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OPTIMAL MONETARY POLICY AND FIRM ENTRY

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This paper characterizes optimal monetary policy in an economy with endogenous firm entry, a cash-in-advance constraint, and preset wages. Firms must make profits to cover entry costs; thus the markup on goods prices is efficient. However, because leisure is not priced at a markup, the consumption–leisure trade-off is distorted. Consequently, the real wage, hours, and production are suboptimally low. Because of the labor requirement for entry, insufficient labor supply also implies that entry is too low. This paper shows that in the absence of fiscal instruments such as labor income subsidies, the optimal monetary policy achieves higher welfare under sticky wages than under flexible wages. The policy maker uses the money supply instrument to raise the real wage—the cost of leisure—above its flexible-wage level, in response to expansionary shocks to productivity and entry costs. This increases labor supply, expanding production and firm entry.

Keywords: Entry, Optimal Monetary Policy, Sticky Wages

1. INTRODUCTION

The creation of new firms and products, also referred to as extensive margin investment, is of importance to macroeconomists and policy makers. It propagates and amplifies shocks; see Bergin and Corsetti (2008). Changes in product diversity have an impact on welfare and, if ignored, lead to mismeasurement of price indices; see Broda and Weinstein (2010). This paper asks whether, in the face of nominal rigidities, monetary policy should be concerned with movements in the number of firms.

The presence or absence of entry costs is crucial for the desirability of price markups and profits. If the number of producers is fixed or entry is costless, profits are regarded as an undesirable distortion. However, if firm entry is costly, profits provide compensation for startup costs and therefore an incentive to enter the market. The markup on goods prices is in fact efficient and not by itself distortionary. Rather, it is the absence of a markup on leisure that leads to a

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distortion of the leisure–consumption trade-off and, as a result, to a misallocation of resources. Hours are too low, and therefore too little is produced both at the intensive margin (production of goods) and at the extensive margin (creation of firms). Instead of removing the markup on goods prices, policy should aim at bringing the markup on the price of leisure to the same level. As shown by Bilbiie et al. (2008), efficiency can be restored through a labor income subsidy that aligns the two markups. The contribution of this paper is to show that under nominal rigidities, monetary policy can be used to mimic the effect of a labor subsidy by manipulating the real wage in response to shocks. The intuition is that the misalignment of the markups on leisure and on consumption goods implies that the real wage, the price of leisure, is inefficiently low.

The analysis is based on a stylized business cycle model with firm entry as the only form of investment. There are three distortions: monopolistic competition, a cash-in-advance (CIA) constraint, and preset wages.[1] Firms have monopoly power over the goods they produce. New firms are established up to the point where monopoly profits just cover entry costs, which are modeled as labor costs. The available (state-contingent) policy instruments are lump-sum taxes, the interest rate, and the money supply. Distortionary fiscal instruments are unavailable. The policy maker commits to state-contingent paths for the model variables that maximize welfare, taking as given the optimal decisions of households and firms. The main result is that optimal monetary policy achieves higher welfare under sticky wages than under flexible wages, through its influence on the real wage. A policy of raising the real wage above its flexible-wage level in response to expansionary shocks increases hours and expands both production and firm entry.

Differently from cashless economy models, money is not determined residually here. The CIA restriction on consumption purchases introduces a monetary friction and thus a role for money. Once the interest rate is set to its optimal level, the money supply can be used as a separate instrument for monetary policy. A zero net interest rate—the Friedman Rule—is optimal. This is a standard result in CIA models. Setting a higher interest rate taxes consumption relative to leisure, thereby worsening the aforementioned allocative distortion. Under the Friedman Rule, the role of the money supply policy depends on whether wages are sticky or flexible; under flexible wages, allocations are given and the money supply pins down the price level. Under sticky wages, given that the net interest rate is set to zero, the money supply policy affects allocations.

A closely related study is that of Adão et al. (2003), hereafter ACT, who consider optimal monetary policy in an economy with sticky prices in which fiscal policy is restricted to lump-sum taxation. They find that the optimal allocation under nominal rigidities and the flexible allocation are not the same when government spending shocks are taken into consideration. There are three notable differences between this paper and ACT. First, investment and entry are absent and monopolistic markups are inefficient in ACT. Second, this paper abstracts from government spending shocks, whereas in ACT, such shocks are the main driver of the result that replicating the flexible-price equilibrium is not welfare-maximizing. This paper instead highlights the role of entry costs in rendering the flexible equilibrium inefficient and shows how this inefficiency can be addressed with monetary policy. Third, ACT assume price stickiness, whereas in this paper, prices are flexible and wages are sticky.

Berentsen and Waller (2009) analyze optimal monetary policy in a model with endogenous entry and a microfounded demand for money. They find that the Friedman Rule is optimal if the entry cost is modeled as a fixed cost.[2] In sticky-price models with endogenous firm entry, Bergin and Corsetti (2008) and Bilbiie et al. (2007) find that it is optimal to stabilize goods prices fully, i.e., to replicate the flexible-price solution, while letting the number of firms fluctuate freely. In both studies, monetary frictions are ignored and appropriate fiscal policies ensure that the flexible-price allocation is efficient. This paper instead considers monetary policy as a tool to stabilize fluctuations around a distorted steady state, i.e., in the absence of short-run fiscal policy. Faia (in press) studies optimal monetary policy in a model of entry and oligopolistic competition. Finally, Bilbiie et al. (2011) describe the implications of endogenous product variety on the optimal long run inflation rate.

2. MODEL

The economy is initially in a state of nature, denoted by s^0. Thereafter, it is hit by a series of stochastic i.i.d. shocks to entry costs and to productivity. Every variable determined at time t is indexed by the history of shocks that have occurred up to t, denoted by s^t. Let S^t be the set of possible state histories. The probability of observing a particular history is denoted by $\Pr(s^t)$.

2.1. Final Goods Sector

There is a mass $N(s^t)$ of differentiated intermediate goods, each produced by a monopolistically competitive firm. A firm is indexed by $f \in [0, N(s^t)]$. A final goods firm bundles these intermediate goods $Y(f, s^t)$, taking as given their price $P(f, s^t)$, and sells the output $Y(s^t)$ to consumers at the competitive price $P(s^t)$. The optimization problem of the final goods firm is to choose the amount of inputs that maximizes profits; i.e., it solves

$$\max_{Y(f,s^t)_{f\in[0,N(s^t)]}} \left[P\left(s^t\right) Y\left(s^t\right) - \int_0^{N(s^t)} Y\left(f, s^t\right) P\left(f, s^t\right) df \right],$$

subject to the Dixit–Stiglitz (1977) production function

$$Y\left(s^t\right) = \left[\int_0^{N(s^t)} Y\left(f, s^t\right)^{\frac{\theta-1}{\theta}} df \right]^{\frac{\theta}{\theta-1}}, \qquad (1)$$

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where $\theta > 1$ is the elasticity of substitution between intermediate goods. The first-order condition gives the input demand function

$$Y\left(f, s^t\right) = \left[\frac{P\left(f, s^t\right)}{P\left(s^t\right)}\right]^{-\theta} Y\left(s^t\right). \tag{2}$$

Substituting the input demand into the production function yields the price index

$$P\left(s^t\right) = \left[\int_0^{N\left(s^t\right)} P\left(f, s^t\right)^{1-\theta} df\right]^{\frac{1}{1-\theta}}.$$

2.2. Intermediate Goods Sector

Intermediate firms use labor $L_c(s^t)$ to produce differentiated goods. They set prices to maximize profits,

$$P\left(f, s^t\right) Y\left(f, s^t\right) - W\left(s^t\right) L_c\left(s^t\right),$$

subject to the demand function given by (2) and the production function

$$Y\left(f, s^t\right) = Z\left(s^t\right) L_c\left(s^t\right), \tag{3}$$

where $W(s^t)$ is the wage rate and labor productivity $Z(s^t)$ is exogenous with positive support. The optimal price is a constant[3] markup over marginal cost,

$$P\left(f, s^t\right) = \frac{\theta}{\theta - 1} \frac{W\left(s^t\right)}{Z\left(s^t\right)}. \tag{4}$$

Profits are a constant fraction of firm revenue,

$$D\left(f, s^t\right) = \frac{1}{\theta} P\left(f, s^t\right) Y\left(f, s^t\right). \tag{5}$$

2.3. Returns to Product Diversity and Marginal Rate of Transformation

Under endogenous firm entry, the Dixit–Stiglitz (1977) aggregator (1) exhibits increasing returns to product diversity. This has implications for how, in the aggregate, inputs are converted into final output. To understand the first-best efficiency conditions presented in the next section, I now derive the marginal rate of transformation (MRT) for this economy.

