ARE RISK PREMIA RELATED TO REAL EXCHANGE RATE SWINGS? EVIDENCE FROM $I(2)$ CVARs WITH SURVEY EXPECTATIONS

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This paper finds strong evidence of a positive relationship between currency risk premia and real exchange rate swings, significant at the 1% if not 0.1% level, for three US$ exchange rate samples. The risk premia are measured using survey data on traders' exchange rate forecasts. This circumvents the need for an auxiliary hypothesis of rationality and enables more direct focus on risk behavior. The analysis is conducted using the $I(2)$ cointegrated VAR, which allows for time-varying trends in the variables. Evidence of persistent changes is found for interest rates, prices, and nominal and real exchange rates. Interest rate shocks appear to drive the system, whereas expectations are correcting only in the longer run.

Keywords: Risk Premium, Survey Data, $I(2)$ CVAR, Real Exchange Rate

1. RISK AND PRICE SWINGS

Survey data have been used extensively in the literature to decompose excess foreign exchange returns into an expected component (interpreted as a risk premium) and an unexpected component (the forecast error).[1] Collectively, these studies find evidence of predictable forecast errors and a risk premium. This has motivated many studies that attempt to explain excess returns through departures from full-information, model-consistent rational expectations, at times abstracting from risk entirely.[2] Very few studies, however, have examined the determinants of the risk premium observed in survey data.

This study will use survey data to test an alternative characterization of risk behavior in the foreign exchange market. Frydman and Goldberg's (2003, 2007) imperfect knowledge economics (IKE) gap model relates risk not to the second

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moment properties of returns, as is the common characterization of risk assessment, but rather to the asset price's divergence (gap) from its perceived benchmark value (for example, purchasing power parity). The intuition is that as a currency becomes more overvalued relative to its benchmark, bulls (with long positions) will become more worried about the likelihood and potential magnitude of a reversal, and the concomitant losses it would generate. Therefore they will raise their required expected return to maintain their positions. Bears, meanwhile, will respond in the opposite fashion, both effects acting to raise the aggregate risk premium.

This alternative view of risk is both intuitive and appealing. Ned Phelps, for example, promoted it as a means of assessing systemic risk in a 2009 open letter to leaders of the G-20.[3] Arguably, though, the IKE gap theory has not yet been sufficiently tested. The initial empirical work of Frydman and Goldberg (2007) possessed an upward endogeneity bias, as this paper will describe, which is problematic given that the alternative hypothesis is of a positive relationship. They also rely on several untested a priori data restrictions, which will be relaxed here.

Further, more recent econometric techniques have found that the exchange rate is actually more persistent than a simple random walk, going back to the Markov-switching work of Engel and Hamilton (1990).[4] This is an important empirical finding, liable to bias estimates of expected excess returns if ignored [Evans and Lewis (1995)], and requires analysis beyond the error-correction model used in Frydman and Goldberg (2007) or the $I(1)$ cointegrated vector autoregression in Stillwagon (2015a). Very few empirical studies, however, incorporate a time-varying trend. Among the exceptions are those using the $I(2)$ cointegrated vector autoregression (CVAR), which Johansen et al. (2010) note is a generalization of the segmented trends model where the stochastic trend can take on a continuum of values and estimation is conducted as a system of simultaneous equations.[5]

This neglect of persistent changes in economic variables likely stems from the common reliance on univariate tests for diagnosis, which Juselius (2012, 2014) demonstrates have limited ability to detect an $I(2)$ trend when the signal-to-noise ratio (the variance of the $I(2)$ component relative to the variance of the $I(1)$ component) is low, as is often found to be the case. In addition to more properly testing and addressing nonstationarity, the $I(2)$ CVAR provides the benefits of more rigorous restriction testing and more elaborate examination of the driving and adjustment dynamics.

Using the $I(2)$ CVAR, this paper tests the main prediction of the IKE gap model against survey data measures of the risk premium for three US\$ exchange rate samples (with the Deutsche mark, British pound, and Japanese yen).[6] Overidentification is achieved by imposing the no-risk-adjusted-arbitrage conditions of the gap theory. The hypothesis that the risk premium is positively related to the real exchange rate finds compelling support even under this more stringent testing. In all three samples, the coefficient representing the gap effect is positive and statistically significant at the 1% if not 0.1% level. Further, the variables form a

stationary cointegrating relation that other studies have struggled to attain [Engel et al. (2008)]. This can be interpreted as an equilibrium between the risk premium and the real exchange rate. The common stochastic trends provide evidence that interest rates are causing the persistent swings of the exchange rate. Of unique interest in this study is the direct observation of expectations, which appear to be largely autonomous over the short run.

2. AN ALTERNATIVE CHARACTERIZATION OF RISK

The IKE gap model incorporates a lost insight of Keynes (1936), examined by Tobin (1958) and then largely overlooked by the discipline.[7] In discussing liquidity preference, or the reason individuals may prefer to hold cash over interest bearing bonds, Keynes remarked,

> [u]nless reasons are believed to exist why future experience will be very different from past experience, a . . . rate of interest [much lower than the benchmark rate], leaves more to fear than to hope, and offers, at the same time, a running yield which is only sufficient to offset a very small measure of fear [of capital loss]. [Keynes (1936, p. 202)]

Frydman and Goldberg draw from this and model the risk premium as a function of the asset price's deviation, or the forecast's deviation, from its (more stable) benchmark value, which they refer to as the "gap." In the case of foreign exchange, a natural candidate for such a benchmark value is the purchasing power parity (PPP) exchange rate, as there is a large literature on mean reversion of the exchange rate back to this level, which equalizes costs across countries [Rogoff (1996), Taylor and Taylor (2004)].

The equilibrium condition, assuming PPP as the benchmark, implies a cointegrating relationship between the risk premium and the gap. Frydman and Goldberg refer to this equilibrium condition as uncertainty-adjusted uncovered interest parity (UAUIP). In particular, Frydman and Goldberg use the Engle and Granger (1987) error-correction model to estimate

$$(s^e_{t+1|t} - s_t + i^*_t - i_t) = \sigma(s^e_{t+1|t} - s^{\text{PPP}}_t) + \rho_t + \varepsilon_t. \tag{1}$$

The log exchange rate s_t is written here in terms of the domestic price of foreign currency. The left-hand side represents the risk premium, which is just the difference between the expected foreign return, which is the foreign interest rate i^*_t, plus any additional expected gain or loss arising from a change in the exchange rate $s^e_{t+1|t} - s_t$, and the domestic return, which is the home interest rate i_t. Frydman and Goldberg measure $s^e_{t+1|t}$ with survey data on the median of traders' point forecasts for the exchange rate one month ahead from Money Market Services International, a common source in this literature [see Dominguez (1986) and Froot and Frankel (1989) for more details about these data].[8] The gap is measured as the difference between the exchange rate forecast and PPP ($s^e_{t+1|t} - s^{\text{PPP}}_t$).

To elaborate on the intuition of the model, the risk premium is defined as the risk premium for bulls minus the risk premium for bears, where the aggregation weights are related to wealth shares [which are assumed to be equal between bulls and bears, as in Delong et al. (1990)].[9] Consider the case where the foreign currency becomes more overvalued relative to PPP, an increase in $s_t - s_t^{\text{PPP}}$. Bulls, although they may expect further appreciation, will become more worried about the possibility and magnitude of a sudden reversal (even if only partial, as is often the case) and will increase their forecast of the potential losses to speculation.[10] Consequently, they will increase their required premium to maintain their long positions. Bears, meanwhile, betting on a reversal of some degree, will become less worried about a further movement away, lowering their premium. Both effects act to raise the aggregate premium, and therefore the foreign currency will need a higher expected return in equilibrium to compensate for the growing "gap risk," implying that $\sigma > 0$.[11] Conversely, a higher expected foreign return will cause the foreign currency to appreciate, until it is sufficiently overvalued relative to PPP to deter further appreciative speculation.

