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ARTICLES

MONEY TAXES, MARKET SEGMENTATION, AND SUNSPOT EQUILIBRIA

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This paper investigates how volatile the general price level can be in an equilibrium where all uncertainty is extrinsic. The government operates a lump-sum redistribution policy using fiat money. An approach to modeling asset market segmentation is introduced in which this tax policy determines how volatile the price level can be, which in turn determines the volatility of consumption. The paper characterizes (i) the set of general price levels consistent with the existence of competitive equilibrium and (ii) the resulting set of equilibrium allocations. The results demonstrate how redistribution policies that are fixed in nominal terms can have a destabilizing effect on an economy, and show how to evaluate the amount of volatility that a particular policy may induce.

Keywords: Segmented Markets, Taxation, Economic Volatility, Indeterminacy

1. INTRODUCTION

General equilibrium models of money often have the property that competitive equilibrium is indeterminate: There is a continuum of equilibrium values of money for a given economy, each leading to a distinct real allocation [see especially Balasko and Shell (1993)]. When extrinsic uncertainty is introduced and asset markets are imperfect, the degree of indeterminacy can expand to be equal to the number of extrinsic states of nature [see Villanacci (1993)]. Since these states simply represent the beliefs of agents in the model and have no effect on the fundamentals of the economy, it seems reasonable to think that their number can be arbitrarily large.[1] Thus, high degrees of indeterminacy are pervasive in monetary sunspots models, even when there are only small imperfections in asset markets.

Aside from the degree of indeterminacy, however, one might be interested in the *volatility* of the set of equilibria, that is, the magnitude of the potential variation of prices and consumption across states. Even when the degree of indeterminacy

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is high, the amount of volatility may be quite low.[2] This paper focuses on the volatility rather than the indeterminacy question. The model presented builds on that of Bhattacharya et al. (1998), a single-commodity model with purely extrinsic uncertainty and lump-sum taxes and transfers denominated in fiat money. The degree of nominal indeterminacy is always maximal; it is equal to the number of states of nature even when asset markets are perfect. The amount of nominal volatility, however, is shown to vary with asset market frictions and with government policy. The objective of this paper is to characterize as precisely as possible the amount of nominal volatility, and to show how the interaction of the policy with asset market frictions determines how much of this nominal volatility is translated into real volatility, that is, volatility of consumption. Consumers are assumed to be risk averse; since all uncertainty is extrinsic, this implies that Pareto-optimal allocations have no real volatility and hence any such volatility that emerges in equilibrium is "excessive." The amount of excess volatility that each policy generates is clearly an important factor in determining which policy to implement.

This paper introduces an approach to studying asset market segmentation where consumers are divided into what are termed *participation groups*. Asset trade between members of different groups is difficult; in the limiting case studied here, it is simply impossible. In this sense, asset markets are segmented, or fragmented, along group lines. One interpretation of these groups is as a model of formal and informal credit markets, where a primary group represents the formal market and excluded consumers may get together in informal market(s). It may be useful to think of a developing country where the formal sector is an urban area and the informal sector is rural,[3] but this is not the only possibility. Segmentation also occurs when different consumers trade assets at different points in time, as in the models of Grossman and Weiss (1983) and Rotemberg (1984).[4] Such segmentation has been used to generate real effects from open-market operations because, at the time of a monetary injection, only a subset of consumers are on the other side of the market to absorb the new money.[5] Participation groups can be thought of as points in time, with each consumer being restricted to trading at one of these times. Note, however, that segmentation plays a different role here than it does in models of the Grossman–Weiss–Rotemberg variety. The purpose of segmentation there is to delay the diffusion of an unexpected increase in the money supply through the economy. In the present model, there are no changes in the nominal sizes of transfers. Instead, segmentation may prevent consumers from obtaining perfect insurance against sunspot-driven variations in the real value of the transfers, an effect that is absent in the other models.

The policies studied here are redistributive in nature, not purely monetary like an open-market operation. Each consumer is either given a lump-sum transfer of fiat money or required to pay a lump-sum tax using money. Such a policy generates a perfectly inelastic supply of money (from the consumers who receive a transfer) and a perfectly inelastic demand for money (by the consumers who must pay a tax). When supply and demand are equal, therefore, many different price levels will clear the market. However, it is not the case that the price level can take on *any* value. If the price of money (the inverse of the general price level) is too high, the real tax on some consumer may be larger than her endowment, so that her tax obligation cannot be met and competitive equilibrium does not exist. Hence, requiring the existence of competitive equilibrium imposes endogenous restrictions on the price of money. A series of articles by Balasko and Shell (1981, 1986, 1993) provides analyses of the set of money prices consistent with equilibrium in static and overlapping-generations models without uncertainty. The properties of the set in these environments are now well understood. Peck (1987), for example, shows that for some economies this set need not be connected, whereas Garratt (1992) shows that whether or not the set is connected can depend on the choice of the numeraire.

When there are multiple states of nature, the general price level is a vector consisting of the value of money in each state. In this case, whether a consumer can meet her tax obligation in every state depends on how she is able to transfer wealth across states, which it turn depends on the asset market conditions she faces. Using the restricted participation approach of Cass and Shell (1983), Bhattacharya et al. (1998) provide the first study of the set of equilibrium money prices in an economy with extrinsic uncertainty. They calculate this set for a family of examples, and also show that even when there are perfect asset markets that eliminate real volatility, sunspots can increase the amount of price-level volatility consistent with the existence of competitive equilibrium. In work subsequent to the present paper, Keister (1998) extends the participation-group model to infinite-horizon economies, focusing on the effects of the resulting no-arbitrage conditions on the set of equilibrium money prices. Both Bhattacharya et al. (1998) and Keister (1998) employ two-state models with consumers having identical, logarithmic preferences. The present paper characterizes the set of equilibrium money prices for the general, n-state, one-commodity case (with heterogeneous preferences). The results here provide a foundation for the log-utility approach by showing that some of the important qualitative properties of equilibrium are preserved under the more general preference structure. This paper shows that if the tax-transfer policy generates a reallocation of wealth across participation groups, then there will exist equilibria in which sunspots affect allocations (i.e., in which there is real volatility). The intensity of intergroup spot market trade determines the potential magnitude of the sunspot-induced variations. This intensity, in turn, is determined by the value of money in each state. Much of the focus of the paper, therefore, is on characterizing the set of money prices for which competitive equilibrium exists. This set is completely characterized for the two extreme cases, where markets are completely segmented and where there is no segmentation whatsoever. These sets are then used to provide bounds for the general case of partial market segmentation.

The analysis in this paper points out that efforts by the government to redistribute wealth may have a destabilizing effect on the economy. As an example, consider an economy in which asset markets are segmented in the following way: Wealthy consumers trade assets in the formal sector while poor consumers trade in an informal sector. Assume for simplicity that no asset trade is possible between sectors. Now suppose that the government implements a tax-transfer policy

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designed to achieve a more equal distribution of wealth. This will necessarily involve a net tax on the formal sector and a net transfer to the informal sector, and hence a reallocation of wealth between the two participation groups. Therefore, a quite natural form of market segmentation combined with a common type of fiscal policy leads to the existence of equilibria in which both prices and consumption are excessively volatile. Furthermore, as the government increases the magnitude of the redistribution plan, more and more volatile equilibria can be introduced. For the government to accurately assess the potential consequences of such a policy, it needs to know the relationship between the policy choice and the amount of volatility in the equilibrium value of money. In other words, it needs to know how the set of equilibrium money prices is determined. Once this set is known, it is possible to calculate the corresponding levels of volatility in consumption, and hence the potential cost of implementing the policy.

The layout of the remainder of the paper is as follows: The next section describes the details of the model, including the structure of asset market segmentation. Section 3 then discusses equilibrium conditions in detail, and Section 4 analyzes the set of money prices consistent with the existence of competitive equilibrium. Section 5 provides a characterization of the set of equilibrium allocations, and Section 6 concludes.

2. MODEL

There are two periods, labeled 0 and 1. Period 0 is for asset trading only; consumption takes place in period 1. At the beginning of period 1, one of a finite number n of extrinsic states of nature is revealed to have occurred. These states are labeled $\{s_1, \dots s_n\}$ and state s_i occurs with probability $\pi(s_i)$. Thus this is an essentially static general equilibrium model enhanced with minimal dynamics so that it can address uncertainty and volatility.

