Robotica (2011) volume 29, pp. 959–965. © Cambridge University Press 2011
doi:10.1017/S0263574711000129

A force analysis of a 3-RPS parallel mechanism by using screw theory

Y. Zhao, J. F. Liu and Z. Huang*

Robotics Research Center, Yanshan University, Qinhuangdao, 066004, P.R. China

(Received in Final Form: Febuary 3, 2011. First published online: March 23, 2011)

SUMMARY

The force analysis of parallel manipulators is one of the important issues for mechanical design and control, but it is quite difficult often because of the excessive unknowns. A new approach using screw theory for a 3-RPS parallel mechanism is proposed in this paper. It is able to markedly reduce the number of unknowns and even make the number of simultaneous equations to solve not more than six each time, which may be called force decoupling. With this method, first the main-pair reactions need to be solved for, and then, the active forces and constraint reactions of all other kinematic pairs can be simultaneously obtained by analyzing the equilibrium of each body one by one. Finally, a numerical example and a discussion are given.

KEYWORDS: Parallel mechanism; Screw theory; Force analysis; Active force; Constraint reaction.

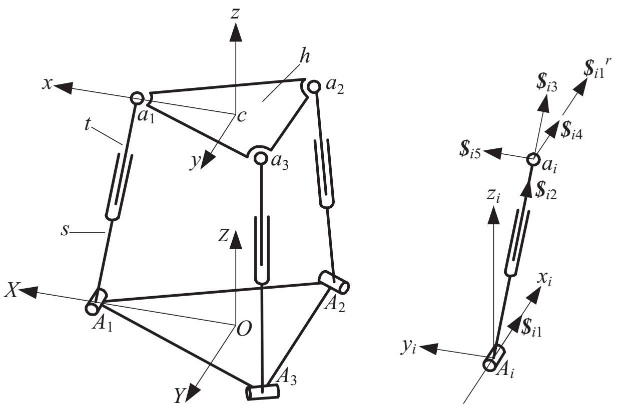
1. Introduction

Force analysis of parallel manipulators (PMs) is one of the important issues for design, simulation, and control of manipulators. Generally, it is necessary to solve for the active forces and constraint reactions of all pairs. Force analysis contains statics and dynamics analyses. Only a statics analysis is needed when the device moves at low speed. However, a dynamics analysis is needed when it moves at high speed, since the inertia force on each link cannot be neglected. The study has been traditionally carried out through different methods, for example, d'Alembert's principle, the principle of virtual work, the Newton–Euler method, and the Lagrange formulation. Kumar and Waldron[1] used the Moore–Penrose pseudoinverse method to analyze the force distribution in closed kinematic chains. Nahon and Angeles[2,3] used Quadratic Programming with constraints to solve torques and the optimization of dynamic forces in mechanical hands. Buttolo and Hannaford[4] set up linear programming to solve torques of a redundantly actuated PM. Dasgupta and Mruthyunjaya[5,6] discussed the force redundancy of PMs and the inverse dynamics of the Stewart platform. Merlet[7] presented the efficient estimation of external articular forces. Kim and Choi[8] analyzed the forward/inverse kinetostatic capabilities of manipulators in terms of an eigenvalue. Merlet[9] discussed the dynamics of PMs. Zhang and Gosselin[10,11] discussed a general kinetostatic model of PMs. Gallardo, Rico, and Frisoli[12] analyzed the dynamics of PMs by screw theory. Firmani and Podhorodeski[13] presented force-unconstrained poses for a redundantly actuated planar PM. Nokleby *et al.*[14] analyzed PM kinematics using screw algebra. Li *et al.*[15] presented the inverse dynamic formulation of a 3-degree-of-freedom (DOF) Tricept robot. Russo *et al.*[16] discussed the static balancing of a parallel robot. Lu, Shi, and Hu[17] and Lu[18] analyzed active forces and several passive forces of some PMs. Ceccarelli[19] also discussed the static force of serial and parallel robots.

The force analysis of PMs generally requires finding the inertia forces, active forces, and constraint reactions of the whole mechanism. The analysis of the constraint reactions of kinematic pairs is terribly complicated. Firstly, since the number of links and kinematic pairs of PMs is quite excessive, higher order matrices have to be processed. For example, a simple 3-RPS mechanism has 42 unknowns. For a 5-DOF 5-5R PM, the number of unknowns is even up to 130, so 130 equations need to be built for the problem. Secondly, the lower mobility PMs can be divided into two types: mechanisms without overconstraints and ones with overconstraints. Dealing with overconstraints is also a new problem. Thirdly, a statically indeterminate problem often arises for the lower mobility PMs. For instance, the previously mentioned 5-DOF 5-5R PM has 21 links, and only 126 equilibrium equations can be built. The number is less than its 130 unknowns. Especially, for the simple 3-DOF 3-RRR spherical PM, its order of static indeterminacy is up to six.

