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NOTES

THE DYNAMICS OF WEALTH INEQUALITY IN A SIMPLE RAMSEY MODEL: A NOTE ON THE ROLE OF PRODUCTION FLEXIBILITY

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It has been shown that the Ramsey growth model with agents that differ in their initial wealth endowments is compatible with a wide range of distributional outcomes, yet it is difficult to characterize under which circumstances the distribution of wealth becomes more or less unequal. In this note, we characterize the steady state distribution of wealth and compare it to the initial distribution, obtaining analytical conditions for one to be more skewed than the other. We show that whether wealth inequality increases or decreases during the transition to the steady state depends on simple and intuitive conditions on parameter values. Standard values for these parameters indicate that it is more likely that wealth inequality decreases as the economy accumulates capital.

Keywords: Growth, Wealth Distribution, Transitional Dynamics

1. INTRODUCTION

The representative-agent Ramsey growth model has been widely used by macroeconomists for almost 80 years, and in recent years, in particular, has become the standard framework for addressing a number of important issues in contemporary macrodynamics and growth theory.[1] In a recent paper, Caselli and Ventura (2000) have characterized relatively mild conditions under which various sources of heterogeneity are nevertheless compatible with viewing the aggregate (average) economy behaving as if it is populated by a single representative consumer (RC).[2] This is an important contribution, because it provides the RC model with a

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much firmer theoretical underpinning and counters some of the criticism to which this framework has been subjected; see, e.g., Kirman (1992), and others. As Caselli and Ventura stress, the RC assumption does not rule out heterogeneity and, indeed, it has important distributional consequences that depend on the evolution of aggregate magnitudes. Their main point is that the RC model permits one to study aggregate behavior without needing to consider the details of distribution, thereby preserving analytical tractability.[3]

In the second part of their paper, Caselli and Ventura address some of the consequences of the RC model for the dynamics of wealth distribution. They provide a number of numerical examples showing that during the transition to the steady state the cross-section of (relative) wealth could become more or less unequal, and illustrating the possibility of nonmonotonic dynamics for wealth inequality if the economy is far from the steady state.[4] The objective of this note is to extend the analysis of the distributional consequences of the RC model, because this too is important. We focus on a single, but important, source of heterogeneity, namely, differences in the initial wealth across individuals.[5] We characterize the steady state distribution of wealth and compare it to the initial distribution, obtaining analytical conditions for one to be more skewed than the other. We derive a simple proposition summarizing the relationship between the taste parameters and production parameters that determine whether wealth inequality increases or decreases during the transition toward the steady state.

For many years now, the Cobb-Douglas production function has come to be accepted as a reasonable specification of aggregate production and, indeed, the bulk of the recent literature on growth theory is based on this formulation; see, e.g., Lucas (1988), Barro (1990), and Jones (1995) among the most influential. As justification for this, Berndt's (1976) early comprehensive study is often cited. Berndt found that as the quality of data construction is improved, the differences between time-series and cross-sectional estimates of the elasticity of substitution can be reconciled, and indeed, estimates of the elasticity of substitution close to unity were obtained. However, recent authors have argued that the treatment of technological change has biased the estimates toward unity, and that modifying the econometric specification leads to significantly lower estimates of the elasticity of substitution, in the range 0.5–0.7, thus rejecting the Cobb-Douglas specification; see, e.g., Antràs (2004), Klump, McAdam, and Willman (2007). Because research in other areas has shown that even small deviations from the Cobb-Douglas benchmark can have profound consequences, it is important to allow for a more general specification of production.[6] Thus, the main objective of this note is to provide a simple characterization of the distributional consequences of the elasticity of substitution in production in a standard workhorse macroeconomic model.

This paper is organized as follows. Section 2 describes the economy and derives the macroeconomic equilibrium. Section 3 characterizes the distribution of wealth, and section 4 derives the main results of the paper pertaining to the conditions under which wealth inequality increases or decreases. Section 5 concludes.

