7 MÉCANIQUE CÉLESTE(CELESTIAL MECHANICS)

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I INTRODUCTION

In the past three years progress of celestial mechanics research was as rapid as in the previous decade. In fact now new theories of planetary and lunar motions which have nearly the same accuracy as those of the most precise observations have become available after a decade effort and new thories of some of natural satellite motions have been developed by several authors. Many papers on artificial satellite motions from several aspects were still published and more precise expressions for relativistic effects were derived.

Several faint satellites which have very interesting dynamical characters were discovered and very detailed structures of Saturnian rings as well as very narrow rings of Jupiter, Saturn and Uranus were disclosed. Therefore, much more papers than before treating resonance problems for satellites, rings, asteroids and so on, their stabilities and dynamical evolutions were published in the past three years, as there are now more resonance problems to be studied in the solar system. Commensurable relations exist not only among revolution periods but also among revolution and rotation periods. And, therefore, rotational motions of the moon and planets attract many authors to study their dynamical evolutions.

As many high-speed computors have become accessible new types of periodic solutions for three and four body problems were found, some of them being periodic even in the three dimensional space. In fact many papers on periodic solutions were published. Some papers treat systems containing not only point masses but also one or more finite bodies and try to find their particular solutions. Mathematical theories to try to understand qualitative properties of solutions of the equations of motion in celestial mechanics made a steady progress and some important contributions on this subject were published.

In spite of the rapid progress still there are so many problems which have not yet been fully solved in celestial mechanics and it is expected that planetary as well as lunar and satellite motion theories will be further improved.

This report does not cover all the papers published or works done in 1979-1981. Unfortunately, the sections on periodic solutions and on mathematical theories had to be made very short and those on natural and artificial satellite theories had to be dropped because of 12 page limitation of the report of the commission. However, please refer the natural satellite theories to the report of Commission 20.

II PLANETARY THEORY

At the Bureau des Longitudes new planetary theories have been developed by two different approaches(25.042.073), namely, by the method of successive approximation and the iterative method. In both cases solutions are put in quasi-periodic functions of time, namely, they are developed into trigonometric series, for which the arguments are linear combinations of the mean longitudes of the eight planets and the coefficients are numerically expressed by polynomials of time.

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By the method of successive approximation perturbations are developed formally in powers of planetary masses by starting from Keplerian orbits and Lagrange planetary equations. The first-order solutions were obtained and published in 1975. Second-order theories are derived for the major planets and the four inner planets, respectively, by Simon and Bretagnon(22.042.041 and 006) and Bretagnon(27.042.057). The aimed accuracies are 0."001 for the inner planets and 0."01 for the major planets after a century. However, comparisons with numerical integrations show that it is 0."01 for the inner planets. Third-order perturbations have been already derived by Simon and Francou and their paper is in the process of publication. By the iterative method the successive solutions of Lagrange planetary equations are obtained by Fourier series manipulations. This method is applied to the major planets. Six iterations are necessary for some cases, particularly for Jupiter-Saturn interaction calculations. Althogh this method has a disadvantage in computing quasi-resonant terms with satisfactory accuracy, the solutions converge at the level of 0."001 for short-periodic terms and that of 0."01 to 0."1 for long-periodic terms. Comparisons with the numerical integrations by Oesterwinter and Cohen(1972) could determine the orbital constants. The comparison shows that except for very long-period terms in the longitudes of Uranus and Neptune estimated errors in the theories are about 0."3 in the mean longitude and 0."1 in other elements after 1 000 years(Bretagnon, *A & A 101* p342 1981). For the inner planets, on the other hand, the comparisons for 25 years give the following accuracies: 0."005 for Mercury, 0."003 for Venus and the earth and 0."005 for Mars. This represents an improvement of a factor 10 to 100 over the theories by Le Verrier and Newcomb.

A method to construct planetary theory that does not allow for any periodic term with time-dependent amplitude is developed by Duriez(25.042.100). This method is applied to the four outer planets up to the second order of the masses and to the seventh degree of the eccentricities and the inclinations. Comparisons with numerical integrations and the theory by Bretagnon show that the secular mean variations of the angular orbital elements are derived with the accuracy of 0."1 per year. Brumberg(22.042.087) proposes a new method to treat perturbed two-body problem in rectangular coordinates. The method is based on reduction of the variational equations of the two-body problem with arbitrary elements of the Jordan system. Pavlov(26.042.028) derives an expression of the coordinates of a planet through the eccentric anomaly of a disturbing planet. Krasinsky, Pitjeva, Sveshnikov and Sveshnikova develop an analytical theory of the motion of the inner planets and compare them with Venus radar observations. Its brief description is given in 22.091.071. Pitjeva(27.092.001) improves the orbital elements of Mercury necessary for the analytical theory by using radar observations in 1964-1965. Kamel and his colleagues are developing a general planetary theory and study the motions of Uranus and Neptune(25.041.105, 26.042.001 and *Ap & Space Sc 78* p3 1981).

Perturbations by Pluto in the other planet motions are evaluated by Piraux(26.091.017). New methods to expand the disturbing function for Pluto-Neptune interactions are devised by Petrovskaya and Ivanova(22.042.122) and Yuasa and Hori(25.042.074). They claim that the convergency for this case is satisfactory by their method. Mayo(25.098.069) derives analytical expressions for the perturbations of planetary orbits due to a thick constant density asteroid belt.

