
Fraction of housing exit driven by moving shock	49.03%	48.04%	N/A	N/A	49.45%	47.76%
Fraction of default driven by moving shock	81.73%	0%	0%	0%	81.56%	91.74%
Annual modification rate	N/A	2.88%	N/A	1.50%	N/A	2.29%
Fraction of negative equity households	29.65%	33.62%	26.48%	34.74%	29.74%	34.31%

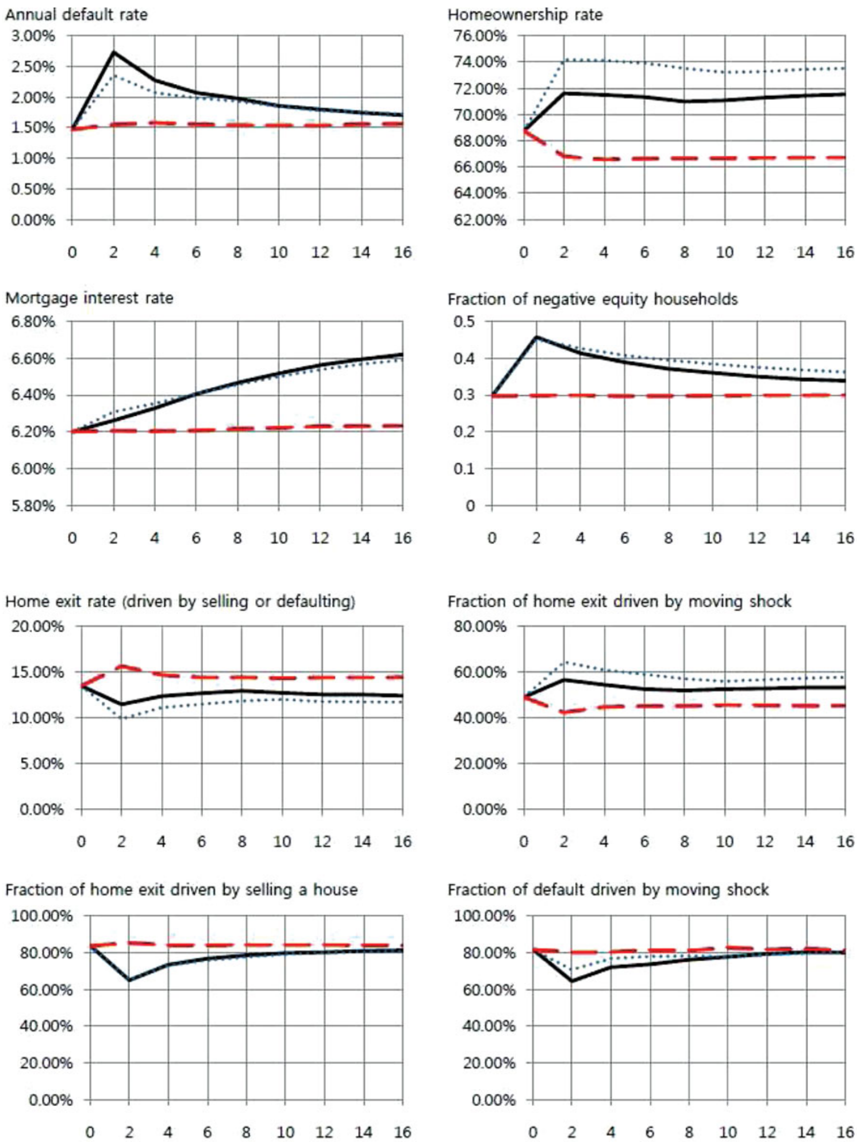

FIGURE 5. Responses to the drop in average housing price, income, and both. The horizontal axis is the year. The solid line is for both shocks together, the dotted line is for the housing price shock, and the dashed line is for the income shock.

the dashed line is for the income shock only. When agents unexpectedly face both shocks, the annual mortgage default rate increases from 1.47% to 2.73% over the next two years. With a negative housing price shock (income shock), the default rate increases from 1.47% to 2.36% (1.55%).

https://doi.org/10.1017/S1365100515000395 Published online by Cambridge University Press

The homeownership rate goes up with the negative housing price shock and goes down with the negative income shock. When both types of shocks hit simultaneously, the response in the homeownership rate is then mixed. When the average housing price decreases, the inflow from renters to homeowners goes up. Households holding few financial assets can suddenly afford to buy houses.[28] Conversely, when average income decreases, holding average housing prices constant, renters find it more difficult to buy a house. Only households with high assets can purchase houses when they face an unexpected income shock.[29] At the same time, financially constrained homeowners start to sell their houses to relieve financial tightness, which leads to a decrease in the homeownership rate. Model-generated panel data commonly show that households that decide to enter owner-occupied houses in period 2 have higher incomes and financial assets and face lower housing prices than those that decide not to buy houses, consistent with steady-state results.[30]

The average interest rate increases over time in both scenarios, especially from the house price shock. Because the mortgage contract is a long-term contract, the average interest rate does not respond quickly. When the housing price drops, low-asset renters start buying houses. This increases the default risk and therefore the average interest rate.

When there is a negative housing price shock, the fraction of negative-equity households among mortgage holders suddenly increases. It jumps from 29.6% to 45.7% when both shocks hit, and to 45.1% with only the house price shock. According to Mayer et al. (2009), the negative equity ratio of subprime loans jumped to more than 50% in California, Florida, Arizona, and Nevada in 2008. In Ohio, Michigan, and Indiana, the ratio jumped to around 30%. The income shock has almost no effect on the fraction of negative equity households.

With a negative income shock, the home exit rate suddenly increases. However, with a negative housing price shock, it suddenly decreases. When average income goes down, financially constrained households start to sell their houses voluntarily, which increases the home exit rate. When the average housing price goes down, homeowners are less likely to sell their homes voluntarily, given the reduced and possibly negative capital gain incurred. Homeowners simply enjoy the housing service utility by staying in their homes.[31] The reduction in home selling dominates the home exit triggered by defaults. Hence, the home exit rate falls on net.

The PSID shows that households have become more likely to move over time. Using two-year probabilities from the PSID, 11.8% of homeowners moved between 1997 and 1999, 12.1% between 1999 and 2001, 15.0% between 2001 and 2003, 16.9% between 2003 and 2005, and 17.1% between 2005 and 2007. But after the housing crisis, the home exit rate decreased to 12.7%. This is consistent with the model-generated numbers following just the negative housing price shock or both shocks. That is, the home exit rate decreases from 13.47% to 11.47% with both shocks and to 9.88% with only a housing price shock.

In the model, home exit occurs in four ways: (1) selling after a moving shock, (2) selling without a moving shock, (3) default after a moving shock, and (4) default without a moving shock. First, with a negative income shock, the fraction of home exits triggered by a moving shock decreases. When there is a negative income shock, households are more likely to sell their houses voluntarily. That is, the fraction of home exits driven by a moving shock goes down. In the case of a negative housing price shock, households seldom sell their houses voluntarily. However, the number of defaults triggered by a moving shock increases. Hence, the fraction of home exits driven by a moving shock rises.

Following a negative housing price shock, the fraction of home exits through (voluntary or involuntary) selling goes down, whereas home exits due to default increase. With a negative income shock, home sales rise, along with a small increase in defaults, so the fraction of exits through selling slightly increases.[32]

Last, households can default on their debt with or without a moving shock. As I showed in Table 2, many defaults are triggered by a moving shock. In Figure 5, the fraction of defaults driven by a moving shock suddenly decreases with a negative house price shock. That is, households are more likely to default on their mortgages voluntarily without being affected by a moving shock.

7. ANALYSIS OF U.S. HOUSING POLICY

In this section, I evaluate the effectiveness of government-driven mortgage modification programs in reducing the mortgage default rate. The U.S. government introduced several foreclosure prevention policies after the outbreak of the housing crisis. The Streamlined Foreclosure and Loss Avoidance Framework, the Federal Deposit Insurance Corporation (FDIC) Loan Modification Program, the Hope for Homeowners (H4H) refinancing program, the Streamlined Modification Program, and the Homeownership Preservation Policy Program were introduced between late 2007 and early 2009 [Gerardi and Li (2010); Robinson (2009)]. However, the success of those programs is still questionable.

In March 2009, the Obama administration launched a new initiative called the Home Affordable Modification Program (HAMP). The preceding model with the modification option generally represents the structure of the HAMP. In particular, it requires lending institutions to calculate the expected net present value of cash inflows with and without modification before deciding whether to provide a loan modification. Mortgage servicers follow the same logic in my model. Financial institutions are also forced to participate in the modification program both in the model and through the HAMP program, after receiving government subsidies.

