and its role in constructing smooth bump functions. Integration is briskly dealt with using upper and lower Riemann sums and the book concludes with chapters on convexity and inequalities (with applications to macroeconomics), trigonometry (via arc length), complex numbers, and complex power series. There is also a glimpse ahead to complex variables, culminating with the Taylor series for analytic functions and the tantalising revelation that (p. 239): "The complex numbers overshadow the elementary calculus of one variable silently pulling its strings.".

The sections in each chapter are short and punchy, interspersed with routine recap exercises and more involved longer exercises – some much more involved, as in the invitation on p. 135 to prove Borel's theorem that any sequence of real numbers can occur as the Maclaurin coefficients of a C^∞ function. Not only do these ask a lot of the intended reader, but some mathematical sophistication (for example in the manipulation of modulus inequalities) is assumed throughout. Several of the thornier proofs would, I think, have benefited from more explanation and there are a number of typos, the most serious of which occurs in the statement of L'Hopital's rule where $\lim_{x \to c} \frac{f'(x)}{g'(x)}$ is rendered as $\lim_{x \to c} \frac{f'(c)}{g'(c)}$ and thus threatens to blur the distinction between the two common forms of this rule. I was also puzzled by the multiple occurrence of some exercises: establishing $e^x.e^y = e^{x+y}$ by multiplying power series occurs on p. 127 and p. 216 and the proof that continuity implies uniform continuity on a closed, bounded interval occurs on p. 89, p. 152 and as Theorem 4.2.2.. And, in a few places, I felt that details were glossed over: for example, on p.125, expanding $\left(1 + \frac{1}{n}\right)^n$ and letting $n \to \infty$ shows that $\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n \leqslant \sum_{j=0}^{\infty} \frac{1}{j!}$, but more is needed to show equality and, on p.164, having f continuous (rather than integrable) in the change of variables formula makes the connection with the preceding results clearer.

In summary then, *Calculus for cranks* is a slightly idiosyncratic introduction to real analysis aimed at a specific target audience—users of calculus with some knowledge of its techniques who want to know why it works. It is written in a direct and rather provocative style which preserves the informality of the classroom and is laced with a wry sense of humour. The author's stress on repeatable techniques (rather than the stepping-stones of 'big' theorems) and the real numbers treated as infinite decimals results in a distinctive excursion through familiar territory, although the brisk pace may leave some readers behind. But I'm still not sure about the title!

Reference

1. J. C. Burkill, *A first course in mathematical analysis*, Cambridge University Press (1962).

10.1017/mag.2022.156

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Introduction to complex variables and applications by Mark J. Ablowitz and Athanassios S. Fokas, pp. 420, £39.99 (paper), ISBN 978-1-10895-972-8, Cambridge University Press (2021)

Introduction to complex variables and applications, in the Cambridge Texts in Applied Mathematics series, is a stylish, well-written and up to date introduction to complex variable methods for undergraduate (or early graduate) students in applied mathematics, science and engineering. As the well-known authors explain in the preface, not only is this material mathematically beautiful and compelling, but it is also useful for solving a wide array of problems. Indeed, a notable feature of the text is the inclusion of applications (for example to ideal fluid flow and electrostatics) to motivate and illustrate the development of the theory.

The book comprises five long chapters covering first the basic building blocks of complex numbers and elementary functions, analytic functions and integration, sequences, series and singularities of complex functions, and then residue calculus and conformal mappings. While there is a usefully full bibliography, the authors have striven to make the book self-contained with a pragmatic approach to proofs showing clearly why results are true without getting too bogged down in minutiae. Some proofs are gathered together in standalone sections and the authors readily acknowledge where providing full details (such as in the discussions of the uniqueness of analytic continuation and the Riemann mapping theorem) would require too much of a detour. The text is interspersed with a magnificent collection of fully worked examples and well-crafted exercises (with answers to selected odd numbered questions) which really help to cement understanding and develop confidence through practising the techniques.