Lestrade(*A & A 100* p143 1981) derives analytical formulas for relativistic corrections in planetary orbits, which give not only secular perturbations but also periodic terms, one of them having 9km amplitude for Mercury. Anatonacopoulos and Tsoupakis(25.066.245) derive expressions of a second-order post-Newtonian approximation for N-body system and improve the formula for the secular motion of the perihelion. Anatonacopoulos(25.066.246) derives the equations of motion for a test particle near one of the triangular points in the field of a heavy body up to second post-Newtonian approximation. Piragas, Zhdanov, Aleksandrov and Piragas(22.066.077) treat a test particle motion in the centrally symmetric gravitational field of general relativity. They claim that the form of the equations and the main results remain valid in the two-body problem of comparable mass in the post-Newtonian approximation. Hiscock and Lindblom(25.042.135) report that the post-Newtonian secular motions of the pericenters of the innermost satellites of Jupiter and Saturn are largest in the solar system, being many times larger than that of Mercury. Brumberg (26.042.035) derives relativistic ephemeris corrections in radar ranging measurements and astrometric observations of inner planets for the case that the earth and one of the inner planets move along circular orbits on a same plane in the solar gravity field described by the generalized three parametric Schwarzschild metric.

Comparisons between existing theories and numerical integrations are also made by several authors. Comparisons between Newcomb's theory and JPL-ephemerides for the earth-moon system are made by Stumpff(*A & A 101* p52 1981) for 1700-2100. When Newcomb's theory is corrected the residuals are reduced to 0."05 for the 400 years by changing the adopted constants. Kinoshita and Nakai(25.097.048) report that the largest discrepancy between Clemence's theory of Mars and their numerical integrations is 0."054 and reformulate the theory by the same way as Clemence and correct some errors. After that the largest difference is reduced to 0."025 in longitude. Numerical theories of Mars for 1961-1972 and those of the major planets in 1950-2150 are developed by Izvekov(25.097.048 and 27.092.001) and Dolgachev, Domozhilova and Rybakov(25.091.009 and 26.091.028). Izvekov(27.091.002 and 27.093.018) claims that Venus motion is known with the accuracy of 10^{-9} AU for 1961-1972.

At US Naval Observatory planetary ephemerides using the IAU 1976 system of astronomical constants and the equator and the equinox of the FK5(J2000.0) are developed by Kaplan, Pulkkinen, Satoro, Van Flandern and Seidelmann with cooperation of Standish and Williams of JPL and Oesterwinter. They find that there are still some systematic differences between the observations and the ephemerides for some of the outer planets. One of the hypotheses being investigated is existing of another planet beyond Pluto. Efforts are made to express coordinates of planets with Tchebyshev polynomials of time by Rocher(27.098.014), by Chapront and Rocher(27.042.092), by Khotimskaya(27.091.024) and by Doggett.

III LUNAR THEORY

The main incentive for developing a more accurate theory of the motion of the moon is the fact that lunar laser ranging observations have achieved now a few centimeter accuracy and in order to interpret these observations any theory which is able to represent the motion of the center of the mass of the moon with the same accuracy must be in hands. Presently only numerical integrations can provide lunar positions with such a high internal consistency. The last two such integrations available are DE-111 of JPL and ECT-18 of CERGA and the University of Texas. Both have been used for the reduction of the lunar laser ranging data. Therefore, several authors have been trying to improve the solutions of the main problem and the expressions due to the shapes of the moon and the earth. Although the solar perturbations are, of course, much larger than any others, many people have suspected that the planetary perturbations in Brown's theory have many errors and, therefore, the weak point in the existing theories is rather in this part. One of the existing theories which are used for calculating the lunar ephemeris was formulated by Eckert and Bellesheim by the same principle as Brown and is called ELE. Gutzwiller (25.094.076) compares ELE with two new theories, ALE by Deprit and SALE by Henrard. About 200 largest terms in each of the polar coordinates are used for comparisons. With a few exceptions the differences are below 0."001 for the longitude and latitude and 0."000 01 for the parallax. ELE is further improved by Vondrák(25.094.062).

Literal solutions of the main problem is derived by Schmidt(25.042.062 and 27.042.035) with use of a computor manipulation method by the same way as Hill and Brown. Lestrade(28.042.065) applies Laplace's idea to use the true anomaly as the independent variable to the lunar theory. A purely analytical approach to solve the equation is made by expressing the solutions in formal power series of the three parameters, namely the ratio of the solar and lunar mean motions, the eccentricity and the inclination. The difficulty arises from the fact that the expansion converges only very slowly and only modern high-speed computors can scope with the huge developments involved that have to be made at least to an order of 25 to 30 in the ratio and also a very high degree of the eccentricity and the inclination. The formal convergence of such a theory is investigated by Bec-Bosenberger(26.094.019) and Kovalevsky(26.094.033) and it is shown that despite the possible presence of small divisors of order 3 it is possible to gain one order more in the accuracy of the solution with a finite number of iterations. Dong(27.042.088) proposes a method to derive an exact solution of Hill's equation and discusses its convergency and its connection to Floquet solutions.