The symmetry of the intermediate firms' output levels implies that the production function of the final goods firm reduces to

$$Y\left(s^t\right) = N\left(s^t\right)^{\frac{1}{\theta-1}} \left[N\left(s^t\right) Y\left(f, s^t\right)\right]. \tag{6}$$

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From (6) we see that $\frac{1}{\theta-1}$ represents the degree of increasing returns to product diversity. If $\frac{1}{\theta-1} > 0$, there are increasing returns to product diversity, which is true for $\theta > 1$, as Dixit and Stiglitz (1977) assume. As $\theta \to \infty$, i.e., as the elasticity of substitution between inputs into final good production increases, the degree of increasing returns to product diversity diminishes. See also Kim (2004). The symmetry of the intermediate goods prices implies that the aggregate price index is

$$P\left(s^t\right) = P\left(f, s^t\right) N\left(s^t\right)^{-\frac{1}{\theta-1}}. \tag{7}$$

The price index is decreasing in the number of differentiated goods. As the number of goods rises, it becomes less costly to produce the same amount of final output.

Next, I derive an aggregate production function for this economy by combining the production function of the intermediate goods firms (3) with the production function of the final goods firm under symmetry (6),

$$Y\left(s^t\right) = N\left(s^t\right)^{\frac{1}{\theta-1}} Z\left(s^t\right) L_C\left(s^t\right), \tag{8}$$

where $L_C(s^t) = N(s^t)L_c(s^t)$ is total labor used in the production of goods. Differentiating (8) with respect to labor, we have

$$\frac{\partial Y\left(s^t\right)}{\partial L_C\left(s^t\right)} = N\left(s^t\right)^{\frac{1}{\theta-1}} Z\left(s^t\right). \tag{9}$$

One additional labor unit is transformed into $N(s^t)^{\frac{1}{\theta-1}} Z(s^t)$ units of the final good. In the standard model with a constant number of firms, the MRT is simply equal to productivity $Z(s^t)$. Thus the aggregate and firm-specific marginal rates of transformation are the same. Here, the aggregate MRT contains the endogenous term $N(s^t)^{\frac{1}{\theta-1}}$, owing to the increasing returns to product diversity. Raising the number of firms by one unit gives rise to a positive externality on final output.

We can rewrite the aggregate production function (8) to express the economy-wide effective labor requirement as

$$Z\left(s^t\right) L_C\left(s^t\right) = N\left(s^t\right)^{-\frac{1}{\theta-1}} Y\left(s^t\right).$$

The reduction in the labor requirement from increasing the number of firms is then obtained by differentiating this expression with respect to $N(s^t)$:

$$\frac{\partial Z\left(s^t\right) L_C\left(s^t\right)}{\partial N\left(s^t\right)} = -\frac{1}{\theta-1} N\left(s^t\right)^{-\frac{1}{\theta-1}-1} Y\left(s^t\right). \tag{10}$$

2.4. Firm Entry

Starting up a firm requires labor services $L_f(s^t)$. Let $F(s^t)$ denote the entry cost in the form of effective labor units $Z(s^t)L_f(s^t)$. The variable $F(s^t)$ is exogenous and has positive support. In nominal terms, the entry cost is

$$\frac{W\left(s^t\right) F\left(s^t\right)}{Z\left(s^t\right)}.$$

Households finance the entry costs incurred by new firms in exchange for claims on those firms' profits. Firms must pay the entry cost anew each period.

2.5. Households

There exists a continuum of households of measure 1. As in Erceg et al. (2000), each household, indexed by $h \in [0, 1]$, supplies a differentiated labor type to a competitive labor packer, who produces a labor bundle subject to the production function

$$L\left(s^t\right) = \left[\int_0^1 L(h, s^t)^{\frac{\phi-1}{\phi}} dh\right]^{\frac{\phi}{\phi-1}}, \quad \phi > 1,$$

and sells it to intermediate firms and to entrants at a price $W(s^t)$.[4]

Households choose paths for consumption $C(s^t)$, wages, and asset holdings to maximize expected lifetime utility,

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \Pr\left(s^t\right) \left\{U\left[C\left(s^t\right)\right] - V\left[L\left(s^t\right)\right]\right\},$$

subject to a sequence of budget constraints explained later, labor demand from the labor packer $L(h, s^t) = [W(h, s^t)/W(s^t)]^{-\phi} L(s^t)$, and a CIA constraint,

$$P\left(s^t\right) C\left(s^t\right) \leq M\left(s^t\right). \tag{11}$$

The parameter β is the households' subjective discount factor. $U(\cdot)$ is strictly increasing and concave, and $V(\cdot)$ is strictly increasing and convex. At the start of period t, households make a portfolio allocation decision in the asset market facing the constraint

$$\mathcal{W}\left(s^t\right) \geq M\left(s^t\right) + B\left(s^t\right) + \sum_{s^{t+1}|s^t} Q\left(s^{t+1}|s^t\right) A\left(s^t, s^{t+1}\right)$$
$$+ \int_0^{N(s^t)} S\left(f, s^t\right) \frac{W\left(s^t\right) F\left(s^t\right)}{Z\left(s^t\right)} df - X\left(s^t\right). \tag{12}$$

Total household wealth in currency terms is denoted by $\mathcal{W}(s^t)$. Households receive a monetary transfer $X(s^t)$ from the government and buy four types of assets. Money holdings are denoted by $M(s^t)$. $B(s^t)$ are one-period nominal risk-free bonds[5] that have a price of one currency unit and a return of $R(s^t) \geq 1$, the gross interest rate, in the next period. $A(s^t, s^{t+1})$ are nominal state-contingent bonds[6] that cost $Q(s^{t+1}|s^t)$ and pay a return of one currency unit in period $t+1$ if and only if the economy is in the state of nature s^{t+1}. A share is denoted by $S(f, s^t)$. Its price is a share of the firm entry cost and its payoff is a share of the entrant's monopoly profits earned at the end of period t and is paid out as dividends at the start of period $t + 1$.

After the closure of asset markets, production takes place and goods markets open. The agents work and use money to make consumption purchases. At the end of the period, they receive labor income and pay a lump-sum tax $T(s^t)$ to the government. At the beginning of period $t + 1$, households have a stock of wealth given by

$$\mathcal{W}\left(s^{t+1}\right) = M\left(s^t\right) + R\left(s^t\right) B\left(s^t\right) + A\left(s^t, s^{t+1}\right) + \int_0^{N(s^t)} S\left(f, s^t\right) D\left(f, s^t\right) df + W\left(s^t\right) L\left(s^t\right) - P\left(s^t\right) C\left(s^t\right) - T\left(s^t\right).$$

Income from asset holdings consists of money carried over from the previous period, interest income on bond holdings, and dividends on share holdings. Initial household wealth is zero, such that $\mathcal{W}(s^0) = 0$.[7] At the beginning of time, $N(s^0)$ firms are given. I rule out Ponzi schemes on asset holdings by assuming that

$$\lim_{T\to\infty} Q\left(s^T|s^0\right)\left[B\left(s^T\right) + \sum_{s^{T+1}|s^T} Q\left(s^{T+1}|s^T\right) A\left(s^T, s^{T+1}\right)\right] \geq 0. \qquad (13)$$

The first-order conditions for asset holdings imply that

$$Q\left(s^{t+1}|s^t\right) = \beta \Pr\left(s^{t+1}|s^t\right) \frac{U_C\left(s^{t+1}\right)}{U_C\left(s^t\right)} \frac{P\left(s^t\right)}{P\left(s^{t+1}\right)}, \qquad (14)$$

$$\frac{U_C\left(s^t\right)}{P\left(s^t\right)} = R\left(s^t\right) \beta \sum_{s^{t+1}|s^t} \Pr\left(s^{t+1}|s^t\right) \frac{U_C\left(s^{t+1}\right)}{P\left(s^{t+1}\right)}, \qquad (15)$$

$$\frac{W\left(s^t\right) F\left(s^t\right)}{Z\left(s^t\right)} = \frac{D\left(f, s^t\right)}{R\left(s^t\right)}. \qquad (16)$$

