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INHERITED TASTES AND ENDOGENOUS LONGEVITY

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This article aims at studying a general equilibrium model with overlapping generations that incorporates inherited tastes (aspirations) and endogenous longevity. The existence of standard-of-living aspirations transmitted between two subsequent generations in a context where the individual state of health depends on public investments in health has some remarkable consequences at the macroeconomic level. First, aspirations allow escaping from the well-known poverty trap scenario described by Chakraborty (2004). Second, the steady-state equilibrium may be destabilized through a super-critical Neimark–Sacker bifurcation when the health tax rate is set at too high or too low a level. This causes endogenous fluctuations in income and longevity.

Keywords: Aspirations, Bifurcations, Endogenous Fluctuations, Longevity, Overlapping Generations

1. INTRODUCTION

The aim of this article is to link two distinct but related issues of the recent macroeconomic literature with overlapping generations, which are endogenous longevity and endogenous preferences (inherited tastes). It is well recognized from both theoretical and empirical perspectives that the reduction in adult mortality is one the reasons why several countries around the world experienced a tremendous growth in gross domestic product (GDP) in the last decades. This is because individuals with low life expectancy save less than individuals with high life expectancy [Chakraborty (2004), Fogel (2004), Nakamura and Mihara (2016), Yasui (2016)].

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This phenomenon is important especially in Western countries since the second part of the 20th century, where it is observed a significant positive correlation between a substantial lengthening of life and sustained economic growth. In addition, aspirations (that is, the standard-of-living of parents affects consumption decisions of children due to habits transmitted intergenerationally)—and more in general consumption habits—are the object of a growing body of studies [de la Croix (1996), de la Croix and Michel (1999), de la Croix (2001), Alonso-Carrera et al. (2004), Artige et al. (2004), Alonso-Carrera et al. (2005, 2007), Schäfer and Valente (2011), Caballé and Moro-Egido (2014), Gómez (2015), Galor and Özak (2016)]. In particular, the recent work of Galor and Özak (2016) provides a well-documented example (by exploring the origins of differences in the rate of time preference observed over time across different regions) about the persistent effect on economic outcomes of inherited tastes. When the health status of an individual affects his intergenerational allocation of consumption, adult mortality may influence the transmission of (external) habits between parents and children. This article tackles this issue by considering an overlapping generations (OLG henceforth) model with endogenous lifetime and aspirations à la de la Croix (1996).

In recent decades, the economic literature has seen a dramatic increase in theoretical and empirical contributions with the aim of explaining the reasons why some countries prosper and others are entrapped in stagnation or poverty. In this regard, one of the most important issues is related to the influence of demography on economic growth and development [Galor (2005), Livi-Bacci (2006), Hall and Jones (2007), Weil (2007), Lorentzen et al. (2008), Cervellati and Sunde (2011), Galor (2011, 2012), Ashraf and Galor (2013), Cervellati and Sunde (2013)]. Economic growth is a concept specifically referred to the growth of an economic variable such as GDP per capita, whereas development is a multidimensional phenomenon related—among other things—to fertility behaviors, life expectancy, happiness, poverty, and the distribution of income. The behavior of fertility and life expectancy over the course of the recent human history is well recognized to be an essential ingredient that has led several economists to consider them as endogenous variables and tackle the issue of economic development in models that have originated the so-called Unified Growth Theory [Galor and Weil (2000), Galor and Moav (2002)].

With specific regard to life expectancy, there are some relevant contributions in the class of OLG models, bringing to light several aspects for which a change in adult mortality may actually affect macroeconomic outcomes. One of the first attempts in this direction is represented by the work of Blackburn and Cipriani (2002), who put together endogenous fertility and endogenous longevity in a unique general equilibrium macroeconomic set up with human capital accumulation through education. The model is able to characterize the existence of two different development regimes leading to high life expectancy, low fertility and high education and human capital accumulation (high development regime), and low life expectancy, high fertility and low education and human capital accumulation (low development regime). Belonging to the high regime or low regime is a matter of initial conditions. Their theoretical findings accord with the empirical evidence on demography and development (Demographic Transition). Another important work is represented by Chakraborty (2004), who includes endogenous lifetime through public investments in health into an otherwise two-period textbook OLG model. This assumption allows him to underline in a clear way how underinvestments in health may cause a vicious circle (by keeping longevity at low levels) that can actually be a source of poverty traps for developing countries. Later, Fanti and Gori (2014) add endogenous fertility to a Chakraborty-like economy and give another explanation to the Demographic Transition based on the interaction between public health investments and public policies on the side of the family. In this regard, the authors show that an adequate use of the tax of children may help escaping from a poverty trap scenario in which some developing countries are relegated. Their results may also represent an explanation of the tremendous development of China due to the application of the Chinese birth planning programme (since the end of the 1970s), recently abrogated.

Contextually to the burgeoning interest in the development economics, there exists a well-recognized literature on endogenous preferences. Amongst others, in a finitely lived agents OLG context, de la Croix (1996) and de la Croix and Michel (1999) introduce standard-of-living aspirations for the current generation based on the consumption experience of parents. Specifically, in their works they accounted for the stylized fact that a child may become habituated to a certain consumption pattern based on historical matters by taking into account a utility function "nonseparable across generations, but still separable across periods of life" [de la Croix and Michel (1999, p. 521)]. This form of habits introduces a negative externality for the current generation that tends to reduce the accumulation of capital in comparison with the absence of aspirations. The presence of this externality may be responsible for the existence of endogenous and persistent fluctuations in income emerging through a Neimark–Sacker bifurcation. However, in both works de la Croix (1996) and de la Croix and Michel (1999) did not clarify whether this bifurcation is supercritical or subcritical. In fact, only when the bifurcation is supercritical and a stable limit cycle is attracting for a discrete-time system, it is possible to refer to oscillations (ever-lasting cycles).

The present article introduces endogenous preferences à la de la Croix (1996) in a Chakraborty-like economy. The existence of aspirations in an economy with exogenous longevity produces the well-known mechanism leading to possible endogenous oscillations. First, the working of aspirations modifies the lifetime consumption bundle by reducing savings, capital accumulation, and income. This is because the relative weight of the effects of past actions on consumption decisions of the current generation (a mechanism displaying constant returns) tends to become larger than the relative weight of the increase in wages (a mechanism displaying decreasing returns due to the hypothesis of constant returns to scale in production). Second, the recession so induced reduces the relative importance of aspirations, which may actually revert the process thus generating a phase of expansion in capital accumulation and income. When longevity is endogenous there exists an additional mechanism affecting economic dynamics. Conflicting forces that may generate or eliminate long-term fluctuations drive this mechanism.

The rest of the article is organized as follows. Section 2 considers an OLG economy with rational individuals, inherited tastes, and endogenous longevity. Section 3 describes the feasible region and shows the existence of a unique positive fixed point. It also presents some results about the local stability of the *unique* interior fixed point and the *supercritical* Neimark–Sacker bifurcation it undergoes (thus generating observable oscillations) when some parameters of the model vary. Section 4 outlines the conclusions. Appendix A provides the proofs of the propositions stated in the main text. Appendix B briefly extends the model developed in the main text to the following cases: (i) taste inheritance transmission implying a comparison of consumption not only to what individuals belonging to the previous generation consumed when they were still living with individuals belonging to the current generation, but also to what was consumed when they were old [Ikefuji and Mino (2009)], (ii) external and internal habits, as in Alonso-Carrera et al. (2007).

