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DEBT DENOMINATION AND DEFAULT RISK IN EMERGING MARKETS

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This paper develops a two-sector small open economy model to analyze the effects of the currency denomination of debt on default risk and interest rates in emerging market economies. Default risk is determined endogenously and depends on the incentives for repayment. The economy can borrow using tradable-denominated nonindexed bonds or bonds whose return is indexed to the domestic price index, which are used as proxies for foreign currency and domestic currency debt, respectively. The model predicts that foreign currency debt leads to lower default risk for high output levels and domestic currency debt reduces the default risk for low output levels. Although the effect of debt denomination on default risk changes with the output level, the default rate of the economy and average interest rates decline as domestic currency borrowing increases. In addition, domestic currency borrowing is found to reduce the countercyclicality of interest rates and the trade balance.

Keywords: Sovereign Default, Debt Denomination, Interest Rates, Real Exchange Rates

1. INTRODUCTION

The inability to borrow in domestic currency in international financial markets is a widespread phenomenon among emerging market economies.[1] The problems associated with foreign currency borrowing have played an important role in many emerging market crises and have been the subject of a vast literature. This issue has been put forward as a factor leading to currency crises, as well as affecting the policy options of governments in responding to crises. Another aspect of foreign currency borrowing, which received more attention after the default of Argentina in December 2001, is its effect on sovereign default risk. In many papers and policy discussions, large amounts of foreign currency debt have been pointed out as one of the factors leading to default in Argentina.[2]

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Although the currency denomination of debt has received considerable attention in the literature, it has not been analyzed in the context of models with endogenous default risk. The quantitative sovereign debt literature, based on Arellano (2008) and Aguiar and Gopinath (2006), follows the approach developed by Eaton and Gersovitz (1981) and studies the relationship between default risk, interest rates, and output. The papers in this literature consider only one kind of debt in modeling default risk—the country borrows with bonds that pay one unit of tradable output regardless of the state of the economy. This kind of borrowing is equivalent to foreign currency–denominated debt, because repayment value is independent of changes in the domestic country price level. Analyzing default behavior and interest rate movements in a context where the country exclusively borrows in foreign currency is relevant, given that emerging markets mostly borrow in foreign currency. However, it is also important to analyze the implications of these models when debt is denominated in domestic currency, in order to see whether default incentives and interest rates would behave differently if emerging markets were able to borrow in domestic currency, and to understand the conditions under which each type of borrowing is better.

This paper contributes to the literature by analyzing how having to borrow in foreign currency affects default incentives and the interest-rate behavior of emerging market economies using a stochastic small open economy model with endogenous default risk. The relationships between default risk, debt denomination, and real exchange rate fluctuations are analyzed in a context where the borrowing country has the option of defaulting on its debt, in which case it will be temporarily excluded from international borrowing and lending, and lose a share of its output. Interest rates, then, are determined endogenously as a function of the default probability of the economy. Differently from the literature on sovereign default, I use a real model with two types of goods, tradables and nontradables, where the endowments of both goods are stochastic. The economy can borrow using two types of bonds: bonds that return one unit of tradable good next period, or bonds whose return is indexed to the domestic consumption–based price index, which is the relative price of aggregate consumption in units of tradable goods. With this formulation, bonds that have a constant return in terms of tradables represent foreign currency debt and bonds that have an indexed return represent domestic currency debt. Modeling foreign and domestic currency debt in this way captures the effect of real exchange rate changes on the relative value of debt denominated in different currencies: real exchange rate depreciation reduces the value of indexed debt, whereas the value of tradable-denominated debt stays constant. Therefore, tradable-denominated debt becomes relatively more costly with real depreciation, just as foreign currency debt becomes more costly compared to domestic currency debt.

The reason that debt denomination affects default risk is that real exchange rate fluctuations become relevant to repayment capacity when debt is denominated in foreign currency. In many emerging markets, real exchange rate depreciations are associated with output declines, increasing the value of foreign currency–denominated debt exactly when the country's repayment ability has deteriorated.[3] Therefore, with a high share of foreign currency debt, debt service becomes even harder during low-output episodes. The real exchange rate depreciations that have been observed in recent default episodes provide evidence for the idea that real depreciations make debt service more difficult. For instance, the quarterly real exchange rate depreciation was 124% during the Russian default of August 1998, 65% in the Ukrainian default of September 1998, and 180% in the Argentine default of December 2001. In line with this, it has been documented by Eichengreen et al. (2003a, 2003b) that credit ratings deteriorate as the share of foreign currency debt increases, reflecting a higher sovereign default risk. Likewise, Bordo et al. (2010) show that foreign currency debt increases the likelihood of currency and debt crises, although the strength of the association depends on the size of a country's reserve base and its policy credibility. On the other hand, Ranciere et al. (2010) show that across financially constrained groups of firms, borrowing in foreign currency reduces the interest rate by two percentage points on the average, controlling for the expected rate of currency depreciation, in a sample of emerging European economies. They also show that an increase in currency mismatch is associated with faster economic growth during tranquil times, but also with a more severe crisis.

The model put forward in the current paper is solved quantitatively using data from the Argentine economy. In the model, the real exchange rate is determined by stochastic shocks to tradable and nontradable endowments. For the type of shocks that generate the observed correlation between real exchange rates and output in emerging market economies, the results show that the relationship between debt denomination and default risk changes depending on the output level. For high output levels, borrowing with nonindexed debt reduces default incentives, whereas for low output levels, indexed debt leads to lower default risk. Interest rates, therefore, are lower with foreign currency borrowing when the economy is doing well, but higher in low-output episodes. In spite of this relationship, simulation results show that the default rate of the economy decreases as it borrows more with indexed bonds. Since defaults mostly occur when output is low and borrowing with indexed debt leads to lower repayment in these states, the default rate of the economy decreases when it borrows more with indexed bonds. In line with this, the volatility and the average level of interest rates also decrease with indexed borrowing. In addition, the results show that the business cycle properties of interest rates and the trade balance change depending on debt denomination. Borrowing with nonindexed bonds increases the countercyclicality of interest rates and the trade balance. In addition, the level of welfare is shown to increase as the economy borrows more with indexed bonds, since this type of borrowing allows a smoother consumption profile.

As mentioned before, most of the papers in this literature consider a model where there is only one good and borrowing is done with bonds that pay one unit of this good regardless of the state of the economy, which is the same as the tradable-denominated bonds in the current model [see Aguiar and Gopinath

(2006); Arellano (2008); Yue (2010); Chatterjee and Eyigungor (2010)]. In Cuadra and Sapriza (2006), there are two goods, importable and exportable, and each unit of bond repays one unit of the importable good. The current paper contributes to the literature on sovereign default risk by introducing tradable and nontradable goods into a standard model of endogenous default and considering the impact of borrowing indexed to the price of domestic consumption. Introducing such a bond allows analyzing debt denomination in the context of sovereign default and investigating how the implications of the model changes.

In addition, there is a large literature that studies foreign currency borrowing, which this paper relates to. One strand in this literature has analyzed the effects of foreign currency debt on the occurrence of currency crises. The emphasis in this set of papers is on the balance sheet effects of a currency depreciation when firms are credit-constrained, in the sense that the amount that can be borrowed depends on the net worth of the firm. With debt denominated in foreign currency, a depreciation has contractionary effects through a reduction in net worth. This, in turn, reduces the amount that can be borrowed and constrains investment. Therefore, the effects of a bad shock are amplified, and this may lead to multiple equilibria where changes in expectations trigger a crisis. The role of foreign currency debt in financial crises through such a channel has been studied by papers such as Krugman (1999) and Aghion et al. (2001, 2004), whereas Schneider and Tornell (2004) use balance sheet effects to study currency and banking crises in relation to boom–bust cycles in developing countries. In this paper, I analyze foreign currency debt in relation to sovereign default risk rather than currency crises, and in this setup, foreign currency debt directly affects the default incentives of the government without the assumption of credit constraints based on net worth.