A remarkable amount of ground is covered, including applications to differential equations (as far as Painlevé transcendents and the contour integral representation of solutions to Airy's equation, $\frac{d^2w}{dz^2} - zw = 0$), Fourier and Laplace transforms, as well as advanced topics such as Mittag-Leffler expansions and the Weierstrass factorisation theorem. And I shall return again and again to the superb chapters on residue calculus and conformal mapping. Residue calculations involving cross-cuts, multivalued function and a wide range of contours showcase the power of this technique and it was also good to see conformal mapping taken beyond the Schwarz-Christoffel transformation for maps of polygons (where maps of a rectangle involve elliptic functions) to analogous formulae for maps of curvilinear polygons (where maps of triangles with zero angles involve elliptic modular functions). By the end, my personal "goody bag" also contained the Chazy differential equation $w''' = 2ww'' - 3(w')^2$ whose solution involves a circular natural boundary which depends on the initial values, together with some nice results on conjugation such as the representation theorem for analytic functions

$$f(z) = u + iv = 2u\left(\frac{z + \bar{z}_0}{2}, \frac{z - \bar{z}_0}{2}\right) - \overline{f(z_0)};$$

the neat formula for the area A enclosed by a contour C, $A = \frac{1}{2i}\oint_C \bar{z}\,dz$; and, above all, the Cauchy-Pompeiu integral formula for a contour C enclosing a region R containing z,

$$f(z) = \frac{1}{2\pi i}\oint_C \frac{f(\zeta)}{\zeta - z}d\zeta - \frac{1}{\pi}\iint_R \frac{\partial f/\partial z}{\zeta - z}\,dA(\zeta),$$

which highlights the fact that non-dependence on $\bar{z}$, $\frac{\partial f}{\partial z} = 0$, is the key to analyticity.

I thoroughly enjoyed reading this book and warmly commend it to anyone seeking a brisk, well-organised account of complex variables with a practical focus on applications and calculational aspects. For those with long memories, it may be worth pointing how it relates to the authors' previous Cambridge University Press book with the similar title, *Complex variables: introduction and applications*, the two editions of which were warmly reviewed in the *Gazette* by Peter Shiu (March 1999, pp.183-184) and this reviewer (July 2004, p.165). The new book is based on the first five chapters of the substantially longer earlier book (which had additional chapters on the asymptotic evaluation of integrals and Riemann-Hilbert problems), but with some rewriting for the undergraduate target audience. For example, Goursat's theorem is now just mentioned in connection with Cauchy's theorem rather than proved.

10.1017/mag.2022.157 © The Authors, 2022
Published by Cambridge University Press on
behalf of The Mathematical Association

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Thinking clearly with data by Ethan Bueno de Mesquita and Anthony Fowler, pp. 432, £25 (paper), ISBN 978-0-69121-435-1, Princeton University Press (2021)

Books that aim to increase statistical literacy are of the utmost importance today, and several good titles for a general readership have been published recently, for example [1], reviewed in the July 2021 *Gazette*. This one is rather more sophisticated; it stems from courses given at the University of Chicago, not only for undergraduate and graduate students but for executives responsible for public and military policy. Based on a large number of real case studies, it goes well beyond the familiar issues. It includes a fair amount of technical information, including equations as well as charts, but not too much, one hopes, to scare a serious audience with an only limited mathematical background.

The first few chapters explore correlation, starting by explaining exactly what it is. The standard trope that 'correlation is not causation' leads to detailed discussion as to what causality is, a discussion that reaches philosophical issues (what is meant by a cause of the First World War?). Focus on the common statistical errors of *p*-hacking (deliberate searching for experiments that give significant results) and what the authors call *p*-screening (the tendency not to report, or not to publish, results that are not statistically significant) are treated alongside a detailed discussion of reversion to the mean, with a good discussion of Galton's arguments, though without explaining why variability of heights remains roughly constant over time. Then comes a section on confounders, including a careful distinction between confounders and mechanisms. The limitations of controlling for confounders lead to randomised experimental design, and the book concludes with three chapters on turning information into decisions. One chapter discusses methods of statistical display, though to nothing like the extent of Tufte's now-classic book [2], and then gives some applications of Bayes' Theorem; then there is a chapter on how to measure the success or otherwise of one's 'mission'. The final chapter, 'On the limits of quantification', reminds us that values have a crucial part in decision-making: 'data and quantitative evidence don't tell us everything we need to know in making decisions'. Each chapter has exercises, mostly for discussion, and a list of references, often to sources of data and to original papers mentioned in the text and exercises.

I learnt a huge amount from the examples discussed. The start is somewhat dispiriting – some of the earliest examples debunk received wisdom in ways that increase one's pessimism – but things rapidly improve. The following cases are only a small sample. 'Selecting on the dependent variable', starts with a discussion of the claim that the highest achievers in some area demanding skill are those who have put in 10 000 hours of serious practice. One is supposed to infer that 'the top is achieved by practice, not talent'. The claim is problematic for two reasons. One is that the data considered are only from the highest achievers; this is what is meant by selecting on the dependent variable (top achievement). It would be necessary to see also what proportion of those who have not become high achievers have put in 10 000 hours of serious practice. The other, very well discussed here, is that someone learning, say,