J. Plasma Physics (1999), *vol.* 62, *part* 5, *pp.* 477–483 Printed in the United Kingdom

X-ray laser gain in a plasma channel

J. PARASHAR

Department of Physics, Samrat Ashok Technological Institute, Vidisha (M.P.) 464 001, India

(Received 24 August 1998 and in revised form 12 June 1999)

Abstract. A preformed plasma channel of light ion species, with a small percentage of high-Z atoms, can be produced by a laser prepulse. When an intense short laser pulse propagates through the channel, high-Z atoms are stripped of most of their electrons. After the passage of the pulse, recombination of electrons with high-Z ions produces population inversion for the generation of coherent X rays. However, the gain is strongly dependent on the radial coordinate r. A paraxial-ray theory of X-ray amplification in such a situation reveals that gain focusing and refraction guiding play an important role in determining the gain and the spot size of the X-ray laser.

1. Introduction

The X-ray laser has evolved into a fascinating concept over the last two decades (Burnett and Corkum 1989; Elton 1990; Leemans et al. 1992; Ma and Tan 1995; Dannelly et al. 1995; Krushelnick et al. 1996). It involves rapid ionization of high-atomic-number (Z) atoms to a very high charge state by an intense short laser pulse. Electron–ion recombination after the passage of the laser pulse causes population inversion in hydrogen-, helium- or neon-like high-Z atoms that eventually leads to stimulated emission of coherent X rays (Murnane et al. 1991; Eder et al. 1992; Midorikawa et al. 1995; Workman et al. 1996). One of the key issues in attempts to achieve rapid tunnel ionization of high-Z atoms is to focus the main laser pulse to a small cross-section and guide it to long distances without it suffering diffraction divergence (Lunney 1986). Durfee et al. (1995) have succeeded in producing a long plasma channel, by employing a laser prepulse, that guides the main pulse to more than 24 Rayleigh lengths. The channel is also capable of guiding X-ray radiation that could be produced in the channel.

In this paper, we examine the issue of X-ray laser gain in a plasma channel. A plasma channel of lighter ion species has a small percentage of high-Z atoms embedded in it (Sprangle et al. 1997). When the main laser pulse propagates through the channel, the high-Z atoms are rapidly ionized to a high charge state. However, the number density of such ionized atoms falls rapidly with r. After the main laser pulse has passed, electron–ion recombination causes population inversion that is maximum on the axis and falls off away from it. The X-ray laser gain in such a situation is strongly dependent on the transverse (radial) coordinate. The non-uniform gain, in conjunction with a refractive index profile having a maximum on the axis, may provide strong guiding for the X-ray laser.

In Sec. 2, we model the X-ray laser gain by an effective complex permittivity

ϵ_{eff} of the plasma. We choose the radial profiles of real and imaginary parts of ϵ_{eff} to be parabolic in the radial coordinate r, in the paraxial-ray approximation. In Sec. 3, we solve the wave equation for the X-ray laser in two dimensions, employing WKB and paraxial-ray approximations (Liu and Tripathi 1994). We obtain two coupled equations for the dimensionless spot size f and the exponential gain factor γ. The set of equations is solved numerically. A discussion of the results is given in Sec. 4.

2. Effective complex permittivity

Consider a situation where a plasma is produced by a succession of two laser pulses, namely a laser prepulse and a main pulse following after a time delay of a few nanoseconds in a gas comprising light atoms and a small fraction of heavy atoms. The prepulse ionizes the light atoms fully and the heavy atoms partially. During the time interval between the pulses, the plasma expands radially, forming a channel with minimum electron density on the axis. The main pulse ionizes the heavy atoms to a much higher charge state. After the passage of the pulse, recombination leads to high-Z ions that can undergo stimulated transitions from a higher excited state to a lower state.

