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NOTE

INFLATION, LIQUIDITY, AND LONG-RUN GROWTH

GE ZHOU
College of Economics and Academy of Financial Research, Zhejiang University

This paper revisits the long-run relationship between inflation and economic growth by exploring the impact of inflation on investment. I illustrate that inflation may have a positive effect on growth by mitigating the liquidity risks of investment projects. Together with the traditional effect of the "inflation tax" on investment, a hump-shaped relationship between inflation and economic growth can be obtained in a calibrated model, which is consistent with the US postwar data. Sensitivity analysis suggests that the degree of financial development and the magnitude of the aggregate liquidity demand help explain the mixed empirical findings.

Keywords: Liquidity Demand, Credit Constraint, Inflation Tax, Endogenous Growth

1. INTRODUCTION

The long-run relation between inflation and economic growth is one of the classic topics in macroeconomics. However, current empirical findings on this topic are mixed. Although there is consensus that economic growth will be hurt by inflation above some threshold level, the effect of inflation below this threshold level is controversial. Actually, there exists some evidence that the effect of low inflation is insignificant, or even positive.[1] Moreover, the threshold inflation itself changes with the sample of countries.[2] In the theoretical literature, however, the negative effect is highly emphasized,[3] but the positive effect just begins to be considered currently. Additionally, theoretical explanations of the mixed empirical findings are rare.

This paper is motivated to provide a rationale for the above-mentioned mixed empirical findings by shedding light on the impact of inflation on investment. The focus on investment is based on the following basic statistics.[4] As Figure 1 illustrates, the inflation–investment association is highly consistent with the inflation–growth association. This implies that there is a possibility of some channel through which inflation affects investment and then the economic growth.

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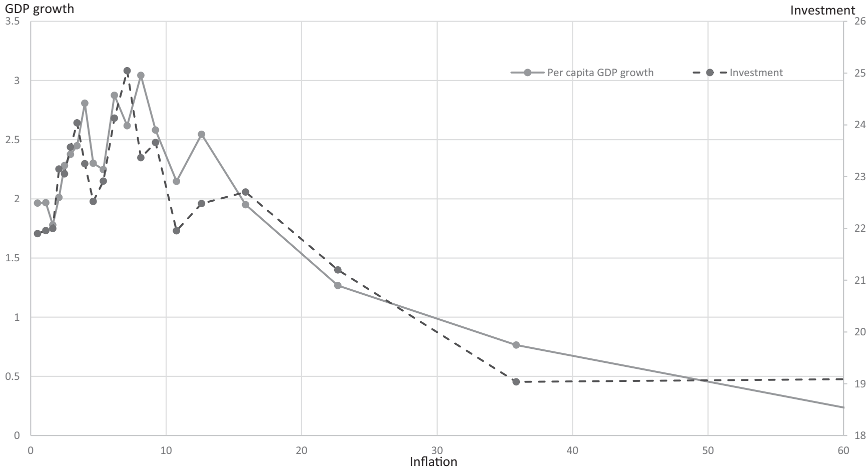

FIGURE 1. Inflation, per capita GDP growth, and investment.

In addition, the key to understanding the mixed findings is to provide a plausible explanation for the positive effect of low inflation. This is because these empirical findings are probably the result of the combination of the positive and negative effects of inflation.

This paper demonstrates a positive channel through which inflation may mitigate the liquidity risks of investment projects, which promotes economic growth. Following the method of Holmstrom and Tirole (1998), I model the liquidity shock to an investment project by the requirement of a nominal additional cost in the middle of production. The nominal additional cost can be understood as the additional circulating funds required to keep the project running. Since goods are perishable, only money can be used as circulating funds. When a project faces a liquidity constraint, the manager has to borrow money outside the firm.[5] Because of moral hazard, the borrowed money has to be secured against the future income of the project. This introduces a credit constraint to the investment project. Under the assumption of zero-interest for intratemporal debt,[6] inflation helps to relax the credit constraint and mitigate the liquidity risk of the project. Therefore, inflation can improve the survival proportion of investment projects and stimulate investment. This in turn boosts economic growth.

The positive channel of inflation introduced in this paper is similar to the liquidity version of bank lending channel suggested by Diamond and Rajan (2006). Both channels are based on the assumption of nominal debt. While Diamond and Rajan (2006) focuses on a banking theory, this paper highlights issues of corporate liquidity. Therefore, the positive channel in this paper can be termed the "corporate liquidity channel" of inflation.

Calibrating a model that includes both the corporate liquidity channel and the conventional "inflation tax" on investment using US postwar data, we have the simulated results reported below. First, there is a hump-shaped relationship between inflation and growth. The result is consistent with the findings of Ghosh and Phillips (1998). Second, the threshold level of inflation is 4%, which is close to the reported values for developed countries by Khan and Senhadji (2001) and Kremer et al. (2013). Third, social welfare also approaches its peak when the inflation rate is around 4%. This implies that the Friedman rule is not optimal in the calibrated model and the model supports the current wisdom about monetary policy with a modest inflation target. Finally, the relationship between inflation and investment is also hump-shaped. It means that there is a "Tobin effect" in the low-inflation region and a "reverse-Tobin effect" when inflation is high. Therefore, no "Fisher effect" exists in the model. The results are consistent with the finding by Ahmed and Rogers (2000), who examine the long-term effects of inflation on the real economy using US data for more than 100 years.

These results can be explained by the following intuition. Inflation mitigates the liquidity risks and improves the survival rate of investment projects so that the expected return of investment increases, which stimulates investment. At the same time, inflation also works as a tax to discourage investment. When the inflation is modest, the positive effect through the corporate liquidity channel can dominate the negative effect from inflation tax on investment. However, with the increase in inflation, the negative effect increases and the positive effect decreases, given the natural upper limit of the survival rate. Finally, the negative effect will dominate the positive effect.

Sensitivity analysis suggests that the degree of financial development and the aggregate liquidity demand explains the mixed empirical findings. As the analysis illustrates, with the development of the financial system, or, with the decrease in liquidity demand, the positive effect of inflation fades and the level of threshold inflation declines. This helps us understand the debate about the effect of low inflation since most empirical research does not consider the implications of financial development and aggregate liquidity demand. Moreover, developing countries, the financial systems of which are undeveloped, usually face worse liquidity shocks as compared to developed countries. Based on the sensitivity analysis, we know that the threshold inflation for developing countries should be higher than that for industrialized countries. This is consistent with the findings of Khan and Senhadji (2001) and Kremer et al. (2013).