This paper proposes an effective approach by using screw theory[20,21] to solve for all the constraint reactions as well as the active forces. The main advantage of the method is that it can significantly reduce the number of unknown constraint reactions and even keep the number of simultaneous equilibrium equations to not more than six each time. That is, it can transform the higher order matrix into a sixth order or less. All of the constraint reactions and active forces can easily be simultaneously obtained independently by analyzing the equilibrium of each body one by one. Compared with the traditional method with higher order matrices, we define this case as force decoupling (see Section 3). Another merit of this method is the actual axes acted about by reaction forces and moments can be clearly determined from screw theory before the numerical calculation, and it is useful for mechanism analysis and design including singularity research.

* Corresponding author. E-mail: huangz@ysu.edu.cn



(a) The 3-RPS mechanism *(b) A limb*

Fig. 1. The 3-RPS mechanism and a limb. (a) The 3-RPS mechanism; (b) a limb.

Here, we only discuss a 3-RPS parallel mechanism without overconstraint to introduce the approach for constraint analysis. At the end of this paper, a numerical example analysis is given. This method is suitable for many similar mechanisms without overconstraint, such as 3T0R mechanisms (symbol 3T0R denotes the mechanisms with three translational mobility and without rotation mobility): 3-5R,[22] 3-RRUR,[23] 3-UPU,[24] 3-RPUR;[25] 3R: 3-UPU,[26] 3-RPR(RR),[27] 3-RRS,[28] 3-RR(RRR), 3-RRR(RR);[29] 2R1T: 3-RRR(RR),[29] 3-RPR(RR);[27] 1T2R: 3-RRR(RR), 3-RR(RRR),[29] etc. These mechanisms are all statically determinate ones.

2. Structure Description

2.1. 3-RPS parallel mechanism

In the 3-RPS mechanism shown in Fig. 1(a), the moving platform and the fixed platform are both equilateral triangles. The axis of the R pair is parallel to the corresponding side of the fixed platform. Link s and link t are connected by the prismatic pair P, which is the active pair. The fixed coordinate system $O - XYZ$, moving coordinate system $c - xyz$, and limb coordinate system $A_i - x_i y_i z_i$ are also shown in Fig. 1.

2.2. Mobility analysis

It is necessary to analyze the mobility for this method, and the Modified Grübler–Kutzbach (G–K) Criterion[30–32] is used. In this mobility analysis, how to deal with overconstraints is a key issue. The method divides overconstraints into two parts: one is the common-constraint factor d and the other is the parallel-constraint factor ν. The Modified G–K Criterion is given as follow:

$$M = d(n - g - 1) + \sum_{i=1}^{g} f_i + \nu, \tag{1}$$

where M denotes the mobility of a mechanism; n is the number of links including the frame; g is the number of kinematic joints; f_i is the number of DOF of the ith joint; ν, named *ν-factor*, is equal to the number of redundant constraints minus the number of common constraints that have been accounted for. The common-constraint factor d is given by

$$d = 6 - \lambda, \tag{2}$$

where λ is the number of common constraints of the mechanism. The common constraint is defined as a screw reciprocal to all the kinematic screws in a mechanism.

In order to analyze the mobility, one of its limbs i is taken out, as shown in Fig. 1(b). Its five single-DOF pairs are expressed in screw Plücker coordinates in system A_i-$x_i y_i z_i$ as follows:[20,21]

$$\begin{aligned} \$_{i1} &= (1 \quad 0 \quad 0; \quad 0 \quad 0 \quad 0), \\ \$_{i2} &= (0 \quad 0 \quad 0; \quad 0 \quad a \quad b), \\ \$_{i3} &= (0 \quad a \quad b; \quad 0 \quad 0 \quad 0), \\ \$_{i4} &= (1 \quad 0 \quad 0; \quad 0 \quad b \quad -a), \\ \$_{i5} &= (0 \quad 1 \quad 0; \quad -b \quad 0 \quad 0), \end{aligned} \tag{3}$$

where the subscripts in $\$_{ij}$ indicate the screw of j kinematic pair in limb i. Components a and b can be arbitrary and inessential variants. Equation (3) has one reciprocal screw

$$\$^{r}_{i1} = (1 \quad 0 \quad 0; \quad 0 \quad b \quad -a), \tag{4}$$

where $\$^{r}_{i1}$ is a constraint force acting on the platform by limb i, parallel to the axis of the revolute pair, and passing through the center of the spherical pair of limb i.

Similarly, the other two limbs also exert two constraints on the moving platform. The three constraint forces are all parallel to the axes of corresponding revolute pairs and also pass through the spherical pair centers. In this case, both the common-constraint and parallel-constraint factors are zero and the mechanism is without overconstraints. Then

$$M = 6(8 - 9 - 1) + 15 + 0 = 3. \tag{5}$$

The platform is equivalently acted upon by a constraint couple and two constraint forces, and the mechanism is able to translate along Z-axis and rotate about the X- and Y-axes.