Another strand of the literature has developed around studying the reasons for the inability of developing countries to borrow in their domestic currency. In the case of private borrowing, the explanations offered are bailout guarantees [Burnside et al. (2000); Schneider and Tornell (2004)], lack of domestic financial development [Caballero and Krishnamurthy (2003)], expectations of a large monetary expansion associated with a risky monetary environment [Jeanne (2003)], and the correlation of default risk with real depreciations coupled with an inability to enforce creditor seniority in foreign debt contracts [Chamon (2003)]. In the case of public debt, this has been explained by the government's ability to reduce the real value of nominal debt by creating inflation [Calvo (1978, 1988)]. In this case, time-inconsistency problems constitute a significant obstacle to issuing domestic-currency public debt. Unlike this literature, the current paper does not aim to explain the inability of developing countries to borrow in their domestic currency, but takes a different perspective, which is to examine the effects of the existing borrowing pattern on the default risk and interest rates, given the real exchange rate–output correlation in developing countries. Therefore, the question that the paper addresses is how different the interest rate behavior in these countries would be if they were able to borrow in their own currency.

The paper is organized as follows: Section 2 presents the model. Section 3 presents the numerical solution of the model with the calibration of the data and the results. A sensitivity analysis is presented in Section 4, and Section 5 concludes.

2. THE MODEL

I analyze a small open economy model with two sectors, a tradable sector and a nontradable sector. There is a benevolent government whose objective is to maximize the households' utility. The government taxes the aggregate endowment of the households and uses the proceeds to finance government consumption, which gives utility to the households. The government can borrow and lend in international financial markets to smooth the fluctuations in government consumption that are due to endowment fluctuations. Debt contracts are not enforceable, as the government has the option to default. When the government defaults, it is temporarily excluded from international financial markets and the aggregate endowment is reduced. The foreign lenders charge a premium on lending based on the expected default probability of the government. Borrowing can be done using two types of bonds: a tradable-denominated bond that delivers one unit of the tradable good next period and an indexed bond that delivers an amount of the tradable good that is indexed to the consumption price index.

2.1. Households

Households are infinitely lived and have preferences over consumption of tradable and nontradable goods and government consumption,

$$E_0 \sum_{t=0}^{\infty} \beta^t U\big(c\big(c_t^T, c_t^N\big), g_t\big), \tag{1}$$

where $0 < \beta < 1$ is the discount factor; c_t^T and c_t^N are respectively consumption of the tradable and nontradable goods; $c(.)$ is the constant elasticity of substitution aggregator; and g_t is government consumption. The period utility function $U(.)$ is assumed to be increasing, strictly concave, and twice continuously differentiable. In each period, households receive stochastic endowment streams of tradable and nontradable goods, y_t^T and y_t^N, and the state vector of endowment shocks is defined as $s_t = (y_t^T, y_t^N)$.[4]

The budget constraint of the household is

$$c_t^T + p_t^N c_t^N = (1-\tau)\left(y_t^T + p_t^N y_t^N\right), \tag{2}$$

where τ is the tax levied on the endowments of tradable and nontradable goods.

p^N is the relative price of the nontradable good in units of the tradable good. Purchasing power parity is assumed to hold for the tradable sector, and the tradable price is normalized to 1. The domestic consumption–based price index, P, is the relative price of aggregate consumption in units of tradables, which is an increasing function of the nontradable price, p^N. Given the CES form of aggregate consumption used in the solution of the model, P corresponds to the CES price index.

2.2. Government

The government finances its consumption by taxing the aggregate endowment of households and borrowing in international financial markets. It is the only agent with access to borrowing and lending, and it has the option of defaulting on its debt. The households obtain utility from government consumption. The government's motive for borrowing and lending is to smooth the provision of government consumption against the fluctuations in tax revenue resulting from stochastic shocks to the endowments.

Borrowing Structure. The government can borrow and lend using one-period bonds. Two types of bonds are available to this economy: a tradable-denominated nonindexed bond (F-bond), which delivers one unit of tradable good next period, and an indexed bond (D-bond), which delivers an amount of tradable good that is indexed to the domestic consumption–based price index, P. When the government sells an F-bond, i.e., purchases an F-bond with a negative face value, it receives q_t^F units of period-t tradable goods and promises to pay 1 unit of period-$(t+1)$ tradable goods next period. On the other hand, if it sells a D-bond , it receives q_t^D units of period-t tradable goods and promises to pay P_{t+1} units of period-$(t+1)$ tradable goods next period. In this setting, borrowing with F-bonds is analogous to borrowing in foreign currency, and borrowing with D-bonds is analogous to borrowing in domestic currency. The repayment value of F-bonds is constant across states, and lenders receive one unit of tradable good in every state unless the government defaults. The repayment value on D-bonds, on the other hand, changes with the price index. The real exchange rate in this context is the quantity of the aggregate consumption good a person has to give up in order to get one unit of the tradable good, i.e., $1/P$. Therefore, a real exchange rate depreciation leads to an increase in the value of repayment on F-bonds relative to D-bonds, $1/P$, capturing the fact that the burden of foreign currency relative to domestic currency debt increases with a real depreciation.[5]

The government borrows in international markets and taxes the total endowment of the households at a constant rate τ to finance government spending. When the government receives its tax revenues from the households, it exchanges the part that is in the form of nontradable goods, τy^N, for $\tau p^N y^N$ units of tradable goods in the goods market, since all final transactions of the government are settled in tradable goods. The markets for tradable and nontradable goods clear with the government's transaction.

The government's budget constraint depends on the tax revenue, the beginning-of-the-period asset position, b_t, the amount of assets chosen in that period, b_{t+1}, and whether it chooses to default. When the government chooses to repay its debt, the budget constraint is as follows:

$$g_t = \tau\left(y_t^T + p_t^N y_t^N\right) - q_t^F \alpha b_{t+1} - q_t^D(1-\alpha)b_{t+1} + \alpha b_t + (1-\alpha)P_t b_t. \quad \textbf{(3)}$$

In this setup $b_{t+1} < 0$ ($b_{t+1} > 0$) means that the government receives (pays) $q_t^F \alpha b_{t+1} + q_t^D(1-\alpha)b_{t+1}$ units of period-t tradable goods and promises to pay (receives) $\alpha b_{t+1} + (1-\alpha)P_{t+1}b_{t+1}$ units of period-($t+1$) tradable goods next period. The shares of F-bonds and D-bonds are assumed to be constant in the portfolio of assets that the government holds. When it borrows or lends, the share of F-bonds is constant at α; the share of D-bonds is $(1-\alpha)$, where $\alpha \in [0, 1]$. This formulation makes it possible to analyze the effects of debt denomination on default risk by solving the model for different α values.

When the government chooses to default, it is excluded from financial markets temporarily and its current debt is erased. Furthermore, there is an additional output loss of defaulting: when the economy is in financial autarky, endowments of both tradables and nontradables decline. The assumption of additional output loss in autarky has been used in all of the recent studies on default, in order to sustain reasonable levels of debt in equilibrium.[6] This assumption follows from the empirical studies that find disruptions in trade and a fall in output following defaults.[7] The budget constraint is then given by

$$g_t = \tau\left(y_{d,t}^T + p_t^N y_{d,t}^N\right), \quad \textbf{(4)}$$

where $y_{d,t}^T = y_t^T - \lambda(y_t^T)$ and $y_{d,t}^N = y_t^N - \lambda(y_t^N)$ are the endowments of tradable and nontradable goods, respectively, when the government chooses to default, and $\lambda(y_t^i) \geq 0$, for $i = T, N$.

In a default, it is assumed that the government defaults fully and on both types of debt. In the literature, the default punishment is formulated as independent of the share of debt that the government defaults on. If the punishment for default does not change with the share or type of debt defaulted on, it is optimal to default fully on all debt. To get partial default as an equilibrium outcome, additional assumptions about default punishment would be needed, which are not introduced here because of the lack of empirical evidence on this issue.