This paper connects the literature on inflation and growth with the literature on finance and growth. Levine (1997, 2005) provides a good literature review. As this paper suggests, modest inflation and financial development are substitutes from the perspective of corporate liquidity. Financial development eases external financing constraints and increases the ability of firms to raise more funds against liquidity risks. Similarly, modest inflation can also mitigate liquidity risks through the corporate liquidity channel. Therefore, when considering the effect of inflation on growth, we cannot omit the implication of financial development, and vice versa.

This paper is related to several existing studies on the relationship between inflation (or, money supply) and growth. Ghossoub and Reed (2010) explain why the effects of monetary policy vary across countries under the assumption that liquidity risks of banks depend on the aggregate capital stock. In contrast to their method, this paper highlights the idiosyncratic liquidity risks based on the standard setting of corporate finance. Miao and Xie (2013) discuss economic growth under money illusion, which is modeled by a nonstandard utility function. However, this paper stresses the channel on investment and uses a standard utility function. Wang and Xie (2013) show that money growth has a positive effect in a cash-in-advance (CIA) economy with labor market frictions. Chu and Lai (2013) analyze the effects of inflation in a research and development growth model with elastic labor supply. Unlike their papers, this paper focuses on the effect of inflation on investment and omits the labor market.

The rest of this paper is organized as follows. Section 2 sets up the model that includes both a corporate liquidity channel of inflation and the conventional inflation tax on investment. Section 3 illustrates long-run effects of inflation from the perspective of capital market equilibrium. Section 4 calibrates the model using US postwar data and presents the basic results. Section 5 conducts the sensitivity analysis. Section 6 concludes.

2. THE MODEL

This section sets up a discrete-time endogenous growth model with money. Money demand is introduced by the CIA constraint. Following Holmstrom and Tirole (1998), I model the corporate liquidity demand by requiring nominal additional cost in the middle of production. With the credit constraint and the assumption of zero-interest for intratemporal debt, the nominal liquidity demand opens a corporate liquidity channel for inflation to affect real economy. The conventional inflation tax on investment also exists. More details are provided as follows.

2.1. Firms

An infinite number of identical and independent firms are continuously and evenly distributed in the area of [0,1]. The firms are owned by the households. Each firm operates one investment project that matures and produces in one period. At the beginning of each period, each firm rents capital from households. In the middle of the production, the investment projects face idiosyncratic liquidity risks. The liquidity shocks may force firms to terminate their projects.

Liquidity risks. In the spirit of Holmstrom and Tirole (1998), I model the liquidity shock by the requirement of a nominal additional cost in the middle of production. The assumption of nominal additional cost is consistent with the basic fact that almost all of transactions are denominated in currency. The nominal cost can be understood as the additional circulating funds required to keep the project running. With the assumption of perishable goods, only money can be used as circulating funds in the economy.

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Specifically, in the middle of period t, firm i, that rents K_t^i units of capital at the beginning of the period, has to pay a random nominal cost, L_t^i. To keep the analysis tractable and not lose the generality, we can assume that the nominal cost is proportional to the scale of firm i's initial capital value,[7] i.e.,

$$L_t^i \equiv \underbrace{\rho_t^i}_{\text{liquidity shock}} \underbrace{P_{t-1} K_t^i}_{\text{capital value}}, \tag{1}$$

where ρ_t^i is the liquidity shock and P_{t-1} is the price level at the end of the period $t-1$. The liquidity shock, ρ_t^i, which modeled by the ratio of nominal cost over the initial capital value, follows a log-normal distribution, i.e.,

$$\rho_t^i \sim LN(\mu_\rho^{i,t}, \sigma_\rho). \tag{2}$$

Here,

$$\mu_\rho^{i,t} \equiv \ln\left[\eta E_{t-1}\left(\frac{Y_t^i}{K_t^i}\right)\right], \tag{3}$$

where $0 < \eta \leq 1$.[8]

If firm i can pay the nominal additional cost, then its investment project will continue and produce $A\mathcal{T}_t^{1-\gamma}(K_t^i)^\gamma$ amount of output. Here, A is the technology level, $\mathcal{T}_t$ is the endogenous productivity level at period t, and $0 < \gamma < 1$. If the firm cannot pay the additional cost, its investment project will be terminated and produce nothing. At the end of period t, a project that successfully produces can collect back the nominal payment for the additional cost and repay the creditors, whereas a terminated project cannot. Irrespective of whether a project fails or not, capital does depreciate.

Credit constraint. Moral hazard leads to no reserve hoarding in firms against liquidity risks as owners of firms are afraid that the managers might run away with money. When liquidity shocks occur, the managers of firms have to borrow funds outside of the firm. For convenience and not losing the generality, I assume that a borrower does not need to pay any interest for intratemporal debt.

Moreover, moral hazard[9] also limits the debt capacity. Each borrower has to face a credit constraint. Specifically, the amount of money that a firm is able to borrow cannot exceed μ fraction of its appraised income.[10] The borrowing multiplier, $0 < \mu \leq 1$, measures the magnitude of financial development. Given the ex ante survival probability of firm i's project, $\mathcal{P}_t^i$, the appraised income of firm, i, is equal to $E_{t-1}[\mathcal{P}_t^i P_t A\mathcal{T}_t^{1-\gamma}(K_t^i)^\gamma]$. Therefore, the maximum amount of money that firm i can borrow is given by

$$\bar{D}_t^i \equiv \underbrace{\mu}_{\text{borrowing multiplier}} \underbrace{E_{t-1}\left[\mathcal{P}_t^i P_t A\mathcal{T}_t^{1-\gamma}(K_t^i)^\gamma\right]}_{\text{appraised income}}. \tag{4}$$