Equation (14) defines the households' stochastic discount factor, the marginal utility growth of nominal wealth, given a particular state of nature in $t + 1$. The period-zero value of consumption in period $t + 1$ must obey $Q(s^{t+1}|s^0) = Q(s^t|s^0)Q(s^{t+1}|s^t)$. Combining (14) and (15) yields an arbitrage condition between risk-free and state-contingent bonds $\sum_{s^{t+1}|s^t} Q(s^{t+1}|s^t) = 1/R(s^t)$. Equation (16) states that the cost of setting up a firm must equal profits discounted by the interest rate. Under flexible wages, (17) equates the real wage to a markup over the marginal rate of substitution between leisure and consumption, adjusted for the cost of holding money,

$$\frac{W\left(s^t\right)}{P\left(s^t\right)} = \frac{\phi}{\phi - 1} \frac{V_L\left(s^t\right)}{U_C\left(s^t\right)} R\left(s^t\right). \qquad (17)$$

An alternative assumption is that wages are predetermined; that is, the wage rate in place in period t, $W(s^t)$, is set in period $t-1$. This particular form of wage stickiness is chosen for simplicity. Under preset wages, the first-order condition for wages is

$$W\left(s^t\right) = \frac{\phi}{\phi - 1} \frac{\sum_{s^t|s^{t-1}} \Pr\left(s^t|s^{t-1}\right) V_L\left(s^t\right) L\left(s^t\right)}{\sum_{s^t|s^{t-1}} \Pr\left(s^t|s^{t-1}\right) \frac{U_C(s^t)}{R(s^t)P(s^t)} L\left(s^t\right)}. \quad \mathbf{(18)}$$

Because of imperfect competition in the labor market, labor supply is reduced and hence leisure is increased relative to the perfectly competitive case. Imperfect competition in the goods market has a similar effect: output and labor hours are lower than under perfect competition. Thus, leisure is sold at a discount, whereas consumption goods are sold at a markup. The theory of optimal taxation tells us that we want markups to be equal across all goods (consumption goods and leisure). Therefore, a markup or tax on leisure, equivalent to a labor subsidy, would be desirable.

There are two reasons for assuming sticky wages instead of sticky prices. First, Lewis (2009) shows that, in a model with endogenous entry, wage stickiness helps to reconcile the model impulse responses of profits and entry to a monetary stimulus with those observed in the data. The second reason is analytical convenience. Under price flexibility, profits are a constant fraction of revenue, which simplifies considerably the optimality condition for share holdings and, as a result, the policy problem.

2.6. Government

The government makes a monetary transfer to the household in the asset market, financed with an expansion of the money stock $M^s(s^t)$ and with lump-sum taxes collected in the goods market. Thus, the government budget constraint is

$$X\left(s^t\right) = T\left(s^t\right) + M^s\left(s^t\right) - M^s\left(s^{t-1}\right).$$

2.7. Market Clearing

Labor is used for production and for firm startups,

$$L\left(s^t\right) = N\left(s^t\right)\left[L_c\left(s^t\right) + L_f\left(s^t\right)\right].$$

Using the respective production functions, labor market clearing requires

$$Z\left(s^t\right) L\left(s^t\right) = N\left(s^t\right)\left[Y\left(f, s^t\right) + F\left(s^t\right)\right]. \quad \mathbf{(19)}$$

The market-clearing conditions for final goods, for the two types of bonds, for shares, and for money are, respectively,

$$Y\left(s^t\right) = C\left(s^t\right), \quad \mathbf{(20)}$$

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$$B\left(s^t\right) = A\left(s^t, s^{t+1}\right) = 0,$$
$$S\left(f, s^t\right) = 1, \tag{21}$$
$$M\left(s^t\right) = M^s\left(s^t\right).$$

DEFINITION 1. *An imperfectly competitive equilibrium is a set of prices, allocations, and policies such that, first, the optimality conditions of the final goods firm, the intermediate goods firms, the labor packer, and the household are satisfied; second, all markets clear.*

3. FIRST-BEST ALLOCATION

The first-best allocation is defined as the allocation chosen by a benevolent social planner who maximizes the utility of the representative household subject to the resource constraint. It is a useful benchmark with which one can compare any constrained-efficient allocation. The first-best problem is as follows:

$$\max_{\left\{[C(s^t),L(s^t),N(s^t)]_{s^t\in S^t}\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty}\sum_{s^t}\beta^t \Pr\left(s^t\right)\left\{U\left[C\left(s^t\right)\right]-V\left[L\left(s^t\right)\right]\right\},$$

subject to the resource constraint

$$Z\left(s^t\right)L\left(s^t\right) = N\left(s^t\right)^{-\frac{1}{\theta-1}}C\left(s^t\right)+N\left(s^t\right)F\left(s^t\right). \tag{22}$$

The resource constraint (22) is derived by substituting the final goods production function under symmetry (6) in the labor market–clearing condition (19). It states that the total amount of (effective) labor is equal to the labor used in the production of goods plus the labor required for firm entry. The first-best allocation satisfies

$$\frac{V_L\left(s^t\right)}{U_C\left(s^t\right)} = N\left(s^t\right)^{\frac{1}{\theta-1}}Z\left(s^t\right), \tag{23}$$

$$F\left(s^t\right) = \frac{1}{\theta-1}N\left(s^t\right)^{-\frac{1}{\theta-1}-1}C\left(s^t\right). \tag{24}$$

Equation (23) is an intrasectoral efficiency condition. It states that the marginal rate of substitution between labor and consumption, $V_L(s^t)/U_C(s^t)$, must equal the aggregate marginal rate of transformation. See (9). Equation (24) is an intersectoral efficiency condition. It states that the cost (in effective labor units) of setting up one additional firm, $F(s^t)$, must equal the reduction in the number of effective labor units required in the production of goods, i.e., the efficiency gain, brought about by this extra firm. See (10).

Let us define two wedges, an intrasectoral wedge $\Theta(s^t)$ and an intersectoral wedge $\Upsilon(s^t)$. The first is the ratio of the marginal rate of substitution between

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labor and consumption to the marginal rate of transformation,

$$\Theta\left(s^t\right) = \frac{V_L\left(s^t\right)/U_C\left(s^t\right)}{N\left(s^t\right)^{\frac{1}{\theta-1}} Z\left(s^t\right)}.$$

The second is the difference between the marginal product of new firms in the goods production sector and their marginal cost in terms of labor units,

$$\Upsilon\left(s^t\right) = \frac{1}{\theta-1}\frac{N\left(s^t\right)^{-\frac{1}{\theta-1}} C\left(s^t\right)}{Z\left(s^t\right)} - \frac{N\left(s^t\right) F\left(s^t\right)}{Z\left(s^t\right)}.$$

In the first-best allocation, the two wedges are constant: $\Theta^{FB}(s^t) = 1$ and $\Upsilon^{FB}(s^t) = 0$.

Assuming log consumption utility and linear labor disutility, such that $U_C(s^t) = C(s^t)^{-1}$ and $V_L(s^t) = 1$, equation (23) becomes

$$\frac{N\left(s^t\right)^{-\frac{1}{\theta-1}} C\left(s^t\right)}{Z\left(s^t\right)} = 1. \qquad \mathbf{(25)}$$

Thus, labor employed in goods production is constant and equal to 1. Substituting this result into the resource constraint (22) and rearranging yields the number of firms as a function of the exogenous variables,

$$N^{FB}\left(s^t\right) = \frac{1}{\theta-1}\frac{Z\left(s^t\right)}{F\left(s^t\right)}. \qquad \mathbf{(26)}$$

The number of firms is proportional to productivity and inversely proportional to the entry cost. Given the number of firms, we can compute consumption using (25),

$$C^{FB}\left(s^t\right) = N^{FB}\left(s^t\right)^{\frac{1}{\theta-1}} Z\left(s^t\right). \qquad \mathbf{(27)}$$

Expressing consumption as a function of exogenous variables only, we have

$$C^{FB}\left(s^t\right) = \left[(\theta-1) F\left(s^t\right)\right]^{-\frac{1}{\theta-1}} Z\left(s^t\right)^{\frac{\theta}{\theta-1}}.$$

Thus, consumption is increasing in productivity, with elasticity $\frac{\theta}{\theta-1}$, and decreasing in the entry cost, with elasticity $-\frac{1}{\theta-1}$. Finally, substituting the number of firms (26) and consumption (27) into the resource constraint (22), we find that labor in the first-best is constant,[8]

$$L^{FB}\left(s^t\right) = \frac{\theta}{\theta-1}. \qquad \mathbf{(28)}$$

To summarize, the equations (26), (27), and (28) describe the first-best allocation.