ρ_t represents the premium on foreign exchange at PPP, which fluctuates over time in the theoretical model because of position sizes relative to wealth. Because these data are not readily available, we will capture this in the model's deterministic components and focus on the gap effect. Frydman and Goldberg (2007) find support for the main prediction of the model that $\sigma > 0$.[12]

At this point it is useful to view some simple graphics of the connection between the expected excess return and the real exchange rate for our samples. To be quite concrete about what is meant by the gap effect, the left column of graphics in Figure 1 (for, from top to bottom, the BP/US$, DM/US$, and JY/US$) shows the expected exchange rate in dotted blue and an estimate of PPP in solid red (based on the Economist's PPP Big Mac Index value in 1993 and extrapolated with consumer price indices). The gap then is the difference between the two series. The current spot exchange rate is also shown in dashed green to illustrate the close co-movement between the expected and current exchange rates (and therefore using either to measure the gap makes negligible difference). The second column shows the gap in dotted blue and our measure of the risk premium based on survey data in solid red.

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A few tentative conclusions can be reached from these graphics, to be more formally tested in what follows. There does appear to be a degree of persistence to expected returns, with some extended periods over which the premium is primarily
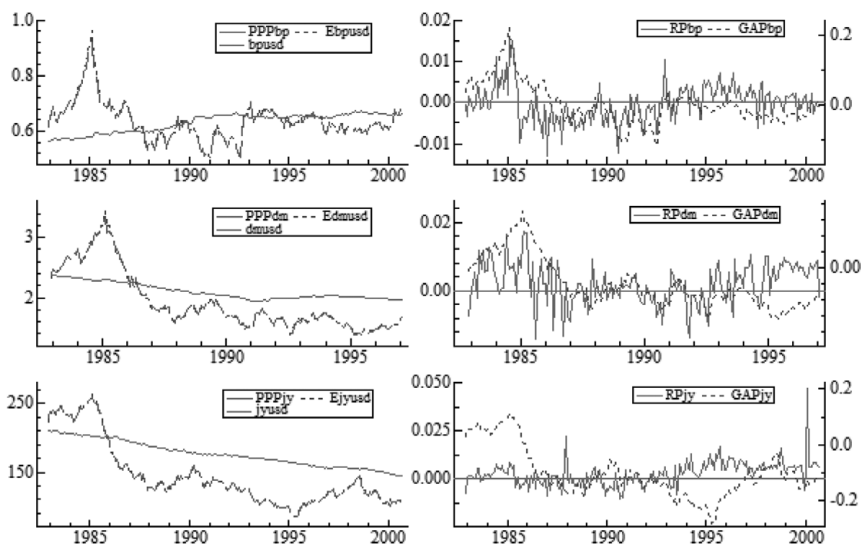
FIGURE 1. The real exchange rate and the risk premium. The first row of panels is for the BP/US\$, the second for the DM/US\$, and the third for the JY/US\$. The three panels on the left show the nominal exchange rate in dashed green, the expected next month exchange rate in dotted blue, and the estimate of PPP in solid red. The panels on the right show the risk premium based on survey data in solid red and our gap measure in dotted blue, which is the difference between the dotted blue and solid red series on the left.

negative or positive for years at a time, as evidenced by it being above or below the black horizontal line. Second, it appears that the premium covaries positively with the gap measure.[13] It also appears that the premium is more volatile than the gap measure, suggesting that there are other factors of relevance in addition to the gap, possibly including some of the $I(2)$ dynamics discussed in the next section.

Last, one can see evidence for a broken trend in the relationship for the yen sample, where the two series would align better if the gap were measured relative to a downward trend initially and an upward trend later in the sample, and to a lesser degree as well toward the end of the mark sample. These broken trends may be capturing fluctuations in ρ_t from changes in bilateral international financial positions, data on which are not available at monthly frequencies for the entirety of the needed time periods, or other perhaps as-yet-undiscovered risk factors extraneous to the gap theory. We can also see a couple of large outliers in the yen premium in late 1987 and early 2000, which may also reflect risk factors outside of the model.

There are a few deficiencies of the original empirical specification used in Frydman and Goldberg (2007), as mentioned in Section 1. Frydman and Goldberg do not test for the stationarity of the individual variables and do not test for the presence of cointegration (the stationarity of the remaining error term). This yields a possibility of spurious regression. Another problem with estimating σ from the

specification in equation (1) is that there is a positive endogeneity bias, because the variable $s_{t+1|t}^e$ enters both sides of the regression with the same sign, so that the regressor is positively correlated with the error term. The estimates of the gap coefficient would tend to have an upward bias then, which is problematic given that our alternative hypothesis is $\sigma > 0$.

Last, the parentheses in equation (1) indicate that the variables have been combined from the outset (to form the premium and gap). By imposing the restrictions a priori, this creates two issues. The empirical model has imposed some untested assumptions, which improve the chances that the model will not be rejected. A more complete examination of the theory would test these restrictions as well to see whether they are truly consistent with the data. Second, these restrictions are liable to muddle the driving and adjustment process. The original results reported that the premium is error-correcting; however, it cannot be discerned whether the adjustment is coming through expectations, the spot rate, or the interest rates, and which variables are weakly exogenously driving the system. By extending analysis to the $I(2)$ model, we can more appropriately address the issue of persistence, and we can relax the data restrictions to provide a more rigorous test of the theory and a better examination of the driving and adjustment dynamics of the relationship. Rearranging equation (1) to collect terms, we have

$$s_t = (1 - \sigma)s_{t+1|t}^e + \sigma s_t^{\text{PPP}} + i_t^* - i_t - \rho_t + \varepsilon_t, \qquad (2)$$

The IKE gap theory predicts a unity coefficient for the interest rate differential and a weighted average of expectations and the benchmark proxied here with PPP. The gap effect hypothesis implies that $\sigma > 0$, and because one would expect that when expectations of a future exchange rate rise, the actual exchange rate would also rise, this implies that $1 > \sigma > 0$. A coefficient less than one on expectations of the price is also consistent with the stability condition implied by most learning models [Evans and Honkapojha (2012)]. These restrictions provide a quite stringent test of the theory with very precise values for the coefficients (aside from the potentially segmented trend ρ_t) of equal coefficients between the exchange rate and interest rate differential, and a weighted average between expectations and PPP. In the estimate, then, there is only one "free parameter," σ, and even it needs to fall between zero and one to accord with the theoretical predictions.

3. THE DATA AND POLYNOMIAL COINTEGRATION

Without loss of generality, the $I(2)$ CVAR can be discussed in terms of acceleration rates, changes, and levels for a VAR(3) specification:

$$\Delta^2 x_t = \Gamma_1 \Delta^2 x_{t-1} + \Gamma \Delta x_{t-1} + \Pi x_{t-1} + \mu_0 + \mu_1 t + \varepsilon_t, \qquad (3)$$

$t = 1982 : 12, \ldots, 2000 : 09$, where $x_t' = [s_t, s_{t+1|t}^e, s_t^{\text{PPP}}, i_t, i_t^*, \Delta p_t^*]$, the information set is composed of data from 1982:12 to 2000:09 (though ending in

1997:02 for the DM/US$ sample) on the spot exchange rate s_t, the forecast of the next period exchange rate $s^e_{t+1|t}$, an estimate of the PPP value of the exchange rate s^{PPP}_t, the domestic interest rate on T-bills i_t, the foreign interest rate i^*_t, and foreign CPI inflation Δp^*_t (to see whether PPP is changing because of domestic or foreign shocks). The survey data report the median respondent's one-month-ahead point forecast of the spot exchange rate and come from Money Market Services International until 1997:02 and then are appended with data from the Currency Forecaster for the remainder of the BP and JY samples. The PPP measure uses the Economist's "Big Mac Index" value in 1993 to capture the level and is then extrapolated with consumer prices. The remaining variables were obtained from the IFS database.