Optimal choice of firms. First, we need to characterize the conditions under which firm i will continue its investment project and the conditions under which the firm will terminate its project. With the assumption of zero-interest for intratemporal debt, the cost of borrowing against liquidity shock is zero. Given the credit constraint, when the additional cost L_t^i, is less than the maximum amount of money that firm i can borrow, $\bar{D}_t^i$, the firm is resistant to the liquidity shock and can continue its project. However, when L_t^i is larger than $\bar{D}_t^i$, the firm cannot obtain enough funds against the liquidity shock and has to terminate its project.[11] Therefore, the ex ante survival probability of firm i's investment project, $\mathcal{P}_t^i$, is equal to the probability that L_t^i is not larger than $\bar{D}_t^i$. That is,

$$\begin{aligned}\mathcal{P}_t^i &\equiv \int_0^{\frac{\bar{D}_t^i}{P_{t-1}K_t^i}} f(\rho_t^i)d\rho_t^i \\ &= \frac{1}{2}\left[1+\text{erf}\left(\frac{\ln \frac{\bar{D}_t^i}{P_{t-1}K_t^i} - \mu_\rho^{i,t}}{\sqrt{2}\sigma_\rho}\right)\right],\end{aligned}$$

where $\text{erf}(x) \equiv \frac{2}{\sqrt{\varpi}}\int_0^x e^{-s^2}ds$ is the error function, and ϖ is the circumference ratio. From the above specifications about the liquidity risk and the credit constraint, we obtain that

$$\begin{aligned}\mathcal{P}_t^i &= \frac{1}{2}\left\{1+\text{erf}\left[\frac{\ln \bar{D}_t^i - E_{t-1}(\ln L_t^i)}{\sqrt{2}\sigma_\rho}\right]\right\} \\ &= \frac{1}{2}\left(1+\text{erf}\left\{\frac{\ln\left[\frac{\mu}{\eta}E_{t-1}(1+\pi_t)\right]}{\sqrt{2}\sigma_\rho}\right\}\right) \equiv \mathcal{P}_t.\end{aligned} \tag{5}$$

This means that an increase in the ex ante inflation will raise the value of $\mathcal{P}_t^i$. This is because the increase in expected price level helps to relax the credit constraint so that projects can raise more funds to pay the nominal additional cost. It is also necessary to mention that the ex ante survival probability of any individual project is identical and only depends on the aggregate variables. Hence, we can denote the survival probability by $\mathcal{P}_t$.

Therefore, we can describe the optimization problem of firm i by

$$\max E_{t-1}\left(\Pi_t^i\right) = E_{t-1}\left[P_t\mathcal{P}_t^i A\mathcal{T}_t^{1-\gamma}(K_t^i)^\gamma - P_t r_t K_t^i\right],$$

where Π_t^i is the profit of firm i at time t and r_t denotes the rental rate. The first-order condition is given by

$$\gamma E_{t-1}\left[\mathcal{P}_t^i A\mathcal{T}_t^{1-\gamma}(K_t^i)^{\gamma-1}\right] = r_t. \tag{6}$$

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Each firm solves the identical optimization question and chooses the same K_t^i. At equilibrium,

$$K_t^i = K_t.$$

Here, K_t denotes the aggregate capital stock. Moreover, we also find that the distributions of the liquidity shocks of individual projects are identical at equilibrium.

Endogenous productivity. The investment projects not only produce goods, but also have the positive externality of improving the endogenous productivity level. Following the standard method in the literature on endogenous growth, I assume that the motion of the endogenous productivity follows the following law:

$$\mathcal{T}_{t+1} = \int_0^1 n_t^i \mathcal{T}_t^{1-\gamma} (K_t^i)^\gamma di,$$

where n_t^i is an indicator variable. When firm i's investment project survives, $n_t^i = 1$; when the project fails, $n_t^i = 0$.

Based on the law of large numbers, there are $\mathcal{P}_t$ fraction of investment projects that can survive in period t. Therefore, the law of motion for the endogenous productivity can be rewritten as

$$\mathcal{T}_{t+1} = \mathcal{P}_t \mathcal{T}_t^{1-\gamma} (K_t)^\gamma. \quad \textbf{(7)}$$

2.2. Households

There are an infinite number of independent and identical households in the economy. Each of them wishes to maximize the sum of time discounted utility values

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma},$$

subject to the budget constraint

$$P_t C_t + P_t[K_{t+1} - (1-\delta)K_t] + M_{t+1} = P_t r_t K_t + M_t + X_t + \int_0^1 \Pi_t^i di, \quad \textbf{(8)}$$

and the CIA constraint on consumption and investment

$$P_t\{C_t + \theta[K_{t+1} - (1-\delta)K_t]\} \le M_t + X_t, \quad \textbf{(9)}$$

where $0 < \beta < 1$ is the time discount factor, $\sigma > 0$ is the parameter of preference, C_t denotes the consumption, $0 < \delta < 1$ is the depreciation rate of capital, M_{t+1} is the money holdings at the end of period t, X_t is the monetary injection, and the parameter θ, the value of which is equal to 1, or, 0, indicates whether there is a CIA constraint on investment or not.

I use $\lambda_{1,t}$ and $\lambda_{2,t}$ to denote the Lagrangian multipliers for the budget constraint and the CIA constraint, respectively. The first-order conditions are given as follows:

$$C_t^{-\sigma} = P_t(\lambda_{1,t} + \lambda_{2,t}), \tag{10}$$

$$\lambda_{1,t} = \beta E_t(\lambda_{1,t+1} + \lambda_{2,t+1}), \tag{11}$$

$$\beta E_t P_{t+1}[\lambda_{1,t+1}(r_{t+1} + 1 - \delta) + \theta\lambda_{2,t+1}(1 - \delta)] = P_t(\lambda_{1,t} + \theta\lambda_{2,t}). \tag{12}$$

2.3. Monetary Authority

The monetary authority is passive and only determines the monetary injections. The timing for the injection at period t, X_t, is just after the realizing of liquidity shocks and before managers' borrowing. The increment in money supply is distributed to households as a lump-sum transfer. We define the growth rate of money, g^M_{t+1}, by X_t/M_t.