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4. OPTIMAL POLICY

This section derives the optimal policy following the approach in Adão et al. (2003). First, I collapse all equilibrium conditions into a single equation—the implementability condition—which, together with the resource constraint, restricts the set of implementable allocations for any given policy sequences. Second, I show that under both flexible and sticky wages, the optimal interest-rate policy is to follow the Friedman Rule. Third, I characterize the optimal allocations under this policy by deriving the optimal intrasectoral and intersectoral wedges under flexible wages and under sticky wages. I show that the flexible-wage optimal allocation coincides with the sticky-wage optimal allocation only if labor supply is inelastic.

4.1. Imperfectly Competitive Equilibrium: Compact Form

DEFINITION 2. *More compactly, we can define a (symmetric) equilibrium as a set of prices,*

$$\left\{\left[P\left(s^t\right), P\left(f, s^t\right), Q\left(s^{t+1}|s^t\right), Q\left(s^{t+1}|s^0\right), R\left(s^t\right), W\left(s^t\right)\right]_{s^t\in S^t}\right\}_{t=0}^{\infty},$$

allocations,

$$\left\{\left[C\left(s^t\right), N\left(s^t\right), L\left(s^t\right)\right]_{s^t\in S^t}\right\}_{t=0}^{\infty},$$

and policies,

$$\left\{\left[T\left(s^t\right), M^s\left(s^t\right)\right]_{s^t\in S^t}\right\}_{t=0}^{\infty},$$

such that

1. the present-value household budget constraint is satisfied,

$$0 \geq \sum_{t=0}^{\infty}\sum_{s^t}\frac{Q\left(s^t|s^0\right)}{R\left(s^t\right)}\left[R\left(s^t\right)P\left(s^t\right)C\left(s^t\right) - W\left(s^t\right)L\left(s^t\right) + T\left(s^t\right)\right]$$
$$+\sum_{t=1}^{\infty}\sum_{s^t}Q\left(s^t|s^0\right)\left[N\left(s^t\right)\frac{W\left(s^t\right)F\left(s^t\right)}{Z\left(s^t\right)} - \frac{1}{\theta}P\left(s^{t-1}\right)C\left(s^{t-1}\right)\right]; \qquad \mathbf{(29)}$$

2. the resource constraint is satisfied,[9]

$$Z\left(s^t\right)L\left(s^t\right) = N\left(s^t\right)^{-\frac{1}{\theta-1}}C\left(s^t\right) + N\left(s^t\right)F\left(s^t\right);$$

3. the following equilibrium conditions are satisfied:

$$P\left(f, s^t\right) = \frac{\theta}{\theta-1}\frac{W\left(s^t\right)}{Z\left(s^t\right)},$$

$$Q\left(s^{t+1}|s^0\right) = Q\left(s^t|s^0\right)Q\left(s^{t+1}|s^t\right),$$

$$\sum_{s^{t+1}|s^t}Q\left(s^{t+1}|s^t\right) = \frac{1}{R\left(s^t\right)},$$

$$P\left(s^t\right) = P\left(f, s^t\right) N\left(s^t\right)^{-\frac{1}{\theta-1}},$$

$$M^s\left(s^t\right) = P\left(s^t\right) C\left(s^t\right),$$

$$\frac{W\left(s^t\right) F\left(s^t\right)}{Z\left(s^t\right)} = \frac{P\left(s^t\right) C\left(s^t\right)}{\theta R\left(s^t\right) N\left(s^t\right)}, \tag{30}$$

as well as

$$\frac{W\left(s^t\right)}{P\left(s^t\right)} = \frac{\phi}{\phi-1} \frac{V_L\left(s^t\right)}{U_C\left(s^t\right)} R\left(s^t\right)$$

under flexible wages, or

$$W\left(s^t\right) = \frac{\phi}{\phi-1} \frac{\sum_{s^t|s^{t-1}} \Pr\left(s^t|s^{t-1}\right) V_L\left(s^t\right) L\left(s^t\right)}{\sum_{s^t|s^{t-1}} \Pr\left(s^t|s^{t-1}\right) \frac{U_C\left(s^t\right)}{R\left(s^t\right) P\left(s^t\right)} L\left(s^t\right)}$$

under sticky wages.

Notice that profits and asset holdings have been eliminated in this equilibrium representation. The present-value household budget constraint (29) and the free entry condition (30) are derived as follows. Using (5), (6), (7), and (20), firm profits can be expressed as a fraction of total consumption expenditure divided by the number of active firms,

$$D\left(f, s^t\right) = \frac{1}{\theta} \frac{P\left(s^t\right) C\left(s^t\right)}{N\left(s^t\right)}. \tag{31}$$

Under the assumption of complete contingent-claims markets, one can write the consumer budget constraint in present-value form. First, we weight each equation (12) by the period-0 value of wealth in state s^t, $Q(s^t|s^0)$. Summing the resulting equations across states and dates and using the no–Ponzi game condition (13) eliminates bond holdings from the budget constraint. Second, we substitute the CIA constraint (11), holding with equality, to eliminate money holdings. Third, we substitute out shares using the market-clearing condition (21). Finally, to derive (29), we eliminate firm profits using expression (31). Substituting (31) into the first-order condition for shares (16) yields the free entry condition (30).

4.2. Implementability Condition and Planner Problem

The objective of the planner is to choose the model variables to maximize the utility of the representative household, taking as given the optimality conditions of the households and the firms, as well as market clearing. The constraints of the planner problem are all the equilibrium conditions given previously. The idea of an implementability condition is that not all of these constraints are restrictive for the planner. When certain variables are substituted out (in particular, the prices), the constraints can be condensed into only one equation, in addition to the resource constraint. The planner then chooses the allocations that maximize utility, given this implementability condition and the resource constraint.

The planner is free to set a path for lump-sum taxes $T(s^t)$ to satisfy (29), whereas the variables $P(f, s^t)$, $Q(s^{t+1}|s^0)$, $Q(s^{t+1}|s^t)$, $P(s^t)$, and $M(s^t)$ adjust to satisfy the first five equilibrium conditions in Section 4.1. The remaining equilibrium conditions restricting the planner problem are the resource constraint (22), the free entry condition (30), and the relevant wage-setting equation: (17) under flexible wages or (18) under sticky wages.

Under *flexible* wages, the set of implementable allocations,

$$\left\{\left[C\left(s^t\right), L\left(s^t\right), N\left(s^t\right)\right]_{s^t\in S^t}\right\}_{t=0}^{\infty},$$

is restricted by the implementability condition,

$$\frac{Z\left(s^t\right)U_C\left(s^t\right)C\left(s^t\right)}{\theta F\left(s^t\right)R\left(s^t\right)^2 N\left(s^t\right)} = \frac{\phi}{\phi-1}V_L\left(s^t\right), \qquad (\mathbf{32})$$

and the resource constraint (22) for any path of the interest rate $R(s^t)$. Equation (32) is derived by combining the free-entry condition (30) with the wage-setting equation (17) to eliminate $W(s^t)$.

Under *sticky* wages, the wage-setting condition is given by (18). Solving the free entry condition for the price level and substituting the result into the wage setting equation to eliminate $P(s^t)$ gives

$$1 = \frac{\phi}{\phi-1}\frac{\sum_{s^t|s^{t-1}}\Pr\left(s^t|s^{t-1}\right)V_L\left(s^t\right)L\left(s^t\right)}{\sum_{s^t|s^{t-1}}\Pr\left(s^t|s^{t-1}\right)\frac{U_C(s^t)Z(s^t)C(s^t)}{\theta F(s^t)R(s^t)^2N(s^t)}L\left(s^t\right)}.$$

Note that we have cancelled $W(s^t)$, which is known in $t-1$. Rearranging and using the law of iterated expectations yields the implementability condition under sticky wages,

$$\sum_{s^t|s^{t-1}}\Pr\left(s^t|s^{t-1}\right)\left[\frac{U_C\left(s^t\right)Z\left(s^t\right)C\left(s^t\right)}{\theta F\left(s^t\right)R\left(s^t\right)^2 N\left(s^t\right)}L\left(s^t\right) - \frac{\phi}{\phi-1}V_L\left(s^t\right)L\left(s^t\right)\right] = 0. \qquad (\mathbf{33})$$

The constraints of the policy problem are the implementability constraint (33) and the resource constraint (22).