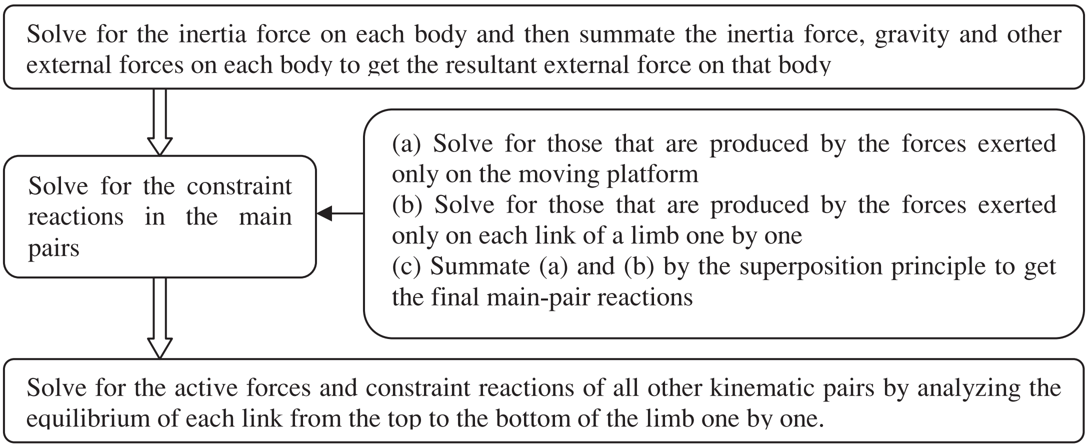
3. Active Forces[1] and Constraint Reactions

The position, velocity, acceleration, and inertia force/moment of the mechanism should be determined before the force analysis. For the analysis process, refer to ref. 30 and many other references.

The kinematic pairs connecting the platform and the limbs are named as the main kinematic pairs or main pairs and the reaction in the main pair is named as the main-pair reaction or main reaction. For a force analysis, the first and key step is to solve for the reactions of the main pairs. Our method is based on the principle of d'Alembert and the flowchart below shows the steps:

[1] If it is unnecessary to calculate the constraint reactions of pairs, the active forces can be directly obtained by the principle of virtual work.

https://doi.org/10.1017/S0263574711000129 Published online by Cambridge University Press



Generally, the number of unknown constraint reactions of a kinematic pair is completely determined by the pair itself. For example, the revolute pair has five unknowns including three constraint forces and two couples perpendicular to the axis of the pair. The translational pair also has five unknowns, three couples, and two forces normal to the axis of the pair. The spherical pair has three constraint forces. The Hooke pair has four reactions including three forces and a couple. Based on the analysis, if the number of the unknowns of the mechanism is more than the number of equilibrium equations, the force analysis would be unsolvable. Since the 3-RPS has eight bodies, three R pairs, three P pairs, three S pairs, and three unknown inputs, the number of equilibrium equations is $(8-1)\times 6 = 42$ and the number of unknowns is $3\times 5 + 3\times 5 + 3\times 3 + 3 = 42$. Then it is solvable.

3.1. *Reactions of main pairs*

In order to simplify the introduction to the method, we first consider only the external force screws $F_h\$_h^F$ and $F_t^1\$_{1t}^F$, which express the six-dimensional force vectors including the inertia force/moment, gravity, and other external forces, and act on platform h and link t of limb 1, respectively.

3.1.1. Main reaction produced by platform forces. One of the main reactions, $\$_{i1}^r$, acting on the main pair at a_i, has already been obtained from Eq. (4). Considering the action of the input force on the limb, there is another reaction force acting on the platform. Its occurrence is equivalent to locking the corresponding prismatic pair. Then there are only four screws in the limb screw system.

$$\begin{aligned} \$_{i1} &= (1 \quad 0 \quad 0; \quad 0 \quad 0 \quad 0),\\ \$_{i3} &= (0 \quad -a \quad b; \quad 0 \quad 0 \quad 0),\\ \$_{i4} &= (1 \quad 0 \quad 0; \quad 0 \quad b \quad a),\\ \$_{i5} &= (0 \quad 1 \quad 0; \quad -b \quad 0 \quad 0). \end{aligned} \tag{6}$$

Two reciprocal screws of those are

$$\begin{aligned} \$_{i1}^r &= (1 \quad 0 \quad 0; \quad 0 \quad b \quad a),\\ \$_{i2}^r &= (0 \quad -a \quad b; \quad 0 \quad 0 \quad 0), \end{aligned} \tag{7}$$

where $\$_{i2}^r$ is also a constraint force and along the axis of the P pair. Let $\$_1^i$ and $\$_2^i$ in $c-xyz$ denote $\$_{i1}^r$ and $\$_{i2}^r$, respectively. f_1^i and f_2^i $(i = 1 \sim 3)$ are the magnitudes of

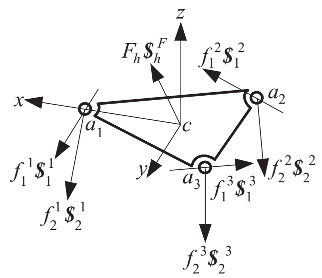


Fig. 2. Free-body diagram of the platform.

the reactions of the S pairs exerted on the platform at a_i by three different limbs and only caused by the applied force $F_h\$_h^F$.