2.4. Market Clearing

The goods market clearing condition can be written as

$$C_t + K_{t+1} - (1 - \delta)K_t = \mathcal{P}_t A \mathcal{T}_t^{1-\gamma} K_t^{\gamma} \equiv Y_t, \tag{13}$$

where Y_t is the gross output in period t. The money market clearing condition is given by

$$M_{t+1} = M_t + X_t. \tag{14}$$

It is necessary to remember that no money is destroyed within the terminated projects since a failed project never borrows.

In addition, there may be some concern about whether the aggregate liquidity is sufficient or not. Given the credit constraint, the aggregate demand for liquidity is not larger than $\bar{D}_t \equiv \mu P_t \mathcal{P}_t A \mathcal{T}_t^{1-\gamma}(K_t)^{\gamma}$. With the assumption of $0 < \mu < 1$ and the CIA constraint on consumption and investment, we have

$$\bar{D}_t < P_t \mathcal{P}_t A \mathcal{T}_t^{1-\gamma}(K_t)^{\gamma} \leq M_t + X_t. \tag{15}$$

This means that the aggregate liquidity is sufficient. However, the financial frictions in the economy limit the debt capacity. Individual projects may not raise enough funds against their liquidity problem.

3. EFFECTS OF INFLATION ON ECONOMIC GROWTH

This section analyzes the effects of inflation on long-run growth from the perspective of the capital market equilibrium. Using a diagram, I illustrate the effects of the corporate liquidity channel of inflation and the inflation tax on investment. The synthetic effect of these two channels is also discussed.

As Appendix A provides, on the balanced growth path (BGP), the law of motion for endogenous productivity (7) can be written as

$$1 + g = \mathcal{P}k^{\gamma},$$

where g is the rate of economic growth and $k \equiv \frac{K}{T}$. This implies that

$$k = \left(\frac{1+g}{\mathcal{P}}\right)^{\frac{1}{\gamma}}.$$

Substituting this into equation (6), we can obtain the capital demand function on the BGP,

$$\gamma A\left[\frac{1}{2}\left(1+\text{erf}\left\{\frac{\ln[\frac{\mu}{\eta}(1+\pi)]}{\sqrt{2}\sigma_{\rho}}\right\}\right)\right]^{\frac{1}{\gamma}}(1+g)^{\frac{\gamma-1}{\gamma}} = r, \qquad \textbf{(16)}$$

where π is the inflation rate on the BGP. From the intertemporal substitution condition (12), we know that the capital supply function on the BGP is given by

$$\beta\left[(1-\delta) + \frac{r}{\theta(1+\pi)(1+g)^{\sigma}/\beta + (1-\theta)}\right] = (1+g)^{\sigma}. \qquad \textbf{(17)}$$

These two functions determine the equilibrium of the capital market on the BGP.

The following proposition summarizes the effect of inflation on economic growth.

PROPOSITION 1. *Consider an economy with a nonnegative inflation rate and nonnegative economic growth,*

(a) when there is no CIA constraint on investment, i.e., $\theta = 0$, *an increase in inflation rate only has a positive effect on economic growth, which is through the corporate liquidity channel;*

(b) when there is a CIA constraint on investment, i.e., $\theta = 1$, *under the parameter restriction of*

$$\frac{1}{\sqrt{2\pi}\sigma_{\rho}}e^{-\left[\frac{\ln\left(\frac{\mu}{\eta}\right)}{\sqrt{2}\sigma_{\rho}}\right]^2} > \gamma\left\{\frac{1}{2}+\frac{1}{2}\text{erf}\left[\frac{\ln\left(\frac{\mu}{\eta}\right)}{\sqrt{2}\sigma_{\rho}}\right]\right\},$$

there is a hump-shaped relation between inflation rate and economic growth rate: the synthetic effect of inflation rate on economic growth is positive when inflation rate is modest, and negative when inflation rate is high.

Proof. See Appendix B.

We can represent the equilibrium of the capital market on the BGP graphically by Figure 2. The gross growth rate, $1+g$, is measured on the horizontal axis, and the rental rate, r, is measured on the verticalaxis. With the assumption of $0 < \gamma < 1$,

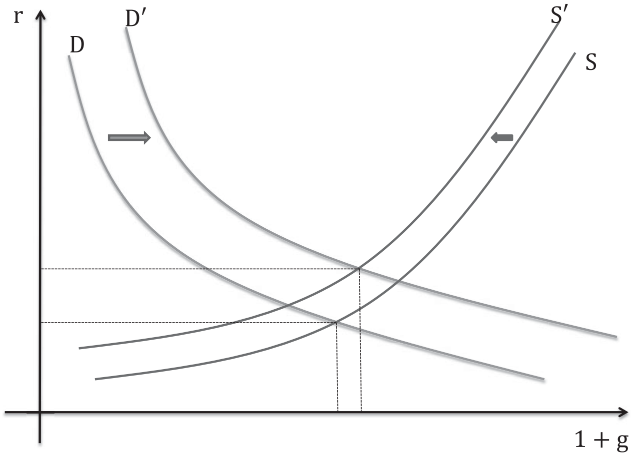

FIGURE 2. Capital market on BGP.

the capital demand curve, D, which is implied by equation (16) is negatively sloped. The capital supply curve, S, which is represented by equation (17), is upward sloping since the values of σ and β are both positive. The intersection of the two curves determines the equilibrium in the capital market.

From the diagram, we can easily determine the effects of inflation on the long-run growth. When $\theta = 0$, there is no CIA constraint on investment. An increase in the inflation rate increases the survival rate of investment projects, which pushes the capital demand curve to the right, from D to D′. The curve S does not shift. The effect of inflation is merely through the corporate liquidity channel. That is, an increase in inflation rate mitigates the liquidity risks of investment projects, which stimulates capital demand. On the other hand, inflation has no effect on capital supply. However, when $\theta = 1$, there is a CIA constraint on investment. In this case, inflation also affects the capital supply by the conventional inflation tax on investment, as argued by Stockman (1981). With an increase in inflation, the capital supply curve shifts to the left, from S to S′. This is because an increase in inflation amplifies the cost of holding money and impedes the capital supply.

As Figure 2 illustrates, in an economy including both the corporate liquidity channel and the inflation tax on investment, it is still possible for an increase in modest inflation to improve the growth rate. This is because the positive effect of inflation through the corporate liquidity channel dominates the negative effect of inflation tax on investment in the low-inflation region. However, with the increase in inflation, the positive effect will diminish given the natural upper limit of the survival probability. Thus, the synthetic effect of inflation can be negative when the inflation rate is high.