2.1. Asset Market Segmentation

The approach to modeling asset market segmentation taken here is a modification of the restricted-participation approach introduced by Cass and Shell (1983).[6] Consumers are divided into a set G of disjoint participation groups, which are indexed by g. At time 0, consumers in group g may trade assets in some set A_g with each other. In addition, all consumers may trade assets in a set A_0. Note that the basic forms of restricted market participation and incomplete financial markets are both special cases of this setup. Restricted participation occurs when there are two groups: A_2 is complete state-contingent claims, and $A_0 = A_1 = \emptyset$. The standard model of incomplete financial markets is a single group with $A_0 = A_1$ being the (incomplete) asset structure. The setup here is, in turn, a specialization of the generalized restricted-participation approach taken by Balasko et al. (1990), where the asset portfolio of consumer h is restricted to lie in some set B_h. The approach here requires that B_h be identical for members of the same group. This requirement

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captures the notion of segmentation in asset markets and places additional structure on the set of equilibria.

In this paper, attention is restricted to a special case of the asset structure, where A_g is complete state-contingent claims for every group g and $A_0 = \emptyset$. This is obviously an extreme case, but it leads to considerable simplification and seems a logical starting point for the analysis of participation groups and segmentation.

2.2. Consumers

Consumers are indexed by h and have preferences represented by utility functions of the von Neumann–Morgenstern form

$$V_h(c_h) = \sum_{i=1}^{n} \pi(s_i) u_h[c_h(s_i)],$$

where $c_h(s_i)$ is the consumption level in state s_i. The function u_h is strictly increasing, smooth, and strictly concave with $\lim_{c \to 0} u'_h(c) = \infty$. A tax of τ_h is levied on consumer h and payable only in units of currency. If τ_h is negative, it is a money transfer. The price of the consumption good in each spot market is normalized to 1; the good is chosen rather than money since money may have no value in equilibrium. The price of money in the state s_i spot market is denoted $p^m(s_i)$. Consumer h in group g has an endowment ω_h, which, because all uncertainty is extrinsic, is independent of the state of nature. She chooses state-contingent commodity purchases $\boldsymbol{c}_h^0$ to solve the problem

$$\begin{aligned} \max \quad & V_h(c_h) \\ \text{s.t.} \quad & \\ & \boldsymbol{p}_g^0 \cdot \boldsymbol{c}_h^0 = 0, \\ & c_h(s_i) = \omega_h - p^m(s_i)\tau_h + c_h^0(s_i) \quad \text{for all} \quad s_i, \\ & \boldsymbol{c}_h \in \mathbf{R}_{++}^n, \end{aligned} \tag{1}$$

where $\boldsymbol{p}_g^0$ is the vector of contingent-claims prices that prevails in the period-0 market in group g. Note that these prices may differ across groups; market segmentation prevents price differences from being arbitraged away. For later reference, note that the first-order necessary condition for $\boldsymbol{c}_h^0$ to solve this problem is that there exist some positive number θ such that

$$\frac{\pi(s_i)}{p_g^0(s_i)} u'_h[c_h(s_i)] = \theta \quad \text{for all} \quad s_i. \tag{2}$$

By defining the consumer's tax-adjusted endowment in each state,

$$\tilde{\omega}_h(s_i) = \omega_h - p^m(s_i)\tau_h,$$

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it is easy to see that the variables ω_h, $\boldsymbol{p^m}$, and τ_h only enter the consumer's demand through their effect on $\tilde{\boldsymbol{\omega}}_{\boldsymbol{h}}$. This follows from the fact that the budget constraints in (1) can be rewritten as the single constraint

$$\boldsymbol{p_g^0} \cdot \boldsymbol{c_h} = \boldsymbol{p_g^0} \cdot \tilde{\boldsymbol{\omega}}_{\boldsymbol{h}}. \tag{3}$$

Prices in the asset market are normalized so that $\sum_{i=1}^{n} \boldsymbol{p_g^0}(s_i) = 1$. The right-hand side of (3) can therefore be written as

$$w_h = \omega_h - \left[\sum_{i=1}^{n} p_g^0(s_i) p^m(s_i)\right] \tau_h. \tag{4}$$

If consumer h is alone in a group, then state-contingent commodity purchases $\boldsymbol{c_h^0}$ must be 0. Equilibrium asset prices in this group must therefore be such that the optimal choice of consumption will be equal to the tax-adjusted endowment in each state:

$$c_h(s_i) = \tilde{\omega}_h(s_i) = \omega_h - p^m(s_i)\tau_h \quad \text{for all} \quad s_i.$$

Such consumers are referred to as being *totally restricted*.

2.3. Fiscal Policy

The vector of taxes across consumers $\boldsymbol{\tau} = (\tau_h)_{h \in H}$ is called the government's *fiscal policy*.[7] Since the government will accept only money for the payment of taxes, feasibility requires that the government give out in transfers at least as much money as it demands in taxes. The set of feasible fiscal policies is therefore taken to be

$$\mathcal{F} = \left\{\boldsymbol{\tau} : \sum_{h \in H} \tau_h \leq 0\right\}.$$

As in Balasko and Shell (1993), a particular subset of $\mathcal{F}$ will prove to be of interest. A fiscal policy is said to be *balanced* if the government takes in exactly as much money as it distributes. The set of balanced fiscal policies is given by

$$\mathcal{F}_{\text{bal}} = \left\{\boldsymbol{\tau} : \sum_{h \in H} \tau_h = 0\right\}. \tag{5}$$

3. EQUILIBRIUM

At several points in the analysis, it will be helpful to examine the relationship between the sunspots economy and the corresponding certainty economy. The certainty economy corresponding to a given sunspots economy is identical in all respects except that there is only one state of nature. This implies that there is no asset market (or, equivalently, purchases in the asset market must be zero), so that each consumer is formally much like a totally restricted consumer in the sunspots economy. This implies that

$$c_h = \omega_h - p^m \tau_h$$

holds for each consumer h.

A sunspots economy is a list $(\{V_h\}, \{\omega_h\}, \{\tau_h\}, G, \boldsymbol{\pi})$. Both consumer preferences and the probability distribution over states are taken as fixed; an economy is therefore denoted by $\mathcal{E}(\boldsymbol{\omega}, \boldsymbol{\tau}, G)$. The corresponding certainty economy is denoted $\mathcal{E}_{CE}(\boldsymbol{\omega}, \tau)$. An equilibrium of the sunspots economy is a set of allocations $\{c_h\}_{h\in H}$ together with a list of prices $\{\boldsymbol{p^m}, \{\boldsymbol{p_g^0}\}_{g\in G}\}$ such that

(i) given prices $\{\boldsymbol{p^m}, \boldsymbol{p_g^0}\}$ for group g, the allocation $\boldsymbol{c_h}$ solves consumer h's optimization problem (1) for every h in g and for every g; and
(ii) markets clear; that is, we have both

$$\sum_{h\in g} c_h(s_i) = \sum_{h\in g} \omega_h - p^m(s_i) \sum_{h\in g} \tau_h \quad \text{for all} \quad s_i \quad \text{and for all} \quad g \tag{6}$$

and

$$\sum_{h\in H} c_h(s_i) = \sum_{h\in H} \omega_h \quad \text{for all} \quad s_i. \tag{7}$$

The set of equilibrium money prices of an economy $\mathcal{E}(\boldsymbol{\omega}, \boldsymbol{\tau}, G)$ is denoted $P^m(\boldsymbol{\omega}, \boldsymbol{\tau}, G)$, or simply P^m. The set of equilibrium allocations for consumer h is denoted $C^h(\boldsymbol{\omega}, \boldsymbol{\tau}, G)$, or simply C^h.