Let the electron density profile of the plasma be parabolic in the radial distance r:

$$n_0 = n_0^0\left(1+\frac{r^2}{a^2}\right),$$

where n_0^0 is the axial density and a is the scale length of density variation, which may be of the order of the spot size of the laser pulses. Let the energy levels of the heavy ions relevant to the X-ray lasing transition be E_2 and E_1. Let the heavy-ion densities in these states be n_2 and n_1 and let the line-shape function be $g(\omega)$, where $\omega = (E_2-E_1)/h$ is the frequency of the X-ray laser. The local gain function γ in such a system is

$$\gamma = B_{21}\, g(\omega)\, (N_2-N_1)\frac{h\omega}{c}\eta_0,$$

where B_{21} is the Einstein coefficient of stimulated downward transition from state E_2 to E_1, η_0 is the refractive index of the medium and c is the velocity of light in vacuum. Physically, γ implies that if the population inversion density N_2-N_1 is uniform and an X-ray signal of intensity I_0 is launched at $z=0$ along $\hat{\mathbf{z}}$ then its intensity at z will be $I = I_0\, e^{\gamma z}$. The electric field of the X-ray laser may be written as

$$\mathbf{E} = \hat{\mathbf{x}} A \exp[-i(\omega t - kz)],$$

where $k = k_r - i\gamma/2$. One may define the effective plasma permittivity as

$$\epsilon_{\text{eff}} = \epsilon_r + i\epsilon_i = \frac{c^2k^2}{\omega^2},$$

with

$$\epsilon_r = 1-\frac{\omega_p^2}{\omega^2},$$

$$\epsilon_i = \frac{\gamma c \epsilon_r^{1/2}}{\omega} \approx \frac{\gamma c}{\omega}.$$

In a medium with non-uniform gain, we choose the γ profile as

$$\gamma = \gamma_0\left(1-\frac{r^2}{b^2}\right) = \gamma_0-\gamma_2 r^2, \tag{1}$$

where $b < a$ and $\gamma_2 = \gamma_0/b^2$. Following Parashar et al. (1997), the width of the γ profile, for a Gaussian intensity profile of the main laser pulse, $E_0 = E_{00}\exp(-r^2/2a^2)$, may be estimated as

$$b = a\exp\left(-\frac{E_a}{E_{00}}\right),$$

where E_a is the Coulomb field of the high-Z atom (Burnett and Corkum 1989; Elton 1990). The ω_p profile may be written as

$$\omega_p^2 = \omega_{p0}^2\left(1+\frac{r^2}{a^2}\right). \tag{2}$$

The gain and ω_p profiles given by (1) and (2) represent the experimental profiles observed by Milchberg et al. (1995) for Ne-like Ar.

3. Self-focusing and gain

The wave equation governing the propagation of an X-ray laser beam in a radially inhomogeneous plasma is

$$\nabla^2\mathbf{E}+\frac{\omega^2}{c^2}\epsilon_{\text{eff}}\mathbf{E} = 0. \tag{3}$$

We write

$$\mathbf{E} = \hat{\mathbf{x}}A(r,z)e^{\int\gamma_1 dz/2}e^{-i(\omega t-k_r z)}, \tag{4}$$

where $\gamma_1(z)$ is an effective gain function to be found later,

$$k_r = \frac{\omega}{r}\epsilon_0^{1/2}, \qquad \epsilon_0 = 1-\frac{\omega_{p0}^2}{\omega^2}.$$

Using (4) in (3), we obtain, in the WKB approximation ($\partial^2 A/\partial z^2 \ll k_z\,\partial A_z/\partial z$),

$$(2ik_r+\gamma_1)\frac{\partial A}{\partial z}+\frac{\partial^2 A}{\partial r^2}+\frac{1}{r}\frac{\partial A}{\partial r}+\frac{\omega^2}{c^2}\left(-\epsilon_2 r^2+i\frac{\gamma_2 c}{\omega}r^2\right)A$$
$$+ik_r(\gamma_1-\gamma_0)A+\frac{1}{2}\frac{d\gamma_1}{dz}A = 0, \tag{5}$$

where $\epsilon_2 = \omega_{p0}^2/\omega^2 a^2$. We introduce an eikonal

$$A = A_0(r,z)e^{iS(r,z)} \tag{6}$$

and separate the real and imaginary parts of (5):