The Π matrix is simply a reformulation of the covariances in the data. If there are unit roots present, the matrix will not be full rank. The hypothesis that x_t is I(1) then is a reduced-rank condition of $\Pi = \alpha\beta'$, where α and β are $p \times r$, where p is the number of variables included in the information set and r is the rank or number of cointegrating relationships. This is the well-known Johansen method or cointegrated VAR [Johansen (1991), Juselius (2006)]. The β vectors describe the linear combinations of the nonstationary variables that form a stationary cointegrating relationship and can be interpreted as representing long-run equilibrium conditions. We can test then whether the restrictions of equation (2) are consistent with any of the β vectors. This will be the primary focus of the empirical work. The α vectors meanwhile describe the error correction following exogenous shocks, signifying which variables are endogenous and how quickly they adjust back to equilibrium.

The remaining components of the $I(2)$ model will allow for additional insights about the dynamics of the system embodied in the α vectors and Γ matrix. The hypothesis that x_t is $I(2)$ is an additional reduced-rank restriction on Γ; $\alpha'_\perp \Gamma \beta_\perp = \xi\eta'$, where ξ, η are $(p-r) \times s_1$ and s_1 is the number of $I(1)$ common stochastic trends. The first condition is related to the variables in levels, whereas the second is related to the variables in first differences. The interpretation of this second $I(2)$ condition is that the differenced series also contain unit roots [Johansen (2006), Juselius (2006)].

It is worth echoing a nuance often noted by Juselius (2006, 2012, 2014) that we are really referring to near-$I(1)$ and near-$I(2)$ type persistence. The statement that a variable is integrated of order two, or even order one, is not a statement about an intrinsic property of that variable; it is simply a useful approximation over the given sample. Even when it is said that a variable is $I(1)$, we are simply saying that the shocks have a highly persistent effect on the levels of the variables, though not necessarily or truly infinite. Similarly, when it is said that a variable exhibits near-$I(2)$ persistence, we are simply saying that the shocks to that variable have a highly persistent impact on both the levels and growth rates of the variable, though again not necessarily or truly infinite. This additional persistence provides important information about the dynamics and interactions between variables that is lost if ignored. For the $I(2)$ case, a remaining large root in the system

alters the interpretation, implying that the variables cointegrate not to $I(0)$ but rather from near order two to near order one. This suggests that the given model, with a remaining large root in the differenced process, has not fully captured the persistence present in the system.

Because the Γ matrix is no longer unrestricted in the presence of $I(2)$ variables, as it is in the $I(1)$ CVAR, we require an alternative parameterization. The cointegration analysis of time series (CATS) software uses the following formulation [see, e.g., Juselius (2006), Juselius and Assenmacher (2015)], which is a variant of Parulo and Rahbek (1999):

$$\Delta^2 x_t = \alpha(\beta' x_{t-1} + \delta' \Delta x_{t-1}) + \zeta \tilde{\tau}' \Delta x_{t-1} + \mu_0 + \varepsilon_t, \quad (4)$$

where $\tilde{\tau}$ represents the relationships in both β' and $\beta_\perp$, δ is a $p \times r$ matrix of polynomially cointegrating parameters, and ζ is a $p \times (p - s_2)$ matrix of medium-run adjustment coefficients, where s_2 is the number of $I(2)$ common stochastic trends. Polynomial cointegration implies a relationship in levels, which becomes $I(1)$, combining with the differenced processes of the $I(2)$ variables to produce a trend-stationary-equilibrium condition, $\beta' x_{t-1} + \delta' \Delta x_{t-1}$. The second reduced-rank condition therefore determines the number of polynomially cointegrating relationships among the r cointegrating relationships and the number of $I(2)$ trends among the common stochastic trends. The trend is restricted to the cointegrating space to avoid undesirable quadratic and cubic effects in the model [Rahbek et al. (1999)].

The moving average representation of the last equation, subject to the two reduced-rank conditions, presents the variables x_t as functions of single and double cumulated errors as well as stationary and deterministic components. It indicates the driving trends of the system, namely which variables' shocks accumulate to alter the equilibrium:

$$x_t = C_2 \sum_{i=1}^{t} \sum_{s=1}^{i} \varepsilon_s + C_1 \sum_{i=1}^{t} \varepsilon_i + C(L)\varepsilon_t + \tau_0 + \tau_1 t. \quad (5)$$

τ_0 and $\tau_1 t$ are functions of the initial conditions and parameters of the VAR, and the coefficient matrices are likewise complex functions of the parameters [Rahbek et al. (1999)]. For the objectives herein, though, focus on C_2 is all that is required. $C_2 = \tilde{\beta}_{\perp 2} \alpha'_{\perp 2}$, where $\tilde{\beta}_{\perp 2} = \beta_{\perp 2}(\alpha'_{\perp 2} \Psi \beta_{\perp 2})^{-1}$. $\beta_{\perp 2}$ and $\alpha_{\perp 2}$ are $p \times s_2$ matrices, which are respectively orthogonal to α, $\alpha_{\perp 1}$ and β, $\beta_{\perp 1}$, where $\alpha_{\perp 1}$ indicates how the unexpected shocks to the variables accumulate into the $I(1)$ trend and $\beta_{\perp 1}$ is the trend's loadings into the variables. The interpretation is that $\alpha'_{\perp 2} \sum_{i=1}^{t} \sum_{s=1}^{i} \varepsilon_s$ provides an estimate of how the unexpected shocks (acceleration rates in the case of $I(2)$ variables) double accumulate into the s_2 second-order common stochastic trends. $C_1 \sum_{i=1}^{t} \varepsilon_i$ represents the standard $I(1)$ common stochastic trends, where the exogenous shocks single accumulate as in the $I(1)$ CVAR [Johansen (1991)].

TABLE 1. β Vector of the polynomially cointegrating relations

BP/US\$ p-value: 0.262				
$s_t + i_t - i_t^*$	$s_{t+1\|t}^e$	s_t^{PPP}	$T(91{:}04)$	Trend
1.000	−0.954	−0.046	-1.105×10^{-4}	-0.906×10^{-4}
[NA]	[−11.661]	[−7.657]	[−4.264]	[18.059]
DM/US\$ p-value: 0.711				
$s_t + i_t - i_t^*$	$s_{t+1\|t}^e$	s_t^{PPP}	$T(91{:}03)$	Trend
1.000	−0.966	−0.034	2.185×10^{-4}	-0.106×10^{-4}
[NA]	[−12.321]	[−5.120]	[3.712]	[1.673]
JY/US\$ p-value: 0.551				
$s_t + i_t - i_t^*$	$s_{t+1\|t}^e$	s_t^{PPP}	$T(93{:}01)$	Trend
1.000	−0.982	−0.018	-1.475×10^{-4}	0.333×10^{-4}
[NA]	[−15.687]	[−3.882]	[−3.917]	[4.299]

Notes: The *p*-value represents the LR tests of the overidentifying restrictions and cointegration for each sample. The coefficient estimates for the first beta vector, associated with the tests of UAUIP, are presented with the *t*-values in brackets.