PROPOSITION 1. *The set of implementable allocations under flexible wages is contained in the corresponding set under sticky wages. Therefore, the optimal allocation under sticky wages makes households at least as well off as under flexible wages.*

Proof. For any interest-rate path $\{R(s^t)_{s^t\in S^t}\}_{t=0}^{\infty}$, the implementability condition under sticky wages (33) is the expected value of the implementability condition under flexible wages (32), and the resource constraint is binding in both cases.

Let $\beta^t \varphi(s^{t-1}) \Pr(s^t)$ be the Lagrange multiplier on (33). Then the planner problem under sticky wages is

$$\max_{\{[R(s^t),C(s^t),L(s^t),N(s^t)]_{s^t\in S^t}\}_{t=0}^{\infty}} \min_{\{[\lambda(s^t),\varphi(s^{t-1})]_{s^t\in S^t}\}_{t=0}^{\infty}} \mathcal{L},$$

where the Lagrangian is

$$\begin{aligned}\mathcal{L} = & \sum_{t=0}^{\infty}\sum_{s^t}\beta^t \Pr\left(s^t\right)\left\{U\left[C\left(s^t\right)\right] - V\left[L\left(s^t\right)\right]\right. \\ & \left. + \lambda\left(s^t\right)\left[Z\left(s^t\right)L\left(s^t\right) - N\left(s^t\right)^{-\frac{1}{\theta-1}}C\left(s^t\right) - N\left(s^t\right)F\left(s^t\right)\right]\right\} \\ & + \sum_{t=0}^{\infty}\sum_{s^{t-1}}\beta^t\varphi\left(s^{t-1}\right)\sum_{s^t}\Pr\left(s^t\right) \\ & \times\left[\frac{U_C\left(s^t\right)Z\left(s^t\right)C\left(s^t\right)}{\theta F\left(s^t\right)R\left(s^t\right)^2 N\left(s^t\right)}L\left(s^t\right) - \frac{\phi}{\phi-1}V_L\left(s^t\right)L\left(s^t\right)\right],\end{aligned}$$

with $\varphi(s^{-1}) = 0$. Under flexible wages, $\varphi(s^{t-1})$ is replaced with $\varphi(s^t)$.

Following Adão et al. (2003), I first derive the optimal interest-rate policy before solving for the optimal allocations under this policy. This method differs from the Ramsey approach to optimal policy of first solving the primal problem for the optimal allocations and then backing out the policies that support these allocations.

4.3. Optimal Interest Rate Policy

The interest-rate policy problem under sticky wages is to choose a path for the interest rate $\{[R(s^t) \geq 1]_{s^t\in S^t}\}_{t=0}^{\infty}$ to maximize $\mathcal{L}$. It is straightforward to show that the Friedman Rule is optimal irrespective of nominal rigidities.

PROPOSITION 2. *The Friedman Rule is optimal; i.e., the optimal gross interest rate is* $R(s^t) = 1$ *for all dates and states.*

Proof. The first derivative of the Lagrangian is

$$\frac{\partial\mathcal{L}}{\partial R\left(s^t\right)} = -2\beta^t \Pr\left(s^t\right)\varphi\left(s^{t-1}\right)\frac{Z\left(s^t\right)U_C\left(s^t\right)C\left(s^t\right)}{\theta F\left(s^t\right)R\left(s^t\right)^3 N\left(s^t\right)}L\left(s^t\right).$$

We have $\frac{\partial\mathcal{L}}{\partial R(s^t)} < 0$ for $\varphi(s^{t-1}) > 0$. Welfare, as summarized by $\mathcal{L}$, decreases as the interest rate increases, given the non-negativity constraint on the net interest rate and the fact that the Lagrange multiplier is strictly positive as long as we are away from the first best. It follows that the nominal interest rate should be as low as possible. Given the lower bound of unity on the gross interest rate, this implies that the Friedman Rule, $R(s^t) = 1$, is optimal.

As can be seen from (17), the money distortion affects the intratemporal consumption–leisure trade-off decision. It drives a wedge between the marginal rate of substitution between consumption and labor and the real wage. The higher the interest rate, the greater is this wedge. The optimality of the Friedman Rule is a standard result in the literature following Ireland (1996) and is shown to hold under more general conditions in Correia et al. (2008). Notice that under the Friedman Rule, the CIA constraint is no longer binding and hence the level of real money holdings is indeterminate. This real indeterminacy at the corner solution can be ignored if we consider limiting equilibria under which the interest rate approaches unity.

4.4. Optimal Allocations under the Friedman Rule

Under the Friedman Rule, the planner problem is written as before, with $R(s^t)$ set equal to 1 in $\mathcal{L}$. Again, under flexible wages, $\varphi(s^{t-1})$ is replaced with $\varphi(s^t)$.

Flexible wages. The first-order conditions for the policy problem under flexible wages imply the intra- and intersectoral optimal wedges

$$\Theta^f\left(s^t\right) = \frac{1+\varphi\left(s^t\right)\frac{Z(s^t)}{\theta F(s^t)N(s^t)}\left[\frac{U_{CC}(s^t)C(s^t)}{U_C(s^t)}+1\right]}{1+\Psi^f\left(s^t\right)}, \tag{34}$$

$$\Upsilon^f\left(s^t\right) = \frac{\varphi\left(s^t\right)\frac{Z(s^t)}{\theta F(s^t)N(s^t)}\frac{U_C(s^t)C(s^t)}{V_L(s^t)}}{1+\Psi^f\left(s^t\right)}, \tag{35}$$

where

$$\Psi^f\left(s^t\right) = \varphi\left(s^t\right)\frac{\phi}{\phi-1}\frac{V_{LL}\left(s^t\right)}{V_L\left(s^t\right)}.$$

Assuming log consumption utility and linear labor disutility, $\Psi^f(s^t) = 1$ and the optimal allocation can be written recursively as follows:

$$N\left(s^t\right) = \frac{\phi-1}{\phi\theta}\frac{Z\left(s^t\right)}{F\left(s^t\right)}, \tag{36}$$

$$C\left(s^t\right) = N\left(s^t\right)^{\frac{1}{\theta-1}} Z\left(s^t\right), \tag{37}$$

$$L\left(s^t\right) = 1 + \frac{\phi-1}{\phi\theta}. \tag{38}$$

Under flexible wages, the Friedman Rule is optimal and implements the unique allocation given by (36) to (38), whereas it does not pin down the price level. Under the Friedman Rule, allocations are unaffected by the money supply policy. The size of the money stock affects only (and indeed pins down) the price level $P(s^t)$ through the CIA constraint. The remaining nominal variables, i.e., firm prices $P(f, s^t)$, wages $W(s^t)$, and profits $D(f, s^t)$, are then determined residually through (4), (17), and (31), respectively. The optimal allocation can thus be implemented with a multiplicity of money supply and price level sequences. This is the nominal indeterminacy under flexible wages explained in Adão et al. (2003).

The wedge between the first best and the optimal allocation under flexible wages is constant and equals the joint markup in goods and labor markets, $\frac{\theta}{\theta-1}\frac{\phi}{\phi-1}$. To see this, compare equations (26)–(28) with equations (36)–(38). Note that the wage markup has an effect similar to that of the goods markup: it makes leisure cheaper relative to consumption. A constant labor income subsidy equal to the joint markup aligns the decentralized allocation with the First Best. The reason, as pointed out in Bilbiie et al. (2008), is that the presence of firm entry costs makes goods markups efficient. The result is a misalignment of markups between consumption and leisure, distorting the labor supply decision. A markup on leisure is needed to restore efficiency. Here, the required markup is higher than that in Bilbiie et al. (2008) because of monopolistic competition in the labor market.

