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**Research Paper**

# Galaxy spin direction distribution in *HST* and SDSS show similar large-scale asymmetry

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## Abstract

Several recent observations using large data sets of galaxies showed non-random distribution of the spin directions of spiral galaxies, even when the galaxies are too far from each other to have gravitational interaction. Here, a data set of $\sim 8.7 \cdot 10^3$ spiral galaxies imaged by *Hubble Space Telescope* (*HST*) is used to test and profile a possible asymmetry between galaxy spin directions. The asymmetry between galaxies with opposite spin directions is compared to the asymmetry of galaxies from the Sloan Digital Sky Survey. The two data sets contain different galaxies at different redshift ranges, and each data set was annotated using a different annotation method. The results show that both data sets show a similar asymmetry in the COSMOS field, which is covered by both telescopes. Fitting the asymmetry of the galaxies to cosine dependence shows a dipole axis with probabilities of $\sim 2.8\sigma$ and $\sim 7.38\sigma$ in *HST* and SDSS, respectively. The most likely dipole axis identified in the *HST* galaxies is at $(\alpha = 78^{\circ}, \delta = 47^{\circ})$ and is well within the $1\sigma$ error range compared to the location of the most likely dipole axis in the SDSS galaxies with $z > 0.15$, identified at $(\alpha = 71^{\circ}, \delta = 61^{\circ})$.

**Keywords:** galaxy: general – galaxies: spiral

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## 1. Introduction

Recently, several experiments using large data sets of galaxies imaged by several different instruments have shown evidence of non-random distribution of the spin directions of spiral galaxies (Slosar et al. 2009; Longo 2011; Shamir 2012, 2013; Hoehn & Shamir 2014; Shamir 2016b, 2017a, 2017b, 2017c; Lee et al. 2019a, 2019b; Shamir 2019, 2020a, 2020b, 2020c). The asymmetry is reflected by differences in the number of galaxies with opposite spin directions (Shamir 2012, 2019, 2020c, 2019b), and it changes with the directions of observation (Shamir 2012) and the redshift (Shamir 2016a, 2019, 2020c). Other experiments showed differences in the brightness of the galaxies (Shamir 2016b, 2017a).

Early experiments used galaxies annotated manually by a large number of volunteers showed no statistically significant difference between the number of galaxies with opposite spin directions (Land et al. 2008). However, it was also found that volunteers annotating the same galaxies tended to classify elliptical galaxies with no apparent spin direction as spiral galaxies that spin clockwise, and therefore leading to a difference in the number of galaxies (Hayes, Davis, & Silva 2017). Another experiment that used manual analysis of the data was based on five undergraduate students annotating $\sim 1.5 \cdot 10^4$ galaxies. In that experiment, the galaxies were also mirrored in an attempt to correct for a possible human bias, and the results showed a difference of ~7% between the number of clockwise and counterclockwise galaxies (Longo 2011).

With the availability of very large astronomical databases, the ability to automate the annotation of the spin directions of spiral galaxies allowed to annotate far larger data sets. These large data sets of galaxies annotated by their spin direction can provide strong statistical signal and profile a possible asymmetry between galaxies with opposite spin directions. It should be noted that the advantage of eliminating the human perception bias is compromised when using machine learning for the annotation, since machine learning algorithms are based on 'ground truth' training data that are annotated manually, and the trained model can therefore still be biased by the data it were trained with.

By using model-driven automatic annotation algorithms (Shamir 2011b), large data sets of galaxies showed asymmetry between the number of galaxies with opposite spin directions and the asymmetry direction and magnitude change based on the direction of observation (Shamir 2012, 2019, 2020b, 2020c) and the redshift (Shamir 2016a, 2019, 2020c). The asymmetry was identified in data collected by the Sloan Digital Sky Survey (Shamir 2012, 2016b) and showed good agreement with the asymmetry identified in data collected by the Panoramic Survey Telescope and Rapid Response System (Shamir 2017a, 2020a, 2020c).

