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One-dimensional macro-scale matter wave dynamics potentially embedded in the Lorentz trajectory

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Abstract. We point the rather unusual existence of a potential embedding in the Lorentz trajectory – charged particle trajectory in a magnetic field – of a one-dimensional matter wave dynamics on the macro-scale as demonstrated through one-dimensional matter wave interference effects on the macro-scale, as well as the observation of a curl-free vector potential in one dimension, also on the macro-scale. Such an embedding is a manifestation of the generation of the quantum modulation of the de Broglie wave of the particle along the magnetic field, arising concomitantly, with a scattering-induced transition across Landau levels in consequence of quantum entanglement between the parallel and perpendicular degrees of freedom.

1. Introduction

The macro-scale has usually been regarded as belonging exclusively to classical dynamics, while quantum dynamics is now known to govern the processes on the micro-scale. The exception to this generalized statement are the low temperature phenomena – the superconductivity, superfluidity and the more recently achieved Bose–Einstein condensation, where the low energy of the particles of the system leads to the de Broglie matter waves acquiring macro-scale dimensions – the macro-scale de Broglie matter waves, characterized by the Planck quantum.

This statement has been generally seen to be true, consistent with the canonical perception regarding the division of physical phenomena into 'macro-classical' and 'micro-quantum' domains. Accordingly, all phenomena existing on the macro-scale – barring the above-mentioned low temperature ones – ought to be explicable in terms of the equations of classical dynamics, and no phenomena characteristic of quantum dynamics, such as the matter wave effects, could exist on the macro-scale. Any departure from this time-honoured and entrenched perception could not ordinarily be expected nor could it be accepted easily.

However, through a series of theoretical and experimental investigations carried out over a number of years, we have discovered essentially a counterexample to the above generalized statement in a rather special system – charged particle dynamics in a magnetic field. We have discovered the existence of phenomena which are identified as one-dimensional matter wave interference effects on the macro-scale (Varma et al. 2002; Varma and Banerjee 2007). The one-dimensionality here refers to one dimension along the magnetic field, and the macro-scale refers to dimensions ~ 5 cm for the wavelength of the associated matter wave for some typical laboratory parameters – the magnetic field strength $B \sim 100$ G and the electron energy $\mathscr{E} \sim 1$ keV. A wavelength of ~ 5 cm for a matter wave under the typical laboratory conditions of temperature could well invite strong scepticism. One must therefore immediately qualify that the macro-scale matter wave referred to here is an entirely distinct object from the de Broglie matter wave. It is, nevertheless, a rather fascinating object, the nature of which will be clarified in what follows. The existence of such an object does lead to the occurrence of a new class of phenomena, even if in a special system, which have a matter wave character on the macro-scale.

As another example of such phenomena, we have also observed the detection of a curl-free vector potential also on the macro-scale and in one dimension (Varma et al. 2012), which is again attributed to this macro-scale matter wave and whose characteristics we shall recount briefly in what follows.

The question that would naturally baffle one is: how are such phenomena possible which exhibit matter wave characteristics on the macro-scale – the one-dimensional matter wave interference effect and the observation of the curl-free vector potential also on the macro-scale and in one dimension, thereby running counter to the canonical view of both these matter wave phenomena. These questions have already been dealt with in a number of papers (Varma 2001, 2012a; Varma et al. 2002, 2012; Varma and Banerjee 2007), where the genesis of these effects has been discussed and it has been shown (Varma 2012a) that these effects are a manifestation of the quantum modulation of the de Broglie wave along the field, which is brought about through a scattering-induced transition across Landau levels.

It needs to be pointed out that all the above-mentioned experimental observations were carried out in macro-scale laboratory conditions with the dimension of the experimental system $L \sim 50$ cm, where the charged particle motion along a magnetic field $B \sim 100$ G and electron energy $\mathscr{E} \sim 1$ keV can well be described as a classical Lorentz trajectory. This paper will be devoted to the discussion of somewhat of a paradoxical fact that these macro-scale matter wave phenomena, which have a quantum origin, as discussed in Varma (2012a), should be associated with the Lorentz trajectories which, by all conventional accounts, are regarded as 'classical'. It is as if these macro-scale matter wave phenomena exist potentially embedded in the Lorentz trajectory. This would appear to be rather enigmatic and bizarre, but true.

However, before such a discussion, it will be useful from the point of view of the general readership to recall very briefly the observation of the most important different macro-scale matter wave phenomena, along with a very brief description of the generation of the quantum modulation, which serves as the macro-scale matter wave to which these phenomena are attributed.