Brown and Eckert give a numerical value to the ratio of the mean motions and solve the equations of motion. However, it is easier to give their approximate numerical values to all the parameters and then to solve the equations, and in order to adjust the values by fitting the observations and to obtain the solutions corresponding to their adjusted values partial derivatives of the solutions with respect to the parameters are computed. Solutions of this type are derived by Chapront-Touzé(27.094.005) at the Bureau des Longitudes and is called ELP. Henrard at Namur adopts a little different method to derive the solutions. At first he derives a solution of the main problem in an analytical way by giving very good numerical values to the parameters. And then he expands the solutions around the nominal values for the parameters and solves the main problem(25.094.075). Therefore, his parameters, in powers of which the solutions are expanded, are the increments to the nominal values. His solution is called SALE. The two authors, Chapront-Touzé and Henrard, (27.094.042) compare their results with each other. The conclusions are that ELP seems to be more precise while SALE seems to be more complete and precise as far as derivatives with respect to the orbital parameters are concerned. The differences are 0."000 94(200cm) in longitude, 0."00023(45cm) in latitude and 120cm in distance, and, therefore, are much larger than their anticipated errors, particularly for the distance. Kinoshita compares the two theories with his numerical integrations for 13 revolution period and finds that the differences in the lunar distance are 100cm for SALE and 1.2cm for ELP. Even after 20 years the difference is as small as 1.5cm for ELP.

Referring to the planetary perturbations Vondrák(25.094.006) reformulates the expressions by Brown's procedure and finds some mistakes in Brown's formulas. And he derives several small terms which were neglected by Brown and publishes a list of the planetary terms. Standaert(28.091.037) computes analytical expressions of the direct planetary perturbations by Lie method using Henrard's solution(SALE) and Bretagnon's planetary theories. The accuracy intended is 0."001 for terms of period up to 2 000 years. Chapront-Touzé and Chapront(28.094.030) compute both direct and indirect planetary perturbations in the frame of ELP. Differences between their solutions and Brown's are as large as 0."005. Common parts of the two solutions by Standaert and Chapront-Touzé and Chapront are compared with each other and it is found that the discrepancies of the values of the coefficients are smaller than 0."000 2.

The perturbations due to the second harmonics of the geopotential, the nutation and the secular variations of the obliquity are computed by Chapront-Touzé and compare satisfactorily with similar computations by Henrard. The perturbations due to the shape of the moon are also computed with the accuracy of 0."000 01 in longitude and latitude and 5 parts in 10^{11} in distance(28.094.036). Relativistic effects and those due to the tides are also computed at the Bureau des Longitudes and Namur, respectively. ELP, more exactly ELP 2000 which is computed by the astronomical constants at 2000, is compared with the numerical integrations by Williams(LE51) after including all the perturbations and it is found that the maximum discrepancies are 20m in longitude and 12m in distance. However, even with such an accuracy it is

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100 times better than that of the current ephemerides such as ILE used in almanacs. ELP will be introduced in the *Connaissance des Temps*.

IV ROTATIONAL MOTION

New theories of the rotation also have been anticipated for the moon to match with the increased accuracy of the lunar laser ranging observations. In fact Eckhardt(*Moon & Planet 25* p3 1981) revises his theory by using more precise models for the gravity potential, the revolution motion and the interior of the moon. Tables which are based on his theory and are truncated at 0."01, are published. Migus(28.094.041) and Moons(Thesis, 1981) also develop their theories. The largest differences among the three theories over several year interval are 0."25. Yoder (26.094.036) improves the rotation theory by adding the torque exerted by an oblate earth, the effect of which is 0."08 in latitude, and by adopting a non-rigid viscous moon model. Cappallo, King, Counselman and Shapiro(*Moon & Planet 24* p281 1981) integrate numerically the equations for the rotation of the moon with revised values for the parameters after determining the initial conditions by fitting the integrations with lunar laser ranging observations at McDonald Observatory with 28cm rms residuals. The results are compared with the numerical theory by Williams(1975) and the theory by Eckhardt(1981), their rms differences in orientation being 0."03 and 0."2, respectively, after removing constant biases. Markov(27.094.036) derives solutions by applying Poincaré's periodic solution theory and by expressing them with osculating elements of Andoyer. Barkin(25.042.003) discusses the stabilty of the solution for the real rotation motion near the periodic solution according to Cassini's law.

The rotation of Mercury and its stability are dicussed by Burns(25.092.009) by taking into account the solar tidal bulge and the solar torque on the permanent tide. For Venus its dynamical evolution is discussed by Beletskij, Levin and Pogorelov (26.093.143, 27.093.016 and *A Zh 85* p198 and p416 1981) by taking into account the torques by the sun and the earth and Lago and Cazenave(26.093.028) investigate the past evolution of the rotation by taking into account the solar tidal torque and mantle-core coupling and show that a thermally driven atmospheric tidal torque can drive the obliquity from a small value to 180° which corresponds to a stable position. Variations of the rotation rate, the nutation and the precession for Mars are discussed by Borderies and Balmino(25.097.049), by Reasenberg and King(26.097.082), by Borderies, Balmino, Castel and Moynot(27.097.061) and by Borderies(27.097.006). Ward(25.097.003) derives the oscillation motion of the obliquity of Mars by using expressions correct up to the fourth degree of the eccentricity and the inclination and an improved value of the moment of inertia and by a linearized theory predicts that the maximum oscillation amplitude is 13.°6 and the center of the oscillation is 24.°4 in the long-term average.

Zhang(27.107.024) obtains primitive periods of the nine planets and its averaged value for asteroids by assuming that all planetesimals and particles were revolving around the sun in circular orbits. Beletskij(*Cel Mech 23* p371 1981) studies how the rotation rate and the inclination of protoplanet had been changed during the first stage of protoplanet formation. Khentov's analysis(22.042.111) on stability conditions of the rotation of planets and satellites discloses why spin-orbit synchronism has not been realized for most of the celestial bodies. Murdock (22.042.072) and Murdock and Robinson(*Cel Mech 24* p83 1981) study some mathematical aspects of spin-orbit resonances.