Experiments with smaller data sets annotated manually also showed patterns of spin directions of galaxies (Slosar et al. 2009), and alignment of spin directions was identified with quasars (Hutsemékers et al. 2014). More recently, consistency in spin directions was also observed with galaxies that are too distant from each other to have any kind of gravitational interactions (Lee et al. 2019b). These links are defined as 'mysterious', leading to the assumption of a link between galaxy rotation and the motion of the large-scale structure (Lee et al. 2019b).

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This paper shows an analysis of the asymmetry between galaxies with opposite spin directions observed when using spiral galaxies from different parts of the sky. The main data set used in this study is taken from *HST*, and the asymmetry in that data set is compared to the asymmetry in a galaxy data set from SDSS used in previous experiments (Shamir 2019, 2020c).

## 2. Data

The data set of spiral galaxies was taken from the Cosmic Assembly Near-infrared Deep Extragalactic Legacy Survey (Grogin et al. 2011; Koekemoer et al. 2011). The initial data set contained 114 529 galaxies taken from the Great Observatories Origins Deep Survey North (GOODS-N), the Great Observatories Origins Deep Survey South (GOODS-S), the Ultra Deep Survey (UDS), the Extended Groth Strip (EGS), and the Cosmic Evolution Survey (COSMOS) fields. The galaxy images were separated from the F814W band FITS images using the *mSubimage* tool included in the *Montage* package (Berriman et al. 2004) and were converted into 122 × 122 TIF (Tagged Image File) images.

The separation of the galaxies into galaxies with clockwise and counterclockwise spin directions was done manually. In previous experiments, automatic analysis was used (Shamir 2013, 2017b, 2019, 2020c). However, while model-driven automatic analysis is unbiased and capable of analysing very large databases, it is limited by its ability to classify all galaxies. Therefore, the spin direction of many galaxies cannot be determined, and these galaxies are excluded from the analysis. In sky surveys such as SDSS, the number of galaxies is high, and therefore sacrificing some of the galaxies still leaves a sufficient number of accurately annotated galaxies and does not affect the analysis as long as the algorithm is fully symmetric. However, the *HST* fields are far smaller than sky surveys such as SDSS, and sacrificing some of the galaxies can reduce the number of galaxies in the data set. Another reason for using manual annotation is to use an accurate method that is different from the methods used in previous experiments.

The annotation was done by first randomly mirroring half of the images and then identifying all galaxies with clockwise spin direction and separating them from the rest of the galaxies. Then, all galaxy images were mirrored, and the clockwise galaxies were again separated from the rest of the galaxies. Each of these two data sets was then inspected to ensure that all galaxies are classified correctly. In the end of the process, 200 galaxies with clockwise spin direction, 200 galaxies with counterclockwise spin direction, and 200 galaxies that their spin direction could not be determined were inspected carefully. The examination showed that all 600 galaxies were annotated correctly. That provided a very clean data set that is also symmetric in the annotations of the galaxies due to the random mirroring and the identification of just clockwise galaxies. But unlike previous data sets, it is also complete in the sense that all galaxies that their spin direction could be determined are indeed annotated. The process was labour-intensive and required ∼250 h of work to complete. It provided a clean data set of 8 690 galaxies with identifiable spin direction. The distribution of the galaxies in the different fields is summarised in Table 1. The Subaru g magnitude and the photometric redshift distribution of these galaxies are shown in Figure 1.

**Table 1.** The number of galaxies in each of the five fields.

| Field | Field centre (degrees) | # all galaxies | # Clockwise galaxies | # Counterclockwise galaxies |
|---|---|---|---|---|
| GOODS-N | 189.23,62.24 | 5 931 | 396 | 373 |
| GOODS-S | 53.12,−27.81 | 5 024 | 276 | 264 |
| COSMOS | 150.12,2.2 | 84 424 | 3 116 | 2 965 |
| UDS | 214.82,52.82 | 14 245 | 323 | 293 |
| EGS | 34.41,−5.2 | 4 905 | 355 | 329 |