The set of equilibrium money prices is of interest because the price of money determines the real value of the reallocation of wealth across groups generated by the fiscal policy. This, in turn, determines the potential magnitude of sunspot-induced variations in consumption across states. A money price vector $\boldsymbol{p^m}$ is consistent with equilibrium if and only if at those prices there is an equilibrium in the asset market of each participation group. One method of finding the set P^m, then, is to determine the set of money prices leading to the existence of an asset market equilibrium for each group g, denoted P_g^m. The set of equilibrium money prices for the entire economy is the intersection of these sets; that is,

$$P^m = \bigcap_{g\in G} P_g^m.$$

3.1. Equilibrium of the Certainty Economy

First consider the certainty economy, where asset market segmentation plays no role. Recall that in the certainty economy we have

$$c_h = \omega_h - p^m \tau_h,$$

so that consumption is positive if and only if either $\tau_h < 0$ or $p^m < \omega_h/\tau_h$ holds. Consumers receiving a transfer will always have an after-tax endowment in the consumption set; the focus is therefore on consumers with $\tau_h > 0$. Let T denote

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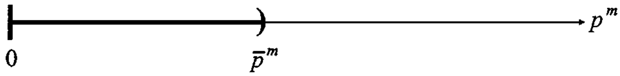

FIGURE 1. P^m_{CE}.

the set of positively taxed consumers. For $h \in T$, money prices in the set $[0, \omega_h/\tau_h)$ allow consumer h to afford a positive consumption level, so that the set of equilibrium money prices is the intersection of all of these sets,

$$P^m_{CE} = \bigcap_{h \in T} \left[0, \frac{\omega_h}{\tau_h}\right),$$

where the CE subscript indicates that this pertains to the certainty economy. Clearly, this is equivalent to defining

$$\bar{p}^m = \min_{h \in T} \frac{\omega_h}{\tau_h}$$

and stating

$$P^m_{CE} = [0, \bar{p}^m)$$

(see Figure 1). Hence, in the certainty economy, the set of equilibrium money prices is a half-open interval with zero as its lowest element.[8]

This nominal indeterminacy translates directly into real indeterminacy, with

$$C^h_{CE} = \left\{ \begin{array}{ccc} \left(\omega_h - \bar{p}^m \tau_h, \omega_h\right] & & \tau_h > 0 \\ \omega_h & \text{if} & \tau_h = 0 \\ \left[\omega_h, \omega_h - \bar{p}^m \tau_h\right) & & \tau_h < 0 \end{array} \right\}.$$

3.2. Equilibrium of the Sunspots Economy

Matters are more complex in the sunspots economy. First, notice that a totally restricted consumer in the sunspots economy is very much like a consumer in the certainty economy. For such a consumer,

$$c_h(s_i) = \omega_h - p^m(s_i)\tau_h$$

must hold in every state in equilibrium. If she is taxed, she can remain in her consumption set if and only if $p^m(s_i)$ is less than ω_h/τ_h in every state. Thus, for any single-member group with positive taxation, we have

$$P^m_g = \left[0, \frac{\omega_h}{\tau_h}\right)^n;$$

that is, the set of possible money prices is the n-fold Cartesian product of the set of numbers less than the consumer's endowment tax ratio.

For consumers who have real trading opportunities, the problem is more involved. Even if a consumer has a negative after-tax endowment in some states, she may still be able to afford positive consumption in every state, depending on asset market prices. The assumptions made on preferences and endowments ensure that equilibrium contingent-commodity prices are always positive and finite. Therefore, a consumer can afford some bundle in her consumption set if and only if her income in the period-0 asset market is positive. From equation (4), therefore, consumer h with $\tau_h > 0$ requires[9]

$$\frac{\omega_h}{\tau_h} > \sum_{i=1}^{n} p_g^0(s_i)\, p^m(s_i). \tag{8}$$

Of course, prices $\boldsymbol{p_g^0}$ in the contingent claims market may depend on $\boldsymbol{p^m}$, and so, the right-hand side of (8) need not be linear in $\boldsymbol{p^m}$. Determining the exact nature of this relationship is the task of Section 4.

3.3. Bonafide Fiscal Policies

For some feasible fiscal policies, the price of money in any equilibrium must be zero in every state. This implies that the economy is essentially nonmonetary; the pre-tax and after-tax endowments are identical. Our interest, therefore, is in fiscal policies that permit equilibria where money has positive value. Balasko and Shell (1993) term such policies *bonafide*, since only with them can the government expect in "good faith" that its fiscal policy will have real effects.

In the certainty economy, Balasko and Shell (1993) show that a fiscal policy is bonafide if and only if it is balanced in the sense of equation (5). In the sunspots economy, there are two possible definitions of bonafidelity, which will be called *weakly* bonafide and *strictly* bonafide.

DEFINITION 1. *The fiscal policy $\boldsymbol{\tau} \in \mathcal{F}$ is weakly bonafide if there is an equilibrium of the economy $\mathcal{E}(\boldsymbol{\omega}, \boldsymbol{\tau}, G)$ with $p^m(s_i) > 0$ for some s_i. The fiscal policy is strictly bonafide if there is an equilibrium with $p^m(s_i) > 0$ for all s_i.*

The following two lemmas demonstrate that in the present model these two definitions coincide. They also demonstrate that a fiscal policy is bonafide (in both senses) if and only if it is balanced, so that the Balasko–Shell result extends to the sunspots economy.

LEMMA 1. *If a fiscal policy is weakly bonafide, then it is balanced.*

Proof. Summing the group market-clearing equation (6) across groups and using the aggregate market-clearing equation (7) yields

$$p^m(s_i) \sum_{h \in H} \tau_h = 0 \quad \text{for all} \quad s_i.$$

Therefore if $p^m(s_i)$ is positive in any state, the fiscal policy must be balanced. ■

LEMMA 2. *If a fiscal policy is balanced, then it is strictly bonafide.*

Proof. Define

$$a = \frac{1}{2}\min_{h\in T}\frac{\omega_h}{\tau_h} > 0$$

and consider $p^m(s_i) = a$ for all s_i. Since τ is balanced, the after-tax endowment

$$\tilde{\omega}_h(s_i) = \omega_h - a\tau_h \quad \text{for all} \quad s_i; \quad \text{for all} \quad h$$

is a feasible allocation and is, in fact, Pareto optimal. Together with the supporting prices $\boldsymbol{p}_g^0 = \boldsymbol{\pi}$ for all g,[10] these constitute an equilibrium. ■

Combining these two lemmas provides an exact characterization of the set of bonafide fiscal policies.

PROPOSITION 1. *A fiscal policy is weakly bonafide if and only if it is strictly bonafide. Such policies are thus referred to as simply as* bonafide. *Furthermore, a policy is bonafide if and only if it is balanced.*

Bearing this result in mind, it is interesting to look back at the consumer's single budget constraint given by equation (3). Each participation group looks like a separate complete-markets economy except in two respects. The first is the obvious remark that the money price vector is common to all groups because all consumers trade in the same spot market in period 1. More importantly, the tax-transfer policy need not be balanced within each group. As a result, each group resembles a complete-markets economy in which the fiscal policy can be bonafide without being balanced. This is the essential contribution of asset market segmentation to the model, and a feature that distinguishes the present approach from the incomplete-markets approach.[11]

4. THE SET OF EQUILIBRIUM MONEY PRICES

This section describes the set of money prices consistent with the existence of competitive equilibrium for a given economy. The task is broken into cases, depending on the segmentation structure. The first two cases studied are the polar ones where markets are completely segmented, so that no asset trade takes place, and where there is no segmentation, so that asset markets are perfect. The labels *CS* and *NS* are used to denote these types of economies. The results from these two cases are then shown to provide bounds for the general case of partial market segmentation.

4.1. Complete Segmentation

Once the state of nature is revealed, the sunspots economy with complete segmentation is identical to the certainty economy. The result of this is that a vector of money prices $\boldsymbol{p}^m$ is consistent with equilibrium in the sunspots economy if and only

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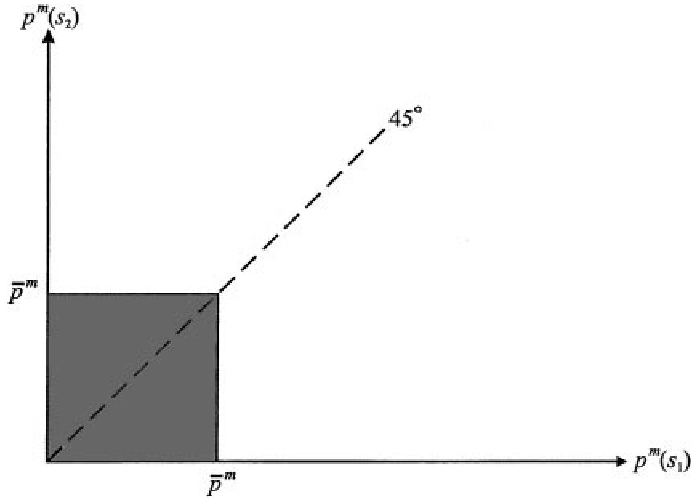

FIGURE 2. P^m_{CS}.

if each element $p^m(s_i)$ is consistent with equilibrium of the certainty economy; hence we have

$$P^m_{CS} = \left[P^m_{CE}\right]^n = [0, \bar{p}^m)^n.$$

Figure 2 depicts this set for the case $n = 2$.