In the model, borrowing and lending are used to smooth government consumption, which enters the utility function of the households as a separate good. In the literature, the papers that study default with similar models have sovereign borrowing and lending being used directly to smooth private consumption. The reason that I introduce a separate government consumption good is to make the price of nontradables and the real exchange rate independent of the amount borrowed/lent and the prices of bonds. Because bonds only affect the government consumption, which is a separate good in the utility function, the real exchange rate is determined solely by the endowment shocks to tradables and nontradables. In this way, it is possible to generate the relationship between real exchange rates and output that is observed in the data, and to preclude international borrowing and lending from introducing additional movements in the real exchange rate.

Government's Problem. The government's objective is to maximize the lifetime utility of households by choosing the amount of borrowing/lending and deciding whether to repay its debt given its level of outstanding assets and the endowment shocks.

The problem of the government can be formulated recursively with the state variables being b_t and s_t, where b_t is the level of outstanding assets at the start of the period and s_t denotes the vector of exogenous state variables, $s_t = (y_t^T, y_t^N)$.

The value function that corresponds to the households' expected lifetime utility when the government has access to credit markets and starts the period with assets b and shocks s is denoted by $V^o(b, s)$. Since the government decides in every period whether to default or repay, the value function satisfies

$$V^o(b, s) = \max \left\{ V^r(b, s), V^d(s) \right\}, \tag{5}$$

where $V^r(b, s)$ is the value of repaying the debt and continuing to have access to the financial markets and $V^d(s)$ is the value of defaulting.

When the government repays its debt, the value function is

$$V^r(b, s) = \max_{b'} \{U(c(c^T, c^N), g) + \beta E V^o(b', s')\} \tag{6}$$

subject to

$$g = \tau(y^T + p^N y^N) - q^F \alpha b' - q^D(1 - \alpha)b' + \alpha b + (1 - \alpha)Pb, \tag{7}$$

$$c^T = (1 - \tau)y^T - \tau p^N y^N, \tag{8}$$

$$c^N = y^N. \tag{9}$$

Equation (7) is the budget constraint of the government when credit markets are open, and equations (8) and (9) are the domestic market–clearing conditions for the tradable and nontradable sectors. Note that in equations (8) and (9) the amount of tradable goods available for consumption is $(1 - \tau)y^T - \tau p^N y^N$ and the amount of nontradable goods is y^N, because the government sells its nontradable tax receipts, τy^N, back to households in exchange for $\tau p^N y^N$ units of tradable goods.

The government decides on its asset holdings for the next period, b', to maximize utility subject to its budget constraint and internalizing the domestic market–clearing conditions, given the level of bond holdings for the current period, b, and the shocks to the tradable and nontradable endowments. Choosing b' will pin down the value of the government consumption for a given level of b because the tax rate is taken as exogenous. Given that the tax revenue fluctuates because of fluctuations in y^T, y^N, and p^N, the government wants to smooth the provision of government consumption by borrowing and lending. The value function under repayment depends on this period's utility and the maximum of next period's value functions for repayment and default, $V^o(b', s')$. The government faces the choice of defaulting or remaining in the credit relationship every period and chooses the option that gives the highest utility. Therefore, the value function for today must account for the decision of the government in the next period, which is captured by $V^o(b', s')$.

When the government defaults, the economy is excluded from credit markets temporarily, and it is assumed that all of its debt is eradicated. It remains in financial autarky for a stochastic number of periods, and the probability of regaining access to credit markets in any given period is θ. As mentioned before, there is also a direct output cost of defaulting. Therefore, the value function under default is

$$V^d(s) = U\left(c\left(c_d^T, c_d^N\right), g_d\right) + \beta E[\theta V^o(0, s') + (1-\theta)V^d(s')], \tag{10}$$

where

$$g_d = \tau\left(y_d^T + p^N y_d^N\right), \tag{11}$$

$$c_d^T = (1-\tau)y_d^T - \tau p^N y_d^N, \tag{12}$$

$$c_d^N = y_d^N. \tag{13}$$

The government's default decision is summarized by the default function, which takes the value 1 for the states in which the government finds it optimal to default. The default function is defined as

$$D(b, s) = \begin{cases} 1 & \text{if } V^d(s) > V^r(b, s) \\ 0 & \text{otherwise.} \end{cases} \tag{14}$$

2.3. International Lenders

The international lenders are assumed to be risk-neutral. They can borrow funds in the international credit markets at the risk-free interest rate r^*. It is also assumed that there is perfect competition among lenders, which drives the expected profits down to zero. Therefore, they will be willing to lend as long as they are promised the risk-free return in expected value. These conditions imply that prices of bonds are as follows:

F-Bonds:

$$q_t^F(b_{t+1}, s_t) = \frac{E_t\left\{(1 - D_{t+1})\right\}}{1 + r^*}. \tag{15}$$

D-Bonds:

$$q_t^D(b_{t+1}, s_t) = \frac{E_t\left\{(1 - D_{t+1})P_{t+1}\right\}}{1 + r^*}. \tag{16}$$

The equilibrium bond prices are consistent with the default probability of the government. For bonds that have a negative face value (government borrowing), bond prices reflect the risk-free rate and a premium for the default probability, whereas bonds with a positive face value (government lending) only reflect the risk-free rate. Bond prices decrease, i.e., the interest rates increase, as the default probability increases. Aside from this, the price of the D-bond also reflects the expected movements in the price index, P, such that a higher P implies a higher

q^D. Because the lenders receive a higher payment next period when P increases, they will be willing to pay a higher price for bonds this period as well.

2.4. Equilibrium

In equilibrium, households choose consumption of tradables and nontradables, taking as given the nontradable price, the government consumption, and the endowment shocks. The government decides on its optimal default policy given the endowment shocks and the initial level of assets, subject to the optimization of the households and the international lenders. In the case of repayment, it also chooses the new level of foreign assets.

A recursive equilibrium for this economy can be defined as follows.

DEFINITION 1. *A recursive equilibrium is a set of functions for (i) consumption of the tradable good* $c^T(b, s)$ *and the nontradable good* $c^N(b, s)$*; (ii) the prices for the nontradable good* $p^N(b, s)$ *and aggregate consumption* $P(b, s)$*; (iii) the government's asset holdings* $b'(b, s)$*, government consumption* $g(b, s)$*, and the default decision* $D(b, s)$*; and (iv) the prices for bonds* $q^F(b', s)$ *and* $q^D(b', s)$ *such that*

1. *Given the government policies,* $c^T(b, s)$*,* $c^N(b, s)$*,* $p^N(b, s)$*, and* $P(b, s)$ *satisfy the household's optimization problem.*
2. *Given the bond prices* $q^F(b', s)$ *and* $q^D(b', s)$ *and the price of aggregate consumption* $P(b, s)$*, the government's asset holdings,* $b'(b, s)$*, government consumption,* $g(b, s)$*, and the default decision,* $D(b, s)$*, satisfy the government's optimization problem.*
3. *Bond prices* $q^F(b', s)$ *and* $q^D(b', s)$ *satisfy the foreign creditors' expected zero-profit condition and are consistent with the government's default probabilities.*
4. *The following domestic market clearing conditions hold:*

Nontradable-sector market-clearing condition:

$$c^N = y^N. \tag{17}$$

Tradable-sector-market clearing condition:

$$c^T = (1 - \tau)y^T - \tau p^N y^N. \tag{18}$$

The following proposition shows that the result about default incentives getting stronger with higher levels of foreign debt for any given realization of the exogenous endowment shocks [Eaton and Gersovitz (1981), Chatterjee et al. (2007), Arellano (2008)] also holds in the current model for all values of α.