Bursa(27.042.041 and 043) derives components of the resulting moment of external gravitational forces caused by a general body and discusses Liouville's equation describing the rotational motion of deformable celestial bodies. It is not intended to include here the rotational motion of the earth.

V DYNAMICAL EVOLUTION AND STABILITY

a) *General Theory and Planets*

Many papers for explaining the present distributions of satellites, asteroids, rings and comets and their dynamical evolutions appeared. For satellites tidal dissipations and planetary encounters may be the most important factors and for comets capture processes by planetary encounters are mostly discussed. Narrow rings which were discovered for the three planets attract much attentions and there are alternative theories for their origins.

Duriez(22.042.010) proves that a secular term of the third order of masses appears in each expression of the semi-major axis of the planets. Barricelli and Aashamar(27.107.023) test by computor simulations whether successive captures followed by planetary fusion could lead to the formation of major planets comparable to Jupiter and Saturn. Yoder(25.041.013) analyzes orbit-orbit and spin-orbit gravitaional resonances using the model of a rigid pendulum subject to a periodic and a constant torques and derives a probability for capture into libration. Hämeen-Antilla and Lukkari(26.042.038) show that the decrease of the orbital inclination of a planetesimal stops when its rms distance from the equatorial plane is twice its radius. Zhou(26.042.015) discusses the two cases of the three-body problem, namely sun-Jupiter-Saturn and sun-Neptune-Pluto cases, and computes the regions of the variations of their orbital inclinations. Carussi and Pozzi(22.042.082 and 083) develop a new method for close encounter computations and investigate close encounters between Jupiter and 3 000 fictitious minor bodies.

b) *Moon and Satellites*

Szebehely and Evans(27.094.006) study a possibility of the lunar capture by assuming that the mass of the sun had been decreased in an early stage and conclude that 38% mass decrease is necessary. Mignard(25.094.067 and 28.094.018) derives a simple equation for studying the tidal evolution of the lunar orbit, discusses qualitative properties of the solutions, integrates it numerically and shows how the inclination and the eccentricity had changed during the close approach to the earth. Lambeck and Pullan(27.094.060) rediscuss the time scale of the lunar evolution as a function of the shape of the moon. Grjebine and Marchal(27.094.030 and 031) argue in favor of a fission hypothesis of the origin of the moon and suggest that the moon stayed for a long interval of time in a geostationary orbit.

Many people believe that Phobos and Deimos, the Martian satellites, were carbonaceous asteroids captured by Mars. Hunten(25.097.007) suggest that the capture took place due to the drag of an extended proto-atmosphere and Van Flandern(26.097.180) adds that collisional and tidal forces could evolve the captured satellites to their present orbits in a sufficiently short time. Lambeck(26.097.031) investigates tidal evolution of the satellites and shows that the tides raised by the planet on them have significant consequences. Cazenave, Dobrovolskie and Lago(27.097.005 and *Icarus 44* p218 1981) show that the Martian satellites could have been captured in Mars's orbital plane and later evolved to their present planes as the tidal effects moved the orbits. Mignard(*MNRAS 194* p365 1981) discusses the tidal evolution of the Martian satellites for a frequency dependent model of the tidal lag for the planet and the satellites and computes the solutions by starting from the presently observed secular accelerations.

Many people are convinced that the tidal dissipation is an important factor for the evolution of Galilean satellites of Jupiter. Peale, Cassen and Reynolds (25.099.003) try to explain the volcanic activities on the surface of Io by the fact that the dissipation of the tidal energy in it is likely to have melted a major fraction of the mass. Yoder(25.097.047) makes a similar argument and discusses how the tidal dissipation in Io and Jupiter controls the resonant configuration among the three inner satellites. Cassen, Reynolds and Peale(*Geophys Res L*

6 p731 and *7* p987 1980 and *Icarus 41* p232 1980) argue that it is possible for the tidal dissipation in the crust on Europa to have preserved a liquid water and report that the tidal dissipation could not have been important for Ganymede for more than 10^8 years and it was never important for Callisto for the formation of the surface. Greenberg(*Icarus 46* p415 1981) treats the tidal evolution of the Galilean satellites in a linearized nine dimensional system of equations, studies small variations around its equilibrium solutions and derives a conclusion in favor of his deep resonance origin hypothesis. Greenberg(in *Satellites of Jupiter* 1981) shows that the orbital motions of the Galilean satellites exert dramatic control over their physical properties through the tidal heating and in turn the tidal dissipation in the satellites and Jupiter has governed the evolution of the orbits and the Laplace resonance relation. Wiesel(*AJ 86* p611 1981) discusses the problem of the creation and subsequent evolution of the great resonance of the three Galilean satellites by using a periodic solution and shows by numerical integrations that the resonant state can be directly entered under the influence of the tidal forces without prior formation of individual 2:1 resonance pairs.

