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Erratum: On Regularity Lemmas and their Algorithmic Applications

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Our paper [3] contained two errors.

We retract Corollary 3.5. We claimed that it follows from a recent algorithmic version [1, 2] of the Frieze–Kannan regularity lemma (see Theorems 3.2 and Theorem 3.3 in [3]). Unfortunately there is an error in our application of these algorithmic results. Although the algorithms in [1, 2] can test whether a *partition* is weakly regular, it is unclear how to apply them to an intermediate step in our proposed algorithm. Specifically, we would need an algorithm that, given

$$G' = d(G) + c_1 K_{S_1,T_1} + \cdots + c_l K_{S_l,T_l},$$

either (1) correctly states that $d_\square(G,G') \leqslant \varepsilon$, or (2) outputs sets S and T such that $|e_G(S,T) - e_{G'}(S,T)| > \varepsilon^{-O(1)}$. In particular, the fact that G' may not have bounded weights presents a challenge in applying results from [1, 2].

A second error is in the proof of Theorem 1.4 in [3], which we also retract. We erred in applying a counting lemma [3, Lemma 4.1] to claim that if G', as above, approximates G in cut norm, then G and G' have similar H-densities. The counting lemma requires the edge-weights to be bounded by 1, which is not necessarily true in this case.

We have been able to fix Corollary 3.5. In our new paper [4], we prove a strengthened version of the algorithm whose running time has improved dependence on ε and n. Specifically, we prove the following theorem to replace Corollary 3.5.

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Theorem 1. *There is a deterministic algorithm that, given $\varepsilon > 0$ and an n-vertex graph G, outputs, in $\varepsilon^{-O(1)}n^2$ time, subsets $S_1, S_2, \dots, S_r, T_1, T_2, \dots, T_r \subseteq V(G)$ and*

$$c_1, c_2, \dots, c_r \in \left\{ -\frac{\varepsilon^8}{300}, \frac{\varepsilon^8}{300} \right\}$$

for some $r = O(\varepsilon^{-16})$, such that

$$d_\square(G, d(G) + c_1 K_{S_1,T_1} + \cdots + c_r K_{S_r,T_r}) \leqslant \varepsilon.$$

We have been able to salvage the following weaker result to replace Theorem 1.4 in [3] (with an improved dependence on n thanks to Theorem 1 above).

Theorem 2. *There is a deterministic algorithm that, given $\varepsilon > 0$, a graph H, and an n-vertex graph G, outputs, in $O(\varepsilon^{-O_H(1)}n^2)$ time, the number of copies of H in G up to an additive error of at most $\varepsilon n^{v(H)}$.*

See [4] for proofs and discussion.

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