Sticky wages. With flexible wages, only one margin is distorted: the leisure–consumption choice. Imperfect competition in product and labor markets gives rise to a constant wedge between the real wage and the marginal rate of substitution between leisure and consumption. The only handle on this wedge is the nominal interest rate, and the optimal policy is to minimize the wedge by implementing the Friedman Rule. In contrast, with sticky wages, shocks create an additional distortion. Through the money supply instrument, monetary policy can use this additional distortion to its advantage by (partially) undoing the distortion that is present under flexible wages whenever a shock hits.

The first-order conditions of the policy problem under sticky wages imply the following intra- and intersectoral optimal wedges:

$$\Theta^s\left(s^t\right)=\frac{1+\varphi\left(s^{t-1}\right)\frac{Z(s^t)}{\theta F(s^t)N(s^t)}\left[\frac{U_{CC}(s^t)C(s^t)}{U_C(s^t)}+1\right]L\left(s^t\right)}{1+\Psi^s\left(s^t\right)},\tag{39}$$

$$\Upsilon^s\left(s^t\right)=\frac{\varphi\left(s^{t-1}\right)\frac{Z(s^t)}{\theta F(s^t)N(s^t)}\frac{U_C(s^t)C(s^t)}{V_L(s^t)}L\left(s^t\right)}{1+\Psi^s\left(s^t\right)},\tag{40}$$

where

$$\Psi^s\left(s^t\right)=\varphi\left(s^{t-1}\right)\left\{\frac{\phi}{\phi-1}\frac{V_{LL}\left(s^t\right)}{V_L\left(s^t\right)}L\left(s^t\right)\right.$$
$$\left.+\left[\frac{\phi}{\phi-1}-\frac{Z\left(s^t\right)}{\theta F\left(s^t\right)N\left(s^t\right)}\frac{U_C\left(s^t\right)C\left(s^t\right)}{V_L\left(s^t\right)}\right]\right\}.$$

Under sticky wages, the Friedman Rule is again optimal. Under the Friedman Rule, there are multiple implementable allocations associated with different money supplies, which all satisfy the implementability condition. Within the set of implementable allocations, the policy maker picks the optimal one, which in general does not coincide with the flexible-wage allocation.

PROPOSITION 3. *In general, the optimal allocation under sticky wages welfare-dominates the optimal allocation under flexible wages.*

Proof. Comparing the sticky-wage optimal wedges (39) and (40) with the flexible-wage optimal wedges (34) and (35), we see that only if $L(s^t) = 1$ does the flexible-wage allocation satisfy the planner's first-order conditions under sticky wages. Then $\Psi_f(s^t) = \Psi_s(s^t)$and the optimal wedges coincide.

This is the case of inelastic labor supply studied by Bilbiie et al. (2008). Intuitively, when labor supply is fixed, policy cannot manipulate the leisure–consumption trade-off to its advantage. Then sticky wages do *not* provide monetary policy with a lever to improve upon the flexible-wage allocation.

The nonoptimality of the flexible solution is reminiscent of the finding in Adão et al. (2003). In their fixed-variety model with preset prices, government spending shocks drive a wedge between the optimal allocations under price stickiness and the flexible-price allocations. They assume, as I do here, that cash is required for goods purchases and that lump-sum taxes are available, whereas distortionary fiscal policy is not. The optimal plan is to follow the Friedman Rule and, in addition, to use the money supply to reduce the cyclical wedges that result from exogenous changes in government spending. In their model, government spending diverts labor away from the production of goods used for private consumption. In this model, entry costs have the same effect. The extra labor needed to produce government consumption or to create firms gives rise to disutility and alters the marginal rate of substitution. Under nominal rigidities and a CIA constraint, the policy maker can influence the amount of consumption (and hence the marginal rate of substitution) through cash injections. There are two important differences between this paper and Adão et al. (2003), however. First, the marginal rate of transformation is different here because of the increasing returns to variety in the aggregate production function, as explained in Section 2.3. Second, in both papers the markups on goods and leisure should be aligned. However, the price markup should be reduced to zero in Adão et al. (2003), whereas in this paper, the markup on leisure should be raised to the markup on goods. This is due to the presence of entry costs, which implies that profits are not distortionary but instead necessary for a firm to enter and produce.

The assumption of a labor requirement for firm startups is important for the results of this paper. Because the wage rate is part of the entry cost, wage stickiness affects the entry decision, and it is through this effect that monetary policy can influence the investment margin. In the Appendix, I present a variant of the model in which entry costs are specified in terms of final output. In that model, wage stickiness does not alter the set of implementable allocations that the policy maker faces. This is because wages are not part of entry costs and any wage-setting restriction therefore does not distort the entry decision. As a result, the optimal allocations are the same under sticky wages as under flexible wages (with price rigidities, the optimal allocations are different, however). Lewis (2009) compares impulse responses to a monetary policy shock to their empirical counterparts, for different variants of the endogenous entry model. Qualitatively, the best-performing model is one in which entry costs are in labor units, rather than in terms of final output, and wages are sticky. This evidence leads me to prefer the benchmark model to the modified version.

5. AN EXAMPLE AND SOME INTUITION

Under log consumption utility and linear labor disutility, the optimal wedges under sticky wages are as follows:

$$\Theta_s\left(s^t\right) = \frac{1}{1+\varphi\left(s^{t-1}\right)\left[\frac{\phi}{\phi-1} - \frac{Z(s^t)}{\theta F(s^t)N(s^t)}\right]},$$

$$\Upsilon_s\left(s^t\right) = \frac{\varphi\left(s^{t-1}\right)\frac{Z(s^t)}{\theta F(s^t)N(s^t)}L\left(s^t\right)}{1+\varphi\left(s^{t-1}\right)\left[\frac{\phi}{\phi-1} - \frac{Z(s^t)}{\theta F(s^t)N(s^t)}\right]}.$$

Together with the resource constraint (22) and the implementability constraint,

$$\sum_{s^t|s^{t-1}} \Pr\left(s^t|s^{t-1}\right)\left[\frac{Z\left(s^t\right)}{\theta F\left(s^t\right)N\left(s^t\right)}L\left(s^t\right) - \frac{\phi}{\phi-1}L\left(s^t\right)\right] = 0,$$

they determine $C(s^t)$, $L(s^t)$, $N(s^t)$, and $\varphi(s^{t-1})$.

In the following, I provide a two-state example[10] for the general case where labor supply is elastic. Consider two states with equal probability and the following calibration. The elasticity of substitution between goods is set to $\theta = 3.8$ as in Bilbiie et al. (2008), which is consistent with the estimate in Lewis and Poilly (2011). Labor types are assumed to be somewhat more substitutable, $\phi = 6$. Productivity in the preshock state $Z(s^0)$ is normalized to 1. As for the calibration of period-0 entry costs, I use the value $F(s^0) = 0.004$, which is close to the one in Barseghyan and DiCecio (2011), according to whom a ratio $\frac{F}{Z} = 0.0038$ matches legal entry fees for the United States as a fraction of output per worker.

Table 1 shows the responses of the number of firms, consumption, and labor to changes in productivity and entry costs. It compares the first-best allocation to the optimal allocation under flexible wages and under sticky wages. Notice that all values have been normalized by the period-0 values in the decentralized economy. For example, in the preshock period, the number of firms in the first-best relative to the number of firms in the decentralized steady state is 1.6. In period 1, the first-best number of firms is 2.4 in the good state and 0.8 in the bad state, again relative to the preshock level in the decentralized economy.

TABLE 1. Two-state example

	Productivity shock[a]					
	Number of firms		Consumption		Labor	
	$t=0$	$t=1$	$t=0$	$t=1$	$t=0$	$t=1$
First-best	1.629	2.443	1.190	2.064	1.113	1.113
	1.629	0.814	1.190	0.465	1.113	1.113
Flex-wage	1	1.5	1	1.734	1	1
	1	0.5	1	0.390	1	1
Sticky-wage	1	1.593	1	1.763	1	1.007
	1	0.472	1	0.385	1	0.997
	Entry cost shock[b]					
	Number of firms		Consumption		Labor	
	$t=0$	$t=1$	$t=0$	$t=1$	$t=0$	$t=1$
First-best	1.629	2.171	1.190	1.319	1.113	1.113
	1.629	1.303	1.190	1.099	1.113	1.113
Flex-wage	1	1.333	1	1.108	1	1
	1	0.800	1	0.923	1	1
Sticky-wage	1	1.357	1	1.113	1	1.001
	1	0.787	1	0.919	1	0.999

[a] Productivity $Z(s^1)$ takes on the values 1.5 and 0.5 with equal probability.
[b] The entry cost $F(s^1)$ takes on the values 0.003 and 0.005 with equal probability.