This nominal volatility also translates directly into real volatility, with

$$C^h_{CS} = \left[C^h_{CE}\right]^n.$$

4.2. No Segmentation

Not surprisingly, the situation is substantially different when all consumers are in the same participation group. It is well known that in this case an equilibrium allocation must be Pareto optimal,[12] which implies that it must be state independent. However, this does not imply that the price of money must be constant across states. To the contrary, Bhattacharya et al. (1998) provide an example in which full market participation leads to an equilibrium with a money price vector that is outside P^m_{CS} and is more volatile than any element of P^m_{CS}.

What matters when there is no segmentation is not the price of money in each individual state, but rather the expected value of the price of money $E[\boldsymbol{p^m}] = \sum_{i=1}^n \pi(s_i) p^m(s_i)$. This statement is formalized in the following proposition, where P^m_{NS} denotes the set of equilibrium money prices of the sunspots model with no segmentation in asset markets.

PROPOSITION 2.

$$P^m_{NS} = \left\{\boldsymbol{p^m} \in \mathbf{R}^n_+ : E[\boldsymbol{p^m}] \in P^m_{CE}\right\}.$$

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Proof. With no segmentation, the equilibrium allocation must be sunspot independent, so that asset market prices are given by $\boldsymbol{p}_{\boldsymbol{g}}^{\mathbf{0}} = \boldsymbol{\pi}$. Substituting this into equation (4) yields the income of taxed consumer h,

$$w_h = \omega_h - E[\boldsymbol{p}^{\boldsymbol{m}}]\tau_h. \tag{9}$$

Since at least one bundle in the consumption set is affordable for consumer h if and only if $w_h > 0$, a consumer with a positive tax requires $E[\boldsymbol{p}^{\boldsymbol{m}}] < \omega_h/\tau_h$. Again letting $\bar{p}^m$ indicate the lowest (positive) endowment tax ratio, we have

$$P_{NS}^m = \left\{\boldsymbol{p}^{\boldsymbol{m}} \in \mathbf{R}_+^n : E[\boldsymbol{p}^{\boldsymbol{m}}] < \bar{p}^m\right\},$$

which is clearly equivalent to the statement of the proposition. ■

An immediate implication of this proposition is that the set of equilibrium money prices when there is complete segmentation is a proper subset of the set when there is no segmentation. This is stated as the following corollary, and shown in Figure 3 for the two-state case.

COROLLARY 1.

$$P_{NS}^m \supset P_{CS}^m.$$

The equilibrium consumption level of each consumer is constant along a locus of money prices where $E[\boldsymbol{p}^{\boldsymbol{m}}]$ is constant, since, from equation (9), income depends on $\boldsymbol{p}^{\boldsymbol{m}}$ only through its expected value. An implication of this is that for each consumer, the set of consumption levels consistent with equilibrium is the same as in the certainty economy; that is, we have

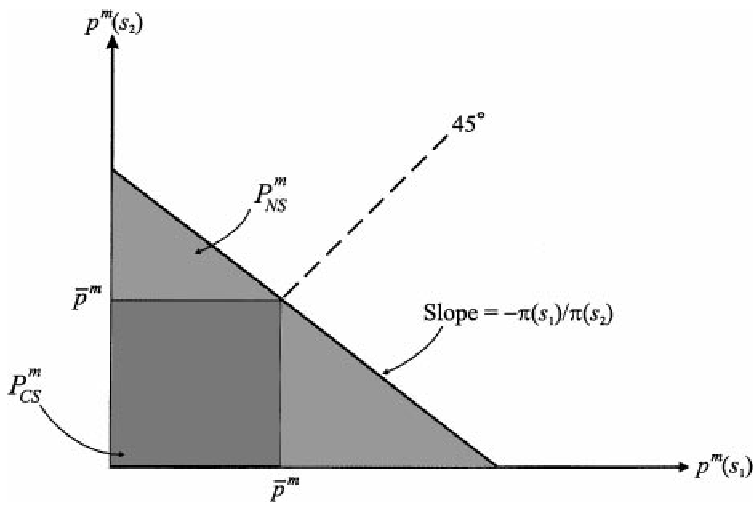

FIGURE 3. $P_{NS}^m \supset P_{CS}^m$.

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$$C_{NS}^{h} = \left\{ \boldsymbol{c_h} \in \mathbf{R}_{++}^{n} : c_h(s_i) = c_h \in C_{CE}^{h} \quad \text{for all} \quad s_i \right\}.$$

There is no real volatility, even though there is high nominal volatility relative to the complete-segmentation case. It is worth noting that Proposition 1 generalizes to the case of many commodities. However, Corollary 1 does not, since the set of equilibrium money prices need not be connected in the many-commodity model.

For the case of only two consumers and two states, the result contained in Proposition 2 can be seen in the Edgeworth box. Suppose consumer 2 is taxed one dollar and consumer 1 receives the corresponding transfer.. The aggregate endowment of the economy is fixed, but the distribution of the endowment after taxes depends on the money price vector. Since the box is square, the contract curve is the minor diagonal. The pre-tax endowment must lie on that line, since uncertainty is extrinsic. This is labelled $\boldsymbol{\omega}$ in Figure 4. The relationship between after-tax endowments and money prices is linear and bijective, so that it suffices to find the set of after-tax endowments $\tilde{\boldsymbol{\omega}}$ consistent with equilibrium. Since equilibrium prices $\boldsymbol{p^0}$ are constant [with $p^0(s_i) = \pi(s_i)$] along the contract curve, this set is the triangular region depicted in Figure 4. The linear transformation of this set into price space yields the set P_{NS}^{m} in Figure 3.

A key element in obtaining these results has been the fact that equilibrium contingent-commodity prices are equal to the respective probabilities of their states regardless of the redistribution generated by the tax-transfer policy.[13] When markets are partially segmented, this is no longer true, as the next section shows.

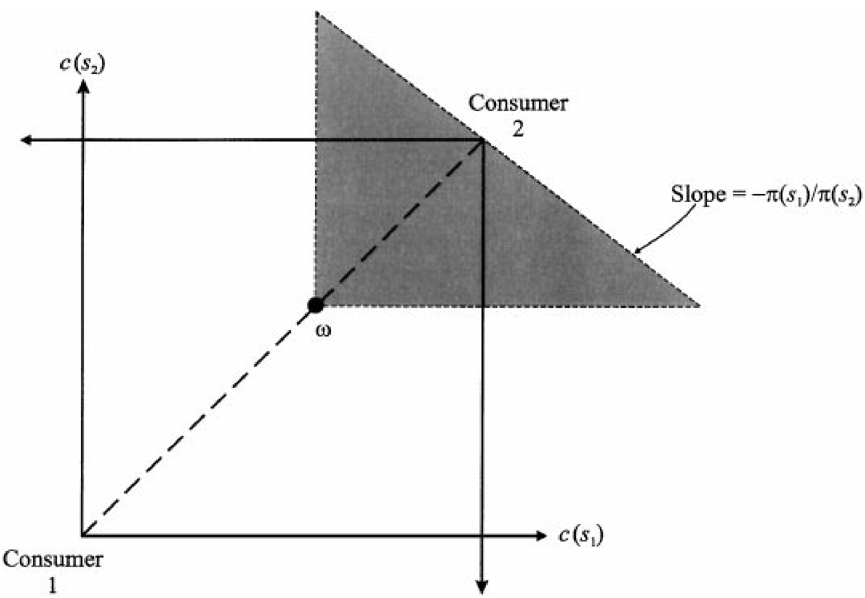

Figure 4. Set of $\tilde{\omega}$ consistent with equilibrium.

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4.3. Partial Segmentation

As mentioned earlier, the method used for determining the set P^m will be to find the set P_g^m for each group and then to take the intersection of all of these sets. For each consumer, the analysis leading to equation (8) still applies. However, the contingent commodity prices are no longer necessarily equal to the probabilities of their respective states. This is a result of the fact that the aggregate after-tax endowment of a group may vary across states, depending on the vector $\boldsymbol{p^m}$ and on the net tax paid by the group. In fact, the situation is qualitatively different, depending on the sign of this net tax; each case is analyzed separately.