From this point of view, each node of the platform has two unknown forces whose directions are known but their magnitudes are unknown. Then the platform is subjected to six unknown reactions. The free-body diagram of the moving platform is shown in Fig. 2.

Since there are six unknowns in six equilibrium equations for the moving platform, the issue is solvable. However, with the traditional approach, each spherical pair has three unknowns and the platform equilibrium with nine unknowns is insolvable. Clearly, the present method reduces the number of unknowns. The six unknowns can be obtained from the following screw equation:

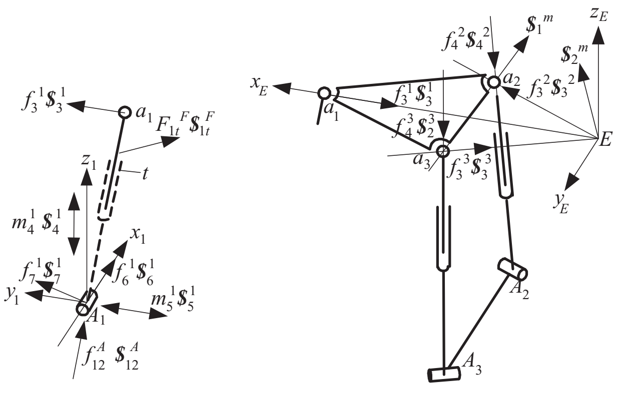
$$\sum_{i=1}^{3}\sum_{j=1}^{2} f_j^i \$_j^i + F_h\$_h^F = 0. \tag{8}$$

3.1.2. Main reaction produced by limb forces.

Main-pair reaction at a_1. In order to analyze the main reactions at a_1 caused by $F_t^1\$_{1t}^F$, we may analyze the equilibrium of link t of limb 1. Then we should analyze the reactions of the S pair at a_1 and the P pair. The free-body diagram of the two-pair link t is shown in Fig. 3(a).

(1) Considering the spherical pair S. The spherical pair at a_1 connects link t and a submechanism, i.e., an RPSSPR mechanism, as shown in Fig. 3(b). In order to determine the reactions of the spherical pair, the mobility of the

https://doi.org/10.1017/S0263574711000129 Published online by Cambridge University Press



(a) Link t in limb 1 (b) Sub-mechanism

Fig. 3. Force analysis. (a) Link t in limb 1; (b) submechanism.

submechanism needs to be determined first. The RPSSPR mechanism can be considered as a generalized pair, and from the Modified G–K Criterion, Eq. (1), its mobility is

$$M = 6(6 - 6 - 1) + 10 + 0 = 4. \quad (9)$$

When its two input pairs are locked, its mobility becomes two. And similarly, there are four constraint forces, $f_3^i\$_3^i$ and $f_4^i\$_4^i$ $(i = 2, 3)$, in total acting on the locked submechanism, as shown in Fig. 3(b). The analysis is similar to that in Section 3.1.1.

Establish the system $E - x_E y_E z_E$ shown in Fig. 3(b), where E is the intersection point of $f_3^2\$_3^2$ and $f_3^3\$_3^3$; the x_E-axis passes through points a_1 and E; the y_E-axis is parallel to side a_2a_3. Two reciprocal screws of the four screws in $E - x_E y_E z_E$, equivalent to the constraint forces, are

$$\begin{aligned} \$_1^m &= (0 \quad 1 \quad 0; \quad 0 \quad 0 \quad d), \\ \$_2^m &= (f \quad 0 \quad g; \quad 0 \quad 0 \quad 0), \end{aligned} \quad (10)$$

where $\$_1^m$ and $\$_2^m$ denote two twists of the subchain. $\$_1^m$ passes the center points of the other two spherical pairs, and $\$_2^m$ passes through both point E and the intersecting point of $f_4^2\$_4^2$ and $f_4^3\$_4^3$.

In addition, the spherical pair at a_1 has another 3 DOF, which can be written in $E - x_E y_E z_E$ as

$$\begin{aligned} \$_3^S &= (1 \quad 0 \quad 0; \quad 0 \quad 0 \quad 0), \\ \$_4^S &= (0 \quad 1 \quad 0; \quad 0 \quad 0 \quad h), \\ \$_5^S &= (0 \quad 0 \quad 1; \quad 0 \quad j \quad 0). \end{aligned} \quad (11)$$

The five-system screws in Eqs. (10) and (11) form an equivalent serial kinematic chain connecting the link t of limb 1 and the base at a_1. And its reciprocal screw, i.e., a constraint, is

$$\$^r = (1 \quad 0 \quad 0; \quad 0 \quad 0 \quad 0), \quad (12)$$

where $\$^r$ simultaneously intersects five twists, $\$_1^m$, $\$_2^m$, $\$_3^S$, $\$_4^S$, and $\$_5^S$; and it is also a constraint force along the x_E-axis of the $E - x_E y_E z_E$ system.