In addition, it is also necessary to mention that an increase of μ and a decrease of η have the same effect on the capital demand as the liquidity channel of inflation. This is obvious from equation (16). We can use the following intuition to explain this finding. An increase of μ implies the enhancement of the financial system. It makes raising funds against liquidity shocks easier. A decrease of η means the average liquidity demand is lower and the expected liquidity shock is weaker. Both changes increase the survival probability of projects and stimulate the capital demand.

4. MODEL SIMULATIONS

This section calibrates the model with the US postwar data and presents the simulated long-run relationship between inflation and growth. The effect of inflation on social welfare, the Tobin effect, and the Fisher effect are also examined.

4.1. Calibration

I choose the standard values for the preference and production parameters. The elasticity of intertemporal substitution is taken to be 1. Therefore, the value of parameter, σ, is 1. The time discount factor, β, is set to 0.99. The capital share in the production function, γ, is 1/3. The depreciation rate of capital, δ, is 0.1. I fix the exogenous technology level, A, at 1/2. Moreover, the parameter, θ, is set to 1. Therefore, there is a CIA constraint on investment.

To match the US postwar data, the borrowing multiplier, μ, is chosen to be 0.8 and the growth rate of money, g^M, is set to 7%. The parameter, η, in the distribution function of liquidity shock, approximately measures the ratio of aggregate liquidity demand over the gross output. Since the data for liquidity demand is not available, I have to use the gross liquidity asset as the proxy. The value of η is assigned to be the average ratio of the total liquidity asset over the output, 0.8. The other parameter in the distribution function of liquidity shock, σ_ρ, is set to 1/80.

Given the above-mentioned set of parameters, we determine that the economic growth rate, g, is 3.3%, the inflation rate, π, is 3.58%, the real interest rate, $r - \delta$, is 5.5%, and the ratio of consumption over output, c/y, is 0.714. All of these predicted results are reasonable for the US economy.

4.2. Numerical Results

The solid line in Figure 3 shows the hump-shaped relationship between inflation and economic growth. The effect of inflation on economic growth is positive in the low-inflation region and becomes negative for higher inflation rate. This is consistent with the findings of Ghosh and Phillips (1998). The threshold level of the inflation is 4%, which is close to the reported values for industrial countries by Khan and Senhadji (2001) and Kremer et al. (2013).

Correspondingly, we can also find a hump-shaped relationship between inflation and social welfare. The solid line with the asterisk marker in Figure 3 represents 1/6,000 of the social welfare. This result suggests that the Friedman rule is not optimal in the calibrated model and supports the current monetary policy with a modest inflation target in developed countries.

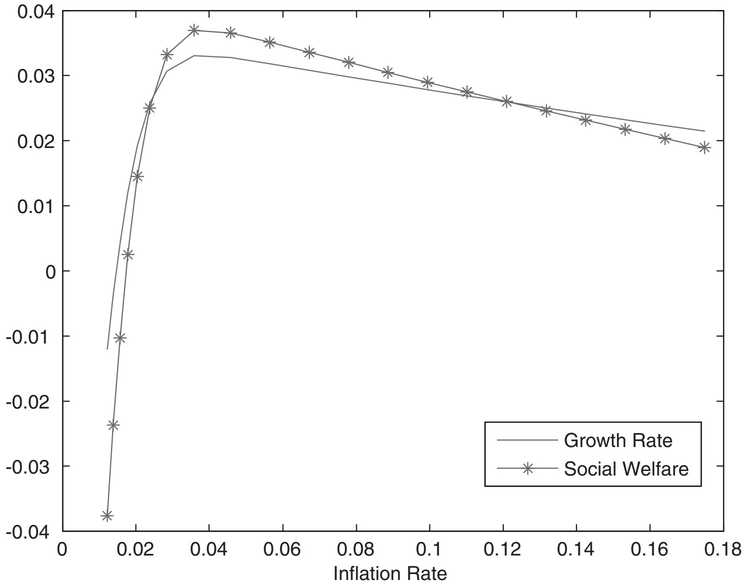

FIGURE 3. Inflation, economic growth, and social welfare.

As Figure 4 illustrates, there is a Tobin effect in the calibrated model but no Fisher effect. The upper right diagram shows that the real interest rate increases with the increase in inflation. The result violates the Fisher effect: the hypothesis that inflation has no effect on the real interest rate. As the two diagrams below illustrate, the Tobin effect exists in the low-inflation region: the effects of modest inflation on output and investment are both positive. It is consistent with the finding by Ahmed and Rogers (2000), who examine the long-term effects of inflation on the real economy using more than 100 years US data. However, when inflation is higher, there is a reverse-Tobin effect, i.e., inflation diminishes output and investment.

The intuition behind these results can be provided from the perspective of the impact of inflation on investment. In an economy with financial frictions, an increase in inflation helps to relax the credit constraint and mitigates the liquidity risks. As the upper left diagram in Figure 4 illustrates, an increase in modest inflation enlarges the survival probability of investment projects. This in turn raises the real interest rate and stimulates investment. When inflation is modest, the positive effect through the corporate liquidity channel dominates the inflation tax on investment from the CIA constraint on investment. Hence, the Tobin effect operates and there is an enhancement of economic growth in the low-inflation region. However, when inflation is higher, the survival probability approaches 1 as time increases infinitely. An increase in inflation has a restricted positive effect on the real interest rate. In this case, the effect of the inflation tax dominates the positive effect. Therefore, we find the reverse-Tobin effect and a decline in economic growth.