Peale, Cassen and Reynolds(28.100.018) argue how eccentricities of Saturnian satellites were decreased by the tidal dissipation and estimate the rigidities and the dissipation function. Peale(22.100.512) discusses that finding Hyperion rotating in the 3:2 spin-orbit resonance like Mercury would imply a primordial origin for the Titan-Hyperion resonance. Bevilacqua, Mench, Milani, Nobili and Farinella (27.100.048) study the resonant case of Titan-Hyperion by numerical integrations, find invariant curves corresponding to low and high eccentricity resonance lockings and show that the observed libration of Hyperion's pericenter lies inside the stable high eccentricity region. Blitzer and Anderson(*Cel Mech 29* p65 1981) investigate a theory of satellite orbit-orbit resonance which is applicable to Titan-Hyperion and Mimas-Tethys pairs. Dermott and Murray(*Icarus* 1981) discuss the properties of the tadpole and horseshoe orbits of the restricted problem of three bodies using analytical and numerical methods, determine the circumstances in which the horseshoe paths rather than the others are expected, and apply their results to the recently discovered co-orbital satellites of Saturn which are shown to be librating in horseshoe orbits. Dermott(25.091.006) computes the present rate of the energy dissipation for Jupiter and Saturn under the assumption that the orbital resonances of their satellites are the results of the orbital evolution due to the tidal dissipation and mentions that approximately the same value is needed for the dissipation factor for all the major planets for such evolutions in spite of the great difference of the energy dissipation rate by the factor greater than 10^4.

Harrington and Van Flandern(25.101.025) show how Pluto and the satellites of Neptune have been originated from a single encounter of Neptune with a massive body. Dormand and Woolfson(28.101.002) argue that Pluto was ejected from Neptune system by an encounter with Triton. Mignard(*A & A 96* p*L*1 1981) favors the idea that Charon was separated from Pluto by a fission process when it was ejected from the Neptune system as Charon is likely tidally locked and the total angular momentum of the system is similar to the one required for a rotational break-up of a fluid body.

Several mathematical papers on the evolution also appeared. Szebehely and McKenzie(22.052.034) compare the results obtained by three methods for stability on satellite motions and derive the simplest formula for the most conservative condition. Dovrak and Marchal(22.042.048) make a simple qualitative analysis of the perturbations on a satellite to derive a lower bound for the duration of escape or capture of a satellite and conclude that it rules out any escape for at least 21 centuries for any satellite. Cline(25.042.084) makes a two-body patched conic analysis for a planetary capture of satellites, in which a gravity assist by a satellite aids in capture. Tanikawa(25.099.058) considers a slow capture process, in which particles approaching the planet lose their energies gradually and fall down into stable orbits around it.

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c) *Rings*

Arguments how Uranian rings were formed are made in several papers. Steigmann (22.101.002 and 25.101.002) suggests that the radii of the α and γ rings of Uranus and perhaps ε 1 and 2 are governed by the resonance with Miranda and Ariel, however, β and δ rings might be associated with an undiscovered satellite with mass equal to 0.4 Miranda's mass. Goldreich and Tremaine(25.101.001 and 26.101.024) suggest that inter-particle collision, radiation drag and differential precession tend to disrupt the rings of Uranus and propose that the rings are confined by gravitational torques from a series of small satellites that orbit within the ring. They suggest that the apse alignment is maintained by the self-gravity. Dermott, Gold and Sinclair(26.101.001) suggest that orbital resonances are involved for the formation of extremely narrow widths and sharply defined edges of Uranian rings and suppose that each ring contains a small satellite which maintains particles in horseshoe orbits around the triangular points. Dermott, Gold and Sinclair(27.099.006) also make a similar argument for narrow rings of Jupiter and Saturn and attempt to account for the origin and the location of the rings.

Dermott and Murray(28.101.035) criticize the argument that the apse alignment of the eccentric ε ring is maintained by the self-gravity alone, consider that it is the close packing of the particles near the pericenters which prevents differential precession and describe how differential precession, particle collisions and self-gravitation together can transform a narrow eccentric ring of uniform width into a ring with a large, positive eccentric gradient. Dermott(*Nature 290* p454 1981) explains the braided appearance of the F-ring of Saturn by an excited wave pattern of equally spaced loops which co-rotate with one of the shepherding satellites of the ring which has one first-order resonance with the ring. Zhou and Zheng(28.042.005) make numerical simulations for a system of colliding bodies to explain a formation of rings.

Cuzzi, Burns, Durisen and Hamil(*Nature 281* p202 1979 and 26.100.034) discuss the vertical structure and thickness of Saturnian rings and describe how solar and satellite perturbations do no significantly affect the vertical thickness but do affect the tilt of the mean ring plane. Hénon(*Nature 293* p33 1981) describes a very simple model of Saturnian rings based on the assumption that the size distribution of particles in the rings is uniform with no preferred value.

d) *Asteroids*

Yoder(26.098.073) points out that the tightly bound population of Trojan asteroids has secularly evolved from less to more tightly bound orbit configuration through some mechanisms including the changes of the Jovian mass or semi-major axis during planetary formation and collisional interactions with external bodies. Bien (22.042.011, 27.042.004 and 27.098.074) derives a solution for Trojans as an example of planar elliptic three-body problem and follows orbits of 18 Trojans with small inclinations and also those with high inclinations. Garfinkel(22.042.075 and 28.042.040) constructs a formal long-periodic solution for Trojan asteroids in the restricted three-body problem and discusses its properties. Érdi(22.042.061, 26.098.004 and *Cel Mech 24* p377 1981) considers the motion of Trojan asteroids using a three-variable expansion method for the elliptical three-body problem, derives an asymptotic solution, conditions for the libration of the perihelion and the periods of variations of the eccentricity and finds that for 20 of 30 cases the perihelion longitudes circulate and for the others they librate. Then Érdi and Presler(28.042.171) test the theory with numerical integrations and find that the periodicity of the eccentricity is about 3 600 years.

Froeschlé and Scholl(25.098.004) make numerical integrations over 10^5 years of fictitious asteroids in the region of 3.6 to 3.9AU and show that a partial depletion of an initially uniform distribution is possible by close encounters with Jupiter. However, Franklin(26.098.071) reports that the truncation outward from 3.4AU of asteroids cannot be strictly the results of perturbations of major planets even over

10^9 years and sets the limit of 0.081 for Jupiter's eccentricity for the stable motions of outer asteroids.