Balanced taxation within a group. When the net tax on the group is zero, that is, when

$$\sum_{h \in g} \tau_h = 0$$

holds, there is no need for spot market trade between this group and any other. In effect, the group can be considered as its own economy and analyzed as in the no-segmentation case studied earlier. Define

$$\bar{p}_g^m = \min_{h \in g \cap T} \frac{\omega_h}{\tau_h}.$$

Then, as in Proposition 1, we have

$$P_g^m = \left\{ \boldsymbol{p^m} \in \mathbf{R}_+^n : E[\boldsymbol{p^m}] < \bar{p}_g^m \right\}.$$

Notice, however, that this is not the same as $\{\boldsymbol{p^m} \in \mathbf{R}_+^n : E[\boldsymbol{p^m}] \in P_{CE}^m\}$, since $\bar{p}_g^m$ may be higher than $\bar{p}^m$. Instead, the set is such that $E[\boldsymbol{p^m}]$ would be consistent with certainty equilibrium if the group g was in fact the entire economy. The set P_g^m is depicted for the two-state case in Figure 5.

Positive taxation of a group. Now consider the case in which the net tax on the group is positive, that is, where

$$\sum_{h \in g} \tau_h > 0$$

holds. First, suppose that the tax burdens are *uniform* within the group, that is, that ω_h / τ_h is the same for every member of the group. In this case, if the money price vector is such that the after-tax endowment of one consumer is negative in some state, then the aggregate after-tax endowment of the group must be negative in that state as well and there can be no asset market equilibrium. Hence, the set of money prices consistent with equilibrium under uniform tax burdens is the same as in the case of complete segmentation: $p^m(s_i)$ must be less than ω_h / τ_h in every state. When tax burdens are not uniform within the group, however, the consumer with the lowest positive endowment tax ratio will always go bankrupt when the

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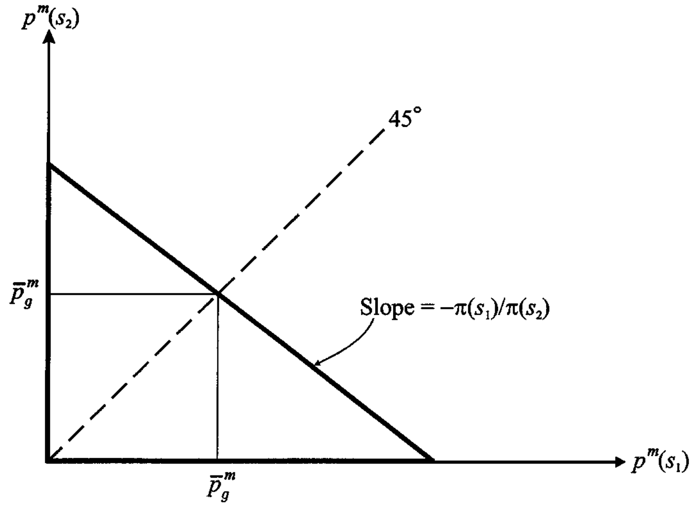

FIGURE 5. P_g^m under balanced taxation.

aggregate after-tax endowment of the group is still positive in every state [and hence $p_g^0(s_i)$ is finite in every state]. This is the more interesting case, and hence it is assumed from here on that tax burdens are nonuniform within each group.

A positive net tax implies that the consumption good is more scarce within the group in states with higher prices of money. The contingent-commodity price for these states must therefore be higher relative to the probability with which the state occurs. This well-known fact is formalized in the following lemma.

LEMMA 3. *When* $\sum_{h\in g} \tau_h > 0$ *holds, for any two states* s_i *and* s_j, *we have*

$$\frac{p_g^0(s_i)}{\pi(s_i)} \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} \frac{p_g^0(s_j)}{\pi(s_j)} \quad as \quad p^m(s_i) \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} p^m(s_j).$$

Proof. Suppose $p^m(s_i) > p^m(s_j)$. Since we have

$$\sum_{h\in g} c_h(s_i) = \sum_{h\in g} \omega_h - p^m(s_i) \sum_{h\in g} \tau_h \quad \text{for all} \quad s_i,$$

it follows that

$$\sum_{h\in g} c_h(s_i) < \sum_{h\in g} c_h(s_j)$$

holds. Therefore, for some $h \in g$, it must be the case that $c_h(s_i) < c_h(s_j)$. From equation (2), the first-order condition of consumer h's optimization problem, it immediately follows that

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$$\frac{\pi(s_i)}{p_g^0(s_i)} < \frac{\pi(s_j)}{p_g^0(s_j)}$$

must hold. The same argument applies to the remaining cases, where $p^m(s_i) < p^m(s_j)$ and where the two money prices are equal. ■

This lemma says that the price of consumption is high in deflationary states, that is, in states where the value of money is high. Notice, however, that these are exactly the states in which a taxed consumer needs to buy a large amount of consumption in order to pay his tax. Hence, when the price of money becomes high in a particular state, asset market prices move *against* a taxed consumer and he is driven bankrupt by lower money price vectors than would be the case if $\boldsymbol{p_g^0} = \boldsymbol{\pi}$ held. Because of this, the set of money price vectors shrinks relative to the balanced taxation case. An outer bound for the set P_g^m is now given by the set of money prices that would be consistent with equilibrium if the members of the group had access to "fair" insurance markets, where $\boldsymbol{p^0} = \boldsymbol{\pi}$ (this set is denoted F_g).

Regardless of asset market prices, a consumer can always meet her tax obligation when the value of money is less than $\overline{p}_g^m$ in every state. Hence, the set of money prices that would be consistent with equilibrium if members of the group were not allowed to trade assets (denoted N_g) provides an inner bound for the set P^m. These bounding results are formalized in the following proposition.

PROPOSITION 3.

$$\textit{When} \sum_{h \in g} \tau_h > 0 \textit{ holds, we have } N_g \subset P_g^m \subset F_g.$$

Proof. Take any $\boldsymbol{p^m} \in N_g$. Then,

$$p^m(s_i) < \overline{p}_g^m$$

holds for all s_i. Therefore, for any nonnegative weights $\{\theta_i\}$ that sum to unity, we have

$$\sum_{i=1}^{n} \theta_i p^m(s_i) < \overline{p}_g^m.$$

Letting $\boldsymbol{\theta} = \boldsymbol{p_g^0}$ yields

$$\sum_{i=1}^{n} p_g^0(s_i) p^m(s_i) < \overline{p}_g^m, \tag{10}$$

so that (8) holds for all consumers in the group and $\boldsymbol{p^m} \in P_g^m$. This establishes $N_g \subseteq P_g^m$. To see that the containment is in fact proper, consider the vector

$$\boldsymbol{p^m} = \begin{Bmatrix} \overline{p}_g^m \\ 0 \end{Bmatrix} \quad \text{for} \quad s_i = \begin{Bmatrix} s_1 \\ s_2, \ldots, s_n \end{Bmatrix}.$$

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This is clearly not in N_g. It is, however, in P_g^m, since $p_g^0(s_i) > 0$ for all s_i implies that $p_g^0(s_1) < 1$, and hence (10) holds.

Now, take any $\boldsymbol{p^m} \in P_g^m$, so that (10) holds. Then, from Lemma 3, we have

$$p^m(s_i) > p^m(s_j) \Rightarrow \frac{p_g^0(s_i)}{p_g^0(s_j)} > \frac{\pi(s_i)}{\pi(s_j)}.$$

Therefore, replacing $\boldsymbol{p_g^0}$ with $\boldsymbol{\pi}$ moves weight from higher values of $\boldsymbol{p^m}$ to lower values of $\boldsymbol{p^m}$, reducing the value of the sum. This implies that

$$\sum_{i=1}^{n} \pi(s_i) p^m(s_i) < \overline{p}_g^m$$

holds, so that $\boldsymbol{p^m} \in F_g$ and $P_g^m \subseteq F_g$ holds.