Using the modification technology introduced earlier, I quantitatively analyze how government-driven modification programs reduce mortgage defaults following a 2007-style house price decline. However, according to the OCC Mortgage Metrics Report, the annual initiated foreclosure rate was around 4.2% in 2009 and 2010. As reported in the preceding section, even in a no-modification environment, the model generates smaller mortgage default rate responses to the unexpected housing price shock (an increase to 2.36%). This motivates me to extend my model

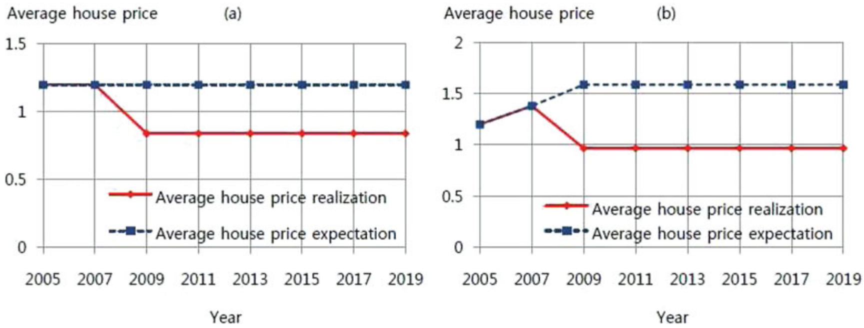

FIGURE 6. Average unit house price ($\bar{p}$) expectation and realization. The dashed line is the expected average unit house price at time 0 *ex ante*. The solid line is the realized average unit house price *ex post*. The horizontal axis is the year. The average unit house price unexpectedly drops by 30% between 2007 and 2009.

in two ways to amplify the simulated mortgage default rate from an unexpected drop in house prices.

7.1. Optimistic House Price Expectations

One possible reason that the model does not generate enough mortgage defaults from an unexpected drop in house price is the steep interest rate schedule in the benchmark model, as shown in Figure 2. Given the interest rate schedule, low-income and low-asset renters face high interest rates or cannot access the mortgage market at all. Hence, these households take out small mortgages or simply never buy houses. Therefore, they are less vulnerable to unexpected shocks.

The slope of the interest rate schedule is endogenously decreased by introducing optimistic expectations about housing prices. In the preceding section, the average unit house price unexpectedly dropped by 30%, as shown in Figure 6a. In contrast to the preceding section, I now assume that the average unit house price is expected to increase ex ante, as shown in Figure 6b. The model expectation and the realization of average unit house prices coincide through 2007. However, every agent expects the average unit house price to continue increasing until 2009. Contrary to the agents' expectation, the average unit house price drops by 30% after the end of 2007. When this optimistic house price expectation is introduced, the interest rate schedules in 2005 and 2007 shift down, reflecting reduced default risks. Hence, households can finance more mortgage debt, taking advantage of the low interest rates, and thus become more vulnerable to negative shocks. This mechanism is consistent with Foote et al. (2012) and Burnside et al. (2015), who argue that the main driver of the foreclosure crisis was optimistic beliefs about house prices.

One issue here is calibrating how people think about future house prices. For simplicity, I assume that the expected percentage changes in the average unit house price from 2005 to 2007 and from 2007 to 2009 are the same. I match the combined loan-to-value ratio of the model in 2007 to the data, which is approximately 0.95 during the housing boom [Keys et al. (2013)]. Although there are no junior liens on mortgages in the model, the combined loan-to-value ratio is the effective level of mortgage burden (or housing equity). An expectation of a 15% increase in the average unit house price per two years is required to match this moment. That is, $\bar{p}$ is the initial average unit house price, and $1.15\bar{p}$ and $1.15^2\bar{p}$ are the expected average unit house prices in 2007 and 2009, respectively.

In the Appendix, I report how households' optimistic expectation about their housing value affects mortgage interest rate schedules. My quantitative exercise shows that the mortgage default rate jumps to 2.55% (2.78%) from an unexpected drop in house price of 30% (along with a drop in income of 10%). This, as discussed, drives the improved quantitative fit of the model.

7.2. Interest Rate Subsidy

Prior to the housing market crash, low-income households could easily access the (subprime) mortgage market with low interest rates, possibly because of the Community Reinvestment Act (CRA). The CRA was designed to encourage financial institutions to extend mortgage, small business, and other types of credit to low- and moderate-income households. Though the CRA was initially introduced in 1977, it may have particularly increased the availability of mortgage financing to low-income households as the housing market boomed. Congressman Paul (2008) said in an interview with CNN that the CRA "[requires] banks to make loans to previously underserved segments of their communities, thus forcing banks to lend to people who normally would be rejected as bad credit risks." Similarly, Roberts (2008) wrote in the *Wall Street Journal* that a policy such as the CRA encourages financial institutions to lend money to low- and moderate-income families.

The loose lending standard used by financial intermediaries before the housing crisis, or alternatively the particularly low financing costs incurred by financial intermediaries precrisis, possibly because of securitization, might be the other force that triggered the foreclosure crisis [Mian and Sufi (2009), Demyanyk and Van Hemert (2011), Purnanandam (2011), Keys et al. (2013)].[33]

Collectively, this motivates me to model low-income households as having easy access to the mortgage market prior to the housing market crash. In particular, when mortgage servicers lent money to low-income households, I assume they could finance these loans at a rate of $r_f - \lambda$. λ captures the mortgage servicers' low financing cost only for low-income households. However, when mortgage servicers lent money to high-income households, the financing cost was the risk-free rate, r_f. Because the financing cost is reduced to $r_f - \lambda$ for lower-income households, the risk-neutral mortgage servicers discount future cash inflows with a rate of $r_f - \lambda$. When mortgage servicers lend money to high-income households, they discount future cash inflows at the risk-free rate. I cannot find a good calibration target for λ. As a quantitative exercise, I choose $\lambda = 0.04$, or 2% per year.

I assume that there are no more interest rate subsidies to low-income households after the housing market crash. This model of subsidies increases the mortgage default rate to 2.9% (3.1%) from an unexpected drop in house prices of 30% (along with a drop in income by 10%).[34] Details about the interest rate subsidy structure and the mortgage servicer's profit function for low-income households are presented in the Appendix.[35]

7.3. Government Mortgage Modification Program

I now analyze the effectiveness of a HAMP-like mortgage modification program introduced in 2009 in reducing the mortgage default rate. The transition timing is as follows. The steady-state economy without modification in Table 2 represents the precrisis economy, or the 2005 economy. At the end of 2005, every agent in the market suddenly expects that average house prices will go up, as shown in Figure 6b. At the same time, unemployed and low-income households receive the interest rate subsidy. At the end of 2007, contrary to general expectation, the average house price declines by 30% and the subsidy to low-income households is terminated. At the end of 2009, the loan modification program is suddenly introduced, which represents the HAMP.

An important issue here is how to set a new cost of modification after introducing the government-driven mortgage modification program. I pick the new cost of modification (α_1) after initiating the government program to match the mortgage modification rate through the HAMP, which is 0.68% in 2011. This yields a post-HAMP cost of modification of $\alpha_1 = 0.18$. That is, the cost of modification before the crisis was 53% ($\bar{\alpha} = 0.53$) of the loan principal. After the crisis, the cost of modification decreases by 35% (= 53% − 18%) of the mortgage principal. This scenario is my benchmark.[36]

I also consider a counterfactual economy where the modification program is not introduced after the housing crisis. That is, the cost of modification remains 0.53 over the transition path. By comparing the benchmark and the counterfactual scenario, I can evaluate the short- and long-run effects of HAMP-like programs in reducing the mortgage default rate.

Figure 7 shows the transition of the default and modification rates from an unexpected drop in average house prices of 30%. The solid line is the benchmark economy and the dotted line is the counterfactual economy where the modification program is absent after the housing crisis.[37] The default rate gap between the two lines after a drop in house prices represents the policy effect. The default rate decreases by 0.63%, 0.54%, and 0.46% in 2011, 2013, and 2015, respectively. The benchmark modification rate is around 0.65% over the transition, whereas the counterfactual modification rate is 0.12%, 0.06%, and 0.03% in 2011, 2013, and 2015, respectively. This suggests that the HAMP mortgage modification program made a significant dent in the mortgage default rate.