2. THE MODEL

This section aims at characterizing a general equilibrium OLG closed economy with endogenous longevity and standard-of-living aspirations à la de la Croix (1996). The economy is comprised of a continuum of two-period lived, identical and rational individuals of measure one per generation. Each generation overlaps for one period with the previous generation and then overlaps for one period with the next generation. Time is discrete and indexed by $t = 0, 1, 2, \ldots$. Life of the typical agent is divided into youth and old age. As is typical in this class of models, an individual works when he is young and retires when he is old. The young member of generation t is endowed with one unit of labor, which is inelastically supplied to firms, receiving the competitive wage $w_t > 0$ per unit of labor. The probability of surviving from youth to old age is endogenous and determined by an individual's state of health, which is, in turn, improved by the public provision of health investments when young (ξ_t). This assumption follows the contributions of Chakraborty (2004) and Fanti and Gori (2014). The survival probability at the end of youth of an individual that belongs to generation t, p_t, depends on the public health expenditure per worker at time t, which is given by $\xi_t \in \mathbb{R}_+$, i.e., $p_t = p(\xi_t)$, where p is a strictly increasing and bounded function. By following Blackburn and Cipriani (2002) and Fioroni (2010), this relationship is modeled as follows:

$$p_t = p(\xi_t) = \frac{p_0 + p_1 \xi_t}{1 + \xi_t}, \quad 0 \le p_0 < p_1 \le 1. \tag{1}$$

We note that p is a continuous and differentiable function such that $p'(\xi_t) = \frac{p_1 - p_0}{(1+\xi_t)^2} > 0$ and $p''(\xi_t) = -2\frac{p_1 - p_0}{(1+\xi_t)^3} < 0$, $\forall \xi_t \in \mathbf{R}_+$. In addition,

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$p(0) = p_0$, whereas $\lim_{\xi_t \to +\infty} p(\xi_t) = p_1$. As is usual in this literature, p_0 can be interpreted as an exogenous measure of the natural rate of longevity irrespective of health investments [Ehrlich (2000), Leung and Wang (2010)]. In contrast, p_1 captures the intensity of the effectiveness of health investments as a device improving longevity (e.g., improvements in scientific research, the spread of vaccination programmes, and so on). The public health expenditure per worker at time t (ξ_t) is financed by levying a (constant) wage income tax at rate $0 \leq \tau < 1$ at a balanced budget. The government health accounting rule, therefore, is given by the following formula:

$$\xi_t = \tau w_t. \tag{2}$$

When young, an individual divides his disposable income, $w_t(1-\tau)$, between consumption ($c_{1,t}$) and saving (s_t). The budget constraint of a young individual of generation t therefore reads as follows:

$$c_{1,t} + s_t = w_t(1-\tau). \tag{3}$$

When old, an individual retires and lives with savings plus the expected interest accrued from time t to time $t+1$ at rate r_{t+1}^e (which will become the realized interest rate at time $t+1$). We also assume the existence of a (perfect) market for annuities, so that the budget constraint at time $t+1$ of an individual of generation t can be expressed as follows:

$$c_{2,t+1} = \frac{R_{t+1}^e}{p_t} s_t, \tag{4}$$

where $c_{2,t+1}$ is consumption when old and $R_{t+1}^e := 1 + r_{t+1}^e$ is the expected interest factor. The assumption of a perfect market for annuities implies that savings of the deceased are spread out by the surviving. Thus, a life annuity represents a form of longevity insurance. From (4) and (3), the lifetime budget constraint of the generation born at time t can be written as

$$c_{1,t} + \frac{p_t c_{2,t+1}}{R_{t+1}^e} = w_t(1-\tau). \tag{5}$$

The individual representative of generation t draws utility from consumption when young and consumption when old. In addition, he evaluates his own consumption when young in comparison with the level of aspirations inherited by his parent (h_t). These are bequeathed or inherited tastes for the individual born at time t representing a reference to compare current consumption. This assumption strictly follows de la Croix (1996) and de la Croix and Michel (1999). Aspirations (or external habits) can be referred to a situation for which an individual weights the utility of his own consumption with a benchmark level corresponding to the consumption experience of the others. Using this notion within a standard OLG model à la Diamond (1965) with a representative agent, allows us conclude that the consumption bundle of the current generation when young is evaluated in comparison with the consumption experience of the past generation when young. This intergenerational mechanism is affected by the individual health status when young, which in turn depends on public investments in health aiming at reducing adult mortality. Then, putting together these ingredients in a unique macroeconomic framework may help linking two distinct but related aspects of the working of an OLG model, as adult mortality affects standard-of-living aspirations transmitted from one generation to the next. Indeed, this may be of importance not only for developed countries, to maintain consumption standards among generations, but also for developing countries, for instance because of social norms associated with specific ethnic groups (ethnicity is primarily an inherited status). The working of this mechanism will be clear later in this article.

More formally, by normalizing the utility from death to zero, the expected lifetime utility function of generation t is given by the following logarithmic formulation:

$$U_t = \ln(c_{1,t} - \gamma h_t) + p_t \ln(c_{2,t+1}), \tag{6}$$

where $0 < \gamma < 1$ captures the intensity of aspirations. The choice of a logarithmic function is essentially made up to keep the model as simple as possible and describe the transmission mechanisms of a change in the degree of aspirations and/or the health tax rate in a clear way. The (expected) utility function given by equation (6) captures the taste inheritance transmission mechanism typical of a basic OLG model with a representative agent [de la Croix (1996)]. In fact, the parental consumption patterns of parents consumed when they were still living with their children is transmitted to the subsequent generation, thus causing a negative externality for the current one.[1] This reference pattern of consumption is therefore contrasted with the effective consumption pattern so that selfish individuals (positive values of γ) will be better off only whether their standard-of-living is sufficiently large. Things may change in the case of altruistic agents (negative values of γ). This case, however, is nonstandard and then it is not account for later in this article. In wider terms, the transmission mechanism of parental consumption patterns from ancestors to descendants may come from an individual perspective or a social perspective. In the former case, each agent's consumption habit (each agent's personal history) affects his current effective consumption. In this case, habits are internal and the assumption about expectations formation mechanism (i.e., perfect foresight or myopic behavior) matters as an individual optimal consumption path may be different depending on whether an individual is rational or not. In the latter case, it is the consumption habit of a social group (in which the agent belongs) or the society as a whole that affects the future consumption behavior of an individual. In this case, habits are an external effect and work exactly out as a standard externality. With specific regard to OLG models with finitely lived individuals, de la Croix and Michel (1999) have pointed out the existence of a third type of habit stock affecting individual's preferences: this is the case of familial capital. In the words of de la Croix and Michel (1999, p. 520): "Under this assumption, children become habituated to a certain standard-of-living while still with their parents. Their past standard-of-living serves as a benchmark to evaluate the level of their own current consumption, once they have become adults and work." [see also Becker (1992) for a discussion on this issue]. In an OLG model with representative agent, the assumption of familial capital affecting individual preferences may be naturally viewed as an externality for the current generation coming from actions of the previous generation.

By taking the wage, the expected interest rate, the longevity rate, the government budget constraint in (2), and the level of aspiration h_t as given, the individual representative of generation t chooses $c_{1,t}$ and $c_{2,t+1}$ to maximize the expected lifetime utility function (6) subject to the lifetime budget constraint (5) and $c_{1,t} > \gamma h_t$. Then, we get

$$\frac{1}{c_{1,t} - \gamma h_t} = \lambda_t \tag{7}$$

and

$$\frac{1}{c_{2,t+1}} = \frac{\lambda_t}{R^e_{t+1}}, \tag{8}$$

where λ_t is the Lagrange multiplier. From (7) and (8), the first-order condition for an interior solution is given by

$$c_{2,t+1} = R^e_{t+1}(c_{1,t} - \gamma h_t). \tag{9}$$

By combining (9) with (3) and (4) gives the saving function of the representative individual and consumption bundles when young and when old, respectively, that is

$$s_t = \frac{p_t}{1 + p_t}[w_t(1 - \tau) - \gamma h_t], \tag{10}$$

$$c_{1,t} = \frac{w_t(1 - \tau) + p_t \gamma h_t}{1 + p_t}, \tag{11}$$

$$c_{2,t+1} = \frac{R^e_{t+1}\,[w_t(1 - \tau) - \gamma h_t]}{1 + p_t}. \tag{12}$$

Firms are identical and markets are competitive. The (constant-returns-to-scale) technology of the representative firm used to produce output at time t (Q_t) combines capital (K_t) and labor (L_t) according to the Cobb–Douglas formulation $Q_t = AK_t^{\alpha} L_t^{1-\alpha}$, where $A > 0$ and $0 < \alpha < 1$. Profit maximization implies that the interest factor and the wage rate are equal to the marginal productivity of capital and the marginal productivity of labor, respectively, that is

$$R_t = \alpha A k_t^{\alpha - 1}, \tag{13}$$

$$w_t = (1 - \alpha) A k_t^{\alpha}, \tag{14}$$

where $k_t := K_t/L_t$ (we have assumed that capital fully depreciates at the end of every period and the price of output is normalized to one).