2. Quantum modulation of the de Broglie wave as a macro-scale matter wave and its observational consequences

We shall recall here very briefly the observations of these very unusual effects that have been reported. These are (i) one-dimensional matter wave interference effects on the macro-scale, as against the standard micro-scale de Broglie matter wave interference effects (Varma et al. 2002; Varma and Banerjee 2007); (ii) even more enigmatic is the observation of a curl-free vector potential on the macro-scale and in one dimension reported in Varma et al. (2012). This is in sharp contradistinction to the corresponding micro-scale effect, known as the Aharonov–Bohm effect. A detailed comparison between the two effects has been recently presented in Varma (2012b). The limitation of space does not permit a detailed description of these effects. Suffice it to say that these are seen to be totally unexpected and contrary to the standard conceptual perception whereby all matter wave phenomena must necessarily be on the micro-scale.

It has been shown in a recent paper (Varma 2012a) how the quantum modulation, which is attributed to these effects, is generated in the process of scattering-induced transition across Landau levels, and the crucial role that the quantum entanglement plays in the process of this generation, whereby the plane wave state of the particle gets modulated concomitantly with the Landau level transitions. It has also been shown that the wavelength of the quantum modulation is the $\hbar$ independent expression $\lambda_n = 2\pi v/n\Omega$, where n is the Landau level interval across which transition occurs and v is the particle velocity parallel to the magnetic field. All observations relating to the macro-scale interference effects have been explained in terms of this wavelength. We wish to emphasize that the 'quantum entanglement' between the parallel and the perpendicular degrees of freedom plays a crucial role in the generation of the quantum modulation as a consequence of transition across Landau levels.

The observation of the curl-free vector potential on the macro-scale referred to above is also attributed to the quantum modulation as macro-scale matter wave. This effect is doubly enigmatic: (a) one, for its being on the macro-scale and (b) for its being one-dimensional as against the Aharonov–Bohm effect, which requires a minimum of two dimensions, being regarded as being of topological origin. The observation of this effect on the macro-scale has been regarded as quite astonishing, because being on the macro-scale it comes into apparent conflict with the Lorentz equation which does not permit it, because the latter involves only the magnetic field which vanishes, by definition, for a curl-free vector potential.

Just as the de Broglie wave dynamics is described by the Schrödinger wave equation, the quantum modulations too have wave equations which describe effects associated with them. These are given below and are, surprisingly, also of Schrödinger form, but one-dimensional because of modulations being one-dimensional, and were obtained in Varma (2001) starting from the Schrödinger wave equation in the path integral representation:

$$\frac{i\mu}{n}\frac{\partial \Psi(n)}{\partial t} = \frac{1}{2m}\left(\frac{\mu}{in}\frac{\partial}{\partial x} - \frac{e}{c}\hat{A}_x\right)^2 \Psi(n) + \mu\Omega\Psi(n); \quad n = 1, 2, 3, \ldots. \tag{1}$$

The macro-scale character of their description is expressed by the fact that this set of equations has a macro-scale action $\mu = N\hbar, N >> 1$ in lieu of $\hbar$ in the Schrödinger equation These equations predict the macro-scale observation of the curl-free vector potential alluded to above, because the curl-free vector potential component $\hat{A}_x$ appears in it in the same manner as a vector potential does in the Schrödinger equation. An algorithm for the manner of the observation of this effect has been discussed in detail in Varma (2010), while its observations have been reported in Varma et al. (2012).

We reproduce in Fig. 1 experimental results depicting the effect on the electron motion of a curl-free vector potential on the macro-scale as reported in the above paper (Varma et al. 2012). The left-hand panel of the figure shows the plate current response in the experiment with the sweep of the current in a toroidal solenoid, which produces a curl-free vector potential field that the electrons, travelling along the magnetic field line, sense as they traverse from the electron gun to the collector plate. The sweeping of the solenoidal current produces a correspondingly varying curl-free vector potential in the space around. The different plots in the left-hand panel, corresponding to different energies $\mathscr{E} = 600, 800, 1000, 1100, 1200$ eV, exhibit an undulatory behaviour, as against the *flat* response expected