I also consider counterfactual economies where the U.S. government's subsidy is doubled (decreases by half).[38] Thus, the new cost of modification decreases

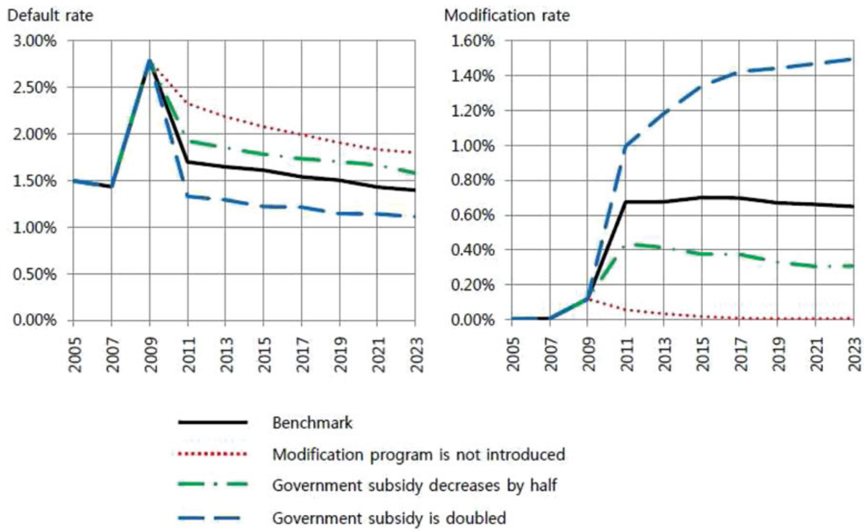

FIGURE 7. Analysis of the government-driven mortgage modification program.

further to $\alpha_1 = 0.06$ (or increases to $\alpha_1 = 0.26$). The dashed line (dot-and-dash line) shows the default/modification rate responses when the government subsidy is doubled (decreases by half). The default rate in 2011 decreases by 0.37 percentage points (increases by 0.23 percentage points), compared with the benchmark, and the modification rate in 2011 increases to 1.00% (decreases to 0.44%).

7.4. Analysis of Households′ Decisions

I continue by analyzing how government mortgage modification programs affect households' optimal decisions. Specifically, I compare the benchmark transition with the counterfactual transition where the cost of modification does not change over the transition path. In Table 9, I compare the financial characteristics of households that are marginal for receiving loan modifications with those of households that fail to receive loan modifications in 2011. As I reported in Section 5.4, households that default on mortgages in 2011 have lower income and fewer financial assets than households that receive loan modifications. Under the benchmark transition, households whose loans were modified held larger mortgages than households that defaulted on their mortgages. However, when I control for households' financial characteristics, through either probit or logit analysis, households having more debt are more likely to default as opposed to receiving modifications. The model-generated data also show that defaulting households have lower consumption and savings than households that receive loan modifications, regardless of the cost of modification. Last, once households obtain modifications, the

https://doi.org/10.1017/S1365100515000395 Published online by Cambridge University Press

TABLE 9. Financial characteristics of households over transition

	Default after a moving shock in 2011		Modification after a moving shock in 2011	
	Benchmark	Counterfactual	Benchmark	Counterfactual
$\frac{\text{Average(modified/defaulted households' income)}}{\text{Average(economywide income)}}$	0.91	0.97	1.34	N/A
$\frac{\text{Average(modified/defaulted households' financial assets)}}{\text{Average(economywide financial assets)}}$	1.02	1.09	1.54	N/A
$\frac{\text{Average(modified/defaulted households' outstanding loans)}}{\text{Average(economywide financial assets)}}$	1.72	1.75	1.95	N/A
$\frac{\text{Average(amount of reduced periodic burden)}}{\text{Average(modified households' income)}}$	N/A	N/A	0.08	N/A
$\frac{\text{Average(modified/defaulted households' consumption)}}{\text{Average(economywide consumption)}}$	0.90	0.98	1.36	N/A
$\frac{\text{Average(modified/defaulted households' saving)}}{\text{Average(economywide saving)}}$	1.04	1.10	1.23	N/A
	Default without a moving shock in 2011		Modification without a moving shock in 2011	
	Benchmark	Counterfactual	Benchmark	Counterfactual
$\frac{\text{Average(modified/xefaulted households' income)}}{\text{Average(economywide income)}}$	0.00	0.26	0.45	0.79
$\frac{\text{Average(modified/defaulted households' financial assets)}}{\text{Average(economywide financial assets)}}$	0.58	0.93	1.09	0.94
$\frac{\text{Average(modified/defaulted households' outstanding loans)}}{\text{Average(economywide financial assets)}}$	1.71	2.32	2.57	2.05
$\frac{\text{Average(amount of reduced periodic burden)}}{\text{Average(modified households' income)}}$	N/A	N/A	0.15	0.07
$\frac{\text{Average(modified/defaulted households' consumption)}}{\text{Average(economywide consumption)}}$	0.08	0.40	0.48	0.65
$\frac{\text{Average(modified/defaulted households' saving)}}{\text{Average(economywide saving)}}$	0.43	0.75	0.84	0.89

periodic mortgage burden decreases by between 8 and 15 percent of their income, depending on their financial status at the time of default.

Defaulting households' income and financial assets under the counterfactual economy are higher than those in the benchmark economy. In the benchmark economy, mortgages are more easily modified. Hence, households that default are more financially troubled. This is also why households that default under the benchmark have lower consumption and savings than in the counterfactual economy.

Under the benchmark transition, the homeownership rate is higher than under the counterfactual transition (not shown in the paper). When mortgages are easily modified, mortgage loan rate schedules shift down. Hence, households can take out loans with low interest rates. Furthermore, financially troubled households do not necessarily move out of their homes once they successfully receive loan modifications. These two forces increase the homeownership rate under the benchmark transition. When I compare financial characteristics of households that enter an owner-occupied house under the benchmark scenario with those of such households under the counterfactual scenario, households in the benchmark case can buy a house even when they have fewer financial assets, or when they face higher house price shocks.[39]

8. CONCLUSION

In this paper, I compare an economy without a loan modification option with an economy with fairly easy modification and evaluate the effect of loan modification on the foreclosure rate. Through loan modification, mortgage servicers can mitigate their losses and households can improve their financial positions without having to walk away from their homes. Because household default imposes costs on both parties, there is room for a mutually beneficial renegotiation of the loan contract. The quantitative results show that the steady-state default rate varies from almost zero with costless modification to a 1.5% default rate when modification is extremely costly.

Motivated by the observed decline in housing prices during the recent recession, I experiment with how the default rate responds to an unexpected drop in house prices by 30%. The default rate increases up to 1.5 percentage points under a no-modification model from unexpected shocks mimicking the recession. I subsequently evaluate the effectiveness of government-driven mortgage modification programs, such as the Home Affordable Modification Program, in reducing mortgage defaults. My quantitative exercise shows that the modeled mortgage modification program reduces mortgage default rates by 0.63 percentage points. I also consider several counterfactual economies where the government's subsidies to promote mortgage modifications are different. Notably, doubling expenditures on subsidies decreases mortgage defaults by an additional 0.37 percentage points. I conclude that government mortgage modification programs have likely reduced mortgage defaults by a significant amount.

NOTES

1. According to the Survey of Consumer Finances (SCF), a 30-year fixed-rate mortgage is the most prevalent type of mortgage product.

2. This means that a renter can buy a house if $(\eta + \chi_{\mathrm{B}})ph + x \leq (1 + r_{\mathrm{f}})a$. The assumption prevents zero-asset/very low asset households from buying a house. According to *How to Buy a Home With a Low Down Payment* from the Federal Citizen Information Center (FCIC), a household that wants to buy a house with a low down payment should have enough cash to cover the down payment, related expenses, and two months of periodic payments. This inequality captures such a constraint.

3. When a homeowner decides to sell a house, the household receives the housing price (net of transaction cost) and vacates the house. The household then moves to a rental house and pays periodic rent. Hence, a household is a homeowner in the first half of a period, and a renter in the last half of a period. I assume that the current utility is $u(c, h_{\mathrm{S,R}})$, which is the utility of the last half of a period.

4. If the model includes information asymmetry between borrowers and lenders, we need to change its structure significantly. For example, suppose mortgage servicers do not observe households' current income. Then mortgage servicers can infer each household's income by observing a household's optimal decisions (saving, default, home purchase, down payment, and selling decisions). That is, though there is information asymmetry between borrowers and lenders, households reveal their types (or hidden information) to mortgage servicers automatically under this model structure with a simple addition of information asymmetry. Future potentially fruitful research in this thread may include a model with information asymmetry between borrowers and lenders involving Bayesian approaches, such as Chatterjee et al. (2011) or D'Erasmo (2011).