By following de la Croix (1996) and de la Croix and Michel (1999), aspirations at time t depend on the standard-of-living of the individual belonging to the previous generation when young (parent), that is young workers within a generation compare their consumption with "what the parents consumed when they were still living with them" [de la Croix (1996, p. 90)]. This assumption of taste inheritance implies that

$$h_t = c_{1,t-1}. \tag{15}$$

We now close the model by considering that the market-clearing condition in the capital market is $k_{t+1} = s_t$. Then, by using (10), (11), and (15) the two-dimensional system that characterizes the dynamics of capital and consumption with endogenous longevity is the following:

$$T : \begin{cases} k_{t+1} = \dfrac{p_t}{1+p_t}[w_t(1-\tau) - \gamma h_t] \\ h_{t+1} = \dfrac{w_t(1-\tau) + p_t \gamma h_t}{1+p_t} \end{cases}, \tag{16}$$

where $p_t = \frac{p_0 + p_1 \tau w_t}{1+\tau w_t}$ and $w_t = (1-\alpha)Ak_t^\alpha$. Now, define $x' := k_{t+1}$, $x := k_t$, $y' := h_{t+1}$, $y := h_t$, and let

$$A(x) = p_0 + m_1 x^\alpha, \quad B(x) = 1 + m_2 x^\alpha, \quad C(x) = m_3 x^\alpha,$$

where $m_1 = p_1 A \tau(1-\alpha) > 0$, $m_2 = \tau(1-\alpha)A > 0$, and $m_3 = (1-\tau)(1-\alpha)A > 0$. Then, after some algebra, the two-dimensional dynamic system described in (16) can be represented by the following continuous and differentiable map in $[0, +\infty) \times [0, +\infty)$:

$$T : \begin{cases} x' = f(x, y) = \dfrac{A(x)[C(x) - \gamma y]}{A(x) + B(x)} \\ y' = g(x, y) = \dfrac{B(x)C(x) + A(x)\gamma y}{A(x) + B(x)} \end{cases}. \tag{17}$$

System (17) is quite complex to be studied given its analytical form. In order to make it more tractable, we use the topological transformation introduced in Fanti et al. (2016) and briefly summarized in Appendix A, thus obtaining the following system:

$$S : \begin{cases} x' = F(x, z) = \dfrac{(p_0 + m_1 x^\alpha)(m_3 x^\alpha + \gamma x - \gamma m_3 z^\alpha)}{(1 + p_0) + (m_1 + m_2)x^\alpha} \\ z' = G(x) = x \end{cases}, \tag{18}$$

which describes the time evolution of capital per workers (x), whereas the dynamics of aspirations (y) can be obtained from the dynamics of z as $y = C(z) - x$.

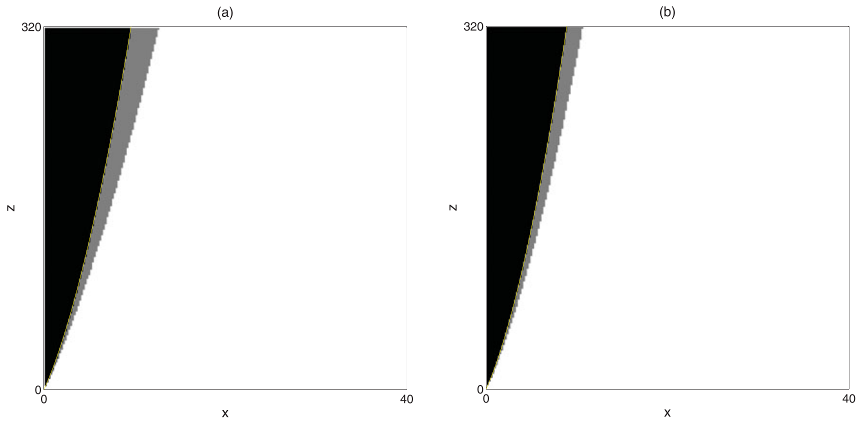

FIGURE 1. (a) The feasible region is depicted in white for the following parameter values: $\alpha = 0.33$, $A = 10$, $p_0 = 0.4$, $p_1 = 0.9$, $\gamma = 0.4$, and $\tau = 0$. (b) If τ increases, the feasible region increases too ($\tau = 0.15$).

3. LOCAL AND GLOBAL DYNAMICS

3.1. Preliminary Properties

Before studying the long-term dynamics generated by system S, we observe that not all trajectories starting from an initial condition $(x(0), z(0)) \in \mathbf{R}^2_+$ are feasible. For instance, if $x(0) = 0$ and $z(0) > 0$, then $x(1) < 1$, that is all initial conditions taken on the y-semiaxes with $z(0) > 0$ produce unfeasible trajectories. Of course, only trajectories that do not exit a suitable (positively invariant) set $D \subset \mathbf{R}^2_+$ are meaningful from an economic point of view. With regard to the properties of the feasible set, the following proposition holds (see Appendix A for the proof).

PROPOSITION 1. *Let S be given by (18). Feasible trajectories can be produced only if, for any given initial value of the stock of capital per worker, aspirations are not too large.*

According to Proposition 1, the feasible region D is a subset of $\mathbf{R}^2_+$. However, Proposition 1 states only a necessary condition for the feasibility of S. In fact, it may occur that a generic trajectory exits set $\mathbf{R}^2_+$ after the first iteration. The feasible region (in which also the utility function given by (6) is well defined) is depicted in white in Figure 1(a) and (b), whereas the yellow curve separating the grey region from the black one is $\theta(x)$, as defined in the proof of Proposition 1 in Appendix A. Points above $\theta(x)$ exit $\mathbf{R}^2_+$ at the first iteration. The grey region describes the whole set of points producing unfeasible trajectories that exit $\mathbf{R}^2_+$ after the first iteration. We observe that the size of the feasible region increases with τ. When $\tau \to 1$, we have $m_3 \to 0$ so that system S always produces feasible trajectories. Then, the following remark holds.

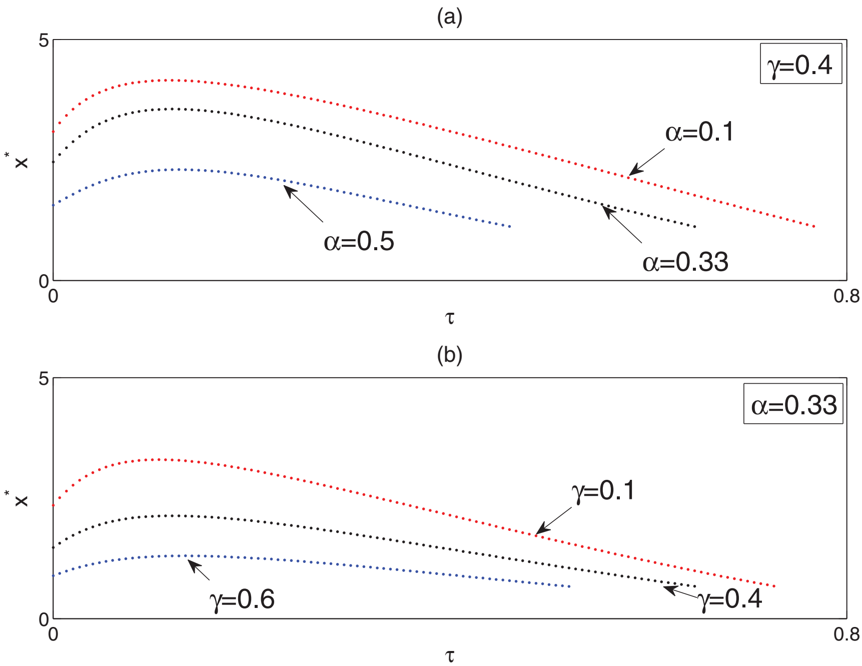

FIGURE 2. The equilibrium value x^* is represented for different values of τ whereas changing, respectively, the values of α [panel (a)] and γ [panel (b)]. The other parameter values are as in Figure 1.