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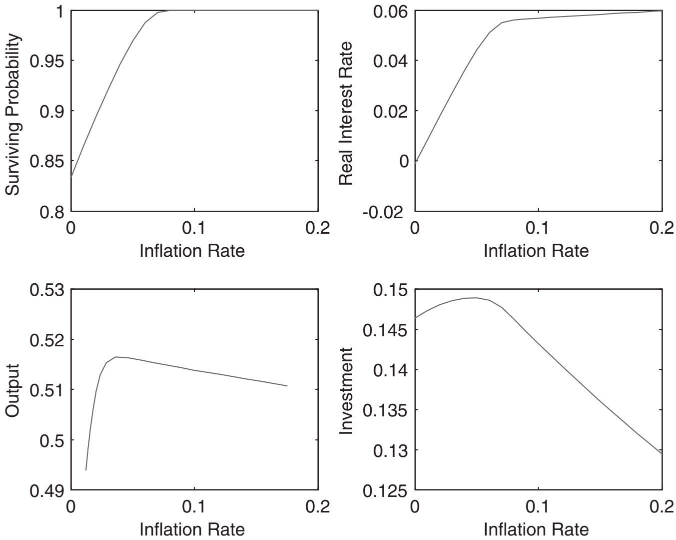

FIGURE 4. The effect of inflation on the real economy.

5. SENSITIVITY ANALYSIS

In this section, I analyze the sensitivity of the long-run relationship between inflation and economic growth to the magnitude of financial development and the average liquidity demand.

5.1. The Degree of Financial Development

As shown in Figure 5, with the increase of μ, from 0.74 to 0.86, the hump-shaped curve that describes the relationship between inflation and growth shifts to left and the positive effect of inflation abates. Since the value of μ indicates the magnitude of financial development, the above-mentioned finding implies that the improvement in the financial system reduces the value of threshold inflation and weakens the positive effect of inflation.

We can explain the result by the following intuition. The development of the financial system reduces financial frictions and makes fund raising easier against liquidity shocks. Similarly, inflation through the corporate liquidity channel can also mitigate liquidity risks by loosening the credit constraint. From this perspective, inflation and financial development are substitutes for each other. With the enhancement of financial development, the positive effects of inflation will fade.

This finding helps to explain why the threshold inflation for developing countries is higher than that for industrial countries. Generally, the financial systems in developing countries are underdeveloped. This leaves more space for the liquidity

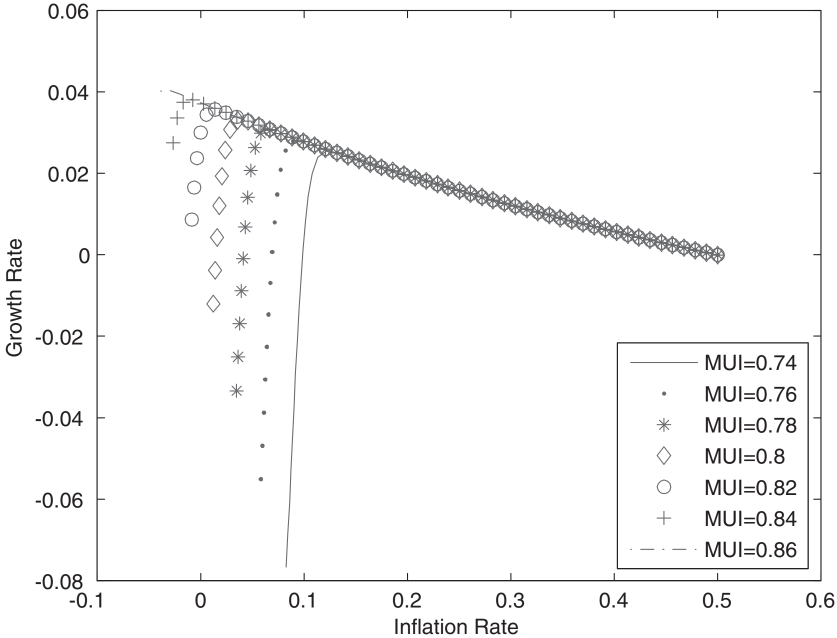

FIGURE 5. Sensitivity analysis on financial development.

channel of inflation to work. Thus, the negative effect of the inflation tax will dominate the positive effect of liquidity channel at a higher inflation rate. Therefore, there is a higher threshold inflation for developing countries. In addition, the analysis also suggests that the different degrees of financial development of individual countries in the same sample are helpful for explaining the mixed findings on the effect of modest inflation. This is because omitting the difference among the degrees of financial development of individual countries will lead to a biased estimation.

5.2. The Liquidity Demand

Figure 6 shows a similar result. With the decrease of η, from 0.86 to 0.74, the positive effect of inflation fades and the threshold inflation declines. The value of η measures the average liquidity demand in an economy. The decrease in the average liquidity demand suggests the attenuation of the liquidity shock. It compresses the space for the corporate liquidity channel of inflation to work. Therefore, the inflation tax would dominate the positive channel at a lower inflation level.

Since developing countries usually face worse liquidity shocks than developed countries, there is ample scope for the corporate liquidity channel of inflation in

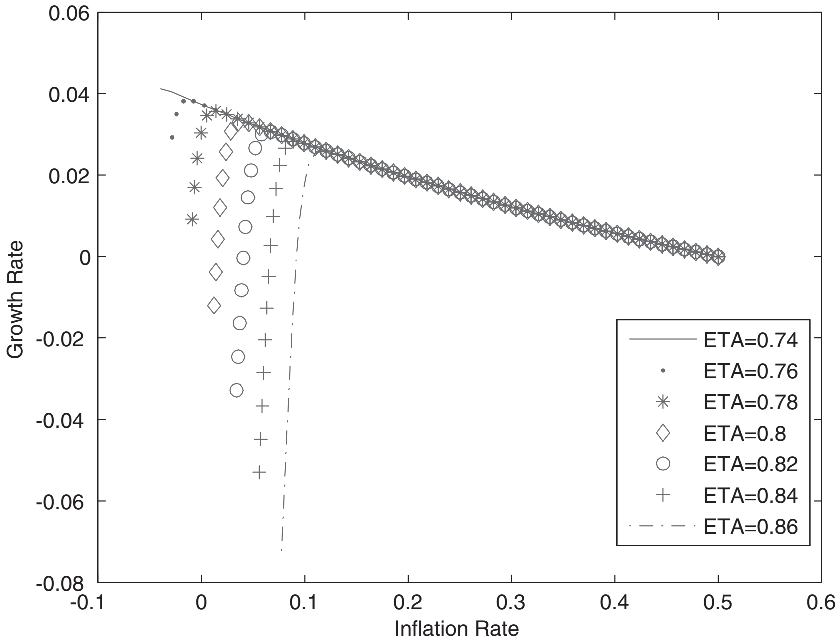

FIGURE 6. Sensitivity analysis on liquidity demand.

developing countries to work. It is helpful to explain the higher threshold inflation in the sample of developing countries. The difference among liquidity demands of individual economies in the same sample is also a suspect for the mixed findings in the low-inflation region.