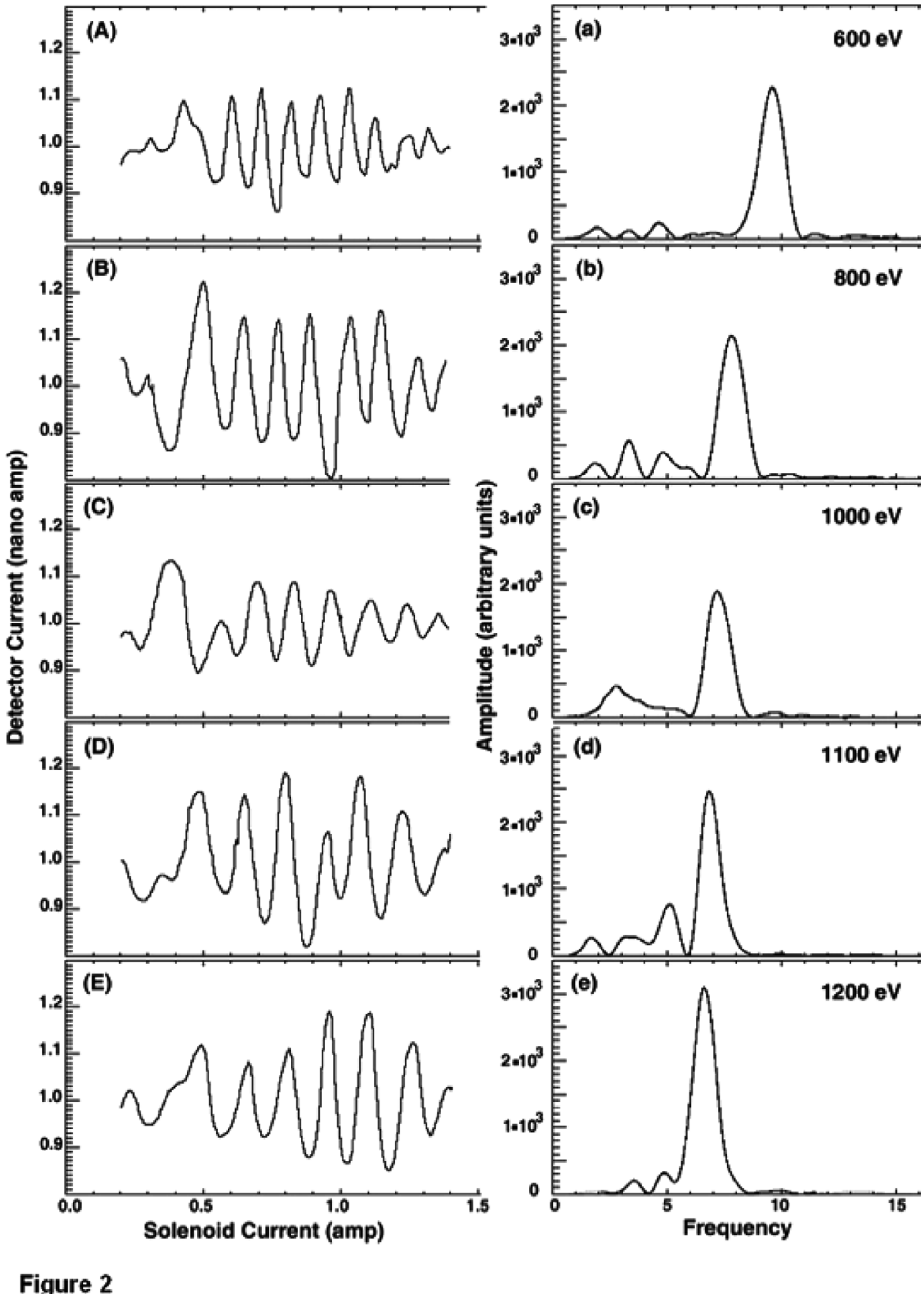

Figure 1. Detector plate current against the current in the toroidal solenoid. Left: panels (A–E) denote detector current in nA against the solenoidal current in amperes (A) for respectively the electron energies $\mathscr{E}$ (eV) = 600, 800, 1000, 1100 and 1200. These have been corrected for the saturating core, which leads to dilation in the interpeak separation in the original plots with increasing solenoidal current. Right: panels (a–e) denote the Fourier plots corresponding to (A–E).

classically, since a curl-free vector potential is not expected to affect the electron dynamics as per the classical Lorentz equation. These plots thus signal the detection of the curl-free vector potential on the macro-scale. Moreover, the energy dependence of the inter-peak separation of the peaks is also found to agree with the theoretical expectation, varying as $\mathscr{E}^{1/2}$. The right-hand panel of the figure, which gives the periodogram of the respective plots in the left-hand panel, shows such a dependence. The original reference (Varma et al. 2012) ought to be looked at for more details. A more comprehensive account of all the different observed phenomena and the associated theoretical framework is presented in a recent review (Varma 2013).

3. Macro-scale matter wave dynamics embedded in the Lorentz trajectory

If we look at the experimental conditions under which these observations have been made, namely the electron

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energy $\mathscr{E} \sim 1$ keV, the magnetic field $B \sim 100$ G and the experimental system's linear dimension $L \sim 50$ cm over which the electron beam traverses in going from the gun to the plate detector, the experiment would essentially be regarded as one in the 'classical' domain, with the charged particle trajectory being regarded as essentially a Lorentz trajectory determined by the Lorentz equation of motion. It would therefore appear quite enigmatic that under such conditions, effects have been observed, as alluded to above, which are not covered by the latter equation, and are, on the other hand, characterized as matter wave effects on the macro-scale, attributed to the quantum modulation.

