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Distance and Speed Errors in ARPA Systems

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Errors of different positioning algorithms for tracked targets used in the ARPA systems and their influence on estimated speed of targets is analysed. Examples show their significance in tracking targets at low ranges.

KEY WORDS

1. ARPA. 2. Radar. 3. Marine Navigation.

1. INTRODUCTION. Different approaches to positioning tracked targets exist in the various ARPA (Automatic Radar Plotting Aids) systems and they are not regulated in any performance requirements. In (Pedersen *et al*., 1999) this problem has been investigated and its influence on the errors of distance (DCPA) and time (TCPA) to the closest point of approach has been discussed. But there are also situations – e.g. ARPA accuracy tests (Lenart, 1989) or tracking stationary targets for auto drift (own ground speed and course made good) – when the speed error has equal or even greater importance, thus this paper is aimed mainly towards this part of the problem.

2. DISTANCE ERRORS. The point of minimum distance to the echo regardless of bearing is commonly used in manual radar plots as the range to the targets – as the most safe – but this is not the truth in automatic tracking as shown in (Pedersen *et al*., 1999). When the aspect angle approaches 90° this point rapidly moves toward the target's aft end and then causes the excessive errors in relative speed and then in DCPA and TCPA on the unsafe side. Therefore, for the analysis in this paper, distance errors are referred to the centre of bearing only. Figure 1 illustrates the situation simplified to two errors:

– the error between minimum distance (point 1) and the centre of radial size of an echo with the pulse length correction (point 2) which is assumed as true centre of the target,
– the error between the centre of radial size of an echo without the pulse length correction (point 3) and the centre with this correction (point 2).

The distance to the point 2 is given by equation:

$$R_2 = R_1 + \frac{R_5 - \Delta R_\tau - R_1}{2} \tag{1}$$

Therefore

$$\varDelta R_{12} = R_1 - R_2 = -\frac{R_5 - \varDelta R_\tau - R_1}{2} = -\frac{R_4 - R_1}{2} \tag{2}$$

and

$$\varDelta R_{32} = R_3 - R_2 = \frac{\varDelta R_\tau}{2} \tag{3}$$

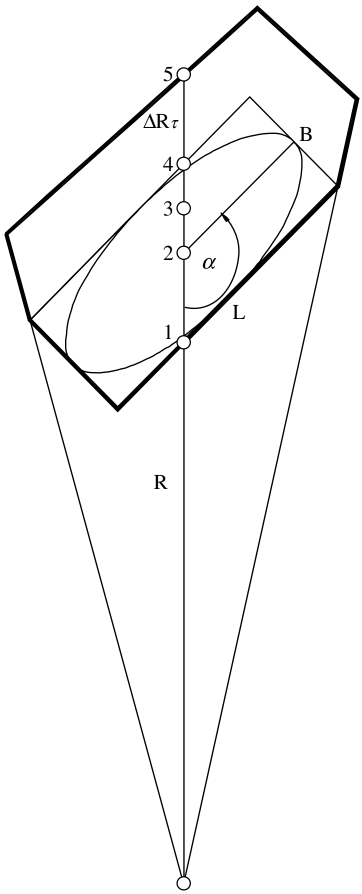

Figure 1.

It should be noted from equation (2) that the error $\varDelta R_{12}$ is equal to the half of the radial size of the target and it does not depend on the pulse length but just on the condition that this size is seen by the radar. This size depends on the aspect angle and can be calculated by a rectangular or an elliptical model.

2.1. *Rectangular model of $\varDelta R_{12}$.* It can be proven that in this model:

$$\begin{aligned} \varDelta R_{12r} &= -\frac{L}{2|cos\alpha|} \quad for \quad \alpha \leqslant tan^{-1}\frac{B}{L} \quad or \quad \alpha \geqslant 180^\circ - tan^{-1}\frac{B}{L} \\ \varDelta R_{12r} &= -\frac{B}{2|sin\alpha|} \quad for \quad \alpha > tan^{-1}\frac{B}{L} \quad or \quad \alpha < 180^\circ - tan^{-1}\frac{B}{L} \end{aligned} \tag{4}$$

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where: α is the aspect angle (for simplification in the range 0–180°), L is the length and B the breadth of the target.