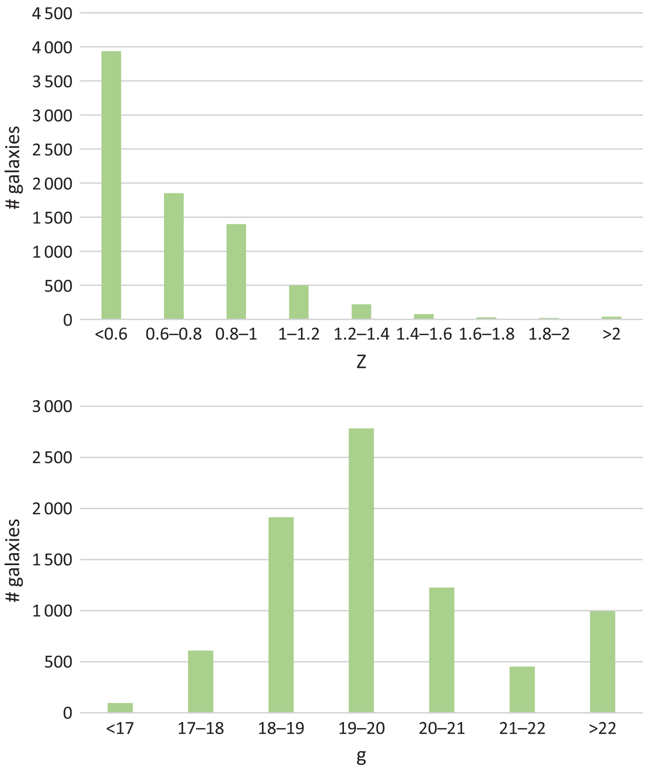

**Figure 1.** The redshift and g magnitude distribution of the *HST* galaxies.

The distribution of spin directions in the *HST* galaxies was compared to data sets of SDSS and Pan-STARRS galaxies that were used in previous experiments (Shamir 2017c, 2017b, 2019, 2020c). These data sets were annotated automatically by the Ganalyzer (Shamir 2011b, 2011a) algorithm. Ganalyzer is a model-driven algorithm that is based on clear and defined rules. It is not based on machine learning or deep neural networks, and therefore cannot be biased by the training set or by complex non-intuitive rules typical to machine learning systems. In addition to the theoretical analysis of the algorithm, it also showed empirical evidence obtained by mirroring a large number of galaxy images. Full details about the galaxy annotation method can be found in Shamir (2017a, 2017b, 2017c, 2020b), and the data set is described in Shamir (2020c).

## 3. Results

The distribution of galaxies in *HST* shows that the number of clockwise galaxies is higher, but the number of galaxies in the different fields is too low to allow statistical analysis. The only exception is the COSMOS field, where the number of galaxies is far higher than in any of the other *HST* fields used in this study. To compare the asymmetry in that field to galaxies imaged by SDSS and Pan-STARRS, the SDSS and Pan-STARRS galaxies in

**Table 2.** Number of clockwise and counterclockwise galaxies in the COSMOS field and in the $10^o \times 10^o$ field of SDSS and Pan-STARRS centred around COSMOS. The *P* value reflects the binomial probability of having asymmetry equal or greater than the observed asymmetry when assuming that a galaxy has 0.5 probability of having clockwise or counterclockwise spin direction. All of these data sets were annotated in an automatic process.

| Survey | # Clockwise galaxies | # Counterclockwise galaxies | *P* value |
|---|---|---|---|
| COSMOS | 3 116 | 2 965 | 0.027 |
| SDSS (Shamir 2017b) | 350 | 295 | 0.017 |
| SDSS (Shamir 2020c) | 461 | 440 | 0.24 |
| Pan-STARRS (Shamir 2020c) | 222 | 190 | 0.06 |

the $10 \times 10$ degrees around the centre of COSMOS were examined. The reason for using a larger field is because COSMOS is far deeper than SDSS and Pan-STARRS, and therefore SDSS and Pan-STARRS have a much smaller number of galaxies in a field of the same size. The difference between the size of the fields naturally makes the comparison indirect, as the fields being compared are different. But although the fields are not identical, such comparison can provide certain information regarding the agreement between the populations of galaxies in that part of the sky.