4. TESTS OF THE GAP MODEL: IDENTIFICATION OF THE LONG-RUN STRUCTURE

The rank diagnostics suggest a rank of three with one $I(2)$ trend (see Appendix). To produce standard errors and estimate the system, $r - 1$ identifying restrictions must be imposed, or two in this case, on each of the cointegrating relations.[14] For the first relation, overidentification is achieved by imposing the restrictions implied by the uncertainty-adjusted UIP equilibrium discussed in Section 2:[15]

$$s_t + i_t - i_t^* - (1-\sigma)s_{t+1\|t}^e - \sigma s_t^{\text{PPP}} - \rho_t = \varepsilon_t. \tag{6}$$

We have one restriction of the interest rate differential, another of an equal coefficient on the interest rate differential and spot rate (which is normalized to one), a zero restriction on the single inflation rate, and the restriction that the coefficients on the expected and PPP exchange rate must be a weighted average of the coefficient on the spot rate. Again, the coefficient on s_t^{PPP} is the estimate of the gap effect, hypothesized to be positive but less than one to ensure that the expected exchange rate also possesses a positive sign. This provides a quite stringent test of the theory, testing the restrictions explicitly now, and allowing in essence only one "free" parameter σ among all of the regressors, and even it must fall between 0 and 1 to corroborate the theory.

ρ_t represents omitted risk factors, such as international financial positions or structural change, possibly from revisions to forecasting, as emphasized by IKE. This term is captured in the deterministics of the model. Each sample contains a trend and a break in the trend denoted by $T(\text{yy:mm})$, where yy represents the year and mm the month of the break.[16] The error term further captures these types of omissions from the models, as well as measurement error in the survey data

(because it is only a sample of traders and is measured with the median forecast rather than according to wealth shares).

Table 1 shows estimates of the β coefficients for equation (6). These can be interpreted as the static, long-run equilibria between the variables, to which they ultimately error-correct. Presented below the coefficient estimates are the t-values in parentheses. Rejection of the restrictions, based on the p-value for each relationship, would imply that cointegration is not present and the relationship cannot be interpreted as an equilibrium condition. These results in Table 1 can be interpreted in a way similar to those of the well-known Johansen method or $I(1)$ CVAR. The $I(2)$ dynamics will be discussed in the following subsections.

In all cases, we cannot reject the very precise restrictions of the IKE gap model, as shown by the p-values of 0.262, 0.711, and 0.551 for the BP, DM, and JY, respectively. This indicates that it forms a stationary cointegrating relationship that can be interpreted as an equilibrium, at least when combined with the differences of the $I(2)$ variables. The gap effect, represented by the coefficient on s_t^{PPP}, is in all cases of the hypothesized sign and statistically significant at the 0.1% level.

Overall there appears to be a great deal of support for the stringent hypotheses of the model, particularly considering that only one risk factor, the gap, is accounting for movements in the premium here, and that measurement error in the survey data or undetectable structural change should lower the p-value of stationarity.

We can see that the gap effect is largest in the BP sample and smallest in the JY sample. This seems congruent with the behavior of the real exchange rates observed in Figure 1. This pattern implies that individuals are attaching a greater weight to the gap in assessing risk for those exchange rates that have tended to exhibit sharper and more frequent reversion back to PPP, which seems quite rational.

Recall that all variables in the preceding relationship are treated as on the left-hand side in equation (6), with only the error term on the right-hand side. We initially observe an upward trend in the BP and DM sample, and a downward trend in the JY. The trends and breaks can be interpreted as the risk premium if at PPP. This suggests an initially growing risk premium (controlling for the gap) for the DM and BP until 1991, and a falling risk premium on the yen until 1993. This initial downward trend is particularly evident for the JY sample in Figure 1.

The trend becomes more positive after 1991:04 for the BP, consistent with a rising risk premium on the BP after controlling for the gap, and reverses sign for the DM and JY. Economic interpretations for the trend and breaks, although speculative, seem readily apparent given the historical context. We see a lower risk assessment for the DM after German reunification, and higher risk for the BP leading up to the breakup of the EMS, particularly as it becomes more imminent. For the JY, we see a greater risk premium as the lost decade begins to take hold and government debt begins to grow, after a falling premium prior likely connected to their substantial net lender status and current account surplus.

Figure 2 shows the actual log exchange rate in red, and its estimated long-run equilibrium value in blue, based on the results in Table 1. We would expect that
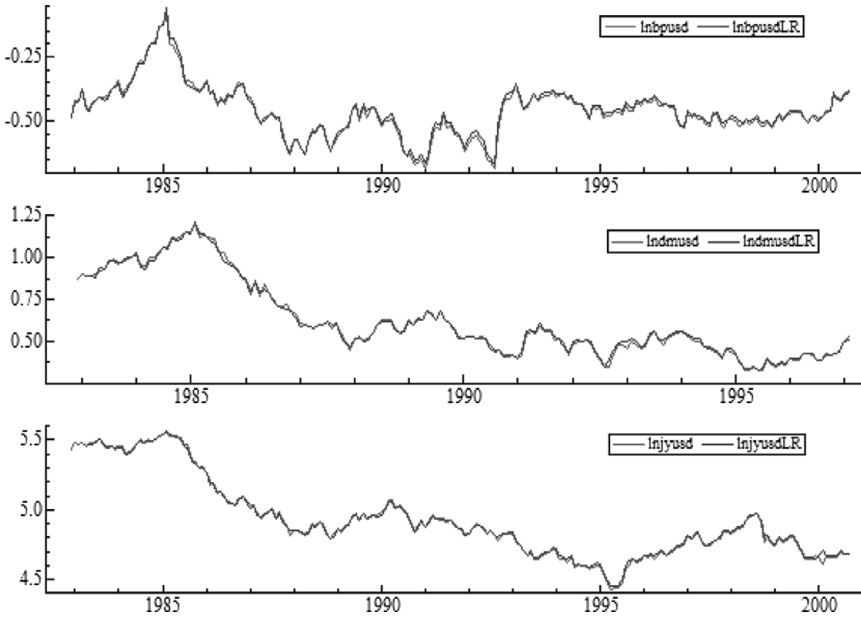
FIGURE 2. The log exchange rate and its estimated long-run equilibrium value.

the actual exchange rate should ultimately error-correct to this equilibrium value, and we can see that they do indeed move together quite closely. The exact nature of the equilibrium adjustment will be described in the following subsection.

In general, the relationships in Table 1 will cointegrate from $I(2)$ to $I(1)$, and then become stationary through combination with the differenced processes of the $I(2)$ variables $[\beta'x_{t-1} + \delta'\Delta x_{t-1} \sim I(0)]$, representing a dynamic, long-run equilibrium. The asymptotic distribution of the δ parameters has become known only more recently [Boswijk (2010)]. The common convention, though [in, e.g., Johansen et al. (2010) and Juselius and Assenmacher (2015)], has been to focus on the δ parameters for the variables that are rejected as at most $I(1)$. In these samples, these include s_t^{PPP} at the 1% level for all samples, i_t for the JY and DM and i_t^* in the BP sample (at least at the 10% level), and s_t and $s_{t+1|t}^e$ in the yen sample. These tests are shown in Table A.5 of the Appendix. The bolded δ coefficients in Table 2 are for those variables rejected as at most $I(1)$ at the 5% level, and italicized represent those rejected at 10%.