5. $I_{\mathrm{OP}}(\Delta) = 1$ if $ph \geq x\frac{1+r_{\mathrm{m}}}{r_{\mathrm{m}}}[1 - \frac{1}{(1+r_{\mathrm{m}})^{N-n}}]$, and 0 otherwise. $I_{\mathrm{OD}}(\Delta) = 1$ if $ph < x\frac{1+r_{\mathrm{m}}}{r_{\mathrm{m}}}[1 - \frac{1}{(1+r_{\mathrm{m}})^{N-n}}]$, and 0 otherwise.

6. I follow the equilibrium pricing concept in Corbae and Quintin (2015).

7. Because mortgage servicers write off mortgage debt up to the point where the values of defaulting and not defaulting are the same, we can interpret this modification scheme as a Nash bargain where the entire bargaining power is held by the mortgage servicer. We can also extend the model to allow households to have some bargaining power, as in Yue (2010). This extension increases computation time dramatically. Within the problem this means that, given mortgage rate schedules, households solve new optimal problems. Mortgage servicers then solve the mortgage rate schedule again, given households' decisions. This procedure iterates until both sets of optimal solutions converge. Because Yue (2010) assumed a one-period bond contract, the computational work was doable. Unlike that in her paper, the model suggested here assumes a multiperiod contract, and in this situation, extending the model to give households some bargaining power makes computation prohibitively time-consuming. Hence, I simply assume that the mortgage servicer holds all bargaining power, which is also consistent with the HAMP structure, as will be explained in Section 7.

8. Young households understand that, though they may receive "modification benefits" as homeowners, they need to pay the tax when they are old. In the model, young households that inhabit owner-occupied houses are suffering from a drop in house prices. Because the goal of the modification program is to rescue such financially vulnerable young households, the tax burden is delayed until these households become old.

9. In computation, I use a very small positive number for the unemployed households' income, $e_{\mathrm{u}} = 10^{-4}$. If a renter does not have enough money to pay the periodic rent, I assume that (s)he can stay in a rental house for free in that period, consuming e_{u} and saving zero.

10. These data were constructed by Robert Shimer. For additional details, please see Shimer (2012) and his Web page, http://sites.google.com/site/robertshimer/research/flows.

11. Because one model period is two years, the average duration of unemployment is calculated by $2/(1 - \pi_{\mathrm{u,u}})(= 2(1 - \pi_{\mathrm{u,u}}) + 4\pi_{\mathrm{u,u}}(1 - \pi_{\mathrm{u,u}}) + 6\pi_{\mathrm{u,u}}^2(1 - \pi_{\mathrm{u,u}}) + ...)$. Given the calibrated value of $1 - \pi_{\mathrm{u,u}}$ as 0.5997, the average duration of unemployment is 3.34 years, which is much longer than the actual unemployment spell. According to the Current Population Reports from the Census, the mean (median) unemployment spell duration per unemployed worker is 1.5 (1.8) months between

2004 and 2007. Hence, the unemployment shock in the model seems to be much severer than the real world shock. However, the model does not include households' expenditure shocks, such as medical expenditure, divorce, or child shocks, as explained in Livshits et al. (2007). The unemployment shock implicitly contains those unmodeled expenditure shocks. There is also a technical reason that I need such a harsh unemployment shock. I initially calculated the model without including an unemployment shock. In that case, defaults are driven almost exclusively by moving shocks, not by income shocks, which I think is not a natural result.

12. Hatchondo et al. (2014) and Campbell and Cocco (2015) calibrate a process for ε, which is correlated with the persistent component of income. In those papers, housing size is uniform, which necessitates a correlation between housing price and income. In this paper, because the household chooses housing size endogenously, I can relax this assumption.

13. When $\alpha \geq 0.53$, given the parameters in Table 1, the steady state modification rate is zero.

14. When $\alpha = 0$, the lump-sum tax paid by old households is given by $\tau = 0.075$ as 12.5%($= \frac{\tau}{e_O} = \frac{0.075}{0.6}$) of the old households' income.

15. According to Guler (2015), the average down payment was 20% in 2002–2006 and 25% in 1992–1995, whereas Haughwout et al. (2008) find that the median combined initial loan-to-value ratio of subprime loans was 0.8 in 2001 and 2002, 0.84 in 2003, 0.85 in 2004, 0.87 in 2005, 0.9 in 2006, and 0.87 in 2007. Keys et al. (2013) report that both the average loan-to-value ratio and the average combined loan-to-value ratio were 0.85 in 2001. Demyanyk and Van Hemert (2011) find that the combined loan-to-value ratio of the first lien subprime loans was 0.794 in 2001, 0.801 in 2002, 0.82 in 2003, 0.836 in 2004, 0.849 in 2005, 0.859 in 2006, and 0.828 in 2007.

16. Bhutta et al. (2010) report that the median borrower does not strategically default until the loan-to-value ratio is 1.62, when defaults induced by job losses and income shocks are distinguished from those induced purely by negative equity. Because they used data that only cover Arizona, California, Florida, and Nevada between 2006 and 2009, the data moment is not exactly comparable to the model-generated statistics. Also, I report the average loan-to-value ratio of defaulted households, without distinguishing defaults driven by job losses and income shocks from those driven purely by negative housing equity. Hence, it is hard to compare the data and the model-generated statistics directly. However, it remains a rough benchmark to compare the data and model moment.

17. The PSID shows that home exit rates by homeowners were 12.10% between 1999 and 2001 and 15% between 2001 and 2003.

18. When $\alpha = 0.265$, the lump-sum tax paid by old households is $\tau = 0.0082$, or 1.4%($= \frac{\tau}{e_O} = \frac{0.0082}{0.6}$) of the old households' income.

19. I also carried out the same econometric exercise using data generated by costly and costless modification economies. The qualitative results are the same.

20. The qualitative results are the same when we use the costly and costless modification models.

21. When I compare the average consumption of households that decide to buy an owner-occupied house with that of those staying in a rental house, the former is higher than the latter. This is because a home buyer has relatively high income and financial assets, despite the need for a down payment.

22. Average income of home entrants is almost the same when the cost of modification changes.

23. This comes from the probit analysis by using the model-generated data.

24. The lump-sum tax τ is 0.075 in a costless modification economy with $\mu = 0$. The only difference between the second and the fourth columns is the moving shock parameter.

25. I also considered the case where the cost of modification is 0.265. It turns out that all statistical moments land between those of the no-modification and costless modification economies.

26. The lump-sum tax τ is 0.075 in a costless modification economy with $\chi_D = 0$. The only difference between the second and sixth columns is the foreclosure cost parameter.

27. I also considered the steady-state economy with $\alpha = 0.265$. All moments are again in the middle between the no-modification and costless modification economies.

28. Because the average house price has fallen, the average consumption of period-2 home buyers increases by 0.4%, compared with the home buyers' consumption under the steady state.

29. Because of a drop in average income, the average consumption of home buyers decreases by 7% compared with the home buyer's average consumption before the income shock.

30. This comes from the probit analysis using model-generated panel data, as carried out in Section 5.3.

31. Model-generated data shows that only households with serious financial troubles choose to sell their homes. After an unexpected drop in house prices, the average income and financial assets of home sellers in period 2 decrease by 0.2% and 8%, respectively, compared with sellers before the house price shock.

32. When house prices fall, home sellers recover less after selling their homes. Hence, their consumption in period 2 decreases by 14%, compared with their consumption before the housing price shock. After the unexpected income shock, consumption of households that sell their homes decreases by 1%, compared with that before the income shock.

33. In particular, Mian and Sufi (2009) suggest that income and mortgage credit growth were negatively correlated between 2002 and 2005, because of the securitization boom.

34. Widespread origination of unconventional mortgages, such as adjusted-rate mortgages, interest-only mortgages, and jumbo loans, might have been an important factor in the foreclosure crisis between 2007 and 2009. Because I model only fixed-rate mortgages, the mortgage default rate responses from an unexpected drop in house prices might be understated.

35. Because the model assumes exogenous house price processes, it cannot account for any possible feedback mechanism between house prices and lending standards. For example, loose lending standards lead to an increase in new home buyers, which fuels a run-up in house prices. This, in turn, again makes it easier for low-income households to finance a mortgage (U.S. Financial Crisis Inquiry Commission 2011). With such a mechanism in the model, the default rate responses from an exogenous shock would likely be amplified again.

36. In the previous version of this paper, I chose the new cost of modification (α_1) to match actual U.S. government spending on the housing program in 2011. According to the U.S. Department of the Treasury, the U.S. government spent approximately 1.9 billion dollars on HAMP and similar housing programs in 2011. Based on this amount, I chose the new cost of modification to match the ratio of government subsidies to household income in the HAMP program, which is around 0.1. In this scenario, the new cost of modification is 0.48.