Remark 2. If $\tau \to 1^-$ then $D \to \mathbf{R}_+^2$.

Therefore, with endogenous longevity, the domain of existence of an OLG economy with production and aspirations is larger than when longevity is exogenous, as in de la Croix (1996). As a direct consequence, when longevity depends on public health investments the system tends to remain feasible with stronger aspirations than when longevity is exogenous.

3.2. Fixed Points, Local Stability, and Bifurcations

We now focus on the question of the existence and number of fixed points of system S. The following proposition proved in Appendix A, holds.

PROPOSITION 3. *System S given by (18) admits two fixed points for all parameter values: the origin $E_0 = (0, 0)$ and the interior fixed point $E^* = (x^*, x^*)$.*

In order to better understand the role of the health tax rate τ on the position of the interior fixed point, we present some numerical evidences. Figure 2 depicts different sequences on (τ, x^*) plane for different values of α or γ. Values of α smaller than (or close to) $1/3$ are in line the most widely recognized empirical estimates on the output elasticity of capital for several countries all around the world [Gollin (2002)], whereas values of α larger than 0.5 "may be rationalized by broadening the concept of capital [Chakraborty (2004, p. 124)]" to include human capital components. The values of γ used in the simulations are in line with de la Croix (1996), de la Croix and Michel (1999), and Alonso-Carrera et al. (2007). We also recall that values of τ belonging to range $[0.1, 0.2]$ imply a ratio of health expenditure to per worker GDP of almost 7–9% [which is an average value for developed countries, see World Health Statistics (2015)]. In addition, values of p_0 and p_1, which represent the probabilities of surviving from youth to old age without and with health investments, respectively, and used in all numerical simulations, are in line with data from World Bank (2015) and World Health Statistics (2015), being proxies for values of (the reciprocal of) adult mortality.

We preliminary observe that if $\tau \to 0^+$, then $x^* \to \left[\frac{p_0 m_3(1-\gamma)}{1+p_0(1-\gamma)}\right]^{\frac{1}{1-\alpha}}$, whereas if $\tau \to 1^-$, then $x^* \to 0$. We note that x^* decreases with γ, in line with the case of exogenous longevity [de la Croix (1996)]. In addition, the sequences depicted in panels (a) and (b) of Figure 2 (that represent the equilibrium value of capital per worker x^* versus τ) are nonmonotonic. In particular, it seems that starting from an initial value $\tau = 0$ (absence of health investments), the steady-state stock of capital increases with τ up to a threshold $\bar{\tau}$ that marks the start for a negative relationship between x^* and τ. This behavior of the capital stock is in line with Chakraborty (2004) and comes from the fact that for small values of τ the positive effect of an increase in public health investment on savings due to a higher longevity more than offsets the reduction in the disposable increase due to higher taxation (vice versa for large values of τ). This suggests that τ should be set at an intermediate value in order to maximize capital per worker at the equilibrium. The negative effect of the tax rate on income (wage) and then on economic growth exceeds (beyond a certain threshold) the positive effect of the financing of health spending that tends to increase savings and, in turn, economic growth through a reduction in adult mortality. Furthermore, since S admits two fixed points for any given $\tau \in (0, 1)$ and it is continuous with respect to τ, then the interior fixed point must converge to the origin if τ becomes large enough.

An investigation of the local stability properties of the equilibria is quite difficult to be performed due to the analytical form of system S. Then, we focus only on some limiting cases and consider other situations by resorting to numerical simulations. Notice that the Jacobian matrix associated with S evaluated at the fixed point $\bar{E} = (\bar{x}, \bar{z})$ is given by

$$JS(\bar{E}) = \begin{pmatrix} F_x(\bar{x}, \bar{z}) & F_z(\bar{x}, \bar{z}) \\ 1 & 0 \end{pmatrix}. \tag{19}$$

With regard to the local stability of E_0, it can be observed that the determinant of the Jacobian matrix tends to $+\infty$ as $x \to 0^+$ and $z \to 0^+$. This implies that the origin cannot be locally stable and the following Proposition holds.

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PROPOSITION 4. *The origin is a locally unstable fixed point.*

As system S in (18) is topologically conjugated to system T in (17), then if $(\bar{x}, \bar{z}) = (0, 0)$ is a locally unstable fixed point of S, then $(\bar{x}, \bar{y}) = (\bar{x}, C(\bar{z}) - \bar{x})|_{(0,0)}$ is locally unstable for map T, thus confirming that Proposition 4 holds for the initial system. In addition, as the origin is locally unstable in a complete neighborhood $I(\underline{0}, \delta)$, then it is also locally unstable in $I(\underline{0}, \delta) \cap D$, where D is the feasible set of T (so that condition $C(x) \geq \gamma y$ for the utility function to be well-defined holds at any iterations). Notice also that Proposition 4 holds for all parameter values, that is also in the case $p_0 = 0$, which replicates the longevity function proposed by Chakraborty (2004). This shows that the existence of inherited living standards (i.e., $\gamma > 0$) is the critical factor that contributes to eliminate the poverty trap scenario thus favoring economic development, although it still remains true that capital accumulation with aspirations is lower than capital accumulation without aspirations, given the direct depressing effect of aspirations on saving. The poverty trap scenario in Chakraborty (2004) happens because of amortization when the capital share in income becomes large: lack of sufficient incentive to save when mortality is high produces a tendency toward underdevelopment and high mortality. Under aspirations ($\gamma > 0$), the origin ceases to be a local attractor. This is true generally, that is, it is not specifically related to some parameter restrictions. As equations (10) and (12) show, aspirations lower saving and future consumption to maintain consumption standards of the new generation [see equation (11)]. That ought to make it harder for the economy to grow out of underdevelopment, that is, easier for a poverty trap to occur, as aspirations favor the beginning of a phase of economic contraction. However, when the strength of aspirations becomes large enough, the contraction is so high to induce a reduction in the aspiration itself, which, in turn, favors the beginning of an expansion. This process, which is an intrinsic characteristic of the presence of aspirations and does not require any restrictions on technological parameters, avoids the appearance of a poverty trap. The reason why aspirations act as a device that eliminates the poverty trap therefore is twofold. There exist (1) the usual effect of capital/income on longevity and saving and youthful consumption, and (2) an intergenerational effect of youthful consumption on habit persistence and hence saving [a high marginal value of (net) consumption at low income levels]. The second effect always dominates and it is general to preferences, i.e., it holds with logarithmic preferences and CIES preferences,[2] and the value of γ. The transmission of standard-of-living may be, therefore, an additional device to avoid stagnation. This also allows to exclude the vicious circle detailed by Chakraborty (2004) of lack of sufficient incentives to save when mortality becomes large. An increase in public investments in health improves the health status of individuals, which, in turn, causes an increase in life expectancy, saving, and capital accumulation (although promoting also a reduction in the disposable income because of the increased labor income taxation[3]). The increased life expectancy causes, ceteris paribus, a reduction in young-age consumption, which generates a reduction in the strength of aspirations of the new generation, providing the basis for a further increase in capital accumulation.[4]

With regard to the local stability of the interior fixed point E^*, the following result can be proved (see Appendix A for the proof).