Under sticky wages, the number of firms, consumption, and labor increase more in response to a positive productivity shock than under flexible wages. In particular, the expansion in labor under sticky wages allows a larger increase in the production of both firms and goods. Similarly, a decrease in the entry cost induces the optimal policy under sticky wages of raising labor, so that the number of firms and consumption increase more than under flexible wages. Importantly, the optimal sticky-wage allocation is more dispersed across states of nature. In response to an expansionary shock, the policy maker exploits the degree of freedom given by wage rigidity to address the undersupply of labor and the underproduction of goods and firms. As labor rises beyond its steady state level, both consumption and the number of firms expand more than in the flexible-wage allocation.[11] The responses of the real wage to the shocks are identical to those of consumption. The real wage increases more and thus leisure becomes more expensive in the sticky-wage allocation than in the flexible-wage allocation.

To summarize, the nominal wage rigidity, combined with the money supply instrument, allows the policy maker to manipulate the real wage in the face of shocks, affecting the consumption–leisure trade-off decision directly. Optimal policy chooses a different allocation than the flexible-wage allocation by introducing a markup on leisure that is absent under flexible wages. Monetary policy can mimic the effect of a labor supply subsidy, which has the same effect of making leisure more expensive relative to consumption.

6. CONCLUSION

This paper investigates the implications of firm entry for optimal monetary policy. The economy has three distortions: product and labor markets are imperfectly competitive, wages are set in advance, and consumption purchases must be made with money. The CIA restriction is undone via the Friedman Rule, which aligns the returns on bonds and money. The markup in the goods market is efficient, because profits are needed to cover the entry cost. However, the absence of a markup on leisure implies that leisure is too cheap relative to consumption goods. Therefore, labor is suboptimally low. Because of the labor requirement for establishing new firms, this has a negative effect on entry. Even though implementing the flexible allocation, i.e., removing the sticky-wage distortion, is feasible, it is not welfare-maximizing. In response to expansionary shocks to productivity and entry costs, the optimal policy implies a larger increase in hours, more consumption, and higher entry than is observed in the flexible economy. Wage rigidity, combined with the money supply instrument, provides the policy maker with a tool to increase the real wage, moving it closer to its efficient level, in response to such shocks. As a result, more labor is employed at both margins: at the intensive margin (production of goods) and at the extensive margin (firm entry).

NOTES

1. Notice that entry itself does not give rise to a (net) externality under monopolistic competition à la Dixit and Stiglitz (1977). In particular, there is a positive externality of entry on consumer surplus through an increase in product diversity. Entry also has a negative externality on firm profits through a "business-stealing effect." These two effects offset each other, so that the net externality is zero. For more details on this, see Bilbiie et al. (2008, 2011) and Lewis (2010).
2. With increasing entry cost due to a congestion externality in entry, deviations from the Friedman Rule are needed to reduce inefficiently high entry levels. Such congestion effects are beyond the scope of this paper.
3. Under oligopolistic competition with strategic interactions, the markup may well be a (negative) function of the number of producers. Although this is a possibly interesting extension, I abstract from markup endogeneity in order to keep the model as simple as possible. See Lewis (2010), which takes up this issue in a nonmonetary model, focusing on optimal taxation.
4. To simplify notation, I drop the h-subscript from here on, given that households are symmetric.
5. Risk-free bonds are needed to define the interest rate.
6. I introduce state-contingent bonds in order to simplify the policy problem. Under complete financial markets one can write the household budget constraint in present-value form, which then becomes a *single* implementability constraint for the policy maker.
7. This is consistent with the result in Chamley (1986) that only *initial* wealth should be taxed, and at a rate of 100%.
8. Note that labor is constant only in the log utility case.
9. If the resource constraint and the household budget constraint are satisfied, the government budget constraint is satisfied by Walras's Law.

10. A closed-form solution does not exist even for the two-period two-state case, which is a nonlinear system of seven equations in seven unknowns.

11. In the Online Appendix, I show (for the case of log linear utility) that if hours, consumption, and entry were instead below their flexible-wage levels, both the intra- and intersectoral wedge would be higher than under flexible wages, which cannot be optimal.

REFERENCES

Adão, Bernadino, Isabel Correia, and Pedro Teles (2003) Gaps and triangles. *Review of Economic Studies* 70(4), 699–713.

Barseghyan, Levon and Riccardo DiCecio (2011) Entry costs, industry structure, and cross-country income and TFP differences. *Journal of Economic Theory* 146(5), 1828–1851.

Berentsen, Aleksander and Christopher Waller (2009) Optimal Stabilization Policy and Endogenous Firm Entry. Manuscript, University of Basel, University of Notre Dame, and FRB-St. Louis.

Bergin, Paul R. and Giancarlo Corsetti (2008) The extensive margin and monetary policy. *Journal of Monetary Economics* 55(7), 1222–1237.

Bilbiie, Florin, Ippei Fujiwara, and Fabio Ghironi (2011) Optimal Monetary Policy with Endogenous Product Variety. CEPR discussion paper 8565.

Bilbiie, Florin, Fabio Ghironi, and Marc Melitz (2007) Monetary policy and business cycles with endogenous entry and product variety. *NBER Macroeconomics Annual* 22.

Bilbiie, Florin, Fabio Ghironi, and Marc Melitz (2008) Monopoly Power and Endogenous Variety in Dynamic General Equilibrium: Distortions and Remedies. NBER working paper 14383.

Broda, Christian and David E. Weinstein (2010) Product creation and destruction: Evidence and price implications. *American Economic Review* 100(3), 691–723.

Chamley, Christophe (1986) Optimal taxation of capital income in general equilibrium with infinite lives. *Econometrica* 54(3), 607–622.

Correia, Isabel, Juan Pablo Nicolini, and Pedro Teles (2008) Optimal fiscal and monetary policy: Equivalence results. *Journal of Political Economy* 116(1), 141–170, 02.

Dixit, Avinash K. and Joseph E. Stiglitz (1977) Monopolistic competition and optimum product diversity. *American Economic Review* 67(3), 297–308.

Erceg, Christopher J., Dale W. Henderson, and Andrew T. Levin (2000) Optimal monetary policy with staggered wage and price contracts. *Journal of Monetary Economics* 46(2), 281–313.

Faia, Ester (in press) Oligopolistic competition and optimal monetary policy. *Journal of Economic Dynamics and Control.*

Ireland, Peter N. (1996) The role of countercyclical monetary policy. *Journal of Political Economy* 104(4), 704–723.

Kim, Jinill (2004) What determines aggregate returns to scale? *Journal of Economic Dynamics and Control* 28(8), 1577–1594.

Lewis, Vivien (2009) Business cycle evidence on firm entry. *Macroeconomic Dynamics* 13(5), 605–624.

Lewis, Vivien (2010) Product Diversity, Strategic Interactions and Optimal Taxation. Manuscript, Ghent University.

Lewis, Vivien and Céline Poilly (2011) Firm Entry, Markups and the Monetary Transmission Mechanism. Manuscript, Ghent University and University of Lausanne.

APPENDIX: ENTRY COST IN TERMS OF FINAL OUTPUT

I now assume that the exogenous entry cost is given in terms of final output instead of effective labor units as in the benchmark model. The new entry cost is denoted by $F_o(s^t)$.