37. The lump-sum tax τ changes over the transition path by covering the cost of modification.

38. In the model, the government's total subsidy is defined by $\int_{\Delta \in \text{Modified households}} (\bar{\alpha} - \alpha_1) A(\Delta)\Psi(\Delta)$.

39. The mortgage modification program also affects the home exit margin, usually through the default, rather than the selling, margin.

40. When I calculated this transition path, I used five unit house price grid points, p_1, p_2, p_3, p_4, p_5. After an unexpected drop in house prices, the unit house price grids declined to $0.7p_1, 0.7p_2, 0.7p_3, 0.7p_4, 0.7p_5$. In my original model, the unit rental price also declined by 30%. In this exercise, I model that the unit rental price is always θ_i over the transition path when a household faces the ith unit house price grid point. That is, the average unit rental price does not decline even after housing prices fall. Only owner-occupied housing prices decline by 30%, not rental rates.

41. The solid line in Figure C.1 is the same as the dotted line in Figure 5.

42. With the relatively higher rental price in this experiment, the default rate, average mortgage interest rate, and home exit rate will all decrease by more, and the homeownership rate will increase by more, than in the scenario with constant average rental prices.

REFERENCES

Adelino, Manuel, Kristopher Gerardi, and Paul S. Willen (2013) Why don't lenders renegotiate more home mortgages? Redefaults, self-cures and securitization. *Journal of Monetary Economics* 60(7), 835–853.

Agarwal, Sumit, Gene Amromin, Itzhak Ben-David, Souphala Chomsisengphet, and Douglas D. Evanoff (2011) The role of securitization in mortgage renegotiation. *Journal of Financial Economics* 102(3), 559–578.

Arslan, Yavuz, Bulent Guler, and Temel Taskin (2015) Joint dynamics of house prices and foreclosures. *Journal of Money, Credit and Banking* 47(S1), 133–169.

Bhutta, Neil, Jane Dokko, and Hui Shan (2010) The Depth of Negative Equity and Mortgage Default Decisions. Finance and economics discussion series, 2010-35, Board of Governors of the Federal Reserve System.

Burnside, Craig, Martin Eichenbaum, and Sergio Rebelo (2015) Understanding Booms and Busts in Housing Markets. Working paper, Kellogg School of Management.

Campbell, John Y. and João F. Cocco (2015) A model of mortgage default. *Journal of Finance* 70(4), 1495–1554.

Chatterjee, Satyajit, Dean Corbae, Makoto Nakajima, and Jose-Victor Rios-Rull (2007) A quantitative theory of unsecured consumer credit with risk of default. *Econometrica* 75(6), 1525–1589.

Chatterjee, Satyajit, Dean Corbae, and Jose-Victor Rios-Rull (2011) A Theory of Credit Scoring and Competitive Pricing of Default Risk. Working paper, Federal Reserve Bank of Philadelphia.

Chatterjee, Satyajit and Burcu Eyigungor (2009) Foreclosures and House Price Dynamics: A Quantitative Analysis of the Mortgage Crisis and the Foreclosure Prevention Policy. Working paper 09-22, Federal Reserve Bank of Philadelphia.

Chatterjee, Satyajit and Burcu Eyigungor (2015) A quantitative analysis of the US housing and mortgage markets and the foreclosure crisis. *Review of Economic Dynamics* 18(2), 165–184.

Corbae, Dean and Erwan Quintin (2015) Leverage and the foreclosure crisis. *Journal of Political Economy* 123(1), 1–65.

Davis, Morris A. and Stijn Van Nieuwerburgh (in press) Housing, finance, and the macroeconomy. In Giles Duranton, J. Vernon Henderson, and William C. Strange (eds.), *Handbook of Regional and Urban Economics*, pp. 753–811. Oxford, UK: Elsevier B. V.

Demyanyk, Yuliya and Otto Van Hemert (2011) Understanding the subprime mortgage crisis. *Review of Financial Studies* 24(6), 1848–1880.

D'Erasmo, Pablo (2011) Government Reputation and Debt Repayment in Emerging Economies. Working paper, Federal Reserve Bank of Philadelphia.

Elul, Ronel (2011) Securitization and Mortgage Default. Working paper 09-21/R, Federal Reserve Bank of Philadelphia.

Foote, Christopher, Kristopher Gerardi, Lorenz Goette, and Paul S. Willen (2009) Reducing foreclosures: No easy answers. In Daron Acemoglu, Kenneth Rogoff, and Michael Woodford (eds.), *NBER Macroeconomics Annual 24*, pp. 89–138. Chicago: University of Chicago Press.

Foote, Christopher, Kristopher Gerardi, and Paul S. Willen (2012) Why Did So Many People Make So Many Ex Post Bad Decisions? The Causes of the Foreclosure Crisis. Public policy discussion paper 12-2, Federal Reserve Bank of Boston.

Gerardi, Kristopher and Wenli Li (2010) Mortgage foreclosure prevention efforts. *Federal Reserve Bank of Atlanta Economic Review* 95(2), 1–13.

Gruber, Joseph and Robert Martin (2003) Precautionary Savings and the Wealth Distribution with Illiquid Durables. Board of Governors of the Federal Reserve System international finance discussion paper 773.

Guler, Bulent (2015) Innovations in information technology and the mortgage market. *Review of Economic Dynamics* 18(3), 456–483.

Harris, Milton and Bengt Holmstrom (1982) A theory of wage dynamics. *Review of Economic Studies* 49(3), 315–333.

Hatchondo, Juan Carlos, Leonardo Martinez, and Juan M. Sanchez (2014) Mortgage Defaults and Prudential Regulations in a Standard Incomplete Markets Model. Federal Reserve Bank of St. Louis working paper 2011-019C.

Haughwout, Andrew, Richard Peach, and Joseph Tracy (2008) Juvenile delinquent mortgages: Bad credit or bad economy? *Journal of Urban Economics* 64(2), 246–257.

Haughwout, Andrew, Ebiere Okah, and Joseph Tracy (2010) Second Chances: Subprime Mortgage Modification and Re-Default. Federal Reserve Bank of New York staff report 417.

Jeske, Karsten, Dirk Krueger, and Kurt Mitman (2013) Housing, mortgage bailout guarantees and the macro economy. *Journal of Monetary Economic* 60(8), 917–935.

Jiang, Wei, Ashlyn Aiko Nelson, and Edward Vytlacil (2014) Securitization and loan performance: Ex ante and ex post relations in the mortgage market. *Review of Financial Studies* 27(2), 454–483.

Keys, Benjamin J., Tomasz Piskorski, Amit Seru, and Vikrant Vig (2013) Mortgage financing in the housing boom and bust. In Edward L. Glaeser and Todd Sinai (eds.), *Housing and the Financial Crisis*, pp. 143–204. Chicago and London: National Bureau of Economic Research.

Levitin, Adam J. (2009) Helping homeowners: Modification of mortgage in bankruptcy. *Harvard Law and Policy Review* 3(2), 1–9.

Livshits, Igor, James MacGee, and Michèle Tertilt (2007) Consumer bankruptcy: A fresh start. *American Economic Review* 97(1), 402–418.

Mayer, Christopher, Edward Morrison, Tomasz Piskorski, and Arpit Gupta (2014) Mortgage modification and strategic default: Evidence from a legal settlement with countrywide. *American Economic Review* 104(9), 2830–2857.

Mayer, Christopher, Karen Pence, and Shane M. Sherlund (2009) The rise in mortgage defaults. *Journal of Economic Perspectives* 23(1), 27–50.

Mian, Atif and Amir Sufi (2009) The consequences of mortgage credit expansion: Evidence from the U.S. mortgage default crisis. *Quarterly Journal of Economics* 124(4), 1449–1496.

Paul, Ron (September 23, 2008) Commentary: Bailouts will lead to rough economic ride. *CNN*.

Pennington-Cross, Anthony (2006) The value of foreclosed property. *Journal of Real Estate Research* 28(2), 193–214.

Piskorski, Tomasz, Amit Seru, and Vikrant Vig (2010) Securitization and distressed loan renegotiation: Evidence from the subprime mortgage crisis. *Journal of Financial Economics* 97(3), 369–397.

Posner, Eric A. and Luigi Zingales (2009) A loan modification approach to the housing crisis. *American Law and Economics Review* 11(2), 575–607.

Purnanandam, Amiyatosh (2011) Originate-to-distribute model and the subprime mortgage crisis. *Review of Financial Studies* 24(6), 1881–1915.

Roberts, Russell (October 3, 2008) How government stoked the mania. *The Wall Street Journal*.

Robinson, Breck (2009) An overview of the home affordable modification program. *Consumer Compliance Outlook*, third quarter, pp. 2, 3, 13–17.