2. THE RAMSEY MODEL WITH HETEROGENEOUS WEALTH ENDOWMENTS

We begin by setting out the components of the model. The economy is populated by L individual consumers, each of whom provides a unit of labor inelastically, so that L is also total labor supply.

2.1. Technology and Factor Payments

Aggregate output is produced by a single representative firm according to a standard neoclassical production function, so that[7]

$$Y = F(K, L) \qquad F_L > 0,\ F_K > 0,\ F_{LL} < 0,\ F_{KK} < 0,\ F_{LK} > 0, \tag{1}$$

where K, L, and Y denote the aggregate stock of capital, labor supply, and output. The wage rate, $w(t)$, and the return to capital, $r(t)$, are determined by their respective marginal physical products,

$$w(t) = F_L(K(t), L) \equiv w(K(t), L) \tag{2a}$$

$$r(t) = F_K(K(t), L) \equiv r(K(t), L), \tag{2b}$$

where $w_K = F_{KL} > 0$; $w_L = F_{LL} < 0$; $r_K = F_{KK} < 0$; $r_L = F_{KL} > 0$.

2.2. Consumers

The L individual consumers are indexed by $i = 1, 2, \ldots L$ and identical in all respects except for their initial endowments of capital, $K_{i,0}$. We shall define the share of capital owned by agent i as $k_i(t) \equiv K_i(t)/[K(t)/L]$. Note that relative capital has mean 1. We denote its initial distribution function by $H_0(k)$, the initial density function by $h_0(k)$, and the initial (given) standard deviation of relative capital by $\sigma_{k,0}$.

Agent i maximizes lifetime utility, assumed to be a function of consumption, in accordance with the isoelastic utility function

$$\max \int_0^\infty \frac{1}{\gamma} C_i(t)^\gamma e^{-\beta t}\, dt, \quad \text{with} \quad -\infty < \gamma < 1, \tag{3}$$

where $\omega \equiv 1/(1-\gamma)$ equals the intertemporal elasticity of substitution. This maximization is subject to the agent's capital accumulation constraint

$$\dot{K}_i(t) = r(t)K_i(t) + w(t) - C_i(t), \tag{4}$$

and yields the familiar Euler equation[8]

$$\frac{\dot{C}_i}{C_i} = \frac{r(K, L) - \beta}{1 - \gamma} \qquad \text{for each } i. \tag{5}$$

The important point about (5) is that each agent, irrespective of capital endowment, faces the same rate of return on capital, and thus chooses the same growth rate for consumption, implying

$$\frac{\dot{C}_i}{C_i} = \frac{\dot{C}_h}{C_h} \qquad \text{for all } i, h. \tag{6}$$

2.3. Macroeconomic Equilibrium and Aggregate Equilibrium Dynamics

We first derive the macroeconomic equilibrium and the dynamics of the aggregate economy. Having determined these, we shall then obtain the dynamics of the distribution of capital. To obtain the macroeconomic equilibrium, we write the individual's accumulation equation, (4), in the form

$$\frac{\dot{K}_i}{K_i} = r(K, L) + \frac{w(K, L)}{K_i} - \frac{C_i}{K_i}. \tag{7}$$

Summing over (7) and noting that $\sum_{i=1}^{L} k_i(t) = 1$, yields the aggregate accumulation equation

$$\frac{\dot{K}}{K} = r(K, L) + \frac{w(K, L)L}{K} - \frac{C}{K}. \tag{7'}$$

In addition, (6) implies that with a fixed population, L, aggregate consumption, $C \equiv \sum_{i=1}^{L} C_i(t)$, also grows at the common individual growth rate, namely

$$\frac{\dot{C}_i}{C_i} = \frac{\dot{C}}{C} \qquad \text{for all } i. \tag{6'}$$

We may then write $C_i = \theta_i\,(C/L)$, where $\sum_{i=1}^{L} \theta_i = L$, and θ_i is constant for each i, and yet to be determined. Thus defined, θ_i denotes agent i's consumption relative to the mean.