$$-2k_r\frac{\partial S}{\partial z}A_0+\frac{\partial^2 A_0}{\partial r^2}+\frac{1}{r}\frac{\partial A_0}{\partial r}-\left(\frac{\partial S}{\partial r}\right)^2 A_0+\gamma_1\frac{\partial A_0}{\partial z}+\frac{1}{2}\frac{d\gamma_1}{dz}A_0-\frac{\omega^2}{c^2}\epsilon_2 r^2 A_0 = 0, \tag{7}$$

$$k_r \frac{\partial A_0^2}{\partial z} + \gamma_0 \frac{\partial S}{\partial z} A_0^2 + \left(\frac{\partial^2 S}{\partial r^2} + \frac{1}{r}\frac{\partial S}{\partial r}\right) A_0^2 + \frac{\partial S}{\partial r}\frac{\partial A_0^2}{\partial r} + k_r(\gamma_1 - \gamma_0) A_0^2 + \frac{\omega \gamma_2}{c} r^2 A_0^2 = 0. \quad (8)$$

We assume a Gaussian ansatz for A_0^2:

$$A_0^2 = \frac{A_{00}^2}{f^2} \exp\left(-\frac{r^2}{r_0^2 f^2}\right). \quad (9)$$

Then (7) and (8) take the forms

$$\frac{\gamma_0}{A_0}\frac{\partial A_0}{\partial z} - 2k_r \frac{\partial S}{\partial z} + \left(-\frac{2}{r_0^2 f^2} + \frac{r^2}{r_0^4 f^4}\right) - \left(\frac{\partial S}{\partial r}\right)^2 + \frac{1}{2}\frac{d\gamma_1}{dz} + \epsilon_2 \frac{\omega^2}{c^2} r^2 = 0, \quad (10)$$

$$\frac{k_r}{A_0^2}\frac{\partial A_0^2}{\partial z} + \gamma_0 \frac{\partial S}{\partial z} + \left(\frac{\partial^2 S}{\partial r^2} + \frac{1}{r}\frac{\partial S}{\partial r}\right) + \frac{\partial S}{\partial r}\left(-\frac{2r}{r_0^2 f^2}\right) + \gamma_2 \frac{\omega}{c} r^2 + k_r(\gamma_1 - \gamma_0) = 0. \quad (11)$$

We expand S, in the paraxial-ray approximation, as

$$S = \psi(z) + \beta(z)\frac{r^2}{2}. \quad (12)$$

Using (12) in (10) and (11), and collecting coefficients of various powers of r, we get

$$-\frac{\gamma}{f}\frac{df}{dz} - 2k_r \frac{d\psi}{dz} - \frac{2}{r_0^2 f^2} + \frac{1}{2}\frac{d\gamma_1}{dz} = 0, \quad (13)$$

$$\frac{\gamma_1}{r_0^2 f^3}\frac{df}{dz} - k_r \frac{d\beta}{dz} + \frac{1}{r_0^4 f^4} - \beta^2 + \frac{\omega^2}{c^2}\epsilon_2 = 0, \quad (14)$$

$$-\frac{2k_r}{f}\frac{df}{dz} + \gamma_1 \frac{d\psi}{dz} + 2\beta + k_r(\gamma_1 - \gamma_0) = 0, \quad (15)$$

$$\frac{2k_r}{r_0^2 f^3}\frac{df}{dz} + \frac{\gamma_1}{2}\frac{d\beta}{dz} - \frac{2\beta}{r_0^2 f^2} + \gamma_2 \frac{\omega}{c} = 0. \quad (16)$$

Multiplying (13) by γ_1 and (14) by $2k_r$ and adding the two, we obtain, in the limit $k_r^2 \gg \gamma_1^2$,

$$\beta = \frac{k_r}{f}\frac{df}{dz} - \frac{1}{2}k_r(\gamma_1 - \gamma_0). \quad (17)$$