The estimated polynomial cointegrating relations for each sample excluding the deterministics, based on Tables 1 and 2, are as follows:[17]

$$\text{BP/US\$} = i_t^* - i_t + 0.954s_{t+1|t}^e + 0.046s_t^{\text{PPP}} - 0.455\Delta s_t^{\text{PPP}} + 0.021\Delta i_t^*, \quad (7)$$

$$\text{DM/US\$} = i_t^* - i_t + 0.966s_{t+1|t}^e + 0.034s_t^{\text{PPP}} - 0.002\Delta s_t^{\text{PPP}} - 0.001\Delta i_t, \quad (8)$$

TABLE 2. α and δ vectors for UAUIP

			BP/US$			
	$\Delta^2 s_t$	$\Delta^2 s_{t+1|t}^e$	$\Delta^2 s_t^{\text{PPP}}$	$\Delta^2 i_t$	$\Delta^2 i_t^*$	$\Delta^3 p^*$
α'	−0.226	0.419	0.006	0.003	**0.006**	−0.005
	[−0.617]	[1.090]	[0.214]	[0.773]	[2.374]	[−0.266]
δ	−0.520	−0.481	**0.455**	0.061	*−0.021*	0.007

			DM/US$			
	$\Delta^2 s_t$	$\Delta^2 s_{t+1|t}^e$	$\Delta^2 s_t^{\text{PPP}}$	$\Delta^2 i_t$	$\Delta^2 i_t^*$	$\Delta^3 p^*$
α'	**−0.873**	−0.012	0.004	0.001	0.002	−0.023
	[−2.871]	[−0.035]	[0.188]	[0.678]	[0.935]	[−1.568]
δ	0.038	0.040	**0.002**	**0.001**	−0.000	0.000

			JY/US$			
	$\Delta^2 s_t$	$\Delta^2 s_{t+1|t}^e$	$\Delta^2 s_t^{\text{PPP}}$	$\Delta^2 i_t$	$\Delta^2 i_t^*$	$\Delta^3 p^*$
α'	−0.374	0.245	−0.043	0.001	0.001	0.021
	[−1.300]	[0.781]	[0.183]	[0.819]	[0.401]	[1.603]
δ	**0.096**	**0.098**	**0.003**	*0.000*	−0.000	−0.000

$$\text{JY/US\$} = i_t^* - i_t + 0.982 s_{t+1|t}^e + 0.018 s_t^{\text{PPP}} - 0.003 \Delta s_t^{\text{PPP}}$$
$$- 0.096 \Delta s_t - 0.098 \Delta s_{t+1|t}^e. \tag{9}$$

The level relationships in equations (7)–(9) are the same as those plotted in Figure 2, though now the effects of the growth rates for the $I(2)$ variables have also been featured. Juselius and Assenmacher (2015) interpret $\beta' x_{t-1}$ as a persistent equilibrium error and $\delta' \Delta x_{t-1}$ as how the growth rates of the variables adjust to this disequilibrium. The economic interpretations for this adjustment will be discussed in the next subsection.

4.1. Dynamics of Adjustment and Positive Feedback

The α and δ coefficients detail how the acceleration rates and changes in the variables respond to disequilibrium. The α' vector demonstrates how the acceleration rates adjust to changes in the other variables. An insignificant coefficient would suggest that the acceleration rates of that variable are not responding to shocks in the system. To interpret the significant adjustment, we also need the δ and β vectors. If $\alpha_{ij}\delta_{ij} < 0$, meaning the coefficients have opposite signs for a given variable in a given cointegrating relation, then the acceleration rates are equilibrium-correcting to the changes (which will be referred to here as short-run adjustment). If the signs are identical, it implies error-increasing behavior, meaning the variable is moving away from equilibrium in the short run. If $\delta_{ij}\beta_{ij} > 0$, then the changes are equilibrium-correcting to the levels (medium-run adjustment), and if $\alpha_{ij}\beta_{ij} < 0$,

the acceleration rates are adjusting to the relationships in levels $\beta' x_{t-1}$ (long-run adjustment). See Juselius and Assenmacher (2015) for another application of the I(2) model describing the multitier error correction.[18]

The largest and most significant α coefficient occurs for s_t in the DM sample with a very large estimate of -0.873 (-1 would be full adjustment each period). It also has a sign that is opposite to both the β and δ vector coefficients, suggesting long- and even short-run adjustment. This is consistent with the gap model, which predicts that the exchange rate should adjust rapidly to restore equilibrium between buying and selling. Expectations do not appear to be significantly adjusting, however, and even the point estimate is very small, which is consistent with the autonomous role for expectations ascribed by IKE.[19]

For the BP and JY samples, the magnitudes of the α coefficients are very large for both $s_{t+1|t}^e$ and s_t, though they are not precisely estimated. This may stem from the high volatility of exchange rates, measurement error noise in the survey data, or vacillating forecasting behavior (due to learning dynamics or peso problems). The peso problem explanation seems particularly relevant for the BP/US\$, given its abrogation of the fixed exchange rate in September 1992 and the estimates that suggest that the exchange rate was not adjusting in the short run, but expectations were adjusting to changes.[20]

For the JY sample, we have the exchange rate adjusting in the short, medium, and longer run (according to the point estimates, though again α is not precisely estimated), but expectations have not been adjusting in the short or medium run, which likely contributed to the very persistent movements observed in Figure 1. The imprecise estimates for Japan may similarly stem from the Bank of Japan's practice of currency intervention.

Focusing on the significant $I(2)$ variables, the δ coefficients on s_t^{PPP} are always opposite their β coefficients (as shown in equations (7)–(9)), suggesting error-increasing behavior in the medium run. This positive feedback may arise from exchange rate pass-through affecting consumer prices. For example, if the U.S. interest rate i_t^* rises, the BP/US\$ rises. Exchange rate pass-through would tend to lead to an increase in s_t^{PPP} from higher U.K. import prices and lower U.S. import prices, which would cause a further appreciation of the US\$ in the medium run (at least until the J-curve dynamics emerged). This effect has been particularly large for the United Kingdom, which may be linked to its current account deficit and greater import dependence than Germany or Japan.

For the interest rates, we observe medium-run adjustment. For example, if the U.K. interest rate rises, the U.S. interest rate also tends to rise (as one would expect via substitution effects), and similarly if U.S. interest rates rise, German and Japanese interest rates also tend to rise. The effect is largest between the United States and United Kingdom, which are both major financial centers, and smallest for Japan, which may be associated with its unique, prolonged experience at or near the zero lower bound, beginning in the 1990s.

The $\alpha_\perp$ vectors in Table 3 illustrate how unexpected shocks to each variable contribute to the common stochastic trends. The variables that are significant have