6. CONCLUSION

This paper demonstrates a corporate liquidity channel through which inflation has a positive effect on the long-run economic growth. Together with the inflation tax on investment, we can obtain a hump-shaped relationship between inflation and growth in a model calibrated using US postwar data. The Tobin effect exists in the low-inflation region. However, when inflation rate is higher, the effect turns to be the reverse-Tobin.

Sensitivity analysis shows that the relationship between inflation and growth depends on the degree of financial development and the aggregate liquidity demand. The different degrees of financial development of individual economies and their different liquidity demands are a potential explanation for the controversial empirical findings. Therefore, in future empirical research on the effect of inflation, we should incorporate indicators of financial development and the liquidity demand as independent variables.

NOTES

1. For example, Sarel (1996) and Khan and Senhadji (2001) suggest that the effect of inflation on economic growth is insignificant, or, even slightly positive when inflation is below the threshold level. On the other hand, Ghosh and Phillips (1998) report that inflation below the threshold level is positively correlated with growth. Kremer et al. (2013) find a positive correlation between inflation and economic growth in the sample of developed countries, but an insignificant correlation in the sample of developing countries when inflation is below the threshold level. In addition, Bullard and Keating (1995) and Ahmed and Rogers (2000) support the conclusion of the positive effect of low inflation on output level.

2. For example, Khan and Senhadji (2001) and Kremer et al. (2013) report that the threshold inflation in the sample of developed countries is lower than that in the sample of developing countries.

3. For example, Gomme (1993) and Gillman and Kejak (2005a). Please refer to Gillman and Kejak (2005b) for a theoretical literature review on the negative effect of inflation.

4. Using the World Economic Outlook Databases from IMF, I perform an exercise similar to the one in Ghosh and Phillips (1995) by plotting the median per capita GDP growth rate and median ratio of investment over GDP, against the median inflation rate for 20 equal-sized subsamples.

5. Moral hazard makes reserve hoarding against the liquidity shock impossible. For example, creditors might worry that managers of projects run away with cash.

6. The main insights in this paper also hold for intertemporal debt with positive interest when considering liquidity effects of money injection. The liquidity effect means that money injection reduces the nominal interest rate at the first stage. This implies no Fisher effect in the short-run. Therefore, inflation still can mitigate the liquidity risks.

7. The main insights of this paper still hold under a general assumption of nominal additional cost. The intuition is simple. Under the assumption of zero-interest for intratemporal debt, an expected inflation will improve firms' fund-raising capacity, which depends on firms' expected revenues. This in turn increases firms' survival probability.

8. $\mu_\rho^{i,t}$ can be a constant. However, it is more plausible that the expected liquidity shock increases with the increase in ex ante average investment return, $E(\frac{Y_t^i}{K_t^i})$. The result for the case of constant μ_ρ^i is very similar to the main result presented in this paper. The relevant analysis is available upon request.

9. Managers might run away with the borrowed money.

10. Firms do not own capital and have no collateral. But, lenders can control some fraction of output. Because output are real goods and not easy to be moved and stolen. Thus, it is nature for lenders to require the loans to be guaranteed by appraised revenue of firms.

11. Actually, the firm would not borrow money since it is helpless to save its project.

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APPENDIX A: DERIVATION OF EQUATIONS ON THE BALANCED GROWTH PATH

Suppose the exogenous technology level is always the constant, A. On the BGP, $c_t \equiv C_t/\mathcal{T}_t, k_t \equiv K_t/\mathcal{T}_t, y_t \equiv Y_t/\mathcal{T}_t, 1+\pi_{t+1} \equiv P_{t+1}/P_t, 1+g_{t+1}^{\mathcal{T}} \equiv \mathcal{T}_{t+1}/\mathcal{T}_t, 1+g_{t+1} \equiv Y_{t+1}/Y_t, \mathcal{P}_t, r_t$, and g_{t+1}^M are all constants.

On the BGP, equation (6) can be written as

$$\gamma \mathcal{P} A k^{\gamma-1} = r,$$

where

$$\mathcal{P} = \frac{1}{2}\left(1+\operatorname{erf}\left\{\frac{\ln[\frac{\mu}{\eta}(1+\pi)]}{\sqrt{2}\sigma_\rho}\right\}\right).$$

Equation (7) implies that

$$1+g^{\mathcal{T}} = \mathcal{P}k^{\gamma}.$$

By the definition of the BGP, we also know that

$$g^{\mathcal{T}} = g.$$

From the goods market clearing condition (13), we have

$$c+(g+\delta)k = \mathcal{P}Ak^{\gamma} = y.$$

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From equations (10) and (11), we obtain

$$\beta E_t \frac{C_{t+1}^{-\sigma}}{P_{t+1}} = \lambda_{1,t},$$

$$\frac{C_t^{-\sigma}}{P_t} - \beta E_t \frac{C_{t+1}^{-\sigma}}{P_{t+1}} = \lambda_{2,t}. \quad \textbf{(A.1)}$$

Thus, equation (12) can be rewritten as

$$\beta E_t P_{t+1} \left\{ \left[\theta \frac{C_{t+1}^{-\sigma}}{P_{t+1}} + (1-\theta)\beta E_{t+1} \frac{C_{t+2}^{-\sigma}}{P_{t+2}} \right] (1-\delta) + \beta E_{t+1} \frac{C_{t+2}^{-\sigma}}{P_{t+2}} r_{t+1} \right\}$$
$$= P_t \left[\theta \frac{C_t^{-\sigma}}{P_t} + (1-\theta)\beta E_t \frac{C_{t+1}^{-\sigma}}{P_{t+1}} \right].$$

On the BGP, we get

$$\beta \left[(1-\delta) + \frac{r}{\theta(1+\pi)(1+g)^\sigma/\beta + (1-\theta)} \right] = (1+g)^\sigma.$$

Given the fact that on the BGP

$$\frac{K_{t+1} - (1-\delta)K_t}{K_t - (1-\delta)K_{t-1}} = 1 + g,$$

a binding CIA constraint (9) implies that

$$(1+\pi)(1+g) = 1 + g^M.$$

However, when the CIA constraint is nonbinding, the value of $\lambda_{2,t}$ is equal to zero. By equation (A.1), we obtain

$$1 + \pi = \beta(1+g)^{-\sigma}.$$

This is the so-called Friedman rule. However, given a positive growth rate, the inflation rate is negative. This is not consistent with our reality. Therefore, this paper focuses on the BGP with a binding CIA constraint.