Recalling (2a), (2b) and the homogeneity of the production function, the equilibrium dynamics of aggregate consumption and capital are then given by

$$\frac{\dot{C}}{C} = \frac{F_K(K, L) - \beta}{1 - \gamma} \tag{8a}$$

$$\dot{K} = F(K, L) - C. \tag{8b}$$

These two equations embody the essence of the RC model. With all individuals following the same Euler equation, the aggregate economy evolves independently of distributional considerations. Under quite general conditions, the economy proceeds as if there is a single representative agent. This is the case as long as the production function has the standard neoclassical properties, and agents have the same tastes represented by a utility function homogeneous in its single argument, consumption.[9] Invoking the standard transversality conditions, these aggregate quantities converge to a steady state characterized by a constant capital stock and consumption level that we denote by $\tilde{K}$ and $\tilde{C}$, respectively, and can be expressed as[10]

$$F_K(\tilde{K}, L) = \beta \tag{9a}$$

$$\tilde{C} = F(\tilde{K}, L). \tag{9b}$$

With no secular growth, the long-run capital stock is determined by equating the marginal product of capital to the rate of time discount. The second equation simply states that in steady state all output is consumed.

Linearizing equations (8a) and (8b) around the steady states (9a) and (9b) yields the local aggregate dynamics

$$\begin{pmatrix} \dot{C} \\ \dot{K} \end{pmatrix} = \begin{pmatrix} 0 & \dfrac{F_{KK}F}{(1-\gamma)} \\ -1 & F_K \end{pmatrix} \begin{pmatrix} C - \tilde{C} \\ K - \tilde{K} \end{pmatrix}, \tag{10}$$

from which we can see that the aggregate dynamics are characterized by saddle-point behavior. The stable paths for K and C can be expressed as

$$K(t) = \tilde{K} + (K_0 - \tilde{K})e^{\mu t} \tag{11a}$$

$$C(t) = \tilde{C} + \frac{F_{KK}F(\tilde{K}, L)}{(1-\gamma)\mu}(K_0 - \tilde{K})e^{\mu t} = \tilde{C} + (\beta - \mu)(K_0 - \tilde{K})e^{\mu t}, \tag{11b}$$

where $\mu < 0$ is the stable eigenvalue and is the solution to

$$\mu^2 - \beta\mu + \frac{F_{KK}F}{1-\gamma} = 0.$$

The slope of the stable arm is positive; accumulating capital is therefore associated with increasing consumption, a standard property of the Ramsey model.

3. THE DYNAMICS OF THE RELATIVE CAPITAL STOCK

To derive the dynamics of individual i's capital stock, relative to the economy-wide average, $k_i(t) \equiv K_i(t)/(K(t)/L)$, we combine (7′) and (7) to obtain

$$\dot{k}_i(t) = \frac{w(K, L)L}{K}(1 - k_i(t)) - \frac{C}{K}(\theta_i - k_i(t)), \tag{12}$$

where K, C evolve in accordance with (11a), (11b) and the initial relative capital $k_{i,0}$ is given from the initial endowments. To solve for the time path of the relative capital stock, we first note that agent i's steady-state shares of capital and consumption satisfy

$$w(\tilde{K}, L)L(1 - \tilde{k}_i) = \tilde{C}(\theta_i - \tilde{k}_i) \qquad \text{for each } i. \tag{13}$$

To analyze the evolution of the relative capital stock, we linearize equation (12) around the steady-state $\tilde{K}$, $\tilde{C}$, $\tilde{k}_i$ defined by (9) and (13). This yields

$$\dot{k}_i(t) = \tilde{r}(k_i(t) - \tilde{k}_i) - (1 - \tilde{k}_i)\frac{w(\tilde{K}, L)L}{\tilde{K}}\left(\frac{C(t) - \tilde{C}}{\tilde{C}}\right) + (1 - \tilde{k}_i)F_{KL}(\tilde{K}, L)L\left(\frac{K(t) - \tilde{K}}{\tilde{K}}\right), \tag{14}$$

which we can write more compactly as

$$\dot{k}_i(t) = \beta(k_i(t) - \tilde{k}_i) + h(\tilde{K})(1 - \tilde{k}_i)\frac{K_0 - \tilde{K}}{\tilde{K}}e^{\mu t}, \tag{14$'$}$$

where

$$h(\tilde{K}) \equiv LF_{KL}(\tilde{K}, L) - \frac{LF_L(\tilde{K}, L)}{F(\tilde{K}, L)}(\beta - \mu). \tag{15}$$