Using (17) in (16), we obtain, in the limit $r_0^2 \ll R_d^2$ (where $R_d = k_r r_0^2$ is the Rayleigh diffraction length) and $\gamma_1^2 \ll (\gamma_1 - \gamma_0)k_r$,

$$\gamma_1 - \gamma_0 \approx -\gamma_2 r_0^2 f^2. \quad (18)$$

Equation (14) can be written as

$$\frac{d^2 f}{d\xi^2} + \frac{\gamma_2 c r_0^2}{\omega}\left(\frac{\omega r_0}{c}\right)^2 \frac{df}{d\xi} = \frac{1}{f^3} - \epsilon_2 r_0^2 \left(\frac{\omega r_0}{c}\right)^2 f - \frac{1}{4}\frac{\gamma_2 c r_0^2}{\omega}\left(\frac{\omega r_0}{c}\right)^2 f^5, \quad (19)$$

where $\xi = z/R_d$. For an initially plane wavefront, the initial conditions at

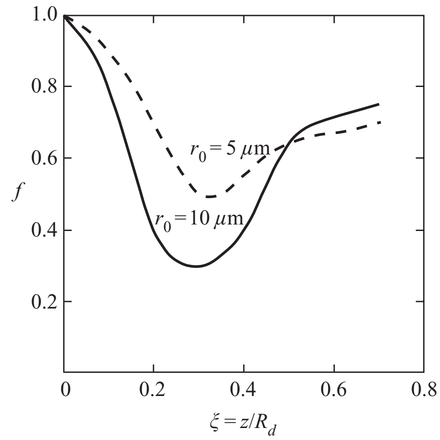

Figure 1. Variation of beamwidth parameter f with ξ.

$\xi = 0$ are $f = 1$ and $df/d\xi = 0$. The first term on the right-hand side of (19) is due to diffraction divergence, the second is due to refraction, and the third is due to gain focusing.

We have solved (19) for the following typical set of parameters (Milchberg et al. 1995):

$$\omega_p = 3.5\times10^{14}\,\mathrm{rad\,s^{-1}} \quad (\text{electron density} \approx 5\times10^{19}\,\mathrm{cm^{-3}}),$$

$$\omega = 8\times10^{16}\,\mathrm{rad\,s^{-1}} \quad (\text{X-ray wavelength} = 247\ \text{Å}),$$

$$\epsilon_2 \approx 25\ \mathrm{cm^{-2}},$$

$$\frac{r_0\,\omega}{c} = 2.6\times10^3 \ \text{ and } \ 1.3\times10^3 \ \ (r_0 = 10\ \mu\mathrm{m}),$$

$$\gamma_2\, r_0^2 = 30 \ \text{ and } \ 7.5 \quad (r_0 = 10\ \mu\mathrm{m} \ \text{ and } \ 5\ \mu\mathrm{m})$$

and plotted f as a function of ξ in Fig. 1. f falls off with ξ more rapidly when r_0 is large. It acquires a minimum $f_{\min}$ and then rises again. At the focus, the radius of the X-ray laser beam, $r_0 f_{\min}$, is nearly 3 μm, for both values of $r_0 =$ 10 μm and 5 μm.

4. Discussion

A preformed plasma channel has two advantages for X-ray laser gain. First, it focuses and guides the intense short-pulse laser. Secondly, the X-ray laser beam is strongly localized near the axial region. Gain focusing appears to be stronger than refraction guiding. The beam spot size at the focus is weakly dependent on the initial radius of the X-ray laser. The axial intensity of the X-ray laser goes with z as

$$I = \frac{I_0}{f^2}\exp\left(\gamma_0\, z - \gamma_2\, r_0^2 \int f^2\, dz\right).$$

In Fig. 2 we have plotted the axial gain

$$G = 10\log_{10}\left(\frac{I}{I_0}\right) = 4\left(\gamma_0\, z - \gamma_2\, r_0^2 \int_0^z f^2\, dz\right) + 20\log_{10} f$$