Let $\$_3^1$ in $c - xyz$ denote $\$^r$, and f_3^1 denote its magnitude. Therefore, the spherical pair has only one constraint, $f_3^1\$_3^1$, in this case.

(2) Considering the prismatic pair P. In order to analyze the reactions of the prismatic pair, it is necessary to consider the kinematic chain consisting of the R pair and the P pair, i.e., the 2-DOF RP chain in limb 1. The kinematic screws of the RP chain are just the first two screws in Eq. (3), and there are four reciprocal screws constraining the link t

$$\begin{aligned} \$_{13}^r &= (0 \quad 0 \quad 0; \quad 0 \quad 0 \quad 1), \\ \$_{14}^r &= (0 \quad 0 \quad 0; \quad 0 \quad 1 \quad 0), \\ \$_{15}^r &= (1 \quad 0 \quad 0; \quad 0 \quad 0 \quad 0), \\ \$_{16}^r &= (0 \quad b \quad -a; \quad 0 \quad 0 \quad 0). \end{aligned} \quad (13)$$

In $c - xyz$, let $\$_4^1 \sim \$_7^1$ denote $\$_{13}^r \sim \$_{16}^r$ of Eq. (13), and m_4^1, m_5^1, f_6^1, and f_7^1 denote their magnitudes, respectively. Therefore, the P pair of limb 1 has four and not five unknown reactions, $m_4^1\$_4^1$, $m_5^1\$_5^1$, $f_6^1\$_6^1$, and $f_7^1\$_7^1$.

(3) The equilibrium of link t. Let $f_{i2}^A\$_{i2}^A$ denote the active force on limb i produced only by forces acting on link t of limb 1$(F_t^1\$_{1t}^F)$. There are six unknowns including the active force $f_{12}^A\$_{12}^A$ in the six equilibrium equations of link t. So it is solvable. The screw equation is

$$\sum_{j=3,6,7} f_j^1\$_j^1 + \sum_{j=4}^{5} m_j^1\$_j^1 + F_t^1\$_{1t}^F + f_{12}^A\$_{12}^A. \quad (14)$$

Main-pair reaction at $a_i (i = 2, 3)$. Now $f_3^1\$_3^1$ has already been known from Eq. (14). In order to solve for the other four main reactions, $f_j^i\$_j^i (i = 2, 3,\ j = 3, 4)$, the equilibrium of the platform shown in Fig. 3b should be considered once again. The screw equation is

$$\sum_{i=2}^{3}\sum_{j=3}^{4} f_j^i\$_j^i + f_3^1\$_3^1 = 0. \quad (15)$$

3.1.3. Resultant main-pair reaction. Finally, the total reactions of main pairs caused by the applied forces $F_h\$_h^F$ and $F_t^1\$_{1t}^F$ can be obtained by the principle of superposition of forces, and they all pass through the corresponding spherical pair centers. From Eqs. (8) and (14) the reaction of the main pair at a_1 is

$$\sum_{j=1}^{3} f_j^1\$_j^1. \quad (16)$$

From Eqs. (8) and(15) the reaction of the main pair at $a_i (i = 2, 3)$ is

$$\sum_{j=1}^{4} f_j^i\$_j^i, \quad i = 2, 3. \quad (17)$$

If other links of the limbs are subject to a known six-dimensional external force, the analysis is similar. And then,

https://doi.org/10.1017/S0263574711000129 Published online by Cambridge University Press

summate them in turn to get the final main-pair reaction on limb i, which is expressed by $f_i^S \$_i^S$ in Eq. (18).

$$f_i^{\mathrm{S}} \$_i^{\mathrm{S}} = f_{i1}^{\mathrm{S}} \$_{i1}^{\mathrm{S}} + f_{i2}^{\mathrm{S}} \$_{i2}^{\mathrm{S}} + \cdots, \quad i = 1 \sim 3, \tag{18}$$

where $f_{ij}^{\mathrm{S}} \$_{ij}^{\mathrm{S}} (i = 1 \sim 3, j = 1, 2 \cdots)$ is the component of $f_i^S \$_i^S$.

3.2. *Active forces and other reactions*

Once the main-pair reactions are obtained, the active forces and constraint reactions of all other kinematic pairs are easily found by solving the equilibrium of each link from the top to the bottom of the limb one by one.

When analyzing link t of limb i, the total main-pair reaction is considered as one of its six-dimensional known external forces. From screw theory, we know that the number of unknown constraint reactions of the prismatic pair may be less than 5 (see Section 5). Then we let

$$f_i^{\mathrm{P}} \$_i^{\mathrm{P}} = f_{i1}^{\mathrm{P}} \$_{i1}^{\mathrm{P}} + f_{i2}^{\mathrm{P}} \$_{i2}^{\mathrm{P}} + \cdots + f_{ij}^{\mathrm{P}} \$_{ij}^{\mathrm{P}}, \quad i = 1 \sim 3, j \leq 5, \tag{19}$$

where $f_i^{\mathrm{P}} \$_i^{\mathrm{P}} (i = 1 \sim 3)$ is the total constraint reaction of the P pair of limb i. $f_{ij}^{\mathrm{P}} \$_{ij}^{\mathrm{P}} (i = 1 \sim 3, j \leq 5)$ is the component of $f_i^{\mathrm{P}} \$_i^{\mathrm{P}}$.