PROPOSITION 1. *Given any* $\alpha \in [0, 1]$ *and an exogenous state* s*, for all* $b^0 < b^1$*, if default is optimal for* b^1*, then default is also optimal for* b^0*. That is,* $D(b^0, s) \geq D(b^1, s)$*.*

Proof. For a given realization of the endowment shocks (y_t^T, y_t^N) and a given $\alpha \in [0, 1]$, if default if optimal for b^1, then

$$U\big(c\big(c_d^T, c_d^N\big), g_d\big) + \beta E[\theta V^o(0, s') + (1-\theta)V^d(s')]$$
$$\geq U(c((1-\tau)y^T - \tau p^N y^N, y^N), g^1) + \beta E V^o(b', s'),$$

where

$$g^1 = \tau(y^T + p^N y^N) - q^F \alpha b' - q^D(1-\alpha)b' + \alpha b^1 + (1-\alpha)Pb^1.$$

Since $g^1 = \tau(y^T + p^N y^N) - q^F \alpha b' - q^D(1-\alpha)b' + \alpha b^1 + (1-\alpha)Pb^1 > g^0 = \tau(y^T + p^N y^N) - q^F \alpha b' - q^D(1-\alpha)b' + \alpha b^0 + (1-\alpha)Pb^0$ for all b',

$$U(c((1-\tau)y^T - \tau p^N y^N, y^N), g^1) + \beta E V^o(b', s')$$
$$> U(c((1-\tau)y^T - \tau p^N y^N, y^N), g^0) + \beta E V^o(b', s').$$

Therefore, if default is optimal for b^1, then it is also optimal for b^0. ■

The value of default is independent of b, whereas the value of repayment is increasing in b. Therefore, if default is optimal for some level of assets b, for a given state s and a given value of α, then the value of repayment will be lower for a lower level of assets, and hence, default will be optimal for this level of assets as well.

3. QUANTITATIVE ANALYSIS

3.1. Data

The model is solved numerically using data from the Argentine economy in order to analyze the default implications of changing the shares of F-bonds and D-bonds. The business cycle statistics for Argentina are presented in Table 1. The data used to compute the statistics for output, real exchange rate, and trade balance are quarterly real series from the first quarter of 1993 to the last quarter of 2007. The interest rate spread, external debt, and debt service data are from the first quarter of 1993 up to the last quarter of 2001, which is the default episode. The real GDP and external debt data are obtained from the Ministry of Finance of Argentina

TABLE 1. Business cycle statistics for Argentina

std(R_s)	3.59	corr(R_s, Y)	−.42
std(TB/Y)	2.95	corr($R_s, TB/Y$)	.68
std(Y)	9.73	corr($TB/Y, Y$)	−.92
std(RER)	15.21	corr(RER, Y)	−.93
Default rate (%)			0.75
Avg. public debt-to-output ratio (%)			27.30
Avg. debt service-to-output ratio (%)			3.47
Avg. spread (%)			8.08

Note: Standard deviations are reported in percentages.

(MECON); the data for exchange rates, consumer price indices, and trade balance are obtained from the IMF's International Financial Statistics (IFS) database; and the data for debt service are obtained from the World Bank. The interest rate spread (R_s) data from 1996 onwards correspond to the Emerging Market Bond Index (EMBI) for Argentina constructed by J. P. Morgan, and are obtained from MECON. The spread data for the period 1993–1995 come from the dataset constructed by Neumeyer and Perri (2005). Real exchange rates are constructed as the ratio of consumer price indices for the United States and Argentina; i.e., a real exchange rate depreciation corresponds to an increase. Output is denominated in terms of tradables in order to make summary statistics from the data consistent with the model's statistics.[8] Output and real exchange rate data are in logs, trade balance is reported as a percentage of output, and all series are seasonally adjusted and HP filtered.

The default rate is from Arellano (2008) and reflects the fact that Argentina defaulted on its debt three times in the last 100 years, which corresponds to an average default rate of 3% annually and 0.75% quarterly. This statistic is based on Beim and Calomiris (2001), who report two episodes of sovereign defaults in Argentina's foreign debt for 1900–2001. Together with the last default episode in 2001, there are three defaults in the last 100 years. The average ratio of external debt of the public sector to total annual output is 27.3% in the period under consideration. The average value of the debt service on external debt as a proportion of GDP is calculated as 3.47%. Because debt in this model corresponds to the external debt of the public sector, the relevant debt service is for the public sector debt as well. However, debt service data are not available for public sector debt separately. Therefore, I compute the average debt service-to-GDP ratio for total external debt, which is 5.59% for the sample period. Multiplying this value by the ratio of external public sector debt to total external debt for the period (62%) gives an approximate value of 3.47% for the public sector debt service to-GDP-ratio.[9]

As demonstrated by the data, interest rate spreads are negatively correlated with aggregate output and positively correlated with the trade balance. The trade balance is also strongly countercyclical. The countercyclicality of interest rates and the trade balance are common features of developing country business cycles that have been documented in many studies. The real exchange rate has a strong negative correlation with output, which is consistent with the observation that many emerging market economies experience real exchange rate depreciations during recessions.[10] The volatility of the real exchange rate is quite high, owing mainly to the large deviations experienced in the default episode.

3.2. Calibration and Functional Forms

For the numerical solution of the model, specific functional forms are assumed for the utility function and the endowment shocks. The functional form of the utility function used in the quantitative solution is

$$U\left(c\left(c_t^T, c_t^N\right), g_t\right) = \frac{\left[c_t^{\zeta} g_t^{1-\zeta}\right]^{1-\sigma}}{1-\sigma}, \tag{19}$$

where $c(c_t^T, c_t^N) = [\omega(c_t^T)^{-\eta} + (1-\omega)(c_t^N)^{-\eta}]^{-1/\eta}$ is the constant elasticity of substitution aggregator.

In equilibrium, the relative price of nontradables is determined by the following condition, which comes from the households' optimization:

$$p_t^N = \left(\frac{1-\omega}{\omega}\right)\left(\frac{c_t^T}{c_t^N}\right)^{\eta+1}. \tag{20}$$

This condition shows that the nontradable price, and hence the price index P_t, is increasing in c_t^T and decreasing in c_t^N. In the data, real exchange rate depreciations are correlated with reductions in both tradable and nontradable output in emerging markets, whereas in an endowment economy model, shocks to nontradable output generate the opposite result. To make the correlations generated by the model consistent with the data, the following endowment structure is assumed: endowments of tradable and nontradable output are characterized by a common stochastic trend, and additionally, the tradable output has transitory shocks around the trend. This formulation provides a simple structure for generating correlations between sectoral output and real exchange rates that are consistent with those observed in the data, using an endowment economy model. The endowment processes are as follows:

$$y_t^T = e^{z_t}\Gamma_t, \tag{21}$$

$$y_t^N = \Gamma_t. \tag{22}$$

The transitory shock follows an AR(1) process,

$$z_t = \mu_z(1-\rho_z) + \rho_z z_{t-1} + \varepsilon_t^z, \tag{23}$$

where $|\rho_z| < 1$.

The trend is characterized as

$$\Gamma_t = \gamma_t \Gamma_{t-1}. \tag{24}$$

The log of the trend growth rate follows an AR(1) process,

$$\log \gamma_t = (1-\rho_\gamma)(\log \mu_\gamma - a) + \rho_\gamma \log \gamma_{t-1} + \varepsilon_t^\gamma, \tag{25}$$

where $|\rho_\gamma| < 1$ and $a = \sigma_\gamma^2/2(1-\rho_\gamma^2)$.

The innovations ε_t^z and ε_t^γ are jointly normally distributed with $E[\varepsilon_t^z] = E[\varepsilon_t^\gamma] = 0$, variances $\sigma_z^2, \sigma_\gamma^2$, and covariance $\sigma_{z\gamma}$.