Noting (11b), together with the equilibrium conditions, (2a), (2b), and (9b), and defining the elasticities $\eta_{wL,K} \equiv (\partial wL/\partial K)/(wL/K), \eta_{C,K} \equiv (\partial C/\partial K)/(C/K)$, we see that

$$\text{sgn}(h(\tilde{K})) = \text{sgn}(\eta_{wL,K} - \eta_{C,K})_{K=\tilde{K}} \tag{16}$$

Whereas both wages and consumption increase with capital, (16) depends on the relative elasticities of labor income, $\eta_{wL,K}$, and consumption, $\eta_{C,K}$ with respect to capital (evaluated at steady state) as the (aggregate) economy evolves along its equilibrium path. As we will see, $h(\tilde{K})$ plays a crucial role in determining the dynamics of wealth distribution.[11] Solving (14$'$) and imposing the condition that the relative share of capital remains bounded, (for each i), the stable solution to this equation is

$$k_i(t) - 1 = \delta(t)(\tilde{k}_i - 1), \tag{17}$$

where

$$\delta(t) \equiv 1 + \frac{h(\tilde{K})}{\beta - \mu}\left(\frac{K(t)}{\tilde{K}} - 1\right). \tag{18}$$

Setting $t = 0$ in (17) and (18), we have

$$k_{i,0} - 1 = \delta(0)(\tilde{k}_i - 1) = \left(1 + \frac{h(\tilde{K})}{\beta - \mu}\left(\frac{K_0}{\tilde{K}} - 1\right)\right)(\tilde{k}_i - 1), \tag{19}$$

where $k_{i,0}$ is given from the initial distribution of capital endowments.

The evolution of agent i's relative capital stock is determined as follows. First, given the time path of the aggregate economy, and the distribution of initial capital endowments, (19) determines the steady-state distribution of capital, $(\tilde{k}_i - 1)$, which, together with (17), then yields the entire time path for the distribution of capital. Using (17)–(19), and equations (11), describing the evolution of the aggregate economy, we can express the time path for $k_i(t)$ in the form

$$k_i(t) - \tilde{k}_i = \left(\frac{\delta(t) - 1}{\delta(0) - 1}\right)(k_{i,0} - \tilde{k}_i) = \left(\frac{K(t) - \tilde{K}}{K_0 - \tilde{K}}\right)(k_{i,0} - \tilde{k}_i) = e^{\mu t}(k_{i,0} - \tilde{k}_i), \tag{20}$$

from which we see that $k_i(t)$ also converges to its steady state value at the rate μ. Then, as has been shown by Caselli and Ventura (2000), the cross-section of wealth converges to a long-run distribution in which wealth is unequally distributed, and the ranking of agents according to wealth is the same as in the initial distribution.

4. THE LONG-RUN CHANGE IN THE DISTRIBUTION OF WEALTH

Having established the existence of a long-run distribution of wealth, we can compare it to the initial distribution. It is convenient to measure distribution by the standard deviation of the capital stock (wealth), although it can be shown that the same analysis applies in terms of more conventional Gini coefficients.[12] Because of the linearity of (17), and (19), we can immediately transform these equations into corresponding expressions for the standard deviation of the cross-sectional distribution of capital. Specifically, $\sigma_k(t) = \delta(t)\tilde{\sigma}_k$ and $\sigma_{k,0} = \delta(0)\tilde{\sigma}_k$, implying

$$\tilde{\sigma}_k - \sigma_{k,0} = (1 - \delta(0))\tilde{\sigma}_k = \frac{h(\tilde{K})}{\beta - \mu}\left(\frac{\tilde{K} - K_0}{\tilde{K}}\right)\tilde{\sigma}_k. \tag{21}$$