PROPOSITION 5. *Let S be given by (18).*

(i) *Assume* $\gamma \to 0^+$. *Then, if* $\alpha \to 0^+$ *or* $\tau \to 0^+$, E^* *is a stable node.*
(ii) *Assume* $\gamma \to 1^-$. *Then,* E^* *is a saddle point.*

Part (ii) of Proposition 5 confirms that the interior equilibrium is locally unstable if aspirations are too high. By considering the interior steady state, we now prove analytically that a *supercritical* Neimark–Sacker bifurcation occurs when α is close to 0.5. Interestingly, we note that the threshold $\alpha = 0.5$ that allows to detect the existence of an observable limit cycle and then generate endogenous fluctuations in income and longevity in a model with aspirations, is the same as the threshold separating the scenario where a unique (stable) steady state exists with the scenario where there are multiple steady states (poverty trap due to underinvestments in health) in Chakraborty (2004).

PROPOSITION 6. *Let* E^* *be the interior fixed point of system* S. *Then,* $\exists \epsilon > 0$ *such that* $\forall \alpha \in I(0.5, \epsilon)$ *there exists a* $\gamma = \gamma_\alpha \in (0, 1)$ *at which a supercritical Neimark–Sacker bifurcation occurs.*

Proposition 6 (the proof is in Appendix A) shows that if α is close to 0.5, then there exists a threshold value γ_α of the intensity of aspirations such that E^* becomes an unstable focus, and an attracting closed invariant curve is created out around it. Hence, fluctuations in both the stock of capital and longevity occur in such a case. It is important to note that Proposition 6 resembles a Chakraborty-like economy when $\gamma = 0$. For $\alpha = 0.5 - \epsilon$, Chakraborty (2004) finds that there exists a unique globally stable interior equilibrium, whereas when $\alpha = 0.5 + \epsilon$ there exist two locally asymptotically stable equilibria (the origin and the interior equilibrium), with the raising of the well-known poverty trap scenario induced by an inadequate financing of public health investments. In contrast, by introducing aspirations in a Chakraborty-like economy allows to have two equilibria (see Proposition 3): the origin and the interior equilibrium. As the origin is always an unstable fixed point, the poverty trap is avoided. This holds because aspirations tend to keep consumption to relatively high levels due to habits purposes and when one person lives longer he also tends to increase savings. However, different from Chakraborty (2004), where the transitional dynamics is always monotonic, the interior equilibrium can be destabilized, thus generating an observable persistent economic cycle.

The occurrence of a Neimark–Sacker bifurcation for economic meaningful values of the capital share in income (i.e., $\alpha \ll 0.5$) cannot be analytically proved as system S is quite complex. However, by resorting to numerical experiments it is possible to show that the neighborhood $I(0.5, \epsilon)$ containing values of α such that an attracting closed invariant curve is created for a given value of γ *can be large enough*. This implies that fluctuations emerge for empirically plausible parameter values (as discussed above). The related numerical evidence is described in the next section.

3.3. Numerical Experiments

In the preceding section, we have shown that an increase in the aspiration intensity is a source of instability and fluctuations in income and longevity (see Proposition 5). In addition, if α is close to 0.5, a bifurcation value of the aspiration intensity exists at which the interior steady state loses its stability and endogenous fluctuations emerge (see Proposition 6). As a value of α close to (or even larger than) 0.5 is not consistent with the most widely recognized empirical estimates about the capital share in income (unless one accounts for a broader concept of capital including human components), the main aim of this section is to present some numerical simulations revealing that endogenous fluctuations may occur with empirically plausible parameter values. The existing OLG literature already identified nonstandard dynamics, including endogenous cycles, in several contexts [e.g., Grandmont (1985), Farmer (1986), Reichlin (1986), Raut and Srinivasan (1994), Grandmont et al. (1998)]. However, the main criticism against that body of work, which is emerged over the years, was that it often required extreme assumptions on parameter values or individuals with static expectations in a typical OLG context [e.g., Michel and de la Croix (2000), de la Croix and Michel (2002), Fanti and Gori (2013)]. In this section, we show that endogenous oscillations in our economy are compatible with parameter values that are in line with empirical estimates as well as with the assumption of perfect foresight.

Before going on with numerical exercises to show the existence of nonmonotonic dynamics also for values of α greatly smaller than 0.5, it would be instructive to understand how the health tax rate affects the bifurcation value γ_α (that crucially depends on α) and infer about its role in determining convergence toward a stable steady state or a limit cycle. As system S cannot be treated in a neat analytical form, we resort to numerical simulations and depict γ_α for different values of τ for values of $\alpha \in (0.3, 0.38)$. Specifically, in Figure 3(a), we consider three different values of α and plot the corresponding bifurcation values γ_α for any $\tau \in (0, 1)$. It can be observed that γ_α exhibits a nonmonotonic shape when τ varies. In fact, for suitable values of α, the curve $\gamma_\alpha(\tau)$ is strictly convex and the turning point is $\tau = 0.5$. These simulations are important as they allow us to extend the results of Proposition 6, showing the existence of observable deterministic long-term cycles with endogenous longevity ($\tau \neq 0$) and Cobb–Douglas preferences also for values of α close to 0.3 [see Figure 3(d)].

Figure 3(b) depicts bifurcation diagrams with respect to τ for $\alpha = 0.33$ and $\gamma = 0.8132$ with the main aim of inquiring about the role of the health tax rate in an economy with aspirations. We observe the existence of two values of τ, namely τ_1 and τ_2 (τ_1 is an empirically plausible value close to 0.2, whereas τ_2 may have only

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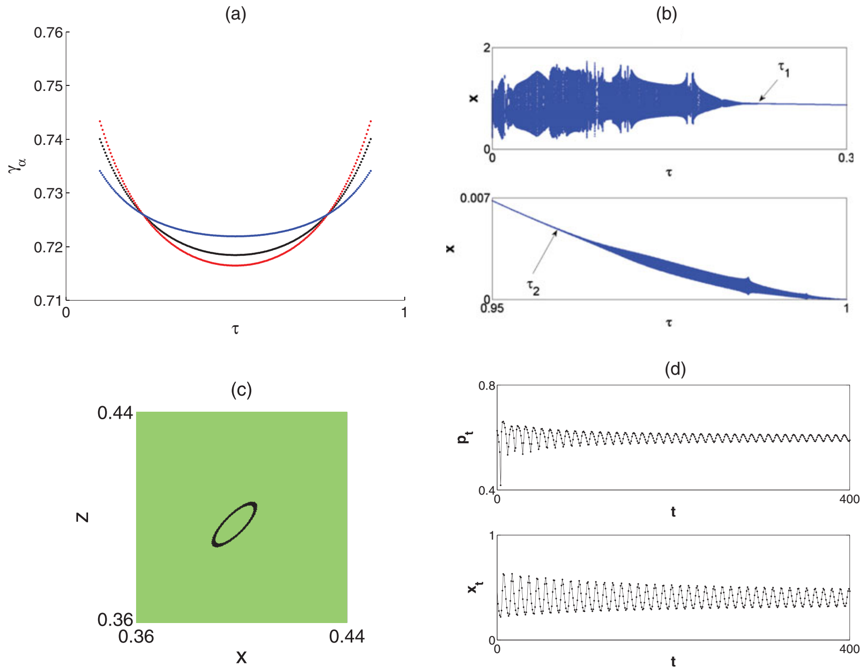

FIGURE 3. $A = 10$, $p_0 = 0.3$, $p_1 = 0.9$. (a) Three bifurcation curves related to the occurrence of the Neimark–Sacker bifurcation are plotted for $\alpha = 0.3$ (red points), $\alpha = 0.38$ (blue points), and $\alpha = 0.33$ (black points). (b) Bifurcation diagrams w.r.t. τ for $\alpha = 0.33$ and $\gamma = 0.8132$: The bifurcation values corresponding to the Neimark–Sacker bifurcation are, respectively, $\tau_1 \simeq 0.226$ and $\tau_2 \simeq 0.959$. (c) For $\tau = 0.2$, $\alpha = 0.33$, and $\gamma_\alpha = 0.8132$ the attracting closed invariant curve is depicted. (d) Fluctuations of capital per capita and longevity for the same parameter values used in (c).