Dermott and Murray(*Nature 290* p664 1981) apply statistical techniques to asteroid orbital data and find that the eccentricity and the inclination increase away from the gaps. They argue that the process responsible for the formation of the gaps has removed those objects near the resonances as there is no significant tendency for low-magnitude objects near the gaps which rejects the collisonal hypothesis of the origin. Froeschlé and Scholl(25.098.061) report that the resonances generally tend to enhance eccentricities and inclinations. Heppenheimer(22.107.014 and 28.042.142) derives conditions for growth of planetesimals in the presence of third-body perturbations and proposes that the gaps are primordial and correspond to region where asteroids failed to form by creation. Gulak(27.091.001) considers that commensurability is a result of dynamical relaxations of spatially restricted mechanical systems with an attracting non-point center. Zhuravlev(27.098.022) concludes by numerical integrations that an observed asymmetry of the gaps relative to the exact commensurability is due to a resonant interaction of asteroids with Jupiter.

Arazov and Gaibov(26.097.005) construct an intermediate orbit for resonant asteroids on the basis of a solution of the internal variant of the generalized three fixed center problem and apply it to 2:1 case. Franklin, Lecar, Lin and Papaloizou (27.098.135) study numerically and analytically the conditions for the truncation at the 2:1 resonance of a disk of infrequently colliding particles surrounding the primary of a binary system and conclude that the truncation and the gaps were produced only if the eccentricity is less than some critical value around 0.08. Dirkis (22.098.009) studies the motion of asteroids near 2:1 resonance by numerical integrations over 2 000 years. Simonenko, Sherbaum and Kruchinenko(26.098.072 and 26.042.046) study the orbital evolution for asteroids near 3:1 resonance by a model calculation for 500 years and derive the condition for libration. Karminskij(26.098.050) studies the real asteroids with 3:1 mean motions.

Danielsson(22.098.042) computes for 1 200 years Toro's orbit which is in resonance with the earth and Venus. Scholl(26.098.074) integrates numerically Chiron's orbit from 6 000BC to 18 000AD and supports the conjecture that the dynamical evolution of Chiron is similar to those of short-periodic comets. Heppenheimer(27.107.003) treats a mechanism for the origin of the eccentricites of asteroids and that of Mars by secular resonance associated with the dissipation of a primitive solar nebula. Williams and Faulkner(*Icarus 46* p390 1981) derive positions of secular resonance surfaces as a function of proper semi-major axis, eccentricity and inclination. Kozai(27.098.003) lists the names of the numbered asteroids for which the eccentricities and the inclinations are changed very much by the secular perturbations. Gradie, Chapman and Williams(27.098.038) and Kozai(27.098.037) restudy the families of asteroids. Simovljevitch(26.042.034) introduces the concept of regular proximity of two asteroids and Lazović lays out a simple method to determine approximate true anomalies of proximity of two elliptic orbits for small minimum distance case(22.042.113).

e) *Comets*

Yabushita(25.102.004) studies the effect of planetary perturbations on long-periodic comets and shows how the distribution of the binding energies of comets varies with time. Nakamura(*Icarus 45* p529 1981) computes the orbital evolution of long-periodic comets to short-periodic ones for 16 representative initial orbits and finds that survival rates of the initial orbits with high inclinations and small perihelion distances are only two or three times smaller than those of the main source orbits. Tomanov(28.102.004) by considering interaction of parabolic comets with Jupiter shows that short-periodic comets must have only direct motions and the deficiency of comets with the semi-major axes of 20 to 30AU is explained by the small probability of capture. Rickman and Froeschlé(26.102.031, 27.102.027 and 008) introduce a fast method to study the orbital evolution of active comets in the inner planetary region and conclude that for each active Mars-crossing comet there are 50 distinct comets on similar orbits. Froeschlé and Rickman(*Icarus 46* p400 1981) derive statistical distributions of Jovian perturbations on short-periodic comets by making numerical integrations with sample of 60 000 comets with low inclinations, perihelion distances between 0 and 7AU and aphelion between 4 and 13AU.

Carussi and Valsecchi(*A & A 94* p226 1981) make a numerical research on dynamics of close approaches of short-periodic comets with Jupiter and confirms that in several cases objects can be captured by the planet as temporary satellites. Carussi, Kresák and Valsecchi(*A & A 99* p262 1981) make a numerical computation of a chain of 80 objects placed along an arc of the pre-encounter orbit of P/Oterma and show that close encounters produce a broad variety of jovicentric and heliocentric orbits including temporary captures by Jupiter over 100 years. Rickman and Malmort(*A & A 102* p165 1981) discuss a possibility of temporary capture of P/Gehrels 3 by Jupiter. Vsekhsvyatskij and Guliev(*A Zh 58* p630 1981) estimate that there are comets which were escaped from the system of Uranus.

VI SOLUTIONS AND THEIR PROPERTIES OF DYNAMICAL SYSTEMS

a) *Periodic Solutions of Three-Body Problem*

Many new families of periodic solutions of the three-body problem are found theoretically and numerically by extending the known solutions usually from bifurcation orbits which are also derived. The extension is made from planar restricted problem to three-dimensional and to the general problems.