To show that this containment is also proper, it suffices to establish that there exists an $\overline{\varepsilon} > 0$ such that $\varepsilon > \overline{\varepsilon}$ implies

$$\boldsymbol{p_\varepsilon^m} = \left[(1-\varepsilon) \frac{1}{\pi(s_1)} \overline{p}_g^m, 0, \ldots, 0 \right] \notin P_g^m.$$

Suppose that this claim is not true. Then there exists a sequence $\varepsilon_1, \varepsilon_2, \ldots$ converging to zero such that $\boldsymbol{p_\varepsilon^m}$ is consistentwith equilibrium at every point in the sequence. For each ε, denote the equilibrium contingent claims price $p_g^0(s_1)$ closest to $\pi(s_1)$ by $p_g^0(s_1; \varepsilon)$.[14] Then, the sequence $p_g^0(s_1; \varepsilon)$ must not be bounded away from $\pi(s_1)$, or else a contradiction to (10) would occur for small enough ε. Therefore, there must exist a subsequence $\varepsilon_1', \varepsilon_2', \ldots$ along which $p_g^0(s_1; \varepsilon)$ converges to $\pi(s_1)$. Notice that this implies that $p_g^0(s_i; \varepsilon)$ converges to $\pi(s_i)$ for all s_i. Along this subsequence, then, optimality requires that each consumer's allocation be converging to state symmetry; that is,

$$\lim_{\varepsilon' \to 0} c_h(s_i) = c_h \quad \text{for every} \quad s_i, \quad \text{for every } h.$$

Summing over consumers, this implies that

$$\lim_{\varepsilon' \to 0} \sum_{h \in g} c_h(s_i) = c \quad \text{for every} \quad s_i,$$

which can be rewritten as

$$\sum_{h \in g} \omega_h - \lim_{\varepsilon' \to 0} p_{\varepsilon'}^m(s_1) \sum_{h \in g} \tau_h = \sum_{h \in g} \omega_h - \lim_{\varepsilon' \to 0} p_{\varepsilon'}^m(s_i) \sum_{h \in g} \tau_h \text{ for } s_i = s_2, \ldots, s_n.$$

Since $\sum_{h \in g} \tau_h > 0$, this implies $\overline{p}_g^m = 0$, a contradiction. ■

This relationship is shown diagrammatically for the case of two states in Figure 6.

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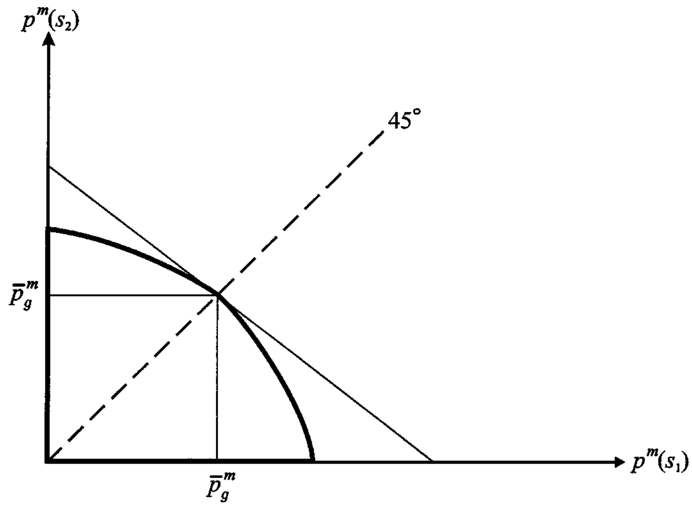

FIGURE 6. P_g^m under positive taxation.

Negative taxation of a group. The results are qualitatively reversed in the remaining case, where

$$\sum_{h\in g} \tau_h < 0$$

holds. Within the group, the consumption good is now more scarce in the states with lower prices of money. The following lemma is exactly symmetric to Lemma 3 and is stated without proof.

LEMMA 4. *When* $\sum_{h\in g} \tau_h < 0$ *holds, for any two states* s_i *and* s_j, *we have*

$$\frac{p_g^0(s_i)}{\pi(s_i)} \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} \frac{p_g^0(s_j)}{\pi(s_j)} \quad as \quad p^m(s_i) \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} p^m(s_j).$$

Consumption is now less expensive in deflationary states, where a taxed consumer must make large purchases in order to pay his tax. When the price of money becomes high in a particular state, asset market prices now move in *favor* of a taxed consumer so that he can withstand higher money price vectors without going bankrupt than would be the case if $\boldsymbol{p}_g^0 = \boldsymbol{\pi}$ held. The result of this change is that the set of money prices consistent with equilibrium for group g is enlarged so that it now contains F_g.

PROPOSITION 4.

$$\textit{When} \sum_{h\in g} \tau_h < 0 \textit{ holds, we have } N_g \subset F_g \subset P_g^m.$$

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Proof. The first relation is essentially that established in Corollary 1. For the second, take any $\boldsymbol{p^m} \in F_g$. Then,

$$\sum_{i=1}^{n} \pi(s_i) p^m(s_i) < \bar{p}_g^m$$

holds. By Lemma 4, replacing $\boldsymbol{\pi}$ with $\boldsymbol{p_g^0}$ moves weight from higher values of $\boldsymbol{p^m}$ to lower values of $\boldsymbol{p^m}$, reducing the value of the sum. Therefore,

$$\sum_{i=1}^{n} p_g^0(s_i) p^m(s_i) < \bar{p}_g^m \tag{11}$$

must also hold, which implies that $\boldsymbol{p^m} \in P_g^m$.

To see that F_g is a proper subset, consider the vector

$$\boldsymbol{p^m} = \left[\frac{1}{\pi(s_1)} \bar{p}_g^m, 0, \ldots, 0\right],$$

which is clearly not in F_g. Since $p^m(s_1) > p^m(s_n)$, the normalization of $\boldsymbol{p_g^0}$ implies that $p_g^0(s_1) < \pi(s_1)$, so that (10) holds and $\boldsymbol{p^m} \in P_g^m$. ■

This relationship is shown for the two-state case in Figure 7. An interesting item of note here is that, in this case, the set P_g^m cannot be convex. The preceding proof demonstrates that the corners of the boundary of F_g, where

$$\boldsymbol{p^m} = \frac{1}{\pi(s_i)} \bar{p}_g^m$$

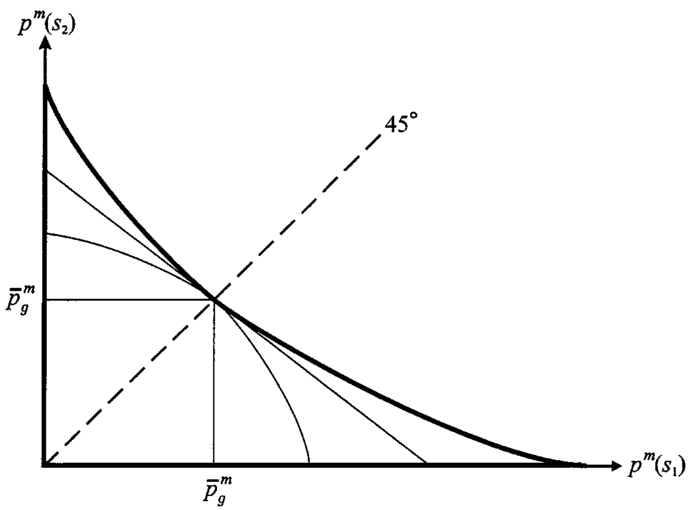

FIGURE 7. P_g^m under negative taxation.

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in the ith element and zero elsewhere, are in P_g^m. However, the unique convex combination of these that is state symmetric, $\boldsymbol{p^m} = (\bar{p}_g^m, \bar{p}_g^m, \ldots, \bar{p}_g^m)$, is not in P_g^m because it would require $\boldsymbol{p_g^0} = \boldsymbol{\pi}$ and thereby bankrupt the consumer with $\omega_h/\tau_h = \bar{p}_g^m$. This fact is stated as a corollary.

COROLLARY 2. *When* $\sum_{h \in g} \tau_h < 0$ *holds,* P_g^m *is not convex.*

5. EQUILIBRIUM ALLOCATIONS

The previous section characterized the set of money prices consistent with equilibrium; the objective of this section is to characterize the resulting set of equilibrium allocations. This is done by providing versions of the two welfare theorems that combine to exactly characterize the set of allocations generated by any feasible fiscal policy $\boldsymbol{\tau} \in \mathcal{F}$. With segmented asset markets, the standard welfare theorems do not hold and equilibrium allocations, as demonstrated above, need not be Pareto optimal. There are two properties that distinguish equilibrium allocations here. The first is that, given the total allocation to each group, the distribution of that allocation must be Pareto optimal within the group. This follows from the assumption that there is a complete asset market in which all group members can trade. The second property is that the total allocation to each group in each state must be attainable through the type of fiscal policy considered here. These two properties are formalized as follows.