a theoretical interest) at which a Neimark–Sacker bifurcation occurs. Notice that if $\tau \in (0, \tau_1)$, then an attracting closed invariant curve is obtained [it is depicted in Figure 3(c)]. When $\tau = \tau_1$ such a curve becomes unstable and the interior fixed point becomes locally attracting; E^* persists to be locally stable until another (supercritical) Neimark–Sacker bifurcation occurs at $\tau = \tau_2$. After the occurrence of such a bifurcation, the long-term evolution of the system converges to a locally attracting closed invariant curve. A first important evidence is that if the economy fluctuates, then the magnitude of the cycles becomes smaller as τ is increased, thus showing a rich dynamic role of the health tax rate. The intensity of economic cycles is larger for small values of τ (i.e., for values of τ that are much more in line with those used in actual markets). This example (and others that we do not report in this article) allows us to conclude that the local stability of the interior fixed point is associated with intermediate values of τ. As a policy consequence, τ should not be fixed either at too small or large values to reduce oscillations or stabilize the economy. This holds even with a large aspirations intensity, which instead would contribute to destabilize the economy under exogenous longevity ($\tau = 0$). The health tax rate therefore can be used as a stabilizing device when it is fixed at intermediate values.

The economic mechanism behind oscillations in the case of exogenous longevity, as clearly detailed by de la Croix (1996), passes through a twofold effect related to savings and the consumption of the previous generation (parent). The former effect displays decreasing returns, whereas the latter one displays constant returns. The presence of standard-of-living aspirations, in fact, causes a negative external effect that is not taken into account by the current generation. The intensity of aspirations in utility, therefore, plays a preeminent role in explaining the working of this externality, as an increase in it causes an increase in the relative importance of life standard aspirations for the young generation. As a consequence, this generation tends to put a larger weight to current consumption than savings. When the degree of aspirations becomes larger enough, it is possible to observe a drop in savings because resources are indeed required to keep life standards at a high level. This implies a reduction in both capital accumulation and income (recession). The recession makes less important the constant returns process associated with the presence of aspirations as compared with the decreasing returns process related to savings and production. This implies a reduction in consumption and an increase in savings thus generating a reverting mechanism leading to an increase in capital accumulation and income (expansion). Then, depending on initial conditions and parameter values, these two conflicting forces may generate endogenous oscillations.

When longevity is endogenous, there exists an additional mechanism affecting the stock of aspirations and the stock of capital per worker that passes through longevity via a change in the health tax rate. In particular, an increase in the health tax rate implies an improvement in the individual health status, which in turn causes an increase in longevity. This leads to higher savings, because the individual life span becomes larger, and lower consumption. As a consequence, aspirations becomes less important and capital accumulation increases through this channel. This may actually explain the reason why in an economy with endogenous longevity stability of the steady-state equilibrium holds even with a large aspirations intensity. However, a higher health rate also reduces the disposable income of the individual. This negative effect tends to cause a reduction in both capital accumulation (through a drop in wages and production) and aspirations. The final effect from the longevity side on economic dynamics is therefore ambiguous and it may work out in the direction of strengthening or weakening oscillations.

To sum up the conflicting forces of health taxation affect the emergence (or the elimination) of economic volatility. In fact, when longevity is endogenous, economic dynamics is affected by a mechanism additional to the ones clearly discussed by de la Croix (1996) for a model with exogenous longevity. In fact, an increase in the health tax rate reduces the disposable income of workers. This causes a reduction in saving (the stock of capital) and consumption (the stock of aspirations). The reduction in the stock of aspirations has positive effects (with constant returns) on capital accumulation. However, the increase in taxation also generates an increase in longevity, which in turn increases savings and the stock of aspirations. The increase in the stock of aspirations has negative effects (with constant returns) on capital accumulation. When the extent of taxation is sufficiently small, the weight of the reduction in the disposable income of individuals is low (this phenomenon displays decreasing returns), whereas the increase in longevity generated by the higher tax rate causes an increase in the stock of aspiration that tends to reduce capital accumulation (this phenomenon displays constant returns). Therefore, the final effect on capital accumulation is ambiguous and may cause oscillations. In contrast, when the extent of taxation is large, the weight of the reduction in the disposable reduces the stock of aspirations. Although this effect displays decreasing returns its effect is strong enough to more than offset the positive effects on aspirations due to the higher longevity (indeed, this effect tends to be negligible when longevity is still high). The reduction in the stock of aspirations tends to increase saving and capital accumulation. The final effect of an increase in taxation on capital accumulation is ambiguous and may generate oscillations also in this case. Finally, when taxation if fixed at intermediate levels, these counterbalancing forces tend to cancel each other out and trajectories tend to be convergent toward the long-term stationary equilibrium.

4. CONCLUSIONS

One of the cornerstone theoretical contributions of OLG economies with endogenous longevity is by Chakraborty (2004), who accounts for individuals whose life span depends on their own state of health which, in turn, is affected by public health investments. Other two relevant works in the OLG literature are de la Croix (1996) and de la Croix and Michel (1999), who study how aspirations (habits transmitted intergenerationally from parents to children) affect macroeconomic outcomes at both decentralized and centralized levels.

This article has concerned with the study of a general equilibrium model with overlapping generations and inherited tastes (aspirations). By accounting for aspirations in a Chakraborty-like economy with logarithmic preferences, Cobb–Douglas technology and rational individuals, it is shown that inherited tastes allow escaping from a poverty trap scenario because the low steady-state equilibrium is always unstable, and the size of the feasible region increases together with the health tax. This is, however, a policy instrument that may actually be used to reduce fluctuations generated by aspirations. In particular, we may observe persistent and strong oscillations when the health tax rate is fixed at relatively small or large values, whereas for intermediate values of it the long-term equilibrium is locally asymptotically stable even for a relatively high degree of aspirations. The analysis is built on the premise that, in the presence of inherited tastes, a permanent change in public health policy, in addition to its direct effect on saving behavior, will have richer implications due to the fact that it also affects the consumption levels of the

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previous generations, thus impinging on the intergenerational externality that is also an important component of optimal saving and capital accumulation.

Possible extensions of the present work may regard the study of endogenous fertility in a model with consumption-based aspirations (this is because when tastes are inherited, endogenous fertility becomes of particular importance given the existing influence on consumption decisions of children caused by the behavior of parents), following the works of Gori and Michetti (2016) and Kaneko et al. (2016), or the analysis of wealth-based aspirations with endogenous longevity, which may become a relevant issue especially when modeling explicitly the willingness of individuals to leave (intended) bequests.

NOTES

1. In Appendix B, we briefly describe a model with taste inheritance affecting the consumption pattern of an individual over the course his whole lifetime.
2. It is possible to show that (0,0) is an unstable fixed point also in the case of CIES preferences. Indeed, numerical experiments not reported in this work show that the interior fixed point is unique for all values of α and the origin is an unstable equilibrium irrespective of the value of the (constant) intertemporal elasticity of substitution of effective consumption.
3. Under the combined effect of taxation on life expectancy and the intergenerational externality that is intrinsic to the presence of inherited living aspirations, public health policy is a crucial factor in determining the emergence of economic volatility (i.e., oscillations of the capital stock and output during the transition toward the steady-state equilibrium). This will be clarified later in this article.
4. It is common in this literature to interchangeably refer to intergenerational habits or status as aspirations. However, intergenerational habits seem to be related to some innate behaviors of people (through upbringing) while aspirations or status-seeking seem to have an element of choice. For example, if people truly had a choice whether or not they want to aspire to their parents' living standards and they choose not to do so at low income levels, the origin could become a local attractor again. Exploring this issue could be of interest for a possible future research.