Data sets that were used in previous studies were examined, all of them were annotated automatically. These included a data set of SDSS (Shamir 2017b) and a data set of Pan-STARRS objects (Shamir 2020c). Because the data set used in Shamir (2017b) contained photometric objects of extended sources, some of the photometric measurements were made from photometric objects inside the same extended source. To avoid the presence of duplicate objects, all objects that had another object within 0.01 degrees or less were removed. Detailed information about these data sets and the distribution of redshift and magnitude of the galaxies they contain are described in the relevant papers (Shamir 2017a, 2017b, 2020c).

Table 2 shows the number of galaxies by their spin directions in each of the three instruments. As the table shows, all data sets show a higher number of clockwise galaxies in that field. The statistical significance is not strong in the Pan-STARRS field, as expected due to the lower number of galaxies compared to COSMOS, but these fields do not conflict with the distribution of galaxy population in COSMOS. Assuming equal probability of having clockwise and counterclockwise galaxies, the probability of having that asymmetry in all of these fields is $2 \times 0.027 \times 0.017 \times 0.06 \simeq 5 \cdot 10^{-5}$.

Previous experiments showed evidence of non-random patterns of the asymmetry between the number of galaxies with opposite spin directions in different parts of the sky (Shamir 2012, 2019, 2020c). That was done by identifying the $(\alpha, \delta)$ at which the asymmetry of the galaxy spin directions had best fit to cosine dependence. The *HST* data used in this experiment include several different fields in different parts of the sky. That allows to fit the distribution of the spin directions of these galaxies to cosine dependence. Fitting the galaxy spin directions to cosine dependence can indicate whether the galaxy spin directions are aligned in a form of a possible dipole axis and can also provide the statistical significance of such axis.

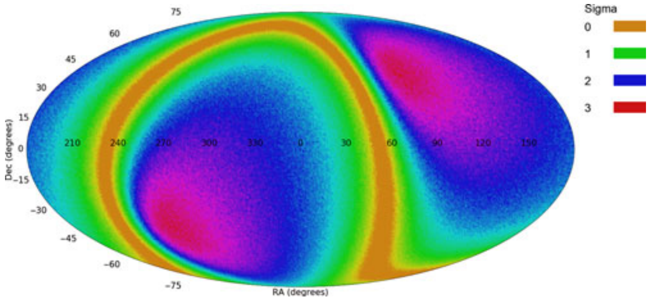

**Figure 2.** Probability of cosine dependence of the spin directions of *HST* galaxies from every possible integer $(\alpha, \delta)$ combination.

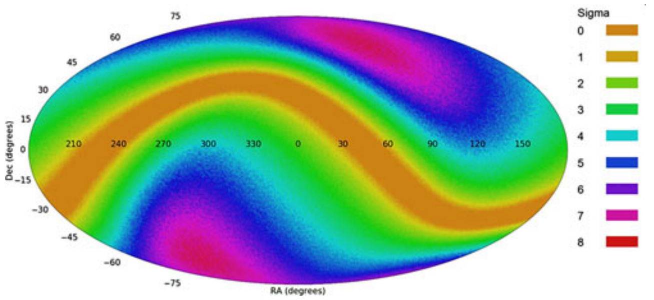

**Figure 3.** Cosine dependence probability of the spin directions of SDSS galaxies from every possible integer $(\alpha, \delta)$ combination.