Shimer, Robert (2012) Reassessing the ins and outs of unemployment. *Review of Economic Dynamics* 15(2), 127–148.

Storesletten, Kjetil, Christopher I. Telmer, and Amir Yaron (2004) Consumption and risk sharing over the life cycle. *Journal of Monetary Economics* 51(3), 609–633.

White, Alan M. (2008) Rewriting contracts, wholesale: Data on voluntary mortgage modifications from 2007 and 2008 remittance reports. *Fordham Urban Law Journal* 36(3), 509–535.

Yue, Vivian Z. (2010) Sovereign default and debt renegotiation. *Journal of International Economics* 80(2), 176–187.

APPENDIX A: OPTIMISTIC HOUSE PRICE EXPECTATIONS AND UNEXPECTED SHOCKS

In this Appendix, I analyze how optimistic beliefs in future housing prices followed by an unexpected drop in house prices amplify mortgage defaults in the model, compared with the scenario without such optimistic beliefs (see Section 7.1).

https://doi.org/10.1017/S1365100515000395 Published online by Cambridge University Press

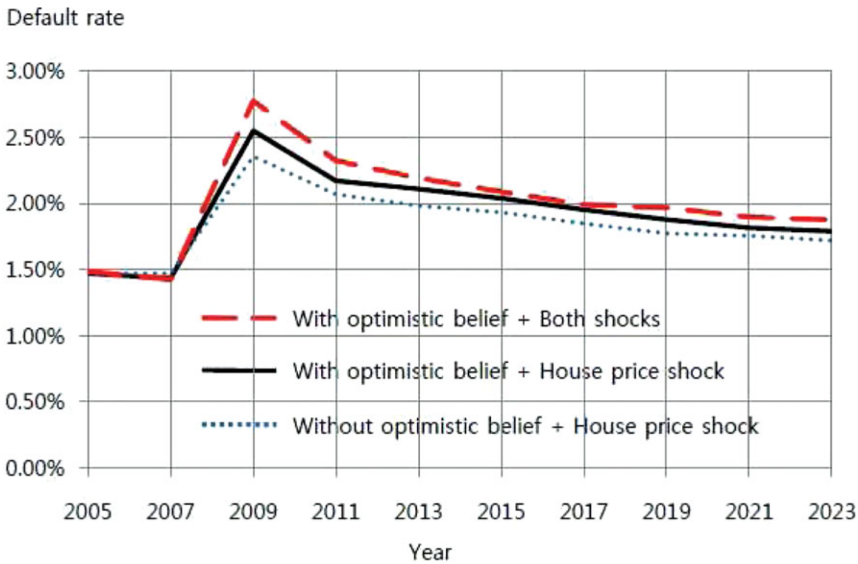

FIGURE A.1. Mortgage default rate with optimistic belief in the housing market.

Consider the following transition scenario. The initial distribution in 2005 is the same as the benchmark distribution, as shown in Table 2. At the end of 2005, every agent in the market suddenly expects that the average unit house price will increase by 15% per two years until 2009, and then stay constant forever, as shown in Figure 6b. Their prior expectation and the realization of average unit house prices coincide in 2007. However, unlike their ex ante expectation, the average house price decreases by 30% in 2009.

Figure A.1 shows the transition of the mortgage default rate under a model without loan modification from the unexpected house price shock. The initial default rate is 1.47%, as shown in Table 2. In 2007, the default rate is almost the same as the rate in 2005. At the end of 2007, the average unit house price unexpectedly decreases by 30% (along with a drop in income of 10%). This pushes the default rate from 1.43% to 2.55% (2.78%).

When agents expect that house prices will increase, they can take out larger amounts of debt at lower interest rates. Figure A.2 shows the ex ante interest rate schedules in 2005, 2007, and 2009. The interest rate schedule becomes steeper over time. In 2005, mortgage servicers expect that households will be less likely to default in the future. This follows because, as they believe the average housing price will go up in the future, households that need to move will sell their houses, rather than default on their mortgage debt. Hence, the interest rate schedule reflects their perception of a smaller default risk. In 2007, the average unit house price is still expected to increase for two more years, which leads to an interest rate schedule steeper than that of 2005 but flatter than that of 2009. Given the period-by-period interest rate schedule, the average loan-to-value ratio in 2007 is 0.95, with an average originating interest rate of 4.35%. Because households take out larger amounts of debt with low interest rates during the housing boom period, they are more vulnerable to the unexpected house price shock.

https://doi.org/10.1017/S1365100515000395 Published online by Cambridge University Press

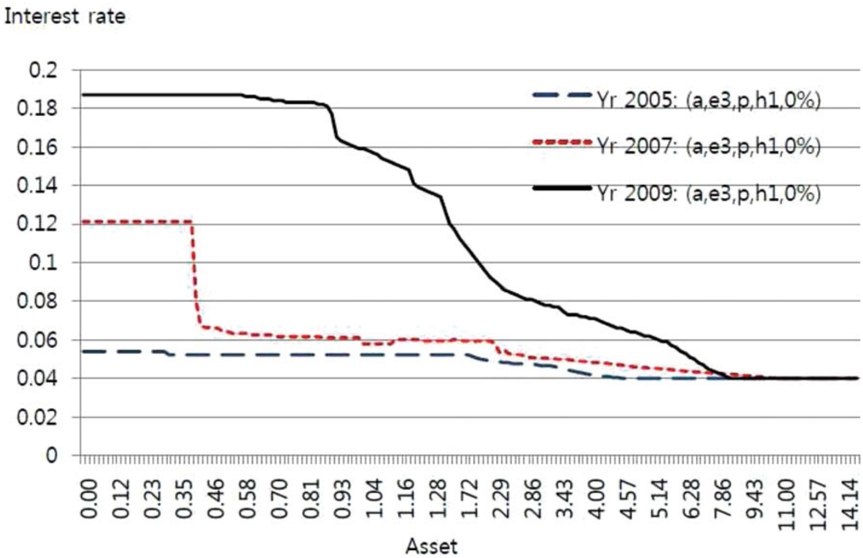

FIGURE A.2. Interest rate schedules with optimistic belief in the housing market. Each schedule is a function of assets (a) conditional on income (e), unit house price (p), house size (h), and down payment (η). $e3$ indicates the third grid among three income grids; $h1$ indicates the first grid among two house size grids; 0% indicates the down payment as a percentage of house price. The unit house price p is fixed.

APPENDIX B: INTEREST RATE SUBSIDY

In Section 7.2, I model mortgage servicers as receiving subsidies when they lend money to low-income (and unemployed) households. That is, when mortgage servicers lend money to those low-income households, their financing cost is reduced to $r_f - \lambda$. However, when mortgage servicers lend money to high-income households, they can only finance money with the risk-free rate. Thus, mortgage servicers' profit function in financing low-income households changes to the following (under the no-modification option):

$$\Pi^0 (a, e_L, p, h, \eta) = -(1-\eta)\, ph + x(a, e_L, p, h, \eta) + \frac{E\Pi_L(a', e', p', h, 1, x, r_m)}{1 + r_f - \lambda},$$

where $\Pi_L(\Delta)$ is defined by

$$\begin{aligned}\Pi_L(\Delta) &= (1-\rho_O)\,\mu I_{MS}(\Delta)\left\{x\frac{1+r_m}{r_m}\left[1-\frac{1}{(1+r_m)^{N-n}}\right]\right\}\\ &+ (1-\rho_O)\,\mu IMD(\Delta)\{(1-\chi_D)\,ph\}\\ &+ (1-\rho_O)(1-\mu)\, I_P(\Delta)\left\{x + \frac{E\Pi(\Delta')}{1+r_f-\lambda}\right\}\end{aligned}$$

https://doi.org/10.1017/S1365100515000395 Published online by Cambridge University Press