APPENDIX B: PROOF OF PROPOSITION 1

Combining equations (17) and (16) to eliminate the variable r, we have

$$\beta \left\{ (1-\delta) + \frac{\gamma A \left[\frac{1}{2} \left(1 + \mathrm{erf}\left\{ \frac{\ln[\frac{\mu}{\eta}(1+\pi)]}{\sqrt{2}\sigma_\rho} \right\} \right) \right]^{\frac{1}{\gamma}} (1+g)^{\frac{\gamma-1}{\gamma}}}{\theta(1+\pi)(1+g)^\sigma/\beta + (1-\theta)} \right\} = (1+g)^\sigma. \quad \textbf{(B.1)}$$

When $\theta = 0$, equation (B.1) can be expressed as

$$\beta\gamma A\left[\frac{1}{2}\left(1+\text{erf}\left\{\frac{\ln[\frac{\mu}{\eta}(1+\pi)]}{\sqrt{2}\sigma_\rho}\right\}\right)\right]^{\frac{1}{\gamma}} = (1+g)^{\sigma-\frac{\gamma-1}{\gamma}} - \beta(1-\delta)(1+g)^{-\frac{\gamma-1}{\gamma}}.$$

Thus, we can obtain

$$\frac{\partial\,(1+g)}{\partial\,(1+\pi)} = \frac{\beta A\left[\frac{1}{2}\left(1+\text{erf}\left\{\frac{\ln[\frac{\mu}{\eta}(1+\pi)]}{\sqrt{2}\sigma_\rho}\right\}\right)\right]^{\frac{1}{\gamma}-1}\frac{1}{\sqrt{\pi}}e^{-\left\{\frac{\ln[\frac{\mu}{\eta}(1+\pi)]}{\sqrt{2}\sigma_\rho}\right\}^2}\frac{1}{\sqrt{2}\sigma_\rho(1+\pi)}(1+g)}{\sigma(1+g)^{\sigma-\frac{\gamma-1}{\gamma}} - (\gamma-1)\,\beta A\left[\frac{1}{2}\left(1+\text{erf}\left\{\frac{\ln[\frac{\mu}{\eta}(1+\pi)]}{\sqrt{2}\sigma_\rho}\right\}\right)\right]^{\frac{1}{\gamma}}} > 0.$$

This means that the effect of inflation on economic growth is positive.

When $\theta = 1$, equation (B.1) can be given by

$$\frac{\beta\gamma A\left[\frac{1}{2}\left(1+\text{erf}\left\{\frac{\ln[\frac{\mu}{\eta}(1+\pi)]}{\sqrt{2}\sigma_\rho}\right\}\right)\right]^{\frac{1}{\gamma}}}{1+\pi} = \frac{\frac{1}{\beta}(1+g)^{\sigma} - (1-\delta)}{(1+g)^{\frac{\gamma-1}{\gamma}-\sigma}}.$$

We can find that

$$\frac{\partial\,(1+g)}{\partial\,(1+\pi)} = \frac{\beta A\left(\frac{1}{2}+\frac{1}{2}\text{erf}\left\{\frac{\ln[\frac{\mu}{\eta}(1+\pi)]}{\sqrt{2}\sigma_\rho}\right\}\right)^{\frac{1}{\gamma}-1}\Omega\,(1+\pi)}{(1+\pi)^2\,\Psi\,(1+g)},$$

where

$$\Omega\,(1+\pi) \equiv \frac{1}{\sqrt{2\pi}\sigma_\rho}e^{-\left\{\frac{\ln[\frac{\mu}{\eta}(1+\pi)]}{\sqrt{2}\sigma_\rho}\right\}^2} - \gamma\left(\frac{1}{2}+\frac{1}{2}\text{erf}\left\{\frac{\ln[\frac{\mu}{\eta}(1+\pi)]}{\sqrt{2}\sigma_\rho}\right\}\right),$$

$$\Psi\,(1+g) \equiv \frac{\sigma}{\beta}(1+g)^{2\sigma-1-\frac{\gamma-1}{\gamma}} - \left(\frac{\gamma-1}{\gamma}-\sigma\right)\times\left[\frac{1}{\beta}(1+g)^{\sigma}-(1-\delta)\right](1+g)^{\sigma-1-\frac{\gamma-1}{\gamma}} > 0.$$

It is easy to find that

$$\frac{\partial\Omega\,(1+\pi)}{\partial\,(1+\pi)} = -\left\{\frac{\ln[\frac{\mu}{\eta}(1+\pi)]}{(\sigma_\rho)^2}+\gamma\right\}\frac{1}{\sqrt{2\pi}\sigma_\rho\,(1+\pi)}e^{-\left\{\frac{\ln[\frac{\mu}{\eta}(1+\pi)]}{\sqrt{2}\sigma_\rho}\right\}^2} < 0.$$

Thus, under the parameter restriction of

$$\frac{1}{\sqrt{2\pi}\sigma_\rho}e^{-\left\{\frac{\ln(\frac{\mu}{\eta})}{\sqrt{2}\sigma_\rho}\right\}^2} > \gamma\left\{\frac{1}{2}+\frac{1}{2}\text{erf}\left[\frac{\ln\left(\frac{\mu}{\eta}\right)}{\sqrt{2}\sigma_\rho}\right]\right\}, \quad \frac{\partial\,(1+g)}{\partial\,(1+\pi)} > 0$$

when π is low, and $\frac{\partial(1+g)}{\partial(1+\pi)} < 0$ when π is high. ■

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