Let $f_i^A \$_i^A$ denote the total active force on the limb i produced by all the external forces, and $F_t^i \$_{it}^F$ denote a known external force acting on link t of limb i. The active forces and constraint reactions of the prismatic pair can be obtained from the screw Eq. (20).

$$f_i^{\mathrm{S}} \$_i^{\mathrm{S}} + f_i^A \$_i^A + f_i^{\mathrm{P}} \$_i^{\mathrm{P}} + F_t^i \$_{it}^F = 0, \quad i = 1 \sim 3. \tag{20}$$

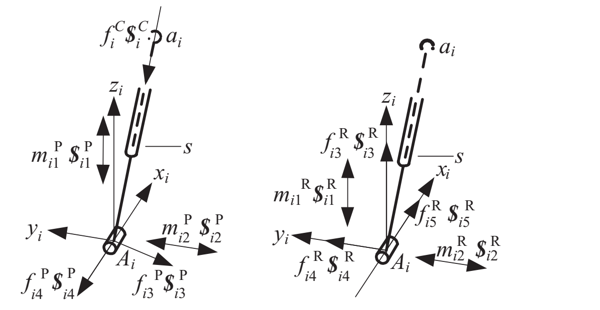
The R pair at A_i has generally five constraint reactions,and we let them be $m_{i1}^{\mathrm{R}} \$_{i1}^{\mathrm{R}}$, $m_{i2}^{\mathrm{R}} \$_{i2}^{\mathrm{R}}$, $f_{i3}^{\mathrm{R}} \$_{i3}^{\mathrm{R}}$, $f_{i4}^{\mathrm{R}} \$_{i4}^{\mathrm{R}}$, and $f_{i5}^{\mathrm{R}} \$_{i5}^{\mathrm{R}}$.When analyzing link s of limb i, the total constraint reactions of the P pair are considered as its known external forces and constraint reactions of the R pair at A_i can be obtained from Eq. (21) below

$$-f_i^A \$_i^A - f_i^{\mathrm{P}} \$_i^{\mathrm{P}} + \sum_{j=1}^{2} m_{ij}^{\mathrm{R}} \$_{ij}^{\mathrm{R}} + \sum_{j=3}^{5} f_{ij}^{\mathrm{R}} \$_{ij}^{\mathrm{R}} + F_s^i \$_{is}^F = 0, \quad i = 1 \sim 3, \tag{21}$$

where $F_s^i \$_{is}^F (i = 1 \sim 3)$ is a known external force acting on link s of limb i.

4. Numerical Example

The numerical results of the 3-RPS mechanism can be easily calculated with our method. Assume that there are the known forces including the inertia force/moment, gravity, and other external forces acting only on platform h and link t of limb 1, and they are expressed as $F_h \$_h^F = (10\ 5\ -78.2;\ 3\ 4\ 2)$, $F_t^1 \$_{1t}^F = (5\ 5\ 6;\ 2\ 3\ 2)$ (N · m and N); the circumradii of the platform and the base are $r = 0.3$ m and $R = 0.4$ m, respectively; the initial distance from the platform to the base is $z_0 = 0.35$ m; the length of the link t, l_t, is 0.3 m. And it is necessary to solve for the active forces and the constraint reactions of kinematic pairs that are broken into their components along three coordinate axes, $\boldsymbol{u}_i$, $\boldsymbol{w}_i$, $\boldsymbol{v}_i$, i.e., the axis of the revolute pair, the axis of the prismatic pair, and the axis perpendicular to the above two axes in every limb.



(a) *Reactions of prismatic pair* (b) *Reactions of revolute pair*

Fig. 4. Link s of limb i. (a) Reactions of prismatic pair; (b) reactions of revolute pair.

Table I. The results of the active forces and constraint reactions of 3-RPS manipulator.

	f_u	f_v	f_w	m_u	m_v	m_w
A_1	–	–	28.52	–	–	–
S_1	−7.46	−12.04	−32.92	–	–	–
P_1	12.46	5.58	–	0	1.31	−1.37
R_1	12.46	5.58	28.52	–	1.31	−1.37
A_2	–	–	28.49	–	–	–
S_2	−10.41	0	−28.49	–	–	–
P_2	10.41	0	–	0	3.79	0
R_2	10.41	0	28.49	–	3.79	0
A_3	–	–	16.48	–	–	–
S_3	11.20	0	−16.48	–	–	–
P_3	−11.20	0	–	0	−4.08	0
R_3	−11.20	0	16.48	–	−4.08	0

The process for solving for the main-pair reactions can be found in Section 3.1.