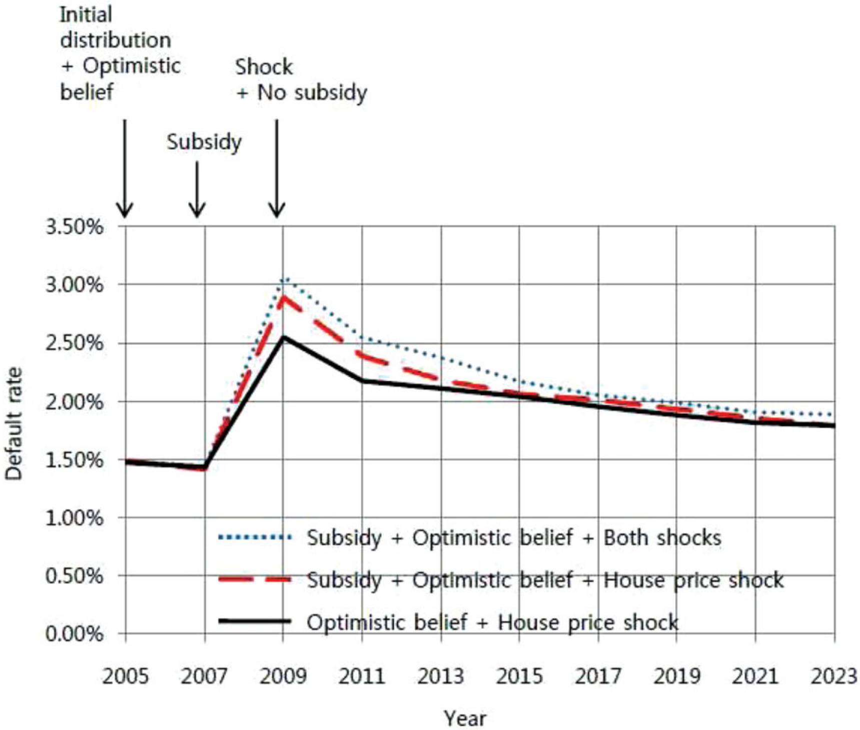

FIGURE B.1. Mortgage default rate with interest rate subsidy.

$$
\begin{aligned}
&+ (1-\rho_{\mathrm{O}})(1-\mu)\, I_{\mathrm{S}}(\Delta)\left\{x\frac{1+r_{\mathrm{m}}}{r_{\mathrm{m}}}\left[1-\frac{1}{(1+r_{\mathrm{m}})^{N-n}}\right]\right\}\\
&+ (1-\rho_{\mathrm{O}})(1-\mu)\, I_{\mathrm{D}}(\Delta)\{(1-\chi_{\mathrm{D}})\, ph\}\\
&+ \rho_{\mathrm{O}} I_{\mathrm{OP}}(\Delta)\left\{x\frac{1+r_{\mathrm{m}}}{r_{\mathrm{m}}}\left[1-\frac{1}{(1+r_{\mathrm{m}})^{N-n}}\right]\right\}\\
&+ \rho_{\mathrm{O}} I_{\mathrm{OD}}(\Delta)\{(1-\chi_{\mathrm{D}})\, ph\}.
\end{aligned}
$$

Because the financing cost is reduced to $r_{\mathrm{f}} - \lambda$, the risk-neutral mortgage servicers discount future cash inflows at a rate of $r_{\mathrm{f}} - \lambda$. When mortgage servicers lend money to high-income households, they discount future cash inflows at the risk-free rate.

This forces me to assume that there is no borrowing/lending interest rate arbitrage opportunity. That is, $r_{\mathrm{m}} \geq r_{\mathrm{f}}$ for every feasible state. (Note that households' saving interest rate is the risk-free rate, r_{f}.) To satisfy the nonarbitrage opportunity, the zero-profit condition cannot always hold. More specifically, because the mortgage servicers' financing cost for low-income households is less than the risk-free rate, some households with low income and significant assets might face an interest rate of $r_{\mathrm{m}} \in [r_{\mathrm{f}} - \lambda, r_{\mathrm{f}})$ under the zero-profit condition. Those households would borrow as much as possible at a low interest rate and save money at the risk-free rate, which is higher than their borrowing rate. A no-arbitrage

https://doi.org/10.1017/S1365100515000395 Published online by Cambridge University Press

assumption rules out such cases by relaxing the zero-profit condition. Hence, I assume that the lower bound for the mortgage interest rate is the risk-free rate, r_{f}. This allows mortgage servicers to possibly make a positive expected profit in some states:

$$\Pi^0\left(a, e_j, p, h, \eta\right)\left\{\begin{array}{ll} \geq 0 & \text{if } e_j = e_u \text{ or } e_{\mathrm{L}} \\ = 0 & \text{if } e_j = e_{\mathrm{H}} \end{array}\right\}. \qquad \textbf{(B.1)}$$

To see the effect of the interest rate subsidy on defaults from the unexpected house price shock, I study the transition in the no-modification model. In 2005, the initial distribution is given by the benchmark distribution. At the end of 2005, every agent in the market suddenly expects that the average house price will go up by 15% per two years until 2009, as shown in Figure 6b. At the same time, low-income households receive the interest rate subsidy, $\lambda = 0.04$. At the end of 2007, the average unit house price unexpectedly decreases by 30%. Also, the interest rate subsidy to low-income households ceases, $\lambda = 0$.

The dashed line in Figure B.1 shows the transition of the default rate under this interest rate subsidy model. I allow both unemployed and low-income households to receive interest rate subsidies. In this case, the default rate increases by 1.5 percentage points from an unexpected drop in house prices of 30%. When agents face an unexpected drop in house prices of 30% along with a drop in income of 10%, the default rate increases by 1.6 percentage points (dotted line). This serves as an improvement to the quantitative fit of the model.

APPENDIX C: ANALYSIS OF HOUSE AND RENTAL PRICES

In the model, I assume that the unit rental price is proportional to the unit housing price. The unit rental price is given by $\theta(p) = \frac{p r_{\mathrm{f}}}{1+r_{\mathrm{f}}}$. Hence, when I calculate the transition path from an unexpected drop in house prices of 30%, the unit rental price also decreases proportionally. However, under an incomplete market structure with nonconvexities, the unit rental price is not necessarily proportional to the unit house price. In this section, I consider a transition path where the average housing price unexpectedly decreases by 30%, whereas the average unit rental price does not change over the transition.[40]

Figure C.1 compares responses to the drop in average housing prices of 30% under changing (solid line) and constant (dotted line) average rental prices.[41] The default rate for the experimental transition (dotted line) is lower than that under the benchmark transition. This happens because holding rental prices constants implies an increase in their relative price when housing prices fall. When the relative rental price goes up, the cost of staying in a rental house increases. This can be interpreted as an increase in default penalty. Because the default cost is now higher, the mortgage interest rate path is lower in this experiment than under the benchmark. Further, the increase in relative rental prices leads to an increase in homeownership rates and a decrease in home exit rates. Therefore, depending on the assumption about the relationship between owner-occupied house prices and rental prices, responses from the unexpected shocks mirroring the 2007 financial crisis are significantly different.[42]

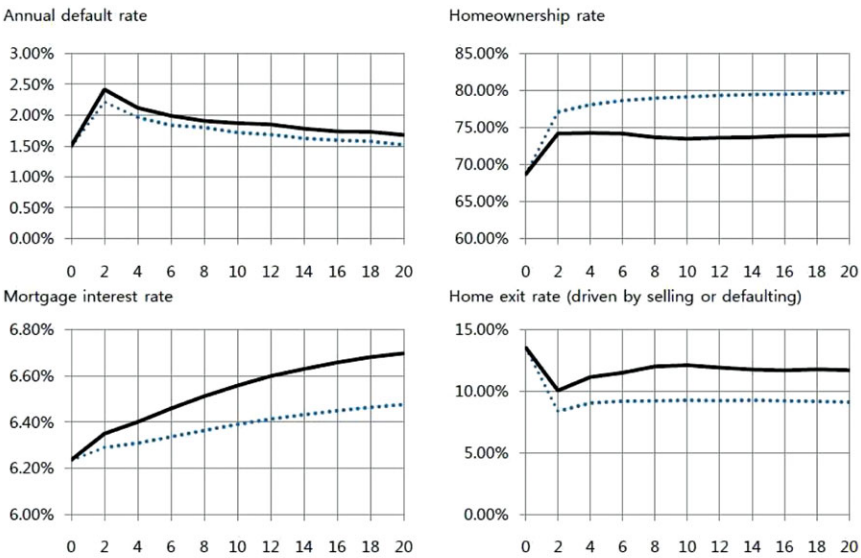

FIGURE C.1. Responses to the drop in housing prices with constant rental prices. The horizontal axis is the year. The solid line is the transition when rental rates change proportionally with housing prices. The dotted line is the transition when rental prices are constant.

APPENDIX D: PROPOSITION PROOF

Let $A(= x\frac{1+r_{\mathrm{m}}}{r_{\mathrm{m}}}[1-\frac{1}{(1+r_{\mathrm{m}})^{N-n}}])$ be the remaining debt principal in state Δ. Also, let $\upsilon(>0)$ be the household's consumption-equivalent default penalty. When a household initially chooses to default after a moving shock, the mortgage servicer decides whether to provide a loan modification. If the mortgage servicer does not modify the loan and lets the household default, the net cash benefit for both parties is

	Household	Servicer
Default	$-\upsilon$	$-A+(1-\chi_{\mathrm{D}})ph$

.