DEFINITION 2. *An allocation* $\boldsymbol{c}$ *is Pareto optimal within groups (POG) if there does not exist another allocation* $\boldsymbol{c}'$ *such that*

$$\sum_{h \in g} c_h'(s_i) = \sum_{h \in g} c_h(s_i) \quad \textit{for all} \quad s_i, \quad \textit{for all} \quad g,$$

and

$$V_h(c_h') \geq V_h(c_h) \quad \textit{for all} \quad h$$

with strict inequality for some h.

This is the standard definition of Pareto optimality, but with the set of possible reallocations restricted to maintain the same aggregate consumption in each group.

DEFINITION 3. *An allocation* $\boldsymbol{c}$ *is a nonnegative proportional transfer (NNTP) from another allocation* $\boldsymbol{c}'$ *if for every group* g, *we have*

$$\sum_{h \in g} [c_h'(s_i) - c_h(s_i)] = \lambda(s_i) k_g \quad \textit{for all} \quad s_i, \tag{12}$$

for some $\boldsymbol{\lambda} \geq 0$ *that is common to all groups and some* k_g *that may vary across groups but is independent of the state of nature.*

Note that if both $\lambda(s_i)$ and $\lambda(s_j)$ are positive and their ratio is given by λ_{ij}, the NNTP conditions reduce to

$$\frac{\sum_{h\in g}[c'_h(s_i) - c_h(s_i)]}{\sum_{h\in g}[c'_h(s_j) - c_h(s_j)]} = \lambda_{ij} \quad \text{for all} \quad g.$$

NNTP characterizes the set of group-level allocations that are attainable through fiscal policy. To see this, take $\boldsymbol{c}'$ to be the endowment point ω, $\lambda(s_i)$ to be $p^m(s_i)$, and k_g to be the net tax on group g. Then equation (12) can be written as

$$\sum_{h\in g} c_h(s) = \sum_{h\in g} \omega_h - p^m(s_i) \sum_{h\in g} \tau_h \quad \text{for all} \quad s_i,$$

which is to say that the aggregate consumption of the group in each state is generated by some nonnegative money prices $\boldsymbol{p^m}$. The NNTP property simply requires that these money prices be the same for each group.

The following propositions verify that POG and NNTP do indeed jointly characterize the set of equilibrium allocations.

PROPOSITION 5. *Any equilibrium allocation of $\mathcal{E}(\omega, \boldsymbol{\tau}, G)$ is (i) Pareto optimal within groups and (ii) a nonnegative proportional transfer from ω.*

Proof. (i) Nothing more is involved that the usual proof of the first welfare theorem. Suppose $\boldsymbol{c}'$ Pareto dominates the equilibrium allocation $\boldsymbol{c}$ in some group g, and that we have

$$\sum_{h\in g} c'_h(s_i) = \sum_{h\in g} c_h(s_i) \quad \text{for all} \quad s_i. \tag{13}$$

Then, at equilibrium prices,

$$\sum_{i=1}^{n} p_g^0(s_i) c'_h(s_i) \geq \sum_{i=1}^{n} p_g^0(s_i) c_h(s_i)$$

holds for all $h \in g$, with strict inequality for some h in g. However, summing across all consumers in the group then yields

$$\sum_{i=1}^{n} p_g^0(s_i) \left[\sum_{h\in g} c'_h(s_i) - \sum_{h\in g} c_h(s_i) \right] > 0,$$

which contradicts equation (13).

(ii) For every group, we have

$$\sum_{h\in g} c_h(s_i) = \sum_{h\in g} \omega_h - p^m(s_i) \sum_{h\in g} \tau_h \quad \text{for all} \quad s_i.$$

Therefore, defining $\boldsymbol{\lambda} = \boldsymbol{p^m}$ and $k_g = \sum_{h\in g} \tau_h$ shows that equation (12) is satisfied. ■

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PROPOSITION 6. *Given* ω *and* G, *any allocation that is Pareto optimal within groups and is a nonnegative proportional transfer from* ω *is an equilibrium of* $\mathcal{E}(\omega, \boldsymbol{\tau}, G)$ *for some* $\boldsymbol{\tau} \in \mathcal{F}$.[15]

Proof. Let $\boldsymbol{c}$ denote such an allocation. Since it is POG, there exists a unique price vector $\boldsymbol{p}_g^0$ supporting it in each asset market. For each consumer and for any $\boldsymbol{p}^m > 0$, set

$$\tau_h = \frac{\sum_{i=1}^n p_g^0(s_i)[\omega_h - c_h(s_i)]}{\sum_{i=1}^n p_g^0(s_i)p^m(s_i)}, \tag{14}$$

so that the boundary of the budget set defined by equation (3) goes through $\boldsymbol{c_h}$. This is then the consumer's optimal choice. Note that $\boldsymbol{\tau}$ is balanced and hence bonafide. It remains to be shown that there exist money prices such that markets clear in each group. Summing equation (14) over consumers in group g yields

$$\sum_{h\in g} \tau_h = \frac{\sum_{i=1}^n p_g^0(s_i)\left\{\sum_{h\in g}[\omega_h - c_h(s_i)]\right\}}{\sum_{i=1}^n p_g^0(s_i)p^m(s_i)}. \tag{15}$$

For any group with $\sum_{h\in g} \tau_h = 0$, $p_g^0(s_i) > 0$ for all s_i implies that $\sum_{h\in g} c_h(s_i) = \sum_{h\in g} \omega_h$ for all s_i, so that the asset market for that group clears regardless of $\boldsymbol{p^m}$. For groups with $\sum_{h\in g} \tau_h \neq 0$, we must have

$$\sum_{h\in g}(\omega_h - c_h(s_i)) \neq 0 \quad \text{for some state} \quad s_i,$$

so that in the NNTP property, $k_g \neq 0$ and $\lambda(s_i) > 0$ for some s_i. Suppose (without loss of generality) that $\lambda(s_1) > 0$. Then we have

$$\frac{\sum_{h\in g}[\omega_h - c_h(s_i)]}{\sum_{h\in g}[\omega_h - c_h(s_1)]} = \frac{\lambda(s_i)}{\lambda(s_1)} \geq 0 \quad \text{for} \quad s_i = s_2, \ldots, s_n$$

for all such groups. Set the vector $\boldsymbol{p^m}$ such that

$$\frac{p^m(s_i)}{p^m(s_1)} = \frac{\lambda(s_i)}{\lambda(s_1)} \geq 0 \quad \text{for } s_i = s_2, \ldots, s_n.$$

Combining this with equation (15) yields

$$\sum_{h\in g} \tau_h = \frac{\sum_{i=1}^n p_g^0(s_i)\frac{p^m(s_i)}{p^m(s_1)}\left\{\sum_{h\in g}[\omega_h - c_h(s_1)]\right\}}{\sum_{i=1}^n p_g^0(s_i)p^m(s_i)},$$

which simplifies to

$$p^m(s_1)\sum_{h\in g} \tau_h = \sum_{h\in g}[\omega_h - c_h(s_1)].$$

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For any other $\lambda(s_i) > 0$, a symmetric argument gives

$$p^m(s_i) \sum_{h \in g} \tau_h = \sum_{h \in g} [\omega_h - c_h(s_i)]$$

and markets clear in every group. If, on the other hand, $\lambda(s_i) = 0$, then $\sum_{h \in g} [\omega_h - c_h(s_i)] = 0$ for every group and setting $p^m(s_i) = 0$ ensures market clearing.

Since markets clear in every group and the tax policy is balanced, there is also aggregate market clearing; that is,

$$\sum_{h \in H} c_h(s_i) = \sum_{h \in H} \omega_h \quad \text{for all} \quad s_i.$$

An equilibrium has thus been constructed. ■

It is worth pointing out that in this proof only the relative values $[p^m(s_i)]/[p^m(s_1)]$ needed to be set, so that an equilibrium with $\boldsymbol{p^m}$ normalized so that, say, $E[\boldsymbol{p^m}] = 1$ could be constructed [cf. Balasko and Shell (1993), Proposition 3.3]. These propositions provide the link between the amount of nominal volatility and the amount of real volatility. Once the set of equilibrium money prices P^m is determined, for each element $\boldsymbol{p^m} \in P^m$, it is easy to calculate the aggregate after-tax resources in each group using equation (6). Asset market equilibrium then delivers an optimal allocation of these resources across the members of the group. Conversely, starting from any POG allocation that satisfies the NNTP property, it is possible to construct a fiscal policy $\boldsymbol{\tau}$ that delivers this allocation as an equilibrium for some $\boldsymbol{p^m} \in P^m$.