In order to solve for the constraint reactions in the P pair, link s of limb i is taken out as a free-body member to analyze. Fig. 4(a) shows four constraint reactions of the prismatic pair, $m_{i1}^{\mathrm{P}} \$_{i1}^{\mathrm{P}}$, $m_{i2}^{\mathrm{P}} \$_{i2}^{\mathrm{P}}$, $f_{i3}^{\mathrm{P}} \$_{i3}^{\mathrm{P}}$, $f_{i4}^{\mathrm{P}} \$_{i4}^{\mathrm{P}}$ and an external force, $f_i^{\mathrm{C}} \$_i^{\mathrm{C}}$, exerted by the actuator. Fig. 4(b) shows five constraints of the R pair $m_{i1}^{\mathrm{R}} \$_{i1}^{\mathrm{R}}$, $m_{i2}^{\mathrm{R}} \$_{i2}^{\mathrm{R}}$, $f_{i3}^{\mathrm{R}} \$_{i3}^{\mathrm{R}}$, $f_{i4}^{\mathrm{R}} \$_{i4}^{\mathrm{R}}$, and $f_{i5}^{\mathrm{R}} \$_{i5}^{\mathrm{R}}$.

The resulting active forces and constraint reactions are listed in Table I, where $A_i, S_i, P_i, R_i (i = 1, 2, 3)$ denote the actuator, spherical pair, prismatic pair, and revolute pair in limb i, respectively. (f_u, f_v, f_w) and (m_u, m_v, m_w) are three components of forces along and moments about three corresponding axes.

5. Conclusion

The paper has presented a new approach by using screw theory for force analysis of statically determinate parallel mechanisms. It is necessary to analyze the main-pair reactions at first, the main-pair reactions can be obtained through solvable equilibrium equations of the platform and

https://doi.org/10.1017/S0263574711000129 Published online by Cambridge University Press

links independently. Once the main reactions are solved, the constraint reactions of other pairs are easily obtained.

An important advantage of the method is that it can markedly reduce the number of unknown reactions, and even keep the number of simultaneous equilibrium equations to not more than six each time. Then all of unknowns can easily be simultaneously obtained independently by analyzing the equilibrium of each body one by one. That is, it can transform the higher order matrix into a sixth order or less, and it can be named as a force-decoupling approach. Another merit of this method is the actual axes acted about by reaction forces and moments can be clearly determined before the numerical calculation, and it is useful for mechanism analysis and design including singularity research.

The approach not only applies to many parallel mechanisms similar to 3-RPS manipulator, but also applies to the parallel mechanisms with special static indeterminacy.[33] However, for the parallel mechanisms with special static indeterminacy, some additional complementary equations are needed.

Acknowledgments

The research work reported here is supported by Nature Science Fund of China under Grant No. 50875227. The authors would like to express their deep gratitude.

References

1. V. Kumar and K. J. Waldron, "Force Distribution in Closed Kinematic Chains," *Proceedings of the 1988 IEEE International Conference on Robotics and Automation*, Philadelphia, PA , USA (1988) pp. 114–119.
2. M. Nahon and J. Angeles, "Real-Time Force Optimization in Parallel Kinematic Chains under Inequality Constraints," *Proceedings of the 1991 IEEE International Conference on Robotics and Automation*, Sacramento, CA, USA (1991) pp. 2198–2203.
3. M. Nahon and J. Angeles, "Optimization of dynamic forces in mechanical hands," *ASME J. Mech. Des.* **113**(2), 167–173 (1991).
4. P. Buttolo and B. Hannaford, "Advantages of Actuation Redundancy for the Design of Haptic Displays," *Proceedings of the 1995 ASME International Mechanical Engineering Congress and Exposition – Part 2*, San Francisco, CA, USA (1995) pp. 623–630.
5. B. Dasgupta and T. S. Mruthyunjaya, "Force redundancy in parallel manipulators: Theoretical and practical issues," *Mech. Mach. Theory* **33**(6), 727–742 (1998a).
6. B. Dasgupta and T. S. Mruthyunjaya, "A Newton-Euler formulation for the inverse dynamics of the stewart platform manipulator," *Mech. Mach. Theory* **33**(8), 1135–1152 (1998b).
7. J. P. Merlet, "Efficient Estimation of the External Articular Forces of a Parallel Manipulator in a Translational Workspace," *Proceedings of the 1998 IEEE International Conference on Robotics and Automation*, Leuven, Belgium (1998) pp. 1982–1987.
8. H. S. Kim and Y. J. Choi, "The Kinetostatic Capability Analysis of Robotic Manipulators," *Proceedings of the 1999 IEEE/RSJ International Conference on Intelligent Robots and Systems*, Kyongju, South Korea (1999) pp. 1241–1246.
9. J. P. Merlet, *Parallel Robots* (Kluwer Academic Publishers, 2000).
10. D. Zhang and C. M. Gosselin, "Kinetostatic modeling of n-dof parallel mechanisms with passive constraining leg and prismatic actuators," *ASME J. Mech. Des.* **123**(3), 375–381 (2001).
11. D. Zhang and C. M. Gosselin, "Kinetostatic modeling of parallel mechanisms with a passive constraining leg and revolute actuators," *Mech. Mach. Theory* **37**(6), 599–617 (2002).
12. J. Gallardo, J. M. Rico, A. Frisoli, D. Checcacci and M. Bergamasco, "Dynamics of parallel manipulators by means of screw theory," *Mech. Mach. Theory* **38**(11), 1113–1131 (2003).
13. F. Firmani and R. P. Podhorodeski, "Force-unconstrained poses for a redundantly-actuated planar parallel manipulator," *Mech. Mach. Theory* **39**(5), 459–476 (2004).
14. S. B. Nokleby, R. Fisher, R. P. Podhorodeski and F. Firmani, "Force capabilities of redundantly-actuated parallel manipulators," *Mech. Mach. Theory* **40**(5), 578–599 (2005).
15. M. Li, T. Huang, J. Mei, X. Zhao, D. G. Chetwynd and S. J. Hu, "Dynamic formulation and performance comparison of the 3-dof modules of two reconfigurable PKM-the Tricept and the TriVariant," *ASME J. Mech. Des.* **127**, 1129–1136 (2005).
16. A. Russo, R. Sinatra and F. Xi, "Static balancing of parallel robots," *Mech. Mach. Theory* **40**(2), 191–202 (2005).
17. Y. Lu, Y. Shi and B. Hu, "A CAD Variation Geometrical Approach of Solving Active/Constrained Forces of Some Parallel Manipulators with SPR-type Active Legs," *Proceedings of the ASME International Design Engineering Technical Conference (DETC2007), Report No. MECH-34308*, Las Vegas, Nevada, USA (2007) pp. 839–845.
18. Y. Lu, "Using virtual work theory and CAD functionalities for solving active and passive force of spatial parallel manipulators," *Mech. Mach. Theory* **42**(7), 839–858 (2007).
19. M. Ceccarelli, *Fundamentals of Mechanics of Robotic Manipulators* (Kluwer Academic Publishers, Netherlands, 2004).
20. R. S. Ball, *Theory of Screws* (Cambridge University Press, London, 1900).
21. K. H. Hunt, *Kinematic Geometry of Mechanisms* (Oxford University Press, Oxford, 1978).
22. Z. Huang and Q. C. Li, "General methodology for type synthesis of lower-mobility symmetrical parallel manipulators and several novel manipulators," *Int. J. Robot. Res.* **21**(2), 131–145 (2002).
23. A. Frisoli, D. Checcacci, F. Salsedo and M. Bergamasco, "Synthesis by screw algebra of translating in-parallel actuated mechanisms," **In**: *Advances in Robot Kinematics* (J. Lenarčič and M. M. Stanišič, eds.) (Kluwer Academic Publishers, 2000) pp. 433–440.
24. Z. Huang and Q. C. Li, "Construction and kinematic properties of 3-UPU parallel mechanisms," *Proceedings of the Design Engineering Technical Conferences (DETC2002), ASME Paper MECH-34321*, Montreal, Canada (2002) pp. 1027–1033.
25. D. X. Zeng and Z. Huang, "Mobility and Position Analysis of a Novel 3-DOF Translational Parallel Mechanism," *Proceedings of the International Design Engineering Technical Conferences and Computers and Information in Engineering Conference, ASME Paper No. DETC2006–99031*, Philadelphia, PA, USA (2006) pp. 957–963.
26. Z. Huang and Q. C. Li, "On the Type Synthesis of Lower-Mobility Parallel Manipulators," *2002 ASME Conference Workshop*, Quebic (2002).
27. Z. Huang and Q. C. Li, "Type synthesis of symmetrical lower-mobility parallel mechanisms using constraint-synthesis method," *Int. J. Robot. Res.* **22**(1), 59–79 (2003).
28. R. Di Gregorio, "The 3-RRS wrist: A new, very simple and not overconstrained spgerical parallel manipulator," *Proceedings of the 2002 ASME Design Engineering Technical Conferences, Report No.* MECH-34344, Montreal, Canada (2002) pp. 1193–1199.
29. Q.C. Li and Z. Huang, "A family of symmetrical lower-mobility parallel mechanisms with spherical and parallel subchains," *J. Robot. Syst.* **20**(6), 297–305 (2003).

30. Z. Huang, L. F. Kong and Y. F. Fang, *Mechanism Theory of Parallel Robotic Manipulator and Control* (China Mechanical Press, Beijing, 1997).
31. Z. Huang and Q. C. Li, "General methodology for type synthesis of lower-mobility symmetrical parallel manipulators and several novel manipulators," *Int. J. Robot Res.* **21**(2), 131–145 (2002).
32. Z. Huang, J. F. Liu and Q. C. Li, "Unified methodology for mobility analysis based on screw theory," **In**: *Smart Devices and Machines for Advanced Manufacturing* (L. Wang and J. Xi, eds.) (Springer-Verlag, London, 2008).
33. Z. Huang, Y. Zhao and J. F. Liu, "Kinetostatic analysis of 4-R(CRR) parallel manipulator with overconstraints via reciprocal-screw theory," *Adv. Mech. Eng.* **2010**, 11 pp. (2010).