To test the probability that the spin direction asymmetry exhibits a dipole axis, the same method used in (Shamir 2012, 2019, 2020c) was applied. Each galaxy was assigned with a value within the set $\{-1, 1\}$. Galaxies with clockwise spin direction were assigned with 1, and galaxies with counterclockwise spin direction were assigned with $-1$. Then, $\chi^2$ statistics was used such that for each possible integer $(\alpha, \delta)$ combination, the angular distance $\phi$ between $(\alpha, \delta)$ and the celestial coordinates of each galaxy in the data set was computed. The $\cos(\phi)$ of the galaxies were then fitted into $d \cdot |\cos(\phi)|$, such that $d$ is the spin direction of the galaxy (a value within the set $\{-1, 1\}$). The $\chi^2$ was computed 1 000 times such that in each time the galaxies were assigned with random spin directions, and the mean and standard deviation were computed for each possible $(\alpha, \delta)$. The mean $\chi^2$ computed with the random spin directions was then compared to the $\chi^2$ computed when $d$ was assigned the real spin directions. The $\sigma$ difference between the $\chi^2$ of the real spin directions and the mean $\chi^2$ when using the random spin directions shows the likelihood of an axis at $(\alpha, \delta)$. When the likelihood of all $(\alpha, \delta)$ was computed, the $(\alpha, \delta)$ of the most likely dipole axis could be identified. Figure 2 shows the probability of a dipole axis in all integer $(\alpha, \delta)$ combinations. The most likely axis was identified at $(\alpha = 78^o, \delta = 47^o)$, with probability of $\sim 2.83\sigma$. The $1\sigma$ error for that axis is $(58^o, 184^o)$ for the right ascension, and $(6^o, 73^o)$ for the declination.

The dipole axis identified in the *HST* galaxies was compared to the dipole axis identified in SDSS galaxies that were annotated automatically (Shamir 2020c). Figure 3 shows the probability of a dipole axis identified in each possible pair of integer $(\alpha, \delta)$ in the SDSS galaxies, when using the galaxies with $z > 0.15$ used in (Shamir 2020c). That data set included 15 863 galaxies annotated automatically by their spin direction. The most likely axis is identified at $(\alpha = 71^o, \delta = 61^o)$, with $\sigma \simeq 7.38$. That most likely axis is close to the most likely dipole axis identified in the *HST* galaxies, and well within the $1\sigma$ error. Figure 4 shows the most likely dipole axis when the galaxies are assigned with random spin directions. As expected, the dipole axis disappears when the galaxy spin directions are random.

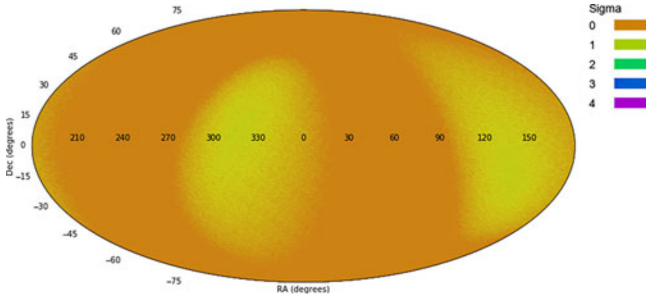

**Figure 4.** Probability of cosine dependence of the spin directions of SDSS galaxies from every possible integer $(\alpha, \delta)$ combination when the galaxies are assigned with random spin directions.

## 4. Conclusion

Results from two different data sets of galaxies imaged by two different instruments show similar asymmetry between galaxies with opposite spin directions. Each data set contains different galaxies, and the galaxies in each data set were annotated using a different method. Both data sets show a statistically significant dipole axis, and the location of the most likely axis is consistent in both data sets. Despite the difference in redshift, the two data sets show fairly similar location of the most likely dipole axis, and well within $1\sigma$ error.

While the observations are clearly provocative, it is difficult to identify an error that could lead to such results. The experiments are based on two different instruments and two different galaxy annotation methods. One of the instruments is space-based, reducing the possibility that the results are driven by an atmospheric effect. These results are consistent with previous similar experiments (Shamir 2013, 2016b, 2017a, 2017b, 2017c, 2019, 2020c). The automatic annotation method is model-driven, does not rely on machine learning, and consistent when the galaxy images are mirrored (Shamir 2017b). Previous experiments also showed that the asymmetry changes in different parts of the sky, which is not expected if the annotation method is biased (Shamir 2017a, 2019, 2020c).