Because the household does not repay its remaining debt A when defaulting, the net benefit of the household is the default penalty, $-\upsilon$. A mortgage servicer loses the household's debt but recovers the house value net of foreclosure costs.

Let ι be the principal reduction from a modification. That is, $A-\iota=\tilde{x}_2(\Delta)\frac{1+r_{\mathrm{m}}}{r_{\mathrm{m}}}[1-\frac{1}{(1+r_{\mathrm{m}})^{N-n}}]$. If the mortgage servicer modifies the loan, the net cash benefit of both parties is

	Household	Servicer
Modification and sell	$-A+\iota+(1-\chi_{\mathrm{S}})ph$	$-\iota$

.

https://doi.org/10.1017/S1365100515000395 Published online by Cambridge University Press

After modifying the loan terms, a household repays the modified amount of debt $A - \iota$, receives the sale price of the house net of transaction costs, and then becomes a renter. The mortgage servicer recovers the original debt but loses the value of the debt reduction.

Because the mortgage servicer reduces the debt principal up to the point where the values of defaulting and not defaulting are equal, the amount of debt reduction ι is determined by

$$-\upsilon = -A + \iota + (1 - \chi_{\mathrm{S}})\, ph.$$

Because υ is the consumption-equivalent utility value, the left- and right-hand sides are comparable. Thus, we have

$$-\iota = -A + \upsilon + (1 - \chi_{\mathrm{S}})\, ph > -A + (1 - \chi_{\mathrm{D}})\, ph.$$

The inequality comes from the assumption that $\chi_{\mathrm{D}} > \chi_{\mathrm{S}}$ and $\upsilon > 0$. Using the original notation, $\tilde{x}_2(\Delta)\frac{1+r_{\mathrm{m}}}{r_{\mathrm{m}}}[1 - \frac{1}{(1+r_{\mathrm{m}})^{N-n}}] = A - \iota$, we have $\max\{(1 - \chi_{\mathrm{D}})ph,\ A - \iota\} = A - \iota$. Hence, a mortgage servicer always provides a loan modification.

APPENDIX E: COMPUTATIONAL METHOD

E.1. STEADY STATE OF NO-MODIFICATION MODEL

1. There are three income grid points, $e \in \{e_1,\ e_2,\ e_3\}$, where $e_3 > e_2 > e_1$. There are five unit house price grid points, $p \in \{p_1,\ p_2,\ p_3,\ p_4,\ p_5\}$, where $p_5 > p_4 > p_3 > p_2 > p_1$. I use a 200-point asset grid. There are 60 equally spaced asset grid points between 0 and e_3, 30 equally spaced asset grid points between e_3 and $h_{\mathrm{L}} p_5$, 40 equally spaced asset grid points between $h_{\mathrm{L}} p_5$ and $3.5 h_{\mathrm{L}} p_5$, and another 70 equally spaced asset grid points from $3.5 h_{\mathrm{L}} p_5$ to the point where asset choice decisions do not bind. (When I refined the grid with additional points, the steady-state statistics did not change.)

2. Solve the old household's problem $V^{\mathrm{O}}(a)$ using value function iteration.

3. Guess a mortgage loan interest rate schedule, $r_{\mathrm{m}}(a, e, p, h, \eta) = r_{\mathrm{f}}$.

4. Guess the renter's value function, $V_{\mathrm{R}}^{\mathrm{Y}}(a, e, p) = 0$.

5. Given $V^{\mathrm{O}}(a)$ and $V_{\mathrm{R}}^{\mathrm{Y}}(a, e, p)$, solve the defaulter's value function, $V_{\mathrm{D}}^{\mathrm{Y}}(a, e, p)$.

6. Given $V^{\mathrm{O}}(a)$ and $V_{\mathrm{R}}^{\mathrm{Y}}(a, e, p)$, solve the value functions for a homeowner without mortgage debt, $V_{\mathrm{F}}^{\mathrm{Y}}(a, e, p, h)$, $V_{\mathrm{FK}}^{\mathrm{Y}}(a, e, p, h)$, $V_{\mathrm{FS}}^{\mathrm{Y}}(a, e, p, h)$.

7. Given $V^{\mathrm{O}}(a)$, $V_{\mathrm{R}}^{\mathrm{Y}}(a, e, p)$, $V_{\mathrm{F}}^{\mathrm{Y}}(a, e, p, h)$, $V_{\mathrm{FK}}^{\mathrm{Y}}(a, e, p, h)$, and $V_{\mathrm{FS}}^{\mathrm{Y}}(a, e, p, h)$, solve the value functions for a homeowner with $(N-1)$-aged mortgage debt, $V_{\mathrm{H}}^{\mathrm{Y}}(a, e, p, h, N-1, x, r_{\mathrm{m}})$, $V_{\mathrm{HP}}^{\mathrm{Y}}(a, e, p, h, N-1, x, r_{\mathrm{m}})$, and $V_{\mathrm{HS}}^{\mathrm{Y}}(a, e, p, h, N-1, x, r_{\mathrm{m}})$.

8. Given $V_{\mathrm{H}}^{\mathrm{Y}}(a, e, p, h, N - 1, x, r_{\mathrm{m}})$, $V_{\mathrm{HP}}^{\mathrm{Y}}(a, e, p, h, N - 1, x, r_{\mathrm{m}})$, and $V_{\mathrm{HS}}^{\mathrm{Y}}(a, e, p, h, N - 1, x, r_{\mathrm{m}})$, solve the life-cycle problem. That is, solve the value functions for a homeowner with $(N-2)$-aged mortgage debt. Then, using those value functions, solve the value functions for a homeowner with $(N-3)$-aged mortgage debt, and so on.

9. Given $V_{\mathrm{H}}^{\mathrm{Y}}(a, e, p, h, 1, x, r_{\mathrm{m}})$, $V_{\mathrm{HP}}^{\mathrm{Y}}(a, e, p, h, 1, x, r_{\mathrm{m}})$, $V_{\mathrm{HS}}^{\mathrm{Y}}(a, e, p, h, 1, x, r_{\mathrm{m}})$, $V_{\mathrm{D}}^{\mathrm{Y}}(a, e, p)$, and $V^{\mathrm{O}}(a)$, solve the renter's value function $V_{\mathrm{R}}^{\mathrm{Y}}(a, e, p)$. Then update the renter's value function and go back to step 4. If every value function converges, go to the next step.

https://doi.org/10.1017/S1365100515000395 Published online by Cambridge University Press

10. Calculate the mortgage servicers' profit function, $\Pi^0(a, e, p, h, \eta)$, using the life-cycle method. If the equilibrium profit is $\Pi^0(a, e, p, h, \eta) < 0$, increase the mortgage loan interest rate slightly, $r_m(a, e, p, h, \eta) = r_m(a, e, p, h, \eta) + \varepsilon$. I chose $\varepsilon = 0.2\%$. Then go back to step 3. If the equilibrium profit is non-negative for every feasible solution, go to the next step.

11. Calculate the stationary distribution.

E.2. TRANSITION DYNAMICS WITH OPTIMISTIC HOUSE PRICE EXPECTATION

1. Let $t = 0$ be the initial period. Every agent expects that the average unit house price will go up for two consecutive periods, $t = 1$ and 2, and then be stable from $t = 3$, as shown in Figure 6b.

2. Solve the optimal policy, interest rate, and value functions at $t = 2$ as I did in the preceding subsection. Let $V^{t=2}(\cdot)$ be the value function at $t = 2$.

3. Given value functions at $t = 2$, solve the value functions and interest rate schedules at $t = 1$. That is, for every value function at $t = 1$, $V^{t=1}$, solve the problem in this way:

$$V^{t=1}(\Delta) = \max u(c, h, \cdot) + \beta E V^{t=2}\left(\Delta'\right).$$

4. Given value functions at $t = 1$, solve the value functions and interest rate schedules at $t = 0$ as I did in the preceding step.

5. Given the initial distribution, every household's decision is ruled by the optimal policy at $t = 0$ and the value functions at $t = 1$, along with the interest rate schedules at $t = 0$. That is, the transition is ruled by the following dynamics:

$$V^{t=0}(\Delta) = \max u(c, h, \cdot) + \beta E V^{t=1}\left(\Delta'\right),$$

where Δ is the initially given distribution (or state).

6. At the end of period 1 (or at the start of period 2), the average unit house price unexpectedly drops. Calculate the new optimal policy, interest rate, and value functions with the new average house price level, as I did in the preceding section.

7. Given the distribution in step 5, every agent's decision follows the optimal policy calculated in step 6, rather than the policy calculated in step 3.

(Because the modification option is absent in this transition, we do not need to consider the tax over the transition path.)