We shall now argue that even under these 'classical' looking conditions, the charged particle motion in a magnetic field still carries its quantum structure of Landau levels and displays it through the rather subtle mechanism (Varma 2012a), in terms of the above-mentioned observed effects as manifestations of the quantum modulation. Astonishingly, it happens to be the case even when the inter-Landau level spacings are as tiny as $\hbar\Omega \sim 10^{-6}$ eV which, in fact, corresponds roughly to the value of the magnetic field used in the experiment. This shows that the Landau levels maintain their well-defined discrete structure even with such tiny interlevel spacings, and over macro-scale linear dimensions of the experiment $L \sim 50$ cm. This would appear very surprising, but is true as evidenced from the observation of phenomena attributed to the transition across such tiny Landau level spacings, leading to the generation of the quantum modulation.

Further, given such tiny inter-Landau level spacing, it may be surmised that one would need only a very weak perturbation to the particle's motion to cause the transition across at least one Landau level, and in fact more. According to the formalism of Varma (2012a), the transition across n Landau level intervals, $n = 1, 2, 3, ...$, would lead to the generation of quantum modulations which correspond to the wavelengths $\lambda_n = 2\pi v/n\Omega$. It has indeed been shown in Varma (2012a) that a weak scattering by the image charges of the particle in the wires of the grid is sufficient to lead to a change in the perpendicular energy of the particle, as expressed by the matrix element for the particular transition, by an amount corresponding to a few Landau level intervals. Thus, even a weak scattering by image charges is adequate to lead to the generation of the quantum modulation. Experiments clearly show this to be the case, because the position of the grounded grid has been found to play an important determining role in all the experimental results reported. It would follow from these considerations that if the magnetic fields were stronger, making the inter-Landau level spacings larger, the observed effects would get weaker, unless the perturbing potentials too would get stronger to enable the transitions to occur across increased energy spacings.

These considerations lead to some very important implications. They show that the generation of quantum modulation in the guiding centre motion of a charged particle has a quantum origin in terms of the discrete quantum structure of the Landau levels. The observations, carried out in a macro-scale experimental setting, show that such quantum-origin effects can occur on the macro-scale. But more significantly, it is also seen that the smaller the magnetic field, the more susceptible the system is to quantum modulation, because with a smaller inter-Landau level spacing even a weak scattering potential can induce transitions across Landau levels. *It is, as if the system has a macro-scale matter wave dynamics potentially embedded in the classical Lorentz trajectory, and that such a dynamics can get activated with a very small perturbation of the particle trajectory.*

Finally, it needs to be emphasized that another quantum property – the quantum entanglement – plays a most crucial role in the generation of the quantum modulation, without which there will be only energy exchange between the parallel and perpendicular degrees of freedom following the scattering, but no quantum modulation.

4. Summarising comments

A brief account is given here of the importance of our investigations carried out over a number of years. First of all, these investigations reveal the existence of phenomena – the observation of one-dimensional matter wave effects along the magnetic field and that of a curl-free vector potential also on the macro-scale, both of which would be normally considered as 'unthinkable', in view of their radical departure from the canonical conception of such phenomena which have been known to exist so far only on the micro-scale.

All these phenomena were discovered, having been motivated by the predictions of their existence through a theoretical formalism (Varma 2001). Further analysis and scrutiny of these phenomena have led to their deeper understanding and greater appreciation. The following points summarise the state of understanding of these phenomena.

(i) These phenomena are a manifestation of the quantum modulation of the de Broglie wave along the magnetic field. (ii) The quantum modulation is generated in consequence of transition across Landau levels induced by a scattering episode, with the discrete structure of levels being germane to the generation. (iii) The quantum property of 'entanglement' plays a crucial role in the generation, without which the modulation of the plane wave state would not occur, leaving only the energy exchange, even if discrete, as the sole consequence of the scattering. (iv) And a very significant one specially highlighted here is that the weaker magnetic fields render the system more susceptible to the generation of quantum modulation. This leads to the conclusion that a Lorentz trajectory has a macro-scale matter wave dynamics, potentially embedded in it, which can get activated even with a weak scattering potential, specially when the magnetic fields are weak.

Acknowledgements

This contribution is dedicated to the memory of my friend Padma Shukla with whom I have had a 40-year-long association, and with whom I spent some of the most enjoyable moments of my life. His sudden demise on the fateful day of January 26, 2013, has left a deep void that will be difficult to fill. I will continue to miss his broad smile, his very sunny disposition and his very enjoyable company.

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