In this formulation, transitory shocks are equivalent to shocks to the ratio of tradable to nontradable output and they generate the relative variation of sectoral

TABLE 2. Parameter values

Risk aversion	$\sigma = 2$
Discount factor	$\beta = 0.88$
Elasticity of substitution	$1/(1+\eta) = 0.5$
Weight of c^T in CES	$\omega = 0.28$
Weight of c in utility	$\zeta = 0.81$
Tax rate	$\tau = 0.19$
Output loss in default	$\varphi = 0.969E(y^i)$
Probability of reentry	$\theta = 0.23$
Nontradable–tradable ratio	$y^N/y^T = 2$
Average endowment growth	$\mu_\gamma = 1.002$
AR(1) coefficient of growth shock	$\rho_\gamma = 0.565$
AR(1) coefficient of transitory shock	$\rho_z = 0.527$
Standard deviation of ε^γ	$\sigma_\gamma = 0.019$
Standard deviation of ε^z	$\sigma_z = 0.032$
Covariance of ε^z and ε^γ	$\sigma_{z\gamma} = 0.00054$
Risk-free interest rate	$r^* = 0.01$

output levels that is observed in the data. The real exchange rate movements in the model are determined by the transitory shocks, because the nontradable price depends on the ratio of tradable consumption to nontradable consumption, as illustrated by equation (20).

Aguiar and Gopinath (2007) show that in emerging markets, output fluctuations are better characterized by shocks to trend growth rather than transitory shocks around a stable trend. Following this, in their studies of default in Argentina, both Aguiar and Gopinath (2006) and Yue (2010) use a stochastic trend to model the output process, since it improves the predictions of this type of model significantly. In particular, it helps generate default levels that are closer to the frequency observed in the data and improves the model's ability to match the countercyclicality of interest rates and trade balance, as well as the positive correlation between the two.

Output is nonstationary with this characterization, because the growth shock has a permanent effect on output. Therefore, in the numerical analysis, the model is detrended by $\mu_\gamma \Gamma_{t-1}$, following Aguiar and Gopinath (2006). The details and the dynamic program for the detrended version are explained in Appendix A.

Table 2 presents the parameter values used in the computational analysis. These parameters either are calibrated to match the empirical regularities of the Argentine economy for the period 1993Q1–2007Q4, or are based on prior empirical studies of Argentina. The coefficient of relative risk aversion is set to 2, which is standard in business cycle studies. Elasticity of substitution between tradable and nontradable consumption is set to 0.5, which is close to the estimate of Gonzalez-Rozada and Neumeyer (2003) of 0.48. The weight on the CES aggregator in the utility function, ζ, is set to 0.81 to match the average government expenditure-to-GDP ratio of 19% in Argentina. Because government expenditure and tax revenue are equalized in autarky, the tax rate of 0.19 is also set to match the government expenditure-to-GDP ratio. The quarterly risk-free interest rate is taken as 1%, which is the average real rate of return on 3-month U.S. Treasury bills.

To calibrate the relative sizes of the tradable and nontradable sectors in Argentina, I use the classification of Arellano (2005), who assesses the degree of tradability of goods by computing the share of total trade (exports plus imports) of each sector as a percentage of total sectoral output. Based on this, the agricultural, manufacturing, and energy sectors are classified as tradable and the share of the tradable sector is 28% of total output. The weight on the tradable consumption in the CES aggregator, ω, is therefore set to 0.28. The mean value of the detrended tradable endowment y^T is normalized to one, and the mean value of the detrended nontradable endowment y^N is set to normalize the consumption price index to be equal to one in the steady state. The mean quarterly growth rate of per capita nontradable output in the data is 0.2%. Therefore the long-run mean of the growth rate of the stochastic trend, μ_γ, is set equal to 1.002. The other parameter values for the endowment processes have been estimated using tradable and nontradable output data.

Following Arellano (2008), the output cost of default is specified as

$$\lambda(y_t^i) = \begin{cases} y_t^i - \varphi & \text{if } y_t^i > \varphi \\ 0 & \text{if } y_t^i \leq \varphi \end{cases} \tag{26}$$

for $i = T, N$. The asymmetric output cost of default enables the model to generate default probabilities consistent with the data, at the same time sustaining higher borrowing levels in equilibrium [see also Chatterjee and Eyigungor (2010)].

The output cost parameter, φ, and the exogenous probability of reentering the markets after default, θ, have been set to match the annual default frequency of 3% and the average debt service-to-GDP ratio of 3.47%. The discount factor is set to 0.88, which is the highest value that enables matching the default rate and the debt service ratio by setting φ and θ.

The model is solved by a value-function iteration algorithm, which is described in Appendix A. The stochastic process for each endowment shock is discretized into a 15-state Markov chain from their joint distribution, using the quadrature-based procedure of Tauchen and Hussey (1991), and bond holdings are discretized into a grid of 200 equally spaced bond levels. The sensitivity of the results to changes in the number of grid points is also analyzed in Appendix A.

3.3. Results

The model is solved numerically for different shares of F-bonds, denoted by α. To illustrate the model's predictions, I report the results for two different α values, $\alpha = 0$ and $\alpha = 1$. The results are monotonic for the range of α values between 0 and 1.

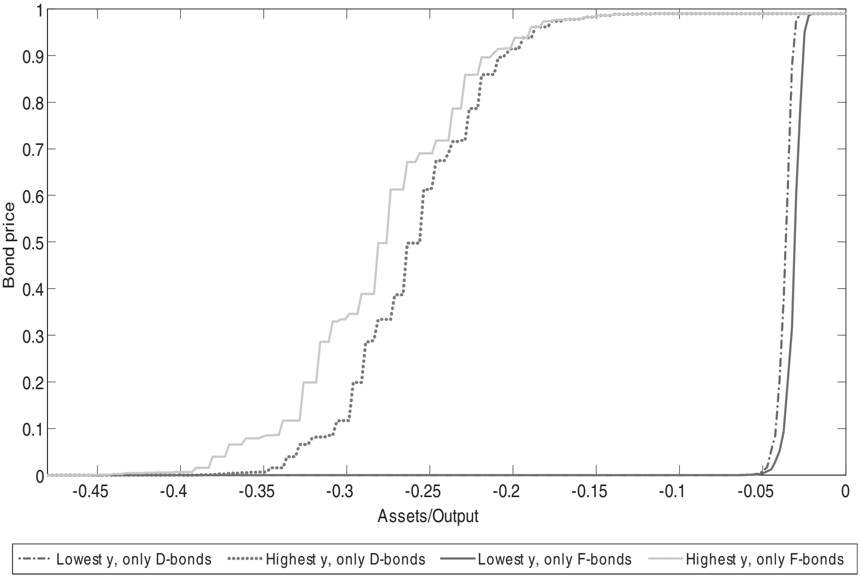

FIGURE 1. Prices of F-bonds.

To analyze how default incentives change with α, I plot the price schedule of nonindexed bonds as a function of the ratio of assets to mean output. The reason for plotting the price of F-bonds rather than the price of D-bonds is that the default risk can be measured directly by the price of F-bonds, as it only depends on the default probability and the risk-free interest rate, whereas the price of D-bonds also reflects movements in the real exchange rate.

Figure 1 shows the equilibrium price schedule of F-bonds for the highest and lowest output levels for the cases where the government is trading only F-bonds, $\alpha = 1$, and only D-bonds, $\alpha = 0$. As the figure illustrates, bond prices are an increasing function of foreign assets; i.e., higher debt levels lead to higher interest rates. When the debt level is low, the government always repays its debt and the bond price is equal to the inverse of the risk-free rate. As the level of debt increases, the default incentive increases and bond prices decrease, reflecting the fact that the government finds it optimal to default for some realizations of endowment. At even higher levels of debt, bond prices fall to zero, since government defaults for all realizations of the endowment shocks. Another point to note is that the model predicts that default is more likely in bad times. For a given level of assets, bond prices are lower for the low-endowment state for both $\alpha = 1$ and $\alpha = 0$, which means that default incentives and interest rates are higher when the output is low.