TABLE 3. $I(1)$ and $I(2)$ common stochastic trends

	$\Delta^2 s_t$	$\Delta^2 s_{t+1|t}^e$	$\Delta^2 s_t^{PPP}$	$\Delta^2 i_t$	$\Delta^2 i_t^*$	$\Delta^3 p^*$
			BP/US$ common trends			
$\alpha'_{\perp 1}(1)$	281.04 [0.47]	181.33 [0.41]	−1188.09 [−0.50]	−48.60 [−0.14]	**−569.52** [−3.50]	405.33 [0.35]
$\alpha'_{\perp 1}(2)$	−280.99 [−0.47]	−181.30 [−0.41]	1187.92 [0.50]	48.55 [0.14]	**569.60** [3.50]	−405.26 [−0.35]
$\alpha'_{\perp 2}(1)$	0.01 [0.54]	−0.00 [−0.24]	**−0.13** [−4.03]	**1.00** [NA]	0.26 [1.12]	**0.10** [2.17]
			DM/US$ common trends			
$\alpha'_{\perp 1}(1)$	−0.02 [−0.62]	0.05 [1.56]	**−0.92** [−2.71]	0.04 [1.58]	**−0.29** [−6.70]	**0.51** [2.10]
$\alpha'_{\perp 1}(2)$	0.05 [0.62]	−0.13 [−1.43]	**2.41** [3.05]	0.03 [0.57]	**0.64** [6.94]	**−1.35** [−2.67]
$\alpha'_{\perp 2}(1)$	−0.00 [−0.38]	0.01 [1.08]	**−0.18** [−3.10]	0.96 [1.34]	**1.00** [NA]	**0.18** [3.73]
			JY/US$ common trends			
$\alpha'_{\perp 1}(1)$	128.60 [1.22]	−183.13 [−1.36]	1146.23 [1.79]	**199.17** [4.85]	**221.56** [5.29]	25.75 [0.08]
$\alpha'_{\perp 1}(2)$	−128.47 [−1.22]	182.94 [1.36]	−1145.03 [−1.78]	**−199.19** [−4.85]	**−221.54** [−5.29]	−25.73 [−0.08]
$\alpha'_{\perp 2}(1)$	−0.00 [−0.42]	0.01 [1.77]	**−0.04** [−3.87]	**−0.88** [−3.79]	**1.00** [NA]	0.01 [0.32]

Notes: The coefficients show the variables' contributions to the common stochastic trends, with t-values presented in brackets. $\alpha'_{\perp 1}(1)$ and $\alpha'_{\perp 1}(2)$ represent the two $I(1)$ trends and $\alpha'_{\perp 2}(1)$ the $I(2)$ trend.

unexpected shocks, represented by the acceleration rates for the $I(2)$ variables, which accumulate over time, driving changes in the equilibrium. The $\alpha'_{\perp 1}(1)$ and $\alpha'_{\perp 1}(2)$ vectors represent how the unexpected shocks to the two $I(1)$ trends accumulate. Of particular interest is the $\alpha'_{\perp 2}(1)$ vector demonstrating how the unexpected shocks doubly accumulate into the $I(2)$ trend, producing the additional persistence beyond $I(1)$ leading to persistent changes in the variables.

A quite robust pattern emerges. Based on the t-values observed in parentheses, the common stochastic trends are generated by unexpected shocks to (acceleration rates of) the interest rates, $\Delta^2 i_t$ and $\Delta^2 i_t^*$, and relative prices $\Delta^2 s_t^{PPP}$. The $I(2)$ trend seems to derive primarily from interest rates and to a lesser extent relative prices (where the coefficient is roughly an order of magnitude lower). This likely reflects the fact that the vast majority of transactions in the foreign exchange market are conducted for financial purposes, as opposed to for trade. Therefore the financial market forces exert a greater influence than the goods market forces. This is roughly consistent with the IKE gap model, which treats the interest rates as exogenously determined in the money markets and models the exchange rate as a flexible asset price within a portfolio balance framework. In particular, for the JY sample, we can interpret the $I(2)$ trend as an effect of the interest rate differential, given that the two interest rates have opposite signs and nearly equal coefficients, which likely embodies the yen's substantial role in the carry trade.

5. CONCLUSION

This paper provides evidence for an alternative characterization of currency risk. It appears that market participants are assessing risk on the basis of how far the exchange rate has moved, in one direction or the other, away from purchasing power parity. The use of survey data has provided a less ambiguous method for discerning investor risk behavior. This suggests that the excess returns puzzle and long swings puzzle can be reconciled through synthesis. The nature of the causation indicates that when the U.S. interest rate increases, for example, this will lead to expected excess returns that cause participants to bid up the US$. This will continue until the deviation from PPP is sufficient to prevent further arbitrage due to the perceived risk of an eventual reversal. In other words, deviations from PPP are driven by deviations from UIP and expected excess returns can exist, even at equilibrium, because they are offset by the risk associated with the price swings they produce.

The results suggest that models that attempt to account for predictable currency returns absent a time-varying risk premium, be it through non-REH forecasting or transaction costs, are at the very least incomplete. All of these models then may benefit from appending a gap risk effect. The use of survey data and this alternative perspective of risk may also prove useful in resolving similar anomalies in other asset markets, for example, the equity premium puzzle or the expectations puzzle of the term structure.

The empirical validity of this conception of risk provides support for the novel policy prescriptions of Frydman and Goldberg (2011). These include announcement guidance about benchmark values (to anchor expectations) and attempting to dampen excessive swings with not only currency interventions but also differential capital requirements for bulls and bears, depending on whether the asset is over- or undervalued. Standard risk premium models were not capable of yielding such insight, because they do not recognize the heterogeneity of expectations and the connection of price swings creating perceived skewness risk. Future research would benefit from assessing the optimal implementation of such policies in the IKE gap framework.

Last, this study also adds to evidence on the importance of the $I(2)$ estimation. It appears that it is necessary to allow for the possibility of persistent trends in nominal price data, including consumer prices, interest rates, and exchange rates, once the more powerful multivariate trace tests are employed. This framework more suitably addresses nonstationarity and allows much richer examination of the driving and adjustment dynamics, which could benefit a very broad scope of time series research.

NOTES

1. See MacDonald (2000) for a survey of survey data studies in three financial markets.
2. Some appeal to irrationality, for example noise-trading [Delong et al. (1990), Mark and Wu (1998)], overconfidence [Burnside et al. (2011)], or ambiguity aversion [Gourinchas and Tornell (2004), Ilut (2012)]. Others, however, suggest non-REH forecasting to be rational, for example through rational

inattention [Bacchetta and van Wincoop (2005)], complexity [De Grauwe and Grimaldi (2006)], or imperfect knowledge [Frydman and Goldberg (2007)].

3. Frydman and Goldberg (2011) outline some policy prescriptions based on this conception of risk.

4. Frömmel et al. (2005) and De Grauwe and Vansteenkiste (2007) also find support for Markov switching in monetary models.

5. See also Johansen (1997, 2006) and for a survey of $I(2)$ models see Haldrup (1998).

6. Frydman and Goldberg (2007) prospect theory-based model also relates a country's risk premium to its international debt position. For CVAR tests of the full model using survey data, see Goldberg et al. (2017).

7. The model also incorporates the loss aversion of Kahneman and Tversky (1979), though this is not connected to the empirical work herein. For a study incorporating ambiguity aversion, see Ilut (2012).

8. For other studies using exchange rate forecasts, see MacDonald and Torrance (1988, 1990), Cavaglia et al. (1993), Chinn and Frankel (1994), Bacchetta et al. (2009), and Stillwagon (2014). Survey data have been used in other contexts including the study of inflation [Ang et al. (2007), Rafiq (2014)], the stock market [Greenwood and Shleifer (2014)], the yield curve [MacDonald and Macmillan (1994), Wright (2011), Stillwagon (2015b)]; and the FOMC's loss function [Pierdzioch et al. (2016)].

9. For further details on the derivation see Frydman and Goldberg (2007, 2013).

10. This greater tendency to reverse as the gap grows is consistent with the nonlinear mean reversion in Taylor et al. (2001), and may explain the long cycles observed in the exchange rates here, where they seldom cross PPP over the sample. Iwata and Wu (2006) also model the foreign exchange risk premium in a nonlinear framework.

11. De Grauwe and Grimaldi (2006) use a similar specification where fundamental traders become less risk-averse the further the exchange rate is from its fundamental value. It is also similar to Brunnermeier et al.'s (2009) "downside" crash risk, here with the elaboration that individuals assess this risk ex ante via the gap.