In this rectangular model ΔR_{12r} changes from $-B/2$ for $\alpha=90^\circ$ to $-\sqrt{L^2+B^2}/2$ for $\alpha=tan^{-1}(B/L)$ *or* $\alpha=180^\circ-tan^{-1}(B/L)$.

2.2. *Elliptical model of* ΔR_{12}. It can be proven that in this model:

$$\Delta R_{12e}=-\frac{LB}{2\sqrt{(Lsin\alpha)^2+(Bcos\alpha)^2}} \tag{5}$$

In the elliptical model ΔR_{12e} changes from $-B/2$ for $\alpha=90^\circ$ to $-L/2$ for $\alpha=0^\circ$ or 180°.

For $L \gg B$ these two models give small differences in results for $\alpha \in (20^\circ, 160^\circ)$, but different models could be used e.g. for the aft and the forward part of the target.

2.3. *Pulse length error* ΔR_{32}. This error is not significant when we use the minimum distance algorithm, but it is significant when we calculate the distance to the target as the distance to the centre of the radial size of the target's echo. When our radar sees the radial size of the target our radar pulse length, irrespective of the aspect angle, extends this size. Since our ARPA can know our radar pulse length this extension can be corrected by subtracting from the radial size of the echo. But if this correction is not done the distance to the target will have the error given by equation (3). Bearing in mind that

$$\Delta R_\tau=150\ \mathrm{m}\cdot\tau \tag{6}$$

where: - τ - pulse length [μs],
substitution into equation (3) yields

$$\Delta R_{32}=75\,\mathrm{m}\cdot\tau \tag{7}$$

For example: $\Delta R_{32}\approx 19$ m for $\tau=0{\cdot}25$ μs and $\Delta R_{32}=75$ m for $\tau=1$ μs.

It should be emphasized that shorter pulse lengths should not be used by ARPAs, even for low range targets, because of the need to maintain the required tracking range for all targets of not less than 12 nm.

3. SPEED ERRORS.

Distance errors are systematic and with the other systematic distance errors (e.g. calibration errors) they influence the bearing component of targets' relative speed estimated by an ARPA system that only tracks targets' consecutive positions (see Figure 2).

For a small change of the tracked target's bearing $\Delta\beta$ the target travels a distance s but the ARPA tracks a distance $s_{\Delta R}$. Hence

$$s=R\cdot\Delta\beta \tag{8}$$

$$s_{\Delta R}=(R+\Delta R)\cdot\Delta\beta \tag{9}$$

$$\Delta\beta=\frac{s}{R}=\frac{s_{\Delta R}}{R+\Delta R} \tag{10}$$

and then

$$s_{\Delta R}=s\cdot\frac{R+\Delta R}{R}=s\cdot\left(1+\frac{\Delta R}{R}\right) \tag{11}$$

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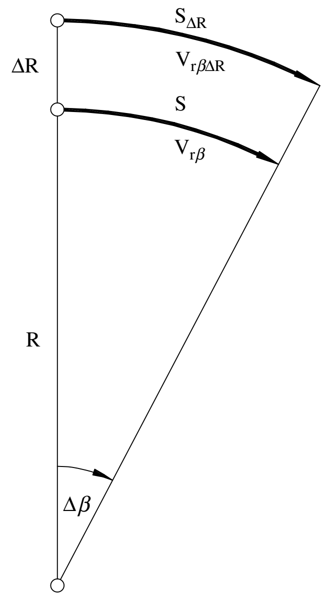

Figure 2.