Families of periodic solutions for the sun-Jupiter case are followed by this way. Kazantzis(22.042.102) first investigates basic families of plane symmetric orbits for restricted problems and studies their horizontal and vertical stabilities and Kazantzis and Zagouras(25.042.106) investigate numerically the bifurcation orbits. Then Kazantzis (25.042.051 and 053, 26.042.040 and 27.042.071) finds new families of three-dimensional periodic orbits with simple and double symmetries of restricted and general problems starting from vertical critical orbits. Zagouras and Kazantzis (25.042.052) derive three-dimensional periodic oscillations about collinear equilibrium points. Robin and Markellos(27.042.046) derive three-dimensional satellite periodic orbits by a similar way. Kasperczuk(26.042.063) and Bien(27.042.034) treat periodic solutions for the sun-Jupiter case.

Michalodimitrakis(25.042.058 and *Ap & Space Sc 78* p27 1981) investigates doubly symmetric vertical critical periodic orbits of Copenhagen problem and finds new families of three-dimensional orbits for the general problem. Ichtiaroglou, Katopodis and Michalodimitrakis(22.042.078,28.042.049 and 066) extend periodic orbits of Copenhagen problem, to restricted planar and then to general problems. Michalodimitrakis (25.042.061, 26.042.039 and 27.042.003) and Itchtiaroglou(27.042.002 and 017 and 28.042.032) derive mono- and bi-parametric families of symmetric three-dimensional periodic orbits and Markellos(22.042.004 and 27.042.038) computes three-dimensional periodic orbits which contain asymmetric ones for restricted and general problems. Also Katopodis(25.042.015) extends periodic solutions in the three-dimension from restricted to general problems. Ichtiaroglou(27.042.002) proves the existence of families of vertical critical periodic orbits of the general planar problem. Robin(*Cel Mech 23* p97 1981) discusses bifurcation of plane with three-dimensional periodic orbits in the elliptic restricted problem and Markellos(*Cel Mech 25* p195 1981) treats the same problem for the general problem. Belbruno(*Cel Mech 25* p195 1981) studies a new family of three-dimensional periodic orbits for the restricted problem which continue off from a consecutive collision orbit. Brjuno(22.042.057 and 058) derives periodic solutions for the restricted problem with consecutive collisions. Markellos and Taylor(22.042.085) derive asymmetric periodic and asymptotic orbits for the restricted problem. Marchal(25.042.085) investigates periodic solutions with very long periods for the general problem.

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Henrard(27.042.030) and Message(27.042.027) prove the existence of Poincaré's periodic orbits of second species and sort, respectively, in the general problem. Ishwar(25.042.001) treats periodic solutions of second genus in the plane restricted problem. A conjecture of Poincaré on the density of periodic orbits of the restricted problem is studied by Gómez and Llibre(*Cel Mech 24* p335 1981). Periodic orbits of Hill's problem in its more general cases are treated by Breakwell and Brown(26.042.038), Michalodimitrakis(27.042.018), Ichtiaroglou(28.042.064 and *A & A 98* p401 1981) and Latyshev(28.042.045). Hénon is studying the evolution of the periodic orbits in the restricted problem when the ratio of the masses tends to zero. Sharma (*Ap & Space Sc 76* p255 1981) derives periodic orbits for the case that the more massive primary is an oblate spheroid for the restricted problem.

c) *Equilibrium Points of Three-Body Problem*

Stabilities of equilibrium points of the three-body problem and periodic orbits around them are also discussed in several papers. Duboshin(25.042.060) investigates solutions of Lagrange and Euler in the general problem in absolute coordinates. The stability of the triangular points for the elliptic restricted problem is discussed by Ivanov, Karimov and Sokol'skij(28.042.016), Meire(28.042.021 and *Cel Mech 23* p89 1981) and motion near the points is discussed by Cheng(25.042.014). The stability of the point for the circular restricted problem is treated by Sokol'skij(25.042.054). Ivanov(26.042.024) discusses the stability in non-restricted problem and McKenzie and Szebehely(*Cel Mech 23* p223 1981) investigate non-linear stability around the point. Mittleman(27.042.070) treats motions about this point for the restricted problem. Bhatnagar and Hallan(22.042.059 and 26.042.006) investigate effects of the perturbations in Coriolis and centrifugal forces on the stability in the restricted problem. Szebehely and McKenzie(*Cel Mech 23* p131 1981) study deformation of a line element in the phase space at the point. Van Velsen(*Cel Mech 23* p383 1981) studies isoenergetic families of quasi-periodic solutions near the point and Broucke(25.042.040) discusses isosceles triangular configurations in the planar general problem.

Puel(26.042.007), Broucke and Walker(27.042.029), Broucke, Anderson, Blitzer, Davoust and Lass (*Cel Mech 24* p63 1981) and Richardson (28.042.036 and 038) study rectlinear problems of the three-body system, periodic orbits about the collinear equilibrium points, rectlinear isosceles orbits and related topics.

d) *Triple Collisions in the Three-Body Problem and Systems Including Finite Bodies*

Losco(22.042.030), Waldvogel(25.042.027), Irigoyen(26.042.030 and 031 and 28.042.001), Marchal and Losco(27.042.040), Simó(27.042.023) and Eschbach(27.042.039) investigate triple collisions in the three-body problem by deriving new systems of equations, studying triple collision manifolds and finding orbits near the collision including triparabolic escape orbits.