12. IKE emphasizes the possibility of structural change. Frydman and Goldberg (2007) find evidence of parameter instability for all three samples proximate to the early 1990s using a CUSUM test and split the samples, though $\sigma > 0$ in the full and subsample estimates. Rossi (2006) also finds evidence of parameter instability in exchange rate relationships. Here we allow for structural change in the deterministic trend ρ if necessary, as is more common in the CVAR [see Johansen et al. (2010)].

13. The simple correlations between the level of the premium and the gap are .433, .298, and −.094 for the BP, DM, and JY, respectively. If we focus instead on the changes, to remove the trending effects, the correlations are .294, .266, and .312. This still ignores the $I(2)$ dynamics and deterministic breaks though.

14. For more on the general-to-specific methodology, see Hendry and Mizon (1993) and Hoover et al. (2006).

15. The other relations are just-identified and then insignificant variables are eliminated. Alternative just-identifying restrictions would not alter the log likelihood, though they seem to indicate inflation equations. Of course to aptly understand inflation's determinants, we would want to expand the information set. That is not the objective here, however.

16. The breaks were diagnosed with Hansen and Johansen's (1999) eigenvalue fluctuation test. Juselius and MacDonald (2004) find an identically timed break in the DM/US$ sample proximate to German reunification. See Hendry (2000) for more on detecting and addressing structural change in the CVAR.

17. An effect of Δi_t significant at 10% for the JY but with an estimated coefficient of 0.000 has been omitted.

18. Johansen et al. (2010) and Juselius and Assenmacher (2015) also conduct $I(2)$ analyses of the exchange rate, for the DM/US$ and CHF/US$, respectively. They focus on the PPP puzzle rather than the excess returns puzzle, though, and do not incorporate survey expectations.

19. Of course, as a matter of induction, this does not necessarily mean one could never find a model where expectations were purely and perfectly adjusting, as predicted by most REH models.

20. For studies incorporating peso problem explanations, see Evans and Lewis (1995) and Burnside et al. (2011).

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APPENDIX

Implementation of the $I(2)$ CVAR, like its $I(1)$ counterpart, begins by achieving a statistically well-specified model. Large standardized residuals (>3.5) are accounted for with

TABLE A.1. Residual diagnostics

Skewness	Δs_t	$\Delta s_{t\|t+1}^e$	Δs_t^{PPP}	Δi_t	Δi_t^*	$\Delta^2 p^*$
BP/US\$	0.303	0.456	−0.430	0.336	−0.203	−0.050
DM/US\$	0.098	0.152	0.276	−0.267	−0.310	−0.088
JY/US\$	−0.363	−0.294	0.086	−0.262	−0.388	0.321

Test for autocorrelation	BP/US\$	DM/US\$	JY/US\$
LM(1) p-value	0.494	0.407	0.062
LM(2) p-value	0.679	0.663	0.322

dummy variables to address serial correlation and skewness (Table A.1), as the cointegration results are generally robust to other types of non-normality [Juselius (2006)]. The lag-length determination indicates a VAR(2) for the DM and BP samples and VAR(3) for the JY sample. Dummy variables are included for the DM sample in 1991:01,1991:10, and 1993:01, and the break in the trend occurs in 1991:03. For the JY sample, we have dummy variables for 1986:03, 1987:12, 1998:10, and 2000:02, and the break in 1993:01. Last, for the BP sample, we have a dummy variable in 1990:04 and a break in 1991:04.

A.1. DETERMINING THE TWO REDUCED-RANK CONDITIONS

The number of cointegrating relations r and the number of $I(2)$ trends s_2 can be inferred using the maximum likelihood procedure of Nielsen and Rahbek (2007). The trace test is computed as a joint hypothesis for all possible combinations of r and s_2. The tests are nested within rows (for a given rank of the Π matrix) and nested in the last column (corresponding to the $I(1)$ model with no $I(2)$ trends). Given the presence of a broken trend in each sample, however, the asymptotic distribution needs to be adjusted, with the adjustment depending on the timing of the break as a proportion of the effective sample. The procedure used follows Kurita et al. (2011) and is estimated with Oxmetrics [see Doornik (2006)].

   The results in Tables A.2–A.4 should be interpreted as beginning with the most restricted model in the upper left ($r = 0$, $s_1 = 0$, $s_2 = p - r$) and continuing until the end of the row, before moving down to the next row and testing from left to right again, proceeding until the first failure to reject. The coefficients represent the trace test statistics; the p-values are presented in brackets. All lower rank tests not reported have been rejected for all choices of s_2.

   The first marginal rejection in Table A.2 for the BP/US\$ occurs at a rank of 2 with no $I(1)$ trends (p-value 0.067). The rejection for rank 3 with one $I(2)$ trend has a substantially higher p-value, however (p-value 0.288). Both choices are roughly consistent with the roots, because a rank of 3 has one sizeable root (0.873) remaining. The unaddressed root may derive from the levels in the Π matrix or from the differences of the Γ matrix. This suggests either that the rank should be two, or that the remaining large root resides in the differenced process of the Γ matrix, indicating an *I(2)* trend. The graphic of the third polynomially cointegrating relation (Figure A.1) looks quite stationary, though, suggesting that we would want to select a rank of at least three. Similarly, the tests for $I(2)$ variables in Table A.5 suggest we would want to select a model with at least one $I(2)$ trend. Based on this collective information, a rank of three with one $I(2)$ trend is selected.

TABLE A.2. BP/US$ I(2) trace test

$p - r$	r	$s_2 = 4$	$s_2 = 3$	$s_2 = 2$	$s_2 = 1$	$s_2 = 0$
4	2	411.338 [0.000]	286.596 [0.000]	183.661 [0.000]	117.619 [0.002]	82.715 [0.067]
3	3		182.416 [0.000]	108.467 [0.001]	60.981 [0.288]	51.639 [0.184]
2	4			55.840 [0.225]	40.786 [0.243]	30.136 [0.230]
1	5				22.444 [0.272]	12.933 [0.283]

Largest roots of the companion matrix					
Unrestricted VAR	0.939	0.939	0.883	0.883	0.478
$r = 4$	1.000	1.000	0.932	0.932	0.468
$r = 3$	1.000	1.000	1.000	0.873	0.473
$r = 2$	1.000	1.000	1.000	1.000	0.499

Notes: The trace test statistics are presented with the p-values in brackets. Also presented are the moduli of the five largest roots of the companion matrix for various choice of rank.
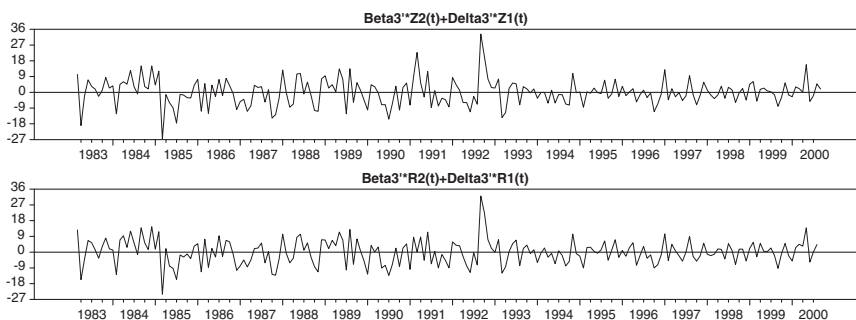

FIGURE A.1. BP/US$ graphic of the third polynomially cointegrating relationship.
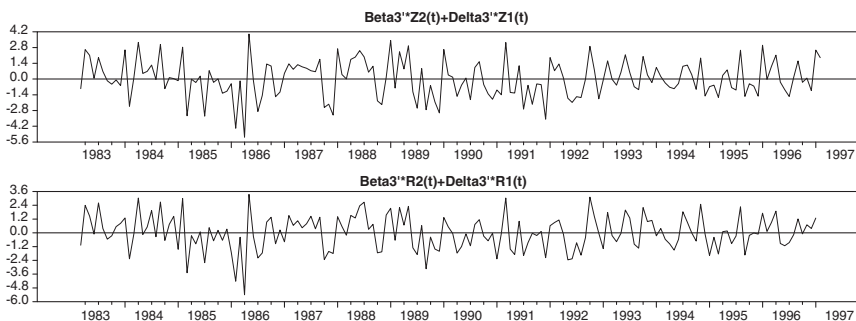

FIGURE A.2. DM/US$ graphic of the third polynomially cointegrating relationship.