Consider a permanent structural change that leads to a change in the aggregate capital stock. Wealth inequality then increases during the transition to the steady state if and only if[13]

$$h(\tilde{K})(\tilde{K} - K_0) > 0. \tag{22}$$

There are then two factors that determine whether inequality increases or decreases during the transition, the initial condition (relative to the long run) and the value of $h(\tilde{K})$. In order to assess the latter, we recall (16). Although both labor income and consumption increase with capital, $h(\tilde{K})$ is positive (negative) depending on whether labor income is more (less) sensitive than consumption to capital. In this case, as the economy accumulates its stock of capital, wealth inequality will increase (decrease). The following intuition applies. Suppose consumption is insensitive to capital, so that labor income is more sensitive to capital than is consumption. As the economy accumulates capital, savings will increase rapidly, and because relatively rich people save more, wealth inequality will increase.

However, both the elasticities, $\eta_{wL,K}$, $\eta_{C,K}$ are endogenous and their relative magnitudes, crucial for determining the relationship between capital accumulation and wealth inequality, depend on the underlying production and taste parameters of the economy. Letting $\varepsilon \equiv (F_K F_L)/(F_{KL} F)$ and $s \equiv F_K K/F$ denote the elasticity

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of substitution and the capital share, respectively, (15) can be rewritten as

$$h(\tilde{K}) \equiv (1-s)\left[\frac{\beta}{\tilde{\varepsilon}} - (\beta - \mu)\right]. \tag{15'}$$

Expressed in this way, $\beta/\tilde{\varepsilon}$ corresponds to the wage elasticity and $(\beta - \mu)$ to the consumption elasticity identified in (16). It is immediately evident that $h(\tilde{K})$ is certainly negative for values of the elasticity of substitution greater than or equal to one, but could be positive for ε less than one. In particular, $h(\tilde{K})$ is positive if and only if $\varepsilon \leq 1$ and the following inequality is satisfied,

$$\beta > -\frac{\varepsilon}{1-\varepsilon}\mu. \tag{23}$$

To proceed further, we assume that the production function is of the CES form

$$Y = F(K, L) = (\alpha K^{-\rho} + (1-\alpha)L^{-\rho})^{-1/\rho}. \tag{24}$$

As demonstrated in the Appendix, we can express (23) as

$$\beta^{1-\varepsilon} > \frac{\varepsilon\alpha^{-\varepsilon}}{1-\gamma(1-\varepsilon)}. \tag{25}$$

We can summarize these results in the following proposition.

PROPOSITION 1. *The dynamics of the distribution of wealth crucially depend on the elasticity of substitution, and the initial condition* K_0.

(i) For $\varepsilon \geq 1$, *an economy that starts below (above) the steady state, i.e.,* $K_0 < \tilde{K}$ $(K_0 > \tilde{K})$, *will experience a reduction (increase) in wealth inequality during the transition.*

(ii) For $\varepsilon < 1$, *an economy that starts below (above) the steady state, i.e.,* $K_0 < \tilde{K}$ $(K_0 > \tilde{K})$, *will experience a reduction (increase) in wealth inequality during the transition if and only if*

$$\beta^{1-\varepsilon} < \frac{\varepsilon\alpha^{-\varepsilon}}{1-\gamma(1-\varepsilon)}$$

holds. If this condition does not hold, then an economy that starts below (above) the steady state, i.e., $K_0 < \tilde{K}$ $(K_0 > \tilde{K})$, *will experience an increase (reduction) in wealth inequality during the transition.*

To understand why the evolution of inequality depends on the initial condition, consider two individuals having different capital endowments. Homothetic preferences imply that they both spend the same share of total wealth at each point in time and have the same rate of growth of total wealth. Total wealth has two components, physical capital and the present value of all future labor income. Because wages are growing at the same rate for both agents but represent a higher share of total wealth for the poorer individual, then his capital must be changing more rapidly than that of the wealthier agent. When the economy is accumulating capital, this means that his capital stock is growing faster and inequality is