The ARPA calculates the speed differentiating a distance in time

$$\frac{ds_{\Delta R}}{dt} = \frac{ds}{dt}\left(1 + \frac{\Delta R}{R}\right) \tag{12}$$

and finally

$$V_{r\beta\Delta R} = V_{r\beta}\left(1 + \frac{\Delta R}{R}\right) \tag{13}$$

or

$$\Delta V_{r\beta\Delta R} = V_{r\beta} \cdot \frac{\Delta R}{R} \tag{14}$$

The above equations reveal that the speed error is the hyperbolic function of distance *R*, and this error rapidly increases for targets tracked at low ranges.

4. EXAMPLES.

4.1. *IMO/IEC tests*. The ARPA systems should conform to IMO performance standards (IMO, 1995) and IEC methods of testing (IEC, 1987). During these tests the accuracy of tracked targets' speeds, courses, DCPA and TCPA is checked (Lenart, 1989).

In scenario 2 (see Figure 3):

Own ship course	$=0^\circ$,
Own ship speed (V)	$=10$ knots,
Target range (R)	$=1$ nm,
Bearing of target	$=0^\circ$,
Relative course of target	$=90^\circ$,
Relative speed of target (V_r)	$=10$ knots.

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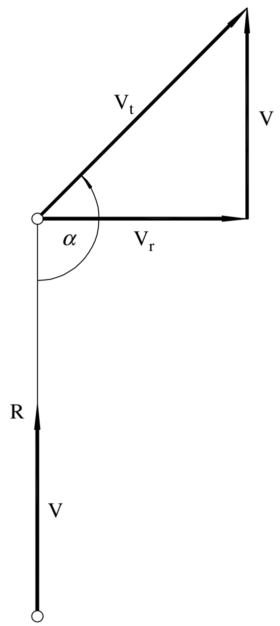

Figure 3.

The required accuracy of relative speed is 0·3 knots i.e. 0·3%. The true speed of target (V_t) is 14·1 knots and the true course of target is 045°. Therefore at CPA:

$R = 1$ nm,
$\alpha = 135°$,
$V_{r\beta} = V_r = 10$ knots.

For a typical target with $L = 200$ m and $B = 30$ m:

from equation (4)	$\Delta R_{12r} = -21{\cdot}2$ m,
from equation (5)	$\Delta R_{12e} = -21{\cdot}0$ m,
from equation (14)	$\Delta V_{r\beta\Delta R_{12}} = -0{\cdot}11\%$,
from equations (7) & (14)	$\Delta V_{r\beta\Delta R_{32}} = 0{\cdot}1$–$0{\cdot}4\%$ for $\tau = 0{\cdot}25$–1 μs.

It is evident that these errors are significant, and perhaps very significant, in comparison to the required accuracy (which is provided for stochastic estimation errors and not for constant systematic errors), moreover it should be appreciated that these errors will not be detected at all when a radar simulator used for these tests utilises the same algorithm for positioning the targets' echoes as the ARPA system under test.

4.2. *Auto drift*. ARPAs should be capable of sea and ground stabilization. The ground stabilization may be provided by tracking one or more stationary targets e.g. buoys or the other navigational marks, lightvessels or other anchored ships. Stationary targets are tracked to compensate own speed errors (offset and drift) to obtain own speed over the ground and course made good. The relative speed of a stationary target estimated by the ARPA is equal to our ground speed on the reciprocal-course.

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When tracking buoys, $\Delta R_{12} \approx 0$ due to their small dimensions, but ΔR_{32} at low ranges (e.g. tracking a buoy abeam 0·2 nm at a fairway) can cause errors $\Delta V_{r\beta} = \Delta V_r = 5\text{–}20\%$.

When tracking lightvessels, or the other anchored ships, there are possible situations when the target is abeam and its aspect angle is 0° or 180° (when ΔR_{12} achieves maximum $L/2$). For $L=200$ m and $R=0{\cdot}2$ nm $\Delta V_{r\beta} = \Delta V_r$ can achieve 27%.

5. CONCLUSIONS. The above examples demonstrate that the problem of ARPAs' distance errors and the resulting speed errors cannot be neglected and should be taken into account at all stages of development of an ARPA system – performance requirements, design, testing and operation. Moreover the same refers to radar simulators for testing and for training purposes.

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