The figure shows that the effect of debt denomination on default incentives and interest rates changes depending on the output level. When the economy is issuing only F-bonds, $\alpha = 1$, the bond price it faces is higher in the highest-output

TABLE 3. Model statistics for Argentina

	F-bonds only ($\alpha = 1$)	D-bonds only ($\alpha = 0$)
std(R_s)	1.61	1.17
std(TB/Y)	0.48	0.44
std(RER)	3.02	3.02
std(Y)	6.43	6.43
std(g)	7.54	7.26
corr(R_s, Y)	−.21	−.16
corr($R_s, TB/Y$)	.42	.25
corr($TB/Y, Y$)	−.29	−.21
corr(RER, Y)	−.78	−.78
Default rate (%)	0.75	0.58
Avg. debt service-to-output ratio (%)	3.47	3.97
Avg. debt-to-output ratio (%)	3.53	4.03
Avg. spread (%)	3.06	2.36
Max spread (%)	7.97	5.90
Increase in welfare with $\alpha = 0$	0.019%	

Note: Standard deviations are reported in percentages.

state, reflecting a lower default probability, compared with the case where it is only issuing D-bonds, $\alpha = 0$. This relationship is reversed in the lowest-output state and the economy faces higher bond prices when it is only issuing D-bonds. Because the amount to be repaid on D-bonds is higher in high-output states and lower in low-output states, borrowing with D-bonds increases default risk when output is high and decreases it when output is low. Even though issuing D-bonds helps achieve a smoother government consumption profile, default incentives increase for $\alpha = 0$ in high-output states. This result shows that the high cost of repayment in the current period outweighs the benefit of having higher consumption smoothing in the future.

As the figure illustrates, the range of bonds that carry a positive and finite risk premium is wider for the highest-output state than for the lowest-output state. In the lowest-output state, default incentives rise rapidly with the level of debt, and therefore there is a very narrow range of bonds with positive and finite risk premium. Because borrowing with F-bonds reduces default risk in the high-output state, it turns out that F-bonds lead to lower default risk and interest rates for a wider range of debt levels. In spite of this, as the simulation results in Table 3 show, the default rate of the economy is lower in the $\alpha = 0$ case. This is because defaults mostly occur in low-output states, and in these states borrowing with D-bonds reduces the default probability, since repayment on D-bonds is lower. Hence, the realized default rate is higher when the country borrows with F-bonds, even though the interest rate it faces is lower for high realizations of the endowment shocks and for a wider range of debt levels.

A related result is that borrowing with F-bonds leads to a looser debt limit for the economy. The level of debt at which the government refuses to repay its debts for all endowment shocks and the bond prices fall to zero increases from 40% of average output in the $\alpha = 0$ case to 47% in the $\alpha = 1$ case. On the other hand, the risk-free debt limit, which is the level of debt up to which the default probability is zero and the country can borrow at the risk-free interest rate for any realization of the endowment shocks, increases slightly when $\alpha = 0$. It increases from 2.2% of output in the case $\alpha = 1$ to 2.7% of output when $\alpha = 0$.

The simulation results from the benchmark calibration of the model are presented in Table 3. The model is simulated for 5,000 periods and the reported statistics are the mean values over 500 simulations of 100 observations each. The simulated data are treated in an identical manner to the empirical data and aggregate output is denominated in terms of tradables.

The simulation results of the model are reported for $\alpha = 1$ and $\alpha = 0$. The model produces a lower default rate in the $\alpha = 0$ case. The quarterly default rate decreases from 0.75% to 0.58% when $\alpha = 0$. As illustrated in Figure 1, borrowing with F-bonds leads to lower interest rates for high realizations of the endowment shocks, whereas D-bonds lead to lower interest rates for low realizations of the endowment shocks. However, since defaults mostly occur in low-output states and D-bonds reduce default incentives in these states, the default rate of the economy decreases when borrowing is done with D-bonds. Related to this, the average debt holding of the government slightly increases, and both the maximum spread and the average spread generated by the model decrease in the $\alpha = 0$ case. The volatility of interest rates also declines when $\alpha = 0$.

The model matches the countercyclicality of the trade balance and interest rates as well as the positive correlation of the two. For the case where $\alpha = 1$, the correlations generated by the model are closer to those in the data than in the $\alpha = 0$ case. Specifically, interest rates and the trade balance become more strongly countercyclical and the correlation of the two becomes stronger as well.

When borrowing is done with D-bonds, holding the default probability constant, the expected repayment value is higher the higher is the output. As a result, the tendency to borrow more when output is high, which gives a countercyclical trade balance, gets weaker and the model generates a less countercyclical trade balance. The countercyclicality of interest rates also decreases when $\alpha = 0$: the repayment on D-bonds decreases in a low-output state because of the lower value of the price index, which leads to a decline in default incentives. Therefore, default probability and interest rates do not increase as much as in the $\alpha = 1$ case when output is low, and vice versa.

In the model, debt is used to smooth fluctuations in government consumption, and simulation results show that borrowing with D-bonds leads to a smoother government consumption profile. Since repayment on D-bonds moves in the same direction as government tax revenue, the fluctuations in government consumption are reduced by this kind of borrowing. This result is also related to the welfare implications of the two types of borrowing: welfare is higher for $\alpha = 0$ than for

TABLE 4. Sensitivity analysis

	Output cost (φ)		Reentry probability		NT–T ratio	
	$0.99E(y^i)$	$0.95E(y^i)$	$\theta = 0.1$	$\theta = 0.3$	$y^N/y^T = 1$	$y^N/y^T = 3$
			Default rate			
$\alpha = 1$	0.81	0.70	0.36	1.11	0.73	0.70
$\alpha = 0$	0.75	0.55	0.26	1.00	0.61	0.62
			Avg. debt/output (%)			
$\alpha = 1$	1.86	6.28	11.51	2.53	4.00	3.94
$\alpha = 0$	2.08	7.10	14.15	2.82	4.51	4.33
			$\text{std}(R_s)$			
$\alpha = 1$	2.24	1.96	0.90	2.73	1.49	1.45
$\alpha = 0$	1.99	1.20	0.45	2.35	1.09	1.20
			$\text{corr}(R_s, Y)$			
$\alpha = 1$	−.04	−.19	−.42	−.12	−.16	−.17
$\alpha = 0$	.01	−.26	−.29	−.09	−.12	−.07
			$\text{corr}(TB/Y, Y)$			
$\alpha = 1$	−.25	−.31	−.41	−.25	−.29	−.28
$\alpha = 0$	−.19	−.24	−.29	−.20	.04	−.37
			Avg spread (%)			
$\alpha = 1$	3.22	2.81	1.35	4.39	2.85	2.69
$\alpha = 0$	2.96	2.24	0.94	3.96	2.37	2.44

$\alpha = 1$, although the difference in the welfare levels is quite small, corresponding to a 0.019% change in permanent consumption. One reason for the small magnitude of the welfare effect is that borrowing is used to smooth only government consumption, which has a lower weight in the utility function. In a model where bonds are used to smooth private consumption as well as government consumption, the effect would be bigger.

4. SENSITIVITY ANALYSIS

Table 4 reports the sensitivity of the results to changes in the output cost of default and the reentry probability, which are the two parameters that have been set to match the default rate and the debt service-to-output ratio from the data. The effects of a change in the tradable share in output are also analyzed.

The first parameter considered is the output cost parameter, φ. As the output cost of default increases, the default rate decreases and the average debt holding increases. At higher default cost, the countercyclicality of the interest rates and the trade balance increase as well. The interest rate–output correlation is especially sensitive to this parameter. Since the output cost of default is higher in high-output episodes, with lower values of this parameter, the difference between low- and high-output states in terms of default incentives decreases. Therefore, with a low default cost, default incentives do not decrease as much in higher-output states, and the countercyclicality of interest rates decreases substantially. In comparing the model's predictions for $\alpha = 0$ and $\alpha = 1$, it is seen that changes in this parameter do not affect the main conclusions. For all values of φ, default rate and average spread are lower and the economy can sustain a higher level of debt when all of the borrowing is done with D-bonds. The business cycle properties of interest rates and the trade balance remain mostly the same, except that for $\varphi = 0.95E(y^i)$ the model generates a higher negative correlation between interest-rate spreads and output in the $\alpha = 0$ case.