Duboshin(22.042.036 and 28.042.018), Troitskaya(27.042.013), Kondurar' and Gamarnik(27.042.059), Vidyakin(27.042.016) and Ipatov (28.042.075) study translatory-rotational motions of three or two rigid bodies in the three-body problem. Ehl'-Sharburi(22.042.110), Jezewski and Donaldson(25.042.016), Sidlichovsky(28.042.020 and 037) and Vidyakin(28.042.047) study translatory-rotational motions of two rigid triaxial or axisymmetric bodies. Stellmacher(26.042.154 and *Cel Mech 23* p145 1981) studies periodic orbits around an oblate spheroid. Barkin(25.042.056), Barkin and El-Sharburi(25.042.006) and Abul'naga and Barkin(26.042.002) investigate motions of a rigid body in the attraction of a sphere.

e) *Several Aspects of Three-Body Problem*

Stability for satellite case is investigated by Williams(25.042.082), Markellos and Szebehely(*Cel Mech 23* p269 1981), Markellos and Roy(*Cel Mech 24* p183 1981) and Marchal and Bozis. Chen, Sun and Luo(22.042.092) and Sun and Luo(28.042.004) analyze the range of the orbital inclinations with respect to the invariable plane for the general problem. Nezhinskij(27.042.080) and Hulkower(27.042.024) study central configurations. Kammeyer(28.042.041) studies linearized mapping associated with the planar problem near a periodic orbit. Chen(22.042.001) discusses the topology of the manifold of the general problem and Nahon(25.042.067) studies collision orbits.

Luk'yanov(22.042.050), Ziglin(22.042.121), Aksenov(25.042.045 and 055), Delmas (25.042.053), Matas(25.042.010), Timoshkova(25.042.007), Delva and Dvorak(26.042.012), Vrcelj(26.042.010), Innanen(27.042.005), Radzievski(27.042.058), Veres(27.042.008), Sokolov and Kholshevnikov(27.042.006 and 069), Waldvogel(27.042.036), Hitzel and Levinson(28.042.007), Veres(28.042.046), Wisdom(28.042.002), Zhuravlev(28.042.014), Degraeve and Pascal(*Cel Mech 24* p53 1981), Gónczi and Froeschlé(*Cel Mech 25* p271 1981), Langebarte(*Ap & Space Sc 75* p437 1981), Pascal(*Cel Mech 24* p53 1981) and many other people investigate several aspects of the three-body problem.

f) *Four and Many Body Problems*

The four body problem is investigated by Simó(22.042.062) for relative equilibrium solution, by Hadjidemetriou(27.042.028) for the motion of a small body under the actions of the sun, Jupiter and Saturn, by Nash and Monaghan(28.042.006) for statistical study of disruptions, by Hadjidemetriou and Michalodimitrakis(*A & A 93* p204 1981) for periodic planetary type orbits, by Michalodimitrakis(*Ap & Space Sc 75* p289 1981) for circular restricted problem, by Zhuravlev and Anikovsky(*Cel Mech 24* p237 1981) for expansion of the disturbing function and by Boigey(Thesis, 1981) for geometrical demonstration of the elimination of the nodes.

N-body problem is investigated by Luk'yanov(22.042.109 and 25.042.046) for symmetric solutions, by Michalodimitrakis(22.042.070 and 077) for the three-dimensional periodic solutions, by Arenstorf(22.042.035) for periodic solutions in rotating coordinates, by Lukkari(22.042.069) and Zheng and Zhou(28.042.005) for numerical simulations of colliding particles, by Marchal(*Acta Astronautica 6* p123 and p1159 and *7* p555 1979) for close approaches and fundamental stability, by Devaney(25.042.083) for specific homothetic collinear solution, by Hadjidemetriou(25.042.064) for planetary type periodic orbits, by Babadzanjanz(26.042.004) for continuation of solutions, by Stellmacher(26.042.044) for the problem of *N* equal masses at each vertex of a regular polygon, by Saari(27.042.021) for central configurations with collision orbits, by Walker, Emslie and Roy(28.042.058) and Walker and Roy(*Cel Mech 24* p227 1981) for stability criteria, by Meyer(*Cel Mech 23* p69 1981) for periodic orbits near infinity, by Smith(*MNRAS 195* p35 1981) for the mean relaxation time of isolated equal mass points, by Carlberg and Hartwick(*AJ 86* p1410 1981) for a dissipationless collapse and by Marchal for possibility of regularizing singularities.

g) *Other Dynamical Systems*

Contopoulos and Michaelides(28.042.059) explain why the characteristics of triple periodic orbits do not bifurcate from the central characteristic. Michaelides(28.151.056) and Contopoulos and Zikides(28.042.033) find that a family of periodic orbits in a 1:1 resonance dynamical system has an infinity of transformations from stability to instability and vice versa. Contopoulos(*Let Nuo Cimento 30* p498 1981) finds systems which do not have universal bifurcation ratio of the conservative systems. Contopoulos(26.042.061 and *Cel Mech 24* p355 1981) finds that near 4:1 or more generally $2n$:1 resonance the central characteristic is broken into two independent families. Contopoulos, Giogilli and Galgani(26.042.064) find the evidence that there are only two quasi-isolating integrals in a system of three degrees of freedom. Then Contopoulos and Zikides(28.042.033) study periodic orbits and ergodic components of resonant dynamical system. Kazantzis(*Ap & Space Sc 78* p27 1981) make numerical integrations of symmetric and asymmetric solutions for a dynamical system. Sun and Froeschlé(*Acta Astronomica Sinica 22* p159 1981) study dependence of Kolmogorov entropy of mappings on coordinate systems.Magnenat(26.042.013) examines asymptotic orbits and instability zone in dynamical systems. Losco(*Cel Mech 25* p159 1981) studies the stability problem with analogy with triple collision.

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