TABLE A.3. DM/US$ $I(2)$ trace test

$p-r$	r	$s_2 = 4$	$s_2 = 3$	$s_2 = 2$	$s_2 = 1$	$s_2 = 0$
4	2	285.753 [0.000]	196.878 [0.000]	140.242 [0.001]	98.494 [0.060]	77.250 [0.145]
3	3		138.469 [0.000]	84.919 [0.106]	43.753 [0.921]	46.154 [0.377]
2	4			32.656 [0.986]	14.326 [1.000]	22.540 [0.659]
1	5				1.945 [1.000]	6.925 [0.835]

Largest roots of the companion matrix					
Unrestricted VAR	0.954	0.954	0.935	0.935	0.448
$r = 4$	1.000	1.000	0.937	0.937	0.420
$r = 3$	1.000	1.000	1.000	0.928	0.441
$r = 2$	1.000	1.000	1.000	1.000	0.558

Notes: The trace test statistics are presented with the *p*-values in brackets. Also presented are the moduli of the five largest roots of the companion matrix for various choice of rank.

TABLE A.4. JY/US$ short rates $I(2)$ trace test

$p-r$	r	$s_2 = 4$	$s_2 = 3$	$s_2 = 2$	$s_2 = 1$	$s_2 = 0$
4	2	299.795 [0.000]	211.545 [0.000]	140.886 [0.001]	96.198 [0.084]	74.593 [0.206]
3	3		139.206 [0.002]	81.719 [0.167]	46.401 [0.858]	37.799 [0.771]
2	4			44.914 [0.682]	28.011 [0.861]	19.437 [0.839]
1	5				13.698 [0.873]	5.032 [0.960]

Largest roots of the companion matrix					
Unrestricted VAR	0.957	0.957	0.949	0.820	0.715
$r = 4$	1.000	1.000	0.930	0.930	0.620
$r = 3$	1.000	1.000	1.000	0.916	0.617
$r = 2$	1.000	1.000	1.000	1.000	0.613

Notes: The trace test statistics are presented with the *p*-values in brackets. Also presented are the moduli of the five largest roots of the companion matrix for various choice of rank.

For the DM/US$ sample, in Table A.3 we have a few marginal rejections of rank two with one $I(2)$ trend (*p*-value 0.060), rank two with no $I(2)$ trend (*p*-value 0.145), and rank three with two $I(2)$ trends (0.106). The *p*-value is again much larger, however, for a rank of three with one $I(2)$ trend (0.921). There is one large root unaddressed for a rank of three (0.928). Therefore the roots of the companion matrix are consistent with a rank of two with no $I(2)$ trends or a rank of three with one $I(2)$ trend. Again, the third polynomially cointegrating relationship looks quite stationary (Figure A.2) and Table A.5 suggests that we would want to select a model with at least one $I(2)$ trend, so given this collective information, a rank of three with one $I(2)$ trend is the preferred choice.

TABLE A.5. Tests of variables being at most $I(1)$

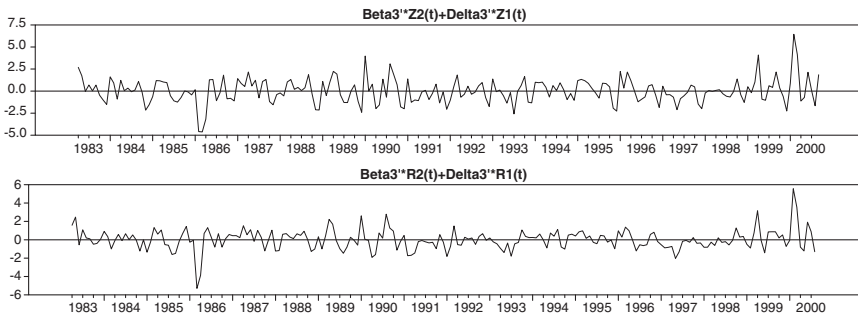| s_t	$s_{t+1\|t}^e$	s_t^{PPP}	i_t	i_t^*	Δp^*			BP	DM	JY
1	0	0	0	0	0		p-value	0.907	0.251	0.001
0	1	0	0	0	0		p-value	0.921	0.254	0.001
0	0	1	0	0	0		p-value	0.000	0.000	0.000
0	0	0	1	0	0		p-value	0.438	0.044	0.084
0	0	0	0	1	0		p-value	0.084	0.353	0.303
0	0	0	0	0	1		p-value	0.913	0.768	0.922
0	1	−1	0	0	0		p-value	0.765	0.402	0.003
0	0	0	1	−1	0		p-value	0.139	0.103	0.008

FIGURE A.3. JY/US$ graphic of the third polynomially cointegrating relationship.

For the JY/US$ sample, in Table A.4 we have a few marginal rejections of rank two with one $I(2)$ trend (p-value 0.084), rank two with no $I(2)$ trend (p-value 0.206), and rank three with two $I(2)$ trends (0.167). The p-value is again much larger, however, for a rank of three with one $I(2)$ trend (0.858). The roots are consistent with a rank of two with no $I(2)$ trends or a rank of three with one $I(2)$ trend. Again, the third polynomially cointegrating relationship looks quite stationary (Figure A.3) and Table A.5 suggests we would want to select a model with at least one $I(2)$ trend, so given this collective information, a rank of three with one $I(2)$ trend is chosen for this sample as well.

A.2. TESTING FOR PERSISTENT CHANGES IN THE VARIABLES: TESTS OF A KNOWN VECTOR IN $\tilde{\tau}$

In order to test whether the variables undergo persistent changes, or exhibit near-$I(2)$ behavior, we can test for a known vector b in $\tilde{\tau}$. The null hypothesis is that the variable being tested is at most trend $I(1)$, conditional on Δx_t. Given that all of the variables were rejected as $I(0)$ in the $I(1)$ model, a failure to reject this hypothesis would imply that the variable is $I(1)$. If the known vector in $\tilde{\tau}$ is rejected, we conclude that the changes in the variable experience significant persistence, that is near-$I(2)$ behavior. Juselius (2012, 2014) demonstrates the importance of conducting tests for near-$I(2)$ persistence in this fashion,

showing that the standard univariate unit root tests have weak ability to discriminate between an *I(1)* and *I(2)* trend when the signal-to-noise ratio is low.

A few common patterns emerge. The first is that relative prices, represented by the PPP exchange rate s_t^{PPP}, are rejected as at most $I(1)$ at very high significance levels, with a *p*-value of 0.000, for all three samples (see Table A.5). The interest rate differential likewise exhibits persistent changes particularly in the yen sample. Not surprisingly, the samples with greater evidence of persistent interest rate differentials also exhibit greater evidence of persistent changes in the expected and spot exchange rate. This near $I(2)$ behavior supports previous empirical work [Engel and Hamilton (1990), Johansen et al. (2010)]. It is also worth noting that Johansen et al. (2010) interpret $I(2)$ persistence in the real exchange rate as decisive evidence against common REH models of exchange rate swings including sticky price and REH bubble models.