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APPENDIX A: TOPOLOGICAL TRANSFORMATION OF SYSTEM T INTO SYSTEM S

Consider system T as given by (17) and observe that

$$x' + y' = f(x, y) + g(x, y) = C(x) \Rightarrow y' = C(x) - x', \qquad \textbf{(A.1)}$$

that is the aspiration level a time $t+1$ can be written in terms of capital per worker in time t and in time $t+1$. Consider now the level of capital per worker at time $t+2$, then from the first equation of system T in (17), we have $x'' = f(x', y')$. Making use of (A.1), the following second-order difference equation describing the evolution of capital per worker is obtained:

$$x'' = f(x', C(x) - x') = F(x', x). \qquad \textbf{(A.2)}$$

According to equation (A.2), given an initial condition $(x(0), x(1))$ (i.e., given the values of the capital per worker at time $t = 0$ and $t = 1$) then by applying function F the evolution of capital per worker can be obtained, as it is given by the following sequence $x(0), x(1), x(2), \ldots, x(n)$, such that $x(i) = F(x(i-1), x(i-2))$, $\forall i = 2, 3, v, n$. It is important to underline that the sequence of the other state variable, i.e., the evolution of aspirations $y(1), y(2), \ldots, y(n)$, can be then obtained by $y(i) = C(x(i-1)) - x(i)$, $\forall i = 1, 2, 3, \ldots, n$.

As it is well known, the second-order equation (A.2) can be written as a system of two difference equations of first order. Define

$$z' = G(x) = x, \qquad \textbf{(A.3)}$$

then (A.2) can be written as follows:

$$x'' = F(x', z') \Rightarrow x' = F(x, z), \qquad \textbf{(A.4)}$$

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and consequently the following system of two first-order difference equations is obtained:

$$S : \begin{cases} x' = F(x, z) = \dfrac{A(x)[C(x) + \gamma x - \gamma C(z)]}{A(x) + B(x)} \\ z' = G(x) = x \end{cases} . \quad \textbf{(A.5)}$$

Proof of Proposition 1

Proof. Consider that if

$$z(0) > \left(\frac{m_3 x(0)^\alpha + \gamma x(0)}{\gamma m_3}\right)^{\frac{1}{\alpha}} = \theta(x(0)),$$

then $x(1) < 0$. Hence, a necessary condition for the feasibility of system S is $z(0) \leq \theta(x(0))$ which corresponds to $y(0) \leq \frac{m_3 x(0)^\alpha}{\gamma}$. ∎

Proof of Proposition 3

Proof. Since $S(0, 0) = (0, 0)$ for all parameter values then the origin is a fixed point. An interior fixed point of S must solve equation $x = F(x, x)$, $x > 0$. After some algebra, we get $x = F(x, x)$ iff $H_1(x) = H_2(x)$, where

$$H_1(x) = x^{1-\alpha}[1 + p_0(1 - \gamma)] - m_1 m_3(1 - \gamma)x^\alpha$$

and

$$H_2(x) = p_0 m_3(1 - \gamma) - [m_1(1 - \gamma) + m_2]x$$

are differentiable for all $x > 0$.

Notice that $H_1(0) > 0$ and $H'(x) < 0$, while $H_2(0) = 0$, $\lim_{x\to+\infty} H_2(x) = +\infty$, and $H_2(x)$ is unimodal and strictly convex. Hence, $H_1(x)$ and $H_2(x)$ intersect only once in point x^*, that is $E^* = (x^*, x^*)$ is a fixed point of S.

∎

Proof of Proposition 5

Proof. By considering the Jacobian matrix in (19), we observe that the trace and the determinant evaluated at E^* are given by $\det(JS(E^*)) = -F_z(x^*, x^*)$ and $\mathrm{tr}(JS(E^*)) = F_x(x^*, x^*)$, respectively.

(i) We observe that

$$F_z(x^*, x^*) = -\frac{\gamma m_3 \alpha (x^*)^\alpha (p_0 + m_1(x^*)^\alpha)}{[(1 + p_0) + (m_1 + m_2)(x^*)^\alpha]^2}, \quad \textbf{(A.6)}$$

hence if $\gamma \to 0^+$, then $\det(JS(E^*)) \to 0$. In such a case the eigenvalues associated to E^* are given by $\lambda_1 = 0$ and $\lambda_2 = F_x(x^*, x^*)$. Hence, E^* is a stable node or it is a saddle point. We focus on $\lambda_2 = F_x(x^*, x^*)$ and observe that $F_x(x^*, x^*)$ is continuous w.r.t. the parameters and x^*. Assume $\gamma = 0$ then, after some algebra, it can be obtained that

$$F_x(x^*, x^*) = \alpha(1 - \alpha)^2 \tau(1 - \tau) A^2 (1 + p_1)(x^*)^{2\alpha - 2} \Theta(x^*)$$

and consequently if $\alpha \to 0^+$ or $\tau \to 0^+$, since x^* and $\Theta(x^*)$ tend to positive values, then $|\lambda_2| < 1$ and E^* is a stable node.

(ii) Assume $\gamma \to 1^-$ then $x^* \to 0^+$ and consequently $F_z(x^*, x^*) \to 0$ [see equation (A.6)]. Furthermore, $F_x(x^*, x^*) \to +\infty$ hence E^* is a saddle point.

■

Proof of Proposition 6

Proof. Let $\alpha = 0.5$, then the interior steady state is given by $\bar{E} = (\bar{x}, \bar{x})$, where

$$\bar{x} = \left(\frac{-a_1 + \sqrt{a_1^2 - 4a_0a_2}}{2a_0}\right)^2$$

being

$$a_0 = 0.5A\tau[(1-\gamma)p_1 + 1],$$

$$a_1 = (1-\gamma)[p_0 - 0.25p_1A^2\tau(1-\tau) + 1],$$

and

$$a_2 = 0.5(1-\gamma)p_0(1-\tau)A.$$

The determinant of the Jacobian matrix evaluated in point $\bar{E}$ is given by

$$\det[JS_1(\bar{x}, \bar{x})] = \frac{0.25\gamma(1-\tau)A(p_0 + 0.5p_1A\tau\bar{x}^{0.5})}{[p_0 + 0.5A\tau(p_1+1)\bar{x}^{0.5} + 1]\bar{x}^{0.5}}.$$

Define $\gamma_{0.5} = \{\gamma \in (0,1) : \det(JS_1(\bar{x}, \bar{x})) = 1\}$ and consider $\bar{x} = \bar{x}(\alpha, \gamma)$, $\alpha \in (0,1)$ and $\gamma \in (0,1)$.

Since $\det(JS_1(\bar{x}(\alpha,\gamma), \bar{x}(\alpha,\gamma)))$ is continuous w.r.t. α and γ then if $\alpha \to 0.5$ and $\gamma \to \gamma_{0.5}$, then $\det(JS_1(\bar{x}(\alpha,\gamma), \bar{x}(\alpha,\gamma))) \to 1$.

As a consequence $\exists I(0.5, \gamma_{0.5}, \epsilon_1)$ such that

$$\det(JS_1(\bar{x}(\alpha,\gamma), \bar{x}(\alpha,\gamma))) \in (1-\epsilon_1, 1+\epsilon_1),$$

for all $(\alpha, \gamma) \in I(0.5, \gamma_{0.5}, \epsilon_1)$.

In particular, inside this neighborhood $\exists \gamma_\alpha \in (0,1)$ such that

$$\det(JS_1(\bar{x}(\alpha,\gamma_\alpha), \bar{x}(\alpha,\gamma_\alpha))) = 1.$$

Using similar arguments, it can be also verified that (i) the trace of the Jacobian matrix evaluated at the fixed point belongs to the interval $(-2, 2)$, (ii) the two nonreal eigenvalues cross the unit circle at a nonzero speed when γ changes, and (iii) none of them may be one of the first four roots of unity (excluding cases of weak resonance).