Another parameter that affects the results is the probability of regaining access to markets after default. This parameter has a greater effect on the results than the output cost parameter. As the reentry probability increases, the default rate of the economy increases substantially, and this leads to a decline in the debt holdings of the economy. In line with this, the volatility and the average level of the spreads increase as well. A decrease in this parameter, on the other hand, makes interest rates and the trade balance more countercyclical. The relationship between the $\alpha = 0$ and $\alpha = 1$ cases, however, is not affected by this parameter either.

In the benchmark calibration of the model, the mean ratio of nontradable output to tradable output is two. Because the share of the tradable good in total output can change over time or vary across countries, I analyze the results for different values of y^N/y^T. However, changing this ratio does not have a significant effect on the results. The only exception is the trade balance–output correlation in the $\alpha = 0$ case. Since the real exchange rate is highly correlated with tradable output in this model, an increase in the share of tradable output affects the borrowing pattern when debt is in terms of D-bonds. Repayment on D-bonds moves in the same direction as the tradable output and with this kind of bonds, the incentive to borrow more in high-output episodes decreases compared to that for F-bonds. As the share of tradable output increases, this effect becomes more pronounced and the correlation turns positive for $y^N/y^T = 1$, whereas it is strongly negative for $y^N/y^T = 3$.

5. CONCLUSIONS

Even though currency denomination of debt has been extensively studied in the literature, it has not been considered in the papers that model sovereign default risk. This paper analyzes the relationship between the share of foreign currency debt and default risk in a real model of a small open economy with stochastic endowments of tradable and nontradable goods. Foreign currency debt is captured by tradable-denominated nonindexed bonds, and domestic currency debt by bonds whose return is indexed to the domestic consumption–based price index.

The results show that the effect of debt denomination on default incentives and interest rates changes depending on the output level: in low-output states borrowing with indexed bonds reduces default risk and in high-output states borrowing with nonindexed bonds reduces default risk. However, the default rate of the economy decreases when the country borrows with indexed bonds, because defaults mostly occur in periods of low output. Simulation results show that debt denomination also affects the business cycle properties of interest rates and the trade balance.

In particular, both the level and the volatility of interest rates increase when the country borrows with nonindexed bonds, while interest rates and the trade balance become more countercyclical. In addition, the level of welfare is shown to increase as the economy borrows more with indexed bonds, since this type of borrowing enables a smoother consumption profile.

The model predicts that domestic currency borrowing reduces the default frequency and improves welfare, whereas in reality emerging market economies almost exclusively borrow in foreign currency. This can be partly explained by one of the predictions of this model, which is that the economy faces lower interest rates when debt is denominated in foreign currency for higher output levels. What makes foreign currency debt harder to repay is the increase in its value relative to the value of domestic output through a real depreciation in times of low output. However, the lower real value of foreign currency debt relative to domestic currency debt in high-output states, which leads to lower default risk and interest rates, may be enticing for governments. Another issue, which is not captured in this model, is the government's ability to affect the value of domestic currency debt. Since the government may resort to reducing the real value of domestic currency debt by creating inflation, foreign lenders may refrain from lending in domestic currency. Because such a channel may affect the relationship between currency denomination of debt and default risk, a possible extension of the current paper would be to incorporate it using a nominal model and to fully analyze the currency denomination issue by also capturing the government's ability to create inflation.

NOTES

1. Table B.1 in Appendix B shows domestic currency debt as a share of total international debt for select country groups. In developing countries, the average share of domestic currency debt is about 2.5% for the period 1993–2001.

2. See Feldstein (2002), Hausmann and Velasco (2002), Calvo et al. (2003), and Perry and Serven (2003).

3. Figure B.1 in Appendix B plots real exchange rate and output in different emerging market economies, and Table B.2 shows the correlation coefficients of the two series. Both series are in logs and HP filtered.

4. The structure of the shocks is explained in Section 3.2.

5. In the literature similar formulations have been used to capture currency denomination of debt in real models. Krugman (1999) and Caballero and Krishnamurthy (2003) characterize foreign and domestic currency debt as borrowing with bonds denominated in foreign and home goods. Chamon (2003) and Schneider and Tornell (2004) have real models with tradable and nontradable goods, similar to the current paper. They model foreign currency debt as borrowing with tradable-denominated bonds and domestic currency debt as bonds whose return is indexed to the relative price of nontradables.

6. See Aguiar and Gopinath (2006), Arellano (2008), Chatterjee and Eyigungor (2010), and Yue (2010).

7. Rose (2005) finds an 8% per year decline in bilateral trade flows following a default. Sturzenegger (2004) estimates the average cumulative drop in output to be 4% over the four years immediately following a default, controlling for other factors that may explain growth performance, for the default episodes of the 1980s.

8. Nominal GDP is deflated by the tradable GDP deflator to convert total output into tradable units. The tradable deflator is calculated as the nominal tradable GDP divided by the real tradable GDP.

9. The ratio of public and publicly guaranteed debt service to GDP, which is available from the World Bank, is 3.12% for the same period. This is the sum of debt service on long-term public and publicly guaranteed debt, and excludes short-term debt.

10. The correlation of the real exchange rate with aggregate output changes from −.63 to −.93 when output is converted to tradable units. The reason for this higher negative correlation is the greater increase in the tradable deflator compared to the GDP deflator with a real depreciation, which leads to a greater decline in output measured in units of tradables while the real exchange rate is depreciating.

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APPENDIX A

In the quantitative analysis, the model is detrended by $\mu_g \Gamma_{t-1}$, following Aguiar and Gopinath (2006). With this normalization, the mean of the detrended tradable endowment, y^T, is one. The Bellman equations recast in detrended form are given hereafter. The detrended counterpart of any variable x_t is represented by $\hat{x}_t = x_t/\mu_g \Gamma_{t-1}$ and the functions in the detrended version are also denoted by a hat.

The value function of a government that has access to credit markets is given by

$$\hat{V}^o(\hat{b}_t, z_t, \gamma_t) = \max\{\hat{V}^r(\hat{b}_t, z_t, \gamma_t), \hat{V}^d(z_t, \gamma_t)\}. \qquad \textbf{(A.1)}$$

The value function when the government repays its debt is as follows:

$$\hat{V}^r(\hat{b}_t, z_t, \gamma_t) = \max_{\hat{b}_{t+1}}\left\{U\left(c\left(\hat{c}_t^T, \hat{c}_t^N\right), \hat{g}_t\right) + \beta E_t \hat{V}^o(\hat{b}_{t+1}, z_{t+1}, \gamma_{t+1})\right\}, \qquad \textbf{(A.2)}$$

TABLE A.1. Grid specifications in the baseline model

	b	z	$\log\gamma$
Number of grid points	200	15	15
Minimum	-0.8	$\mu - 5.4\sigma$	$\mu - 6.9\sigma$
Maximum	0	$\mu + 5.4\sigma$	$\mu + 6.9\sigma$

Note: μ and σ denote the means and the standard deviations of the respective processes.