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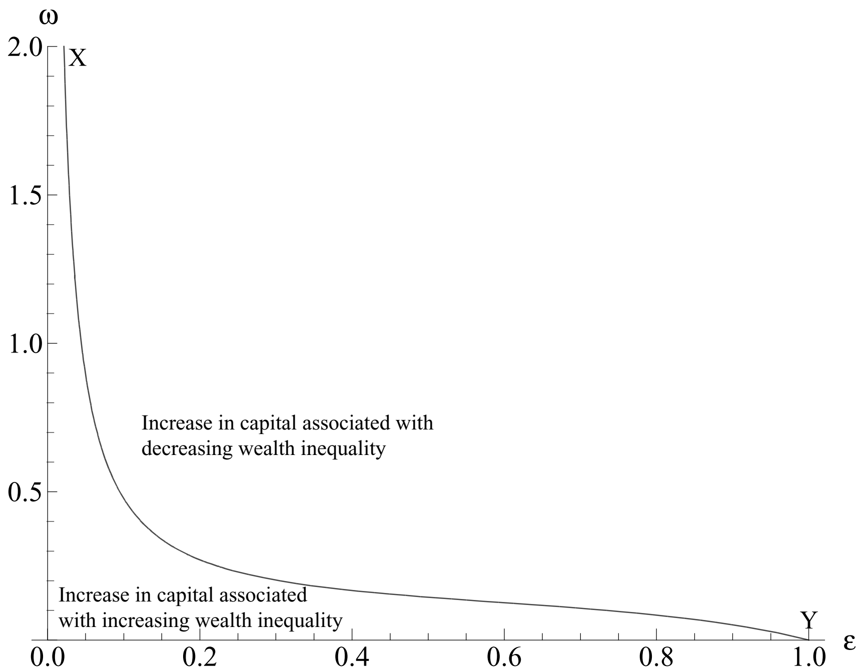

FIGURE 1. Relationship between capital accumulation and wealth inequality.

diminishing. When the economy is converging from above, i.e., when the stock of capital is falling, he will disave faster and inequality will increase.

This result may, however, be reversed if the elasticity of substitution is less than 1, and the distribution of wealth could widen as the economy converges from below. The reason for this is that a low elasticity of substitution implies fast wage growth as the economy accumulates capital. Consumers calculate their total wealth and choose a constant consumption-to-total-wealth ratio. If wages are growing slowly, poor consumers will need a high rate of capital accumulation to sustain their consumption path. However, if wages are growing fast, a lower rate of capital accumulation is optimal. With sufficiently high wage growth, poor consumers may choose to disave early in their life-times and finance current consumption with their (high) future wages. As a result, the distribution of capital becomes more unequal.

The striking feature of (25) is that it provides a criterion for whether wealth inequality increases or decreases with an accumulation of the aggregate capital stock, expressed in terms of four fundamental parameters, α, β, ε, γ, for which empirical evidence abounds. Empirical evidence on the elasticity of capital in production and the rate of time preference are consistent at around $\alpha = 0.4$, $\beta = 0.04$, respectively. In contrast, empirical estimates of the elasticity of substitution in production, ε, and the intertemporal elasticity of substitution in consumption, $\omega \equiv (1/1 - \gamma)$, are much more far-ranging.[14] Figure 1 plots the trade-off between ε and ω, for

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fixed values of $\alpha = 0.4$, $\beta = 0.04$, that would make (25) hold with equality. For points lying below the XY locus, an increase in the aggregate capital stock is associated with an increase in the inequality of wealth, whereas for points lying above this locus wealth inequality decreases. For values of the elasticity of substitution just below unity (say around 0.8), the intertemporal elasticity of substitution must be (ε, α) very small (less than 0.2) for wealth inequality to increase, whereas for higher values of the intertemporal elasticity of substitution (say around 0.75), the elasticity of substitution in production would have to be extremely low (less than 0.1). Overall, this figure suggests that decreasing wealth inequality associated with increasing capital stock is the more plausible outcome.