It should be noted that while the vast majority of spiral galaxies are trailing, in some rare cases, galaxies are counter-winding (Grouchy et al. 2008). A small number of counter-winding galaxies can therefore lead to difference between the number of galaxies with opposite spin directions. However, if counter-winding galaxies are equally distributed between galaxies that spin clockwise and galaxies that spin counterclockwise, no statistically significant difference between the galaxies is expected. Therefore, to explain the observation with counter-winding galaxies, such galaxies need to have a certain preference based on the actual spin direction of the galaxy.

Analysis of the distribution of galaxies is limited by the fluctuations in large-scale galaxy population, known as 'cosmic variance' (Moster et al. 2011). However, here the measurement is a comparison between the number of galaxies with opposite spin directions identified inside the same exposures and same fields. It is therefore expected that fluctuations in galaxy population that affect the number of galaxies with a certain spin direction have the same impact on galaxies with the opposite spin direction. That reduces the possibility that the asymmetry is driven by changes in galaxy population, as any such change is expected to affect both clockwise and counterclockwise galaxies. This relative measurement is different from some other probes that are based on absolute measurements, such as the frequency of short GRBs or Ia supernovae. The use of a relative measurement can also handle effects such as Milky Way obstruction, as any obstruction that affects the ability to detect clockwise galaxies is expected to have a similar effect on the ability to detect counterclockwise galaxies in the same field.

It is naturally difficult to identify an immediate explanation for the observations. Lee et al. (2019b) identified consistency of spin directions of galaxies even if the galaxies are too far to interact gravitationally and defined the observation as 'mysterious' (Lee et al. 2019b). Explanations of the asymmetry can be related to parity-breaking gravitational waves, which can affect galaxy shape during inflation (Biagetti & Orlando 2020), and can provide an explanation to the asymmetry without violating the basic cosmological assumptions. Cosmological-scale anisotropy has been observed in the past with cosmic microwave background (Cline, Crotty, & Lesgourgues 2003; Gordon & Hu 2004; Zhe, Xin, & Sai 2015). These observations also challenge the basic cosmological assumptions and led to theories that differ from the standard cosmological models (Feng & Zhang 2003; Piao, Feng, & Zhang 2004; Rodrigues 2008; Piao 2005; Jiménez & Maroto 2007; Bohmer & Mota 2008). These observations also led to the model of ellipsoidal universe (Campanelli, Cea, & Tedesco 2006; Campanelli, Cea, & Tedesco 2007; Gruppuso 2007), as well as a rotating universe (Gödel 1949; Ozsváth & Schücking 1962; Ozsvath & Schücking 2001; Sivaram & Arun 2012; Chechin 2016).

Cosmological isotropy and homogeneity are basic assumptions used in most standard cosmological theories, although spatial homogeneity is an assumption that cannot be verified directly (Ellis 1979). Some evidence of cosmological isotropy violation has been observed through other messengers such as radio sources (Bengaly, Maartens, & Santos 2018), luminosity–temperature ratio (Migkas et al. 2020), short gamma ray bursts (Mészáros 2019), Ia supernova (Javanmardi et al. 2015), distribution of galaxy morphology (Javanmardi & Kroupa 2017), and cosmic microwave background (Aghanim et al. 2014; Hu & White 1997; Cooray, Melchiorri, & Silk 2003; Ben-David, Kovetz, & Itzhaki 2012; Eriksen et al. 2004). Future instruments such as the Earth-based Rubin observatory and the space-based Euclid can be used to validate whether the asymmetry is observed also in other instruments and provide better profiling of the asymmetry.

Given the multiple reports on anomaly in the distribution of galaxies with opposite spin patterns (Longo 2011; Shamir 2012, 2019; Lee et al. 2019b; Shamir 2020c), it is important to continue the examination of such observations, verifying and profiling the distribution, and identifying whether the reported observations can have non-astronomical explanations.

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