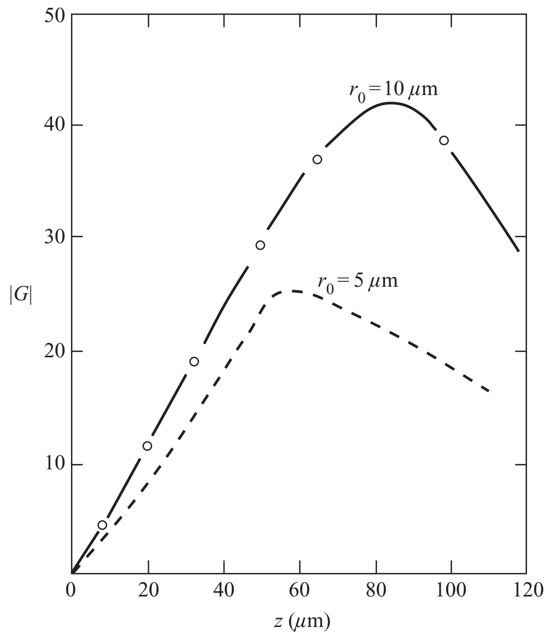

Figure 2. Variation of axial gain G with propagation z distance.

of the X-ray laser. Collisions can be ignored as long as the self-focusing length is longer than the absorption length

$$k_i^{-1} \approx \left(\frac{\nu}{2c}\frac{\omega_p^2}{\omega^2}\right)^{-1}.$$

The scattering can be ignored as long as the level of density fluctuations is low. Here we have assumed the density of high-Z atoms to be less than a few percent of the majority ions of the plasma. The latter have a parabolic density profile, and cause focusing of the X-ray laser.

Acknowledgement

This work is supported by the Department of Science and Technology, Government of India under the Young Scientist Scheme.

References

Burnett, N. H. and Corkum, P. B. 1989 *J. Opt. Soc. Am.* **B6**, 1195.

Dannelly, T. D., Da Silva, L., Lee, R. W., Mrowka, S., Hofer, M. and Falcone, R. W. 1996 *J. Opt. Soc. Am.* **B13**, 185.

Durfee, C. G. III, Lynch, J. and Milchberg, H. M. 1995 *Phys. Rev.* **E51**, 2368.

Eder, D. C., Amendt, P. and Wilks, S. C. 1992 *Phys. Rev.* **A45**, 6761.

Elton, R. C. 1990 *X-Ray Lasers*. Academic Press, San Diego.

Krushelnick, K. M., Tighe, W. and Suckewer, S. 1996 *J. Opt. Soc. Am.* **B13**, 306.

Leemans, W. P., Clayton, C. E., Mori, W. B., Marsh, K. A., Kaw, P. K., Dyson, A. and Joshi, C. 1992 *Phys. Rev.* **A46**, 1091.

Liu, C. S. and Tripathi, V. K. 1994 *Interaction of Electromagnetic Waves with Electron Beams and Plasmas*. World Scientific, Singapore.

Lunney, J. G. 1986 *Appl. Phys. Lett.* **48**, 891.

Ma, G.-B. and Tan, W.-H. 1995 *Opt. Commun.* **119**, 125.

Midorikawa, K., Nagata, Y., Kubodera, S., Obara, M. and Toyoda, K. 1995 *IEEE J. Select. Top. Quantum Electron.* **1**, 931.

Milchberg, H. M., Durfee, C. G. III and Lynch, J. 1995 *J. Opt. Soc. Am.* **B12**, 731.

Murnane, M. M., Kapteyn, H. C. and Falcone, R. W. 1991 *Phys. Fluids* **B3**, 2409.

Parashar, J., Pandey, H. D. and Tripathi, V. K. 1997 *Phys. Plasmas* **4**, 3040.

Sprangle, P., Esarey, E. and Hafizi, B. 1997 *Phys. Rev. Lett.* **79**, 1046.

Workman, J., Maksimuchuk, A., Liu, X., Ellenberger, U., Coe, J. S., Chien, C.-Y. and Umstadter, D. 1996 *J. Opt. Soc. Am.* **B13**, 125.