Caselli and Ventura (2000) provide numerical examples based on a logarithmic utility function, and they identify three cases: (i) a Cobb-Douglas production function, when the distribution of wealth always becomes more compressed (dispersed) during the transition to the steady state from below (above); (ii) a CES technology having an elasticity of substitution less than 1 and a low rate of time discount, when the distribution of wealth becomes more dispersed during the transition to the steady state from above, while dispersion first increases and then decreases during the transition from below; (iii) a CES technology having an elasticity of substitution less than 1 and a high rate of time discount, when the distribution of wealth becomes more dispersed during the transition to the steady state from below, while dispersion first increases and then falls during the transition from above.

These examples can be easily identified in Figure 1 as follows. First, the Cobb-Douglas production function, $\varepsilon = 1$, lies above the XY curve for any $\omega > 0$ and is therefore associated with less wealth inequality as wealth increases. Second, because an increase in the rate of time preference, β, shifts the XY curve up, it is clear that a higher rate of time preference will raise the likelihood of an increase in wealth being associated with more inequality. Our discussion here indicates that for plausible parameter values, capital accumulation is associated with declining inequality if the economy is not too far from the steady state. However, our linearization implies that this conclusion is only valid in the vicinity of the steady state, and, hence, if initial capital were low, wealth dispersion could first increase and then decline.

The results summarized in Proposition 1 generalize Theorem 4 of Obiols-Homs and Urrutia (2005), in which they show that for the Cobb-Douglas technology and logarithmic utility the coefficient of variation of assets declines over time as wealth accumulates. Setting $\varepsilon = 1$ leads to case (i) of Proposition 1, which we have seen yields a similar implication. The results are also consistent with the numerical simulations performed by Glachant and Vellutini (2002), who, also using a Cobb-Douglas production function, find that an increase in the income tax rate leads to a reduction in the capital stock accompanied by an increase in wealth inequality.

Our criterion (25) is expressed in terms of two production parameters (α, ε) and two preference parameters (β, γ). This contrasts with Chatterjee (1994) who expresses the conditions for wealth inequality to decline over time, in terms of consumption and savings behavior, independent of production characteristics.[15] Although we find our criterion, (25), expressed in terms of basic underlying parameters to be convenient, we also may note that as $\beta \to 0, h(\bar{K}) < 0$ irrespective of the production characteristics. This is illustrated by the XY curve in Figure 1 coinciding with the axes, in which case the region for an increase in wealth to be associated with less inequality extends over all values of ε.

5. CONCLUDING COMMENTS

The objective of this note has been to provide a simple condition determining the distributional consequences of the representative consumer optimal growth model. It has been previously shown that the Ramsey model is compatible with a wide range of distributional outcomes, yet it is difficult to characterize under which circumstances the distribution of wealth becomes more or less unequal. Our analysis of the transitional dynamics of individual wealth allows us to derive a simple set of conditions under which wealth inequality increases or decreases in a growing economy.

We show how both production and preference parameters affect the distributional outcome. In particular, we find that either a low elasticity of substitution in production or a small intertemporal elasticity of substitution in consumption is required for wealth inequality to increase during the growth process. Existing estimates of these parameters indicate that the most likely scenario is one in which the distribution of wealth becomes more compressed as capital accumulates. The existence of substantial changes in the degree of wealth inequality has been recently documented by Piketty, Postel-Vinay, and Rosenthal (2006), who show that there was a substantial reduction of wealth inequality during the 20th century. Part of it was a result of "accidental" causes—such as war and taxation—but in the postwar period, the reduction in capital incomes was a major equalizing force. Our analysis indicates that this is a likely outcome once an economy's capital markets are well-functioning and agents are not constrained in their saving and investment decisions.

NOTES

1. The model dates back to Ramsey (1928) and is discussed in a number of contemporary macroeconomics textbooks beginning with Blanchard and Fischer (1989).