6. CONCLUDING REMARKS

This paper introduces the participation-group approach to modeling asset market segmentation and provides a characterization of the set of money prices consistent with the existence of competitive equilibrium in a single-commodity general equilibrium model. It shows how fiscal policy and asset market segmentation interact to determine the amount of sunspot-induced volatility that can occur in the equilibrium price level and in equilibrium consumption. The results provide a foundation for the earlier work of Bhattacharya et al. (1998) and the subsequent work of Keister (1998) by showing that important qualitative properties derived using two states of nature and identical, logarithmic preferences are preserved in the general case.

The results indicate how redistribution policies can have a destabilizing effect on an economy. If markets are segmented in such a way that there is a nonzero net tax on some groups, then consumption by the members of these groups will depend on the value of money. Because this value can always vary with the realization of a sunspot variable, consumption is made excessively volatile. Knowledge of the set of equilibrium money prices (Section 4) and the way in which nominal volatility translates into real volatility (Section 5) allows one to analyze the level of destabilization that a particular policy may cause. This resembles the trade-off between equality and efficiency that arises in models in which the only available tax instruments are distortionary.[16] In both cases a movement away from the Pareto frontier accompanies any redistribution attempt. The most notable difference is that taxes in this model are lump sum, so that there are no distortionary effects involved. Instead, the redistribution combines with asset market segmentation to create equilibrium asset prices that are "distorted" from their perfect-market values.

An obvious generalization of this analysis that may prove interesting is the extension to models with many commodities. This would introduce the possibility of wealth effects of the type demonstrated by Peck (1987), which can affect the qualitative properties of a set of equilibrium money prices. Another extension that might prove fruitful is a two-country model, which would be capable of addressing issues related to exchange-rate volatility. In such an environment, the vector $\boldsymbol{p}^{\boldsymbol{m}}$ would include the value of each country's currency in each state of nature. The set P^m then determines not only how volatile the value of each money can be in equilibrium, but also how volatile the exchange rate can be. This is left for future research.

NOTES

1. See Cass (1989) and Mas-Colell (1992) for arguments to this effect.
2. Cass (1993) discusses situations where small asset market frictions lead to high levels of indeterminacy but a "small" set of equilibria.
3. Hoff and Stiglitz (1990) provide an overview of the problems of rural credit markets in developing countries.
4. See also Alvarez and Atkeson (1997).
5. The models of Chatterjee and Corbae (1992) and Alvarez et al. (1999) study monetary policy with "endogenously segmented" markets, where there is a fixed cost of trading that some consumers choose to pay and others do not. This points to a possible extension of the present setup in which a consumer faces different costs of participating in the different groups. As an example, one could imagine that in some situations the cost of participating in a market depends on the distance that one lives from it.
6. See also Balasko et al. (1995).
7. The terminology in this section follows that of Balasko and Shell (1993).
8. This need not be the case when there are many commodities, as shown by Peck (1987) and Garratt (1992).
9. In the infinite-horizon model of Keister (1998), this same basic condition appears and is termed the *no-bankruptcy constraint*. The primary change caused by the longer time horizon is the introduction of additional constraints imposed by arbitrage considerations.
10. That these prices support any state-symmetric allocation is a property of Von Neumann–Morgenstern utility functions easily seen from equation (2).
11. Antinolfi and Keister (1998) use the equivalence of bonafidelity and balancedness to show that even when asset markets are very incomplete, if there is no segmentation, then introducing the right set of option contracts eliminates all real volatility. With segmented markets, this result clearly would no longer hold.
12. See Cass and Shell (1983) and, for the monetary model, Bhattacharya et al. (1998).
13. See Garratt et al. (in press) for a detailed discussion of the colinearity of prices and probabilities in sunspot models.

14. There may be multiple equilibria of the contingent claims market, but Lemma 3 and the normalization of $\boldsymbol{p}_g^0$ imply that each must have $p_g^0(s_1) > \pi(s_1)$.

15. It is worth noting that any purely Pareto-optimal allocation is in fact a nonnegative proportional transfer from $\boldsymbol{\omega}$ since both are state symmetric.

16. The seminal reference is Mirrlees (1971).

REFERENCES

Alvarez, F. & A. Atkeson (1997) Money and exchange rates in the Grossman-Weiss-Rotemberg model. *Journal of Monetary Economics* 40, 619–640.

Alvarez, F., A. Atkeson & P. Kehoe (1999) Money and Interest Rates with Endogenously Segmented Markets. Staff report 260, Federal Reserve Bank of Minneapolis.

Antinolfi, G. & T. Keister (1998) Options and sunspots in a simple monetary economy. *Economic Theory* 11, 295–315.

Balasko, Y. & K. Shell (1981) The overlapping generations model, II: The case of pure exchange and money. *Journal of Economic Theory* 24, 112–142; Erratum. *Journal of Economic Theory* 25, 471.

Balasko, Y. & K. Shell (1986) Lump-sum taxes and transfers: Public debt in the overlapping generations model. In W. Heller, R. Starr & D. Starrett (eds.), *Essays in Honor of Kenneth Arrow, Vol. II: Equilibrium Analysis*, pp. 121–153. New York: Academic Press.

Balasko, Y. & K. Shell (1993) Lump-sum taxation: The static economy. In R. Becker, M. Boldrin, R. Jones, & W. Thomson (eds.), *General Equilibrium, Growth and Trade: The Legacy of Lionel McKenzie, II*, pp. 168–180. New York: Academic Press.

Balasko,Y., D. Cass & K. Shell (1995) Market participation and sunspot equilibria. *Review of Economic Studies* 62, 491–512.

Balasko,Y., D. Cass & P. Siconolfi (1990) The structure of financial equilibrium with exogenous yields: The case of restricted participation. *Journal of Mathematical Economics* 19, 195–216.

Bhattacharya, J., M. Guzman & K. Shell (1998) Price level volatility: A simple model of money taxes and sunspots. *Journal of Economic Theory* 81, 401–430.

Cass, D. (1989) Sunspots and incomplete financial markets: The leading example. In G. Feiwel (ed.), *The Economics of Imperfect Competition and Employment: Joan Robinson and Beyond*, pp. 677–693. London: Macmillan.

Cass, D. (1993) Real indeterminacy from imperfect financial markets: Two addenda. In R. Becker, M. Boldrin, R. Jones, & W. Thomson (eds.), *General Equilibrium, Growth and Trade: The Legacy of Lionel McKenzie, II*, pp. 56–72. New York: Academic Press.

Cass, D. & K. Shell (1983) Do sunspots matter? *Journal of Political Economy* 91, 193–227.

Chatterjee, S. & D. Corbae (1992) Endogenous market participation and the general equilibrium value of money. *Journal of Political Economy* 100, 615–646.

Garratt, R. (1992) The connectedness of the set of equilibrium money prices depends on the choice of the numeraire. *Journal of Economic Theory* 56, 206–217.

Garratt, R., T. Keister, C.-Z. Qin & K. Shell (in press) Equilibrium prices when the sunspot variable is continuous. *Journal of Economic Theory.*

Grossman, S. & L. Weiss (1983) A transactions-based model of the monetary transmission mechanism. *American Economic Review* 58, 1–17.

Hoff, K. & J. Stiglitz (1990) Introduction: Imperfect information and rural credit markets—Puzzles and policy perspectives. *World Bank Economic Review* 4, 235–250.

Keister, T. (1998) Money taxes and efficiency when sunspots matter. *Journal of Economic Theory* 83, 43–68.

Mas-Colell, A. (1992) Three observations on sunspots and asset redundancy. In D. Gale & P. Dasgupta (eds.), *Economic Analysis of Markets and Games: Essays in Honor of F. Hahn*, pp. 465–474. Cambridge, MA: MIT Press.

Mirrlees, J. (1971) An exploration in the theory of optimum income taxation. *Review of Economic Studies* 38, 175–208.

Peck, J. (1987) Non-connectedness of the set of equilibrium money prices: The static economy. *Journal of Economic Theory* 43, 348–354.

Rotemberg, J. (1984) A monetary equilibrium model with transaction costs. *Journal of Political Economy* 92, 40–58.

Villanacci, A. (1993) Real indeterminacy, taxes and outside money in incomplete financial market economies: I. The case of lump-sum taxes. In F. Gori, L. Geronazzo, & M. Galeotti (eds.), *Nonlinear Dynamics in Economics and Social Sciences*, pp. 331–363. Berlin: Springer-Verlag.