$$\hat{g}_t = \tau\left(\hat{y}_t^T + p_t^N \hat{y}_t^N\right) - q_t^F \alpha \hat{b}_{t+1}\gamma_t - q_t^D(1-\alpha)\hat{b}_{t+1}\gamma_t + \alpha\hat{b}_t + (1-\alpha)P_t\hat{b}_t, \qquad \textbf{(A.3)}$$

$$\hat{c}_t^T = (1-\tau)\hat{y}_t^T - \tau p_t^N \hat{y}_t^N, \qquad \textbf{(A.4)}$$

$$\hat{c}_t^N = \hat{y}_t^N. \qquad \textbf{(A.5)}$$

The value function of a government that chooses to default is given by

$$\hat{V}^d(z_t,\gamma_t) = U\left(c\left(\hat{c}_{t,d}^T, \hat{c}_{t,d}^N\right), \hat{g}_{t,d}\right) + \beta E_t[\theta\hat{V}^o(0, z_{t+1},\gamma_{t+1}) + (1-\theta)\hat{V}^d(z_{t+1},\gamma_{t+1})], \qquad \textbf{(A.6)}$$

where

$$\hat{g}_{t,d} = \tau\left(\hat{y}_{t,d}^T + p_t^N \hat{y}_{t,d}^N\right), \qquad \textbf{(A.7)}$$

$$\hat{c}_{t,d}^T = (1-\tau)\hat{y}_{t,d}^T - \tau p_t^N \hat{y}_{t,d}^N, \qquad \textbf{(A.8)}$$

$$\hat{c}_{t,d}^N = \hat{y}_{t,d}^N, \qquad \textbf{(A.9)}$$

The solution algorithm is as follows:

1. Start with initial guesses for the bond price functions $q_0^F(\hat{b}_{t+1}, z_t, \gamma_t)$ and $q_0^D(\hat{b}_{t+1}, z_t, \gamma_t)$, which correspond to a default probability of zero for each point in the state space.
2. Using these initial prices and initial guesses for $\hat{V}^r(\hat{b}_t, z_t, \gamma_t)$ and $\hat{V}^d(\hat{b}_t, z_t, \gamma_t)$, iterate on the Bellman equations to solve for the optimal value and policy functions.
3. Given the optimal default decision, update the prices of bonds using equations (15) and (16). Using these prices, repeat steps 2 and 3 until the bond prices converge.

Table A.1 reports the grid specifications for the bonds, the transitory shock z_t, and the log of the trend shock γ_t used in the computational analysis. For the bond holdings, equally spaced grid points are used and the limits of the asset space are set to ensure that they do not bind in equilibrium. Setting the maximum bond-holding level to 0 implies that the government is not allowed to accumulate assets, and this constraint never binds in the simulations. The stochastic processes followed by the endowment shocks are specified in Section 3.2. Each shock is discretized from their joint distribution using the quadrature-based procedure of Tauchen and Hussey (1991). Therefore, the grid points are not equally spaced and are set using Gauss–Hermite nodes.

The effects of changing the number of grid points separately for the bonds and the two types of shocks are analyzed in Table A.2. Changing the grid sizes slightly affects the statistics generated by the model, although the main results in terms of the comparison of the $\alpha = 0$ and $\alpha = 1$ cases are not affected. The biggest effect of changing the number of grid points of the transitory shock is on the correlation of interest rates and output; however, $\alpha = 1$ still leads to a more countercyclical interest rate than $\alpha = 0$. Changing the grid size

TABLE A.2. Changes in the number of grid points

	Number of grid points for					
	Bonds		Trend shocks		Transitory shocks	
	$N_b = 300$	$N_b = 400$	$N\gamma = 25$	$N\gamma = 35$	$N_z = 25$	$N_z = 35$
			Default rate			
$\alpha = 1$	0.72	0.73	0.69	0.70	0.75	0.81
$\alpha = 0$	0.66	0.64	0.65	0.65	0.64	0.68
			Avg. debt/output (%)			
$\alpha = 1$	3.60	3.60	3.45	3.40	2.38	2.02
$\alpha = 0$	4.07	4.07	3.92	3.85	2.71	2.27
			$\text{std}(R_s)$			
$\alpha = 1$	1.76	1.86	1.27	1.40	1.69	1.81
$\alpha = 0$	1.30	1.10	1.16	1.22	1.32	1.60
			$\text{corr}(R_s, Y)$			
$\alpha = 1$	−.25	−.25	−.17	−.24	−.13	−.04
$\alpha = 0$	−.21	−.17	−.16	−.14	−.08	−.01
			$\text{corr}(TB/Y, Y)$			
$\alpha = 1$	−.29	−.29	−.28	−.30	−.28	−.26
$\alpha = 0$	−.22	−.22	−.22	−.22	−.21	−.20
			Avg. spread (%)			
$\alpha = 1$	2.88	2.92	2.73	2.82	3.03	3.27
$\alpha = 0$	2.63	2.55	2.57	2.59	2.58	2.75

of the trend shock affects the differences between the two α values slightly more than the other grid changes. However, for all grid changes, the differences in the statistics for the two α values do not change much with further increases in the number of grid points.

APPENDIX B

TABLE B.1. Share of domestic currency debt in total international bonded debt

	1993–1998	1999–2001
Major financial centers[a]	52.6%	68.3%
Euroland	23.2%	56.8%
Other developed countries	17.6%	9.6%
Developing countries	**2.3%**	**2.7%**

Source: Eichengreen et al. (2003b).
[a] Major financial centers are the United States, the United Kingdom, Japan, and Switzerland.

TABLE B.2. Correlations of output and real exchange rate

Argentina	−.6328
Brazil	−.3952
Korea	−.7255
Malaysia	−.5958
Mexico	−.5861
Philippines	−.3064
Thailand	−.6061
Turkey	−.5980

Source: IMF International Financial Statistics (real GDP data for Brazil are obtained from OECD). The data cover 1991 Q1–2007 Q4 for Brazil, 1990 Q1–2006 Q3 for the Philippines, 1993 Q1–2007 Q4 for Thailand, 1990 Q1–2006 Q2 for Turkey, and 1990 Q1–2007 Q4 for the rest of the countries. All series are seasonally adjusted, logged, and HP filtered. Real exchange rates are calculated as $RER_i = NER_i \times CPI_{US}/CPI_i$ for country i.

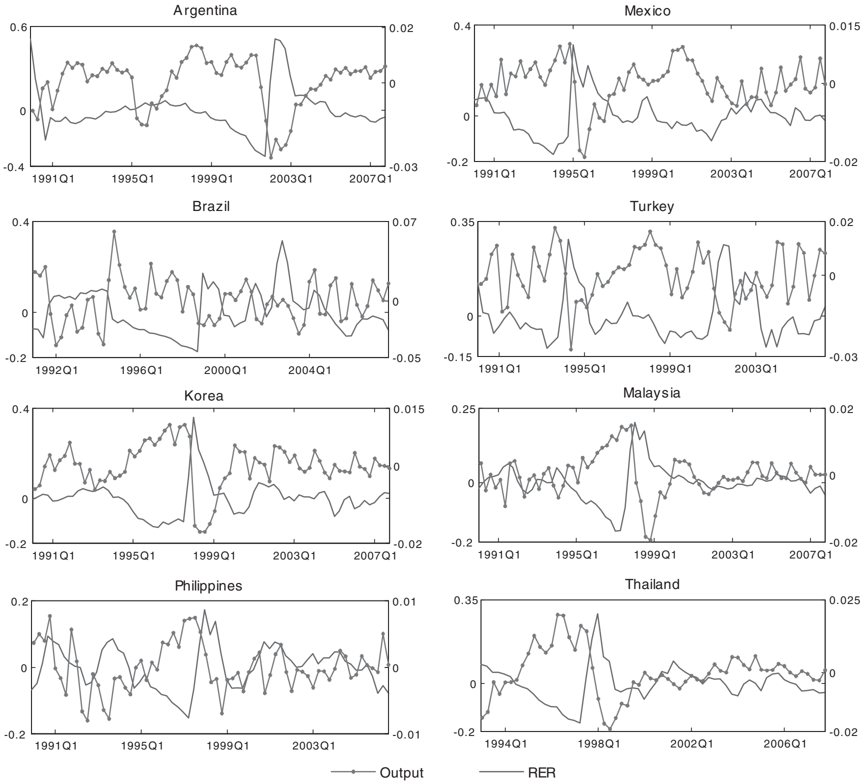

FIGURE B.1. Output and real exchange rates in emerging market economies.