Finally, at $\gamma = \gamma_\alpha$ the local stability of $\bar{E}$ is lost and an attracting closed invariant curve is created. ■

APPENDIX B

This appendix extends the model developed in the main text to two distinct cases: (1) old agents inherit consumption behavior of their parents when they are old; this assumption follows Ikefuji and Mino (2009), who study an OLG model with taste inheritance and

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endogenous growth, and (2) there exist both external and internal habits, as in Alonso-Carrera et al. (2007). The expected lifetime utility function equation (6) modifies to become the following:

$$U_t = \ln(c_{1,t} - \gamma_1 h_{1,t}) + p_t \ln(c_{2,t+1} - \gamma_2 h_{2,t+1}), \quad \textbf{(B.1)}$$

where $0 < \gamma_1 < 1$ weights the intensity of aspirations when young and $0 < \gamma_2 < 1$ weights the intensity of aspirations when old, and $c_{1,t} > \gamma_1 h_{1,t}$ and $c_{2,t+1} > \gamma_2 h_{2,t+1}$ must hold at every t. This represents a generalization under which the set-up developed in the main body of this work is the limiting case when $\gamma_2 = 0$. With this formulation, in fact, aspirations not only affect the individuals' preferences during their youth, but also parents' lifestyles during maturity are a source of taste inheritance for the current generation, as will be clear later. Contrary to the effect of aspirations on saving through $\gamma_1 > 0$, the effect of aspirations under $\gamma_2 > 0$ is to increase the marginal utility of consumption when old, thus inducing individuals to save more, rather than less.

As a child, an individual neither consumes nor works. The sole activity is to observe parents' consumption behavior and then inherit their tastes (exactly as in the model developed in the main text). Then, the habit stock when young, $h_{1,t}$, is an increasing function of parents' consumption when they were young and it is given by

$$h_{1,t} = h_1(c_{1,t-1}) = c_{1,t-1}. \quad \textbf{(B.2)}$$

The habit stock when old, $h_{2,t+1}$, is an increasing function of the parents' old-age consumption, $c_{2,t}$, and the agent's own consumption in the previous period, $c_{1,t}$. The former dependency captures a taste externality (external habit) that works out also in the second period of life. The latter dependency, instead, is the typical case of internal habits studied by Alonso-Carrera et al. (2007). As habits enter additively in the utility function, we consider the following formulation with regard to the habit stock when old:

$$h_{2,t+1} = h_2(c_{1,t}, c_{2,t}) = (1-\eta)c_{2,t} + \frac{\eta}{\gamma_2} c_{1,t}, \quad \textbf{(B.3)}$$

where $\eta = \{0, 1\}$ is a dummy variable that denotes the strength of the intragenerational consumption habits (denoting the internal habit formation) relative to the intergenerational taste externality effect. When $\eta = 0$ (resp. $\eta = 1$) the habit stock when old depends only on the intertemporal taste inheritance (resp. internal habits). In the former (resp. latter) case, habits are an externality (resp. a control variable).

Case 1. Taste inheritance ($\eta = 0$). The constrained maximization of the expected lifetime utility function (B.1), where $c_{2,t}$ is taken as given by the representative individual, yields

$$s_t = \frac{p_t \left[w_t(1-\tau) - \gamma_1 h_{1,t} + \gamma_2 \frac{h_{2,t+1}}{R^e_{t+1}} \right]}{1+p_t}, \quad \textbf{(B.4)}$$

$$c_{1,t} = \frac{w_t(1-\tau) + p_t \left(\gamma_1 h_{1,t} - \gamma_2 \frac{h_{2,t+1}}{R^e_{t+1}} \right)}{1+p_t}, \quad \textbf{(B.5)}$$

$$c_{2,t+1} = \frac{R^e_{t+1} \left[w_t(1-\tau) - \gamma_1 h_{1,t} \right] + \gamma_2 h_{2,t+1}}{1+p_t}. \quad \textbf{(B.6)}$$

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Under this generalization, old-age aspirations—weighted by parameter γ_2—positively affect saving and consumption when old and negatively affect consumption when young, in contrast with the effects of young-age aspirations—weighted by parameter γ_1. By taking into account the market-clearing condition in the capital market, $k_{t+1} = s_t$, and knowing that equations $h_{1,t+1} = c_{1,t}$ and $h_{2,t+1} = c_{2,t}$, where $c_{2,t}$ is given by the one-period backward of (B.6), characterize the dynamics of consumption, the intertemporal equilibrium condition implies:

$$k_{t+1} = \frac{p_t}{1+p_t}[w_t(1-\tau) - \gamma_1 h_{1,t}] + \frac{p_t}{1+p_t}\gamma_2 \frac{h_{2,t+1}}{R^e_{t+1}}, \quad \textbf{(B.7)}$$

$$h_{1,t+1} = \frac{w_t(1-\tau) + p_t\gamma_1 h_{1,t}}{1+p_t} - \frac{p_t}{1+p_t}\gamma_2 \frac{h_{2,t+1}}{R^e_{t+1}}, \quad \textbf{(B.8)}$$

$$h_{2,t+1} = \frac{R_t}{1+p_{t-1}}\left[w_{t-1}(1-\tau) - \gamma_1 h_{1,t-1}\right] + \gamma_2 \frac{h_{2,t}}{1+p_{t-1}}. \quad \textbf{(B.9)}$$

Equations (B.7)–(B.9) can be used to characterize the dynamic system of the economy, which is a three-dimensional system comprised of nonlinear second-order difference equations, where $p_{t-1} = \frac{p_0 + p_1 \tau w_{t-1}}{1+\tau w_{t-1}}$, $w_{t-1} = (1-\alpha)Ak^\alpha_{t-1}$, and $R_t = \alpha A k_t^{\alpha-1}$. In addition, from (B.7) and (B.8) it emerges that, different from the case of the model outlined in the main text, the expected interest factor affects economic dynamics so that an explicit assumption about the expectation formation mechanism of the representative individual about future variables is indeed required. To this purpose, it is well known that $R^e_{t+1} = \alpha A k_t^{\alpha-1}$ if individuals have static expectations or $R^e_{t+1} = \alpha A k_{t+1}^{\alpha-1}$ if individuals have perfect foresight. In this case, the local and global dynamics of the model can be studied explicitly only under the assumption of static expectations. Generally speaking, the use of this kind of expectations in OLG macroeconomic models may be controversial, as it implies that individuals do not adjust information errors over time. In this particular case, the assumption of static expectations would render the results of the model developed in this appendix almost not comparable, both conceptually and in terms of intuition, to the ones analyzed and discussed in the main text, where individuals are assumed to have perfect foresight.

Case 2. Internal habits ($\eta = 1$). The constrained maximization of the expected lifetime utility function (B.1), where $c_{1,t}$ is a control variable, yields

$$s_t = \frac{p_t}{1+p_t}\left[\frac{1+p_t+R^e_{t+1}}{p_t+R^e_{t+1}} w_t(1-\tau) - \gamma_1 h_{1,t}\right], \quad \textbf{(B.10)}$$

$$c_{1,t} = \frac{R^e_{t+1} w_t(1-\tau) + p_t(p_t+R^e_{t+1})\gamma_1 h_{1,t}}{(1+p_t)(p_t+R^e_{t+1})}, \quad \textbf{(B.11)}$$

$$c_{2,t+1} = \frac{R^e_{t+1}}{1+p_t}\left[\frac{1+p_t+R^e_{t+1}}{p_t+R^e_{t+1}} w_t(1-\tau) - \gamma_1 h_{1,t}\right]. \quad \textbf{(B.12)}$$

Equations (B.10)–(B.11) can be used to characterize the dynamic system of the economy with both internal and external habits, which is a two-dimensional system comprised of nonlinear first-order difference equations. Similar to Case 1 above, however, we have to specify individual expectations. In particular, to study the dynamics of the model explicitly one has to account for the hypothesis of static expectations, which is subject to the critique outlined above.

For the previous reasons, we do not proceed further into the analysis of the models related to Case 1 and Case 2, even if we conjecture that in either cases the dynamics are more likely to become monotonic given that old-age aspirations and internal habits work out in the opposite of direction as young aspirations do on economic dynamics.