2. The sources of heterogeneity they consider are: (i) initial endowments of capital, (ii) tastes, and (iii) skills.

3. One important assumption of Caselli and Ventura (2000) is to follow the Ramsey model and assume an inelastic labor supply. Sorger (2000) endogenizes labor supply and shows how this requires the distribution and aggregate behavior to be determined simultaneously, rendering the explicit solution for the dynamics intractable. Turnovsky and García-Peñalosa (2008) show how the RC model can be restored with endogenous labor supply provided the utility function is homogeneous in leisure and consumption.

4. Obiols-Homs and Urrutia (2005) have employed a similar framework, although they restrict their analytical results to logarithmic utility and a Cobb-Douglas production function.

5. Bertola, Foellmi, and Zweimüller (2006) survey the literature on the Ramsey model with heterogeneous capital endowments.

6. See, e.g., Turnovsky (2002).

7. That is, both factors of production have positive, but diminishing, marginal physical products and the production function exhibits constant returns to scale, with $F_{KL} > 0$ being a consequence of the assumption of the latter.

8. Time-dependence of variables will be omitted whenever it causes no confusion.

9. In Turnovsky and García-Peñalosa (2008), we address the issue in the more general case in which the agent also faces a labor-leisure choice. The conditions for the RC model to apply in that case become more complex.

10. The transversality conditions are $\lim_{t\to\infty} \lambda_i K_i(t)e^{-\beta t} = 0$ for each i.

11. We write $h = h(\tilde{K})$ to reflect the fact that in general it is evaluated at the steady-state aggregate stock of capital.

12. See Turnovsky and García-Peñalosa (2008).

13. For example, (9a) implies that a decrease in the rate of time preference leads to an increase in $\tilde{K}$ and, hence, if the economy starts from an initial steady state, implies $\tilde{K} - K_0 > 0$. It is also clear that $\dot{\sigma}_k(t) = \dot{\delta}(t)\tilde{\sigma}_k = (h(\tilde{K})/(\beta - \mu))(\dot{K}(t)/K(t))\tilde{\sigma}_k$, implying that the distribution of wealth evolves monotonically during the transition. This is, however, because we have linearized the system around the steady state. When the economy is far from the steady state, it is possible for wealth distribution to evolve nonmonotonically, as Caselli and Ventura (2000) show in one of their examples. Glachant and Vellutini (2002) provide an alternative expression of the relationship between aggregate capital accumulation and wealth inequality.

14. The preponderance of empirical evidence suggests that ω is relatively small, certainly below unity. Guvenen (2006) reconciles the estimates derived from consumption data, which are typically smaller (often around 0.2) with larger estimates based on financial data (often around 0.75). Estimates of $\omega > 1$ also exist; see, e.g., Vissing-Jørgensen and Attanasio (2003). We have discussed previously Berndt's (1976) early influential study reconciling alternative estimates of ε obtained using different functional forms and data sets. He finds that for his preferred data sets and estimation methods the estimates are tightened and generally lie in the range 0.7 to 1.3. More recent studies obtain estimates at the lower end of this range, or below.

15. In drawing this comparison, we should note that the wealth measure employed by Chatterjee (1994), is somewhat different from ours, being expressed in terms of discounted future distributed profits.

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APPENDIX

DERIVATION OF (25)

To derive the condition (25), note that (**10**) implies that the eigenvalue is given by

$$\mu = \frac{F_K}{2}\left[1 - \sqrt{1 - \frac{4F(\tilde{K})F_{KK}}{(1-\gamma)F_K^2}}\right]. \tag{A.1}$$

Now, the CES production function implies

$$\frac{F_{KK}}{F_K} = -\frac{1-s}{\varepsilon K}, \tag{A.2}$$

and

$$\frac{1-s}{s} = \frac{1-\alpha}{\alpha}\left(\frac{L}{K}\right)^{-\rho}. \tag{A.3}$$

Also, the steady-state condition (9a) implies that the steady state capital-labor ratio is defined by

$$(1-\alpha)\left(\frac{L}{\tilde{K}}\right)^{-\rho} = \left[\left(\frac{\beta}{\alpha}\right)^{-(1-\varepsilon)} - \alpha\right]. \tag{A.4}$$

These expressions together with (A.4) allow us to write (23) in the form (25).

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