Goods-market clearing is no longer $Y(s^t) = C(s^t)$ but instead

$$Y\left(s^t\right) = C\left(s^t\right) + N\left(s^t\right) F_o\left(s^t\right). \tag{A.1}$$

Total output comprises consumption purchases and entry costs. Combining (A.1) with firm profits (5), the final goods firm's production function under symmetry (6), and the price index (7), real firm profits are

$$\frac{D\left(f, s^t\right)}{P\left(s^t\right)} = \frac{1}{\theta}\left[\frac{C\left(s^t\right)}{N\left(s^t\right)} + F_o\left(s^t\right)\right]. \tag{A.2}$$

The household budget constraint becomes

$$\mathcal{W}\left(s^t\right) \geq M\left(s^t\right) + B\left(s^t\right) + \sum_{s^{t+1}|s^t} Q\left(s^{t+1}|s^t\right) A\left(s^t, s^{t+1}\right)$$
$$+ \int_0^{N\left(s^t\right)} S\left(f, s^t\right) P\left(s^t\right) F_o\left(s^t\right) df - X\left(s^t\right).$$

The first-order condition for shares is therefore

$$P\left(s^t\right) F_o\left(s^t\right) = \frac{D\left(f, s^t\right)}{R\left(s^t\right)}.$$

Rearranging and replacing real profits using (A.2), we get the free entry condition,

$$F_o\left(s^t\right) R\left(s^t\right) = \frac{1}{\theta}\left[\frac{C\left(s^t\right)}{N\left(s^t\right)} + F_o\left(s^t\right)\right]. \tag{A.3}$$

Combining the symmetric final goods production function $Y(s^t) = N(s^t)^{1+\frac{1}{\theta-1}} Y(f, s^t)$ with the intermediate firms' production function $N(s^t)Y(f, s^t) = Z(s^t)L(s^t)$, we have the economy's aggregate production function,

$$Y\left(s^t\right) = N\left(s^t\right)^{\frac{1}{\theta-1}} Z\left(s^t\right) L\left(s^t\right). \tag{A.4}$$

Differentiating (A.4) with respect to $N(s^t)$, we can derive the marginal product, in terms of final output, of one additional firm,

$$\frac{\partial Y\left(s^t\right)}{\partial N\left(s^t\right)} = \frac{1}{\theta - 1} N\left(s^t\right)^{\frac{1}{\theta-1}-1} Z\left(s^t\right) L\left(s^t\right). \tag{A.5}$$

Combining equations (A.1) and (A.4) yields the aggregate resource constraint,

$$N\left(s^t\right)^{\frac{1}{\theta-1}} Z\left(s^t\right) L\left(s^t\right) = C\left(s^t\right) + N\left(s^t\right) F_o\left(s^t\right). \tag{A.6}$$

The remaining equilibrium conditions are as in the benchmark model.

A.1. FIRST BEST ALLOCATION

The first best problem is as follows:

$$\max_{\{[C(s^t),L(s^t),N(s^t)]_{s^t\in S^t}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty}\sum_{s^t} \beta^t \Pr\left(s^t\right)\left\{U\left[C\left(s^t\right)\right]-V\left[L\left(s^t\right)\right]\right\},$$

subject to the resource constraint (A.6). The first-order conditions are

$$\frac{V_L\left(s^t\right)}{U_C\left(s^t\right)} = N\left(s^t\right)^{\frac{1}{\theta-1}} Z\left(s^t\right), \tag{A.7}$$

$$F_o\left(s^t\right) = \frac{1}{\theta-1} N\left(s^t\right)^{\frac{1}{\theta-1}-1} Z\left(s^t\right) L\left(s^t\right). \tag{A.8}$$

The intrasectoral efficiency condition (A.7) is the same as in the benchmark model: the marginal rate of substitution between labor and consumption must equal the aggregate marginal rate of transformation. Neither the MRS nor the MRT depends on the specification of the entry cost. The intersectoral efficiency condition (A.8) is, however, different from that in the benchmark model. It states that the cost (in terms of consumption units) of setting up an additional firm, $F_o(s^t)$, must equal the gain in consumption output that the extra firm gives rise to, i.e., the marginal product of a firm (A.5). Under log consumption utility and linear labor disutility, we can derive the recursive system

$$N_o^{\mathrm{FB}}\left(s^t\right) = \left[\frac{1}{\theta-2}\frac{Z\left(s^t\right)}{F_o\left(s^t\right)}\right]^{\frac{\theta-1}{\theta-2}},$$

$$C_o^{\mathrm{FB}}\left(s^t\right) = N_o^{\mathrm{FB}}\left(s^t\right)^{\frac{1}{\theta-1}} Z\left(s^t\right),$$

$$L_o^{\mathrm{FB}}\left(s^t\right) = \frac{\theta-1}{\theta-2}.$$

Labor is constant in the first best allocation and unambiguously higher here than in the benchmark model,

$$L_o^{\mathrm{FB}}\left(s^t\right) = \frac{\theta-1}{\theta-2} > \frac{\theta}{\theta-1} = L^{\mathrm{FB}}\left(s^t\right).$$

When entry costs are specified in units of consumption, the number of firms in the first best responds more to productivity shocks and to entry cost shocks than in the benchmark model. The elasticities $\frac{\theta-1}{\theta-2}$ and $-\frac{\theta-1}{\theta-2}$ are greater (in absolute terms) than 1 and -1, respectively. In the steady state, the number of firms and consumption are higher than in the benchmark model.

A.2. IMPLEMENTABILITY CONDITION AND PLANNER PROBLEM

The set of implementable allocations for $\{[C(s^t), L(s^t), N(s^t)]_{s^t\in S^t}\}_{t=0}^{\infty}$ is restricted by the free entry condition (A.3) and the resource constraint (A.6) for any path of the interest rate $R(s^t) \geq 1$.

Notice that here the wage-setting scheme does not matter for the optimal allocations. If there is wage stickiness, this does not restrict the set of implementable allocations for the policy maker. This is because the wage rate no longer enters the free entry condition and

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therefore does not affect the investment margin. Because wage stickiness does not matter, monetary policy cannot be used to select allocations. The absence of a labor requirement to set up a firm removes the power of the monetary policy instrument under sticky wages to affect the investment margin. Lump-sum taxes must adjust to satisfy the household budget constraint and the money stock must adjust to satisfy the CIA constraint.

In the Online Appendix, I explore the case where entry costs are specified in terms of final output and *prices* are set in advance. Although the flexible-price allocation is attainable under sticky prices, it is not optimal. Unlike wage stickiness, price stickiness does affect the optimal allocations: it affects the real value of firm profits and hence the free-entry condition. A full characterization of the optimal allocations under sticky prices is, however, beyond the scope of this paper.

Let $\beta^t \Pr(s^t)\varphi(s^t)$ be the Lagrange multiplier on the free-entry condition. The planner problem is

$$\max_{\{[R(s^t),C(s^t),L(s^t),N(s^t)]_{s^t\in S^t}\}_{t=0}^{\infty}} \min_{\{[\lambda(s^t),\varphi(s^t)]_{s^t\in S^t}\}_{t=0}^{\infty}} \mathcal{L}_o,$$

where

$$\mathcal{L}_o = \sum_{t=0}^{\infty}\sum_{s^t}\beta^t \Pr\left(s^t\right)\Bigg(U\left[C\left(s^t\right)\right] - V\left[L\left(s^t\right)\right]$$
$$+\lambda\left(s^t\right)\left[N\left(s^t\right)^{\frac{1}{\theta-1}} Z\left(s^t\right) L\left(s^t\right) - C\left(s^t\right) - N\left(s^t\right) F_o\left(s^t\right)\right]$$
$$+\ \varphi\left(s^t\right)\left\{\frac{1}{\theta}\left[\frac{C\left(s^t\right)}{N\left(s^t\right)} + F_o\left(s^t\right)\right] - F_o\left(s^t\right) R\left(s^t\right)\right\}\Bigg).$$

A.3. OPTIMAL INTEREST-RATE POLICY

The interest-rate policy problem is to choose a path for the interest rate $\{(R(s^t) \geq 1)_{s^t\in S^t}\}_{t=0}^{\infty}$ to maximize $\mathcal{L}_o$. The first-order condition is

$$\frac{\partial \mathcal{L}_o}{\partial R\left(s^t\right)} = -\beta^t \Pr\left(s^t\right)\varphi\left(s^t\right) F_o\left(s^t\right).$$

Because this expression is negative, the Friedman Rule is optimal.

A.4. OPTIMAL ALLOCATIONS UNDER THE FRIEDMAN RULE

Under the Friedman Rule, we can derive and rearrange the first-order conditions of the policy problem to express the allocation as

$$N_o\left(s^t\right)^{\frac{\theta-2}{\theta-1}} = \left(\frac{\theta-1}{\theta+1}\right)\frac{1}{\theta-2}\frac{Z\left(s^t\right)}{F_o\left(s^t\right)},$$

$$C_o\left(s^t\right) = \theta N_o\left(s^t\right) F_o\left(s^t\right),$$

$$L_o\left(s^t\right) = \frac{\theta-1}{\theta-2}.$$

Labor is constant and equal to its first-best level. The number of firms in the optimal allocation is smaller than in the first best.

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