# THE ROAD NOT TAKEN: WHAT IS THE "APPROPRIATE" PATH TO DEVELOPMENT WHEN GROWTH IS UNBALANCED?

## AHMED S. RAHMAN

United States Naval Academy

This paper develops a model that endogenizes both directed technologies and demography. Potential innovators decide which technologies to develop after considering available factors of production, and individuals decide the quality and quantity of their children after considering available technologies. This interaction allows us to evaluate potentially divergent development paths. We find that unskilled labor–biased technological growth can induce higher fertility and lower education, exerting downward pressure on growth in per-person income. Despite this, for most plausible developing-country scenarios, unskilled intensive growth produces more per-person income than skill-intensive growth. This result is robust to a variety of growth modeling assumptions.

Keywords: Directed Technical Change, Demography, Human Capital, Fertility

## 1. INTRODUCTION

The last half century has seen great divergence in living standards among the countries of the developing world; whereas rich nations have maintained fairly consistent rates of growth (2% or 3% per annum), poorer nations have traversed widely different growth paths (between -1% and 7%). This paper suggests a possible source of this divergence by producing a model emphasizing the interdependence between directed technical change and demography. In this model, potential innovators decide which technologies to develop after considering available factors of production, and individuals decide the quality and quantity of their children after considering available technologies. This interaction allows us to analyze the macroeconomic effects of "unbalanced growth," where a country *either* develops labor-intensive techniques and expands the pool of unskilled labor, *or* develops skill-intensive techniques and expands the pool of human capital. Which path will lead to greater overall prosperity is the primary focus of this paper.

The model emphasizes how economic growth can often be an unbalanced process, where choices are made between alternative modes of production.<sup>1</sup> A

Address correspondence to: Ahmed S. Rahman, Department of Economics, U.S. Naval Academy, Annapolis, MD 21402, USA; e-mail: rahman@usna.edu.

farm can be maintained either with uneducated farmers wielding hand tools, or with farmers skilled in using agronomic instruments and automated machinery. A factory can be structured as an assembly line run mainly by unskilled workers supervised by a few skilled ones, or as a computer-controlled facility mainly run by skilled workers with a few unskilled janitors.<sup>2</sup> A road can be built using lots of manual labor physically laying down stone and brick by hand, or construction workers trained in using bulldozers and steamrollers. These examples highlight not only that technologies can be directed toward particular factors, but also that each country can take its own unique development path, producing similar things in very different ways [Owen et al. (2009)].

This paper boils down all these considerations into a simple question—would greater aggregate wealth be generated with skilled labor–biased technological growth (the "skill-intensive path") or unskilled labor–biased technological growth (the "unskilled-intensive path")? The answer of course is that it depends. It depends on how *productive* or *abundant* skilled and unskilled labor are. And it depends on how technological changes can affect *future* supplies of skilled and unskilled labor.

By exploring the simultaneity between technological changes directed toward particular factors and the factors themselves, we can explore some of these issues. This approach constitutes a notable departure from the existing literature on technologies that augment specific factors or sectors. These works often either highlight the "inappropriateness" of growth in technologies that can be implemented by only a small portion of the economy, or demonstrate the effects of biased technical growth on factor inequality. For the former case, Basu and Weil (1998), Zeira (1998), and Acemoglu and Zilibotti (2001) illustrate how technologies designed for capital-intensive (physical or human) societies that diffuse to capital-scarce regions are used ineffectually there, if at all. Other works counter by highlighting the indispensability of investment in general, and education in particular, for growth [Barro (2001), Ehrlich and Murphy (2007)]. For the latter case, works such as Katz and Murphy (1992), Acemoglu (1998), Kiley (1999), Xu (2001), and Acemoglu (2002) show that when factors are grossly substitutable, skilled-biased technological growth will raise skill premia and thus factor inequality, whereas unskilled-biased technological growth will lower them.

But these works do not take into account that factors can evolve, and will adjust to changing economic circumstances.<sup>3</sup> If factors do change in these models, they typically do so exogenously. But this partial equilibrium approach may mislead us, particulary when it comes to *long-run* growth (Acemoglu 2010). Allowing for the co-evolution of factors and technologies can alter our perspective on the "appropriate" technological path—that is, the path that generates more macroeconomic growth. Two new considerations emerge with this approach. The first is that factor-composition shifts from unbalanced growth can have different effects on subsequent technological progress. The other consideration is that different technological paths can produce different rates of population growth; long-term living standards are thus affected through both the numerator (effects on income) and the denominator (effects on capita).<sup>4</sup>

With simulations of the model, we discover a number of things. First, by raising the returns to education, skilled-biased technological growth can induce higher education and (through quality–quantity trade-offs in child-rearing) lower fertility; this provides an additional boost to per-person income. This case shows that catering technologies for the larger sectors or more abundant factors may produce dynamically harmful effects such education decreases.

However, we also find that increases in the overall workforce caused by unskilled intensive technological growth can generate faster technological progress (by raising the scale of the market for innovations, or by generating greater knowledge spillovers from a larger pool of skilled workers); this can actually induce more income growth than can the alternative, skill-intensive path. Indeed as we will see, a falling population can have pernicious technological effects in the context of endogenous growth. Thus we see that to answer our titular question, the declines in education generated by unskilled-intensive growth must be weighted against its effects on subsequent technological progress.

This paper heavily borrows from Acemoglu's important work on directed technological change (Acemoglu 1998, 2002). But this work departs from that literature in two fundamental ways. First, the literature relies almost exclusively on analyzing long-run *balanced* growth paths, whereas here we look solely on the *unbalanced* case (where technological growth occurs only in one sector of the economy), implicitly assuming that countries often face a choice in overall growth direction as they transition to balanced growth.<sup>5</sup> Second, as already mentioned, the literature almost always treats factors of production as exogenously determined, whereas here they are endogenous in the model.<sup>6</sup>

The rest of the paper is organized as follows. Section 2 motivates the paper by looking at some cross-country data. Section 3 presents the model in steps, first presenting a model of semiendogenous biased technological growth, and then merging this with a simple theory of demography. This model then motivates our simulation experiments in Section 4. Section 5 provides some concluding remarks.

## 2. SOME DATA

## 2.1. A Cross Section of Factor-Specific Technologies

We begin by taking account of estimated factor-specific productivities of a cross section of countries. Consider the following production function for country i:

$$Y = [(A_{l,i}L_i)^{\sigma} + (A_{h,i}H_i)^{\sigma}]^{1/\sigma},$$
(1)

where Y is aggregate GDP. Here we specify production as one, with constant elasticity of substitution between the skilled and unskilled labor aggregates  $H_i$  and  $L_i$  (this elasticity being  $1/(1-\sigma)$ ).  $A_{l,i}$  and  $A_{h,i}$  are the efficiency levels of unskilled and skilled labor in country i.<sup>7</sup>

If factors of production are paid their marginal products, the "skill premium" can be written as

$$\frac{w_{h,i}}{w_{l,i}} = \left(\frac{A_{h,i}}{A_{l,i}}\right)^{\sigma} \left(\frac{H_i}{L_i}\right)^{\sigma-1}.$$
 (2)

Caselli and Coleman (2006) note that one can study cross-country productivity differences using equations (1) and (2), for these represent two equations with two unknowns. That is, given data on  $Y_i$ ,  $L_i$ ,  $H_i$ , and  $\frac{w_i^h}{w_i^l}$ , we can back out each country's implied pair of technological coefficients and compare them.<sup>8</sup>

Key to this exercise is our parameter choice for  $\sigma \leq 1$ . Careful empirical labor studies such as Autor et al. (1998) and Ciccone and Peri (2005) have found that the elasticity of factoral substitution between more and less skilled workers most likely lies between 1 and 2.5 (consistent with a value of  $\sigma$  between 0 and 0.6). Both for this exercise and the simulations in section 4, we choose a benchmark value of  $\sigma = 0.5$  for a proxy elasticity parameter most applicable for a wide range of countries and for a wide variety of skilled and unskilled labor categories.

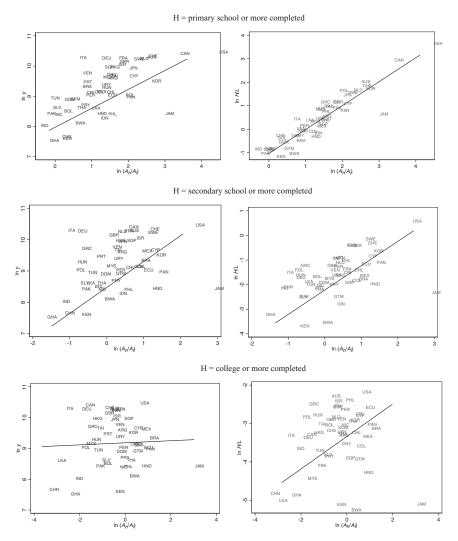
Figure 1 depicts the relationships between relative technical skill bias  $(A_h/A_l)$ , relative skill-endowments (H/L), and income per capita across a broad array of countries. Immediately clear are the positive associations between technical skill bias and skill endowment, and between technical skill bias and income levels. These positive relationships hold whether we consider a skilled worker as someone with primary schooling, or someone with secondary schooling, or even someone with a college education. This was precisely one of the main points behind Caselli and Coleman's study. Not only do wealthy nations enjoy large pools of skilled labor, but also they enjoy relatively higher levels of skilled-biased technology.

But from these static pictures it is not clear which technological path would produce more output for any particular country *over time*. On the one hand, a country with a relative abundance of unskilled labor should greatly benefit by making it more productive. On the other hand, unskilled labor's *level* of productivity may already be fairly low; unskilled-biased technical change that induces a rise in *L* and a fall in *H* would then lower the relatively more productive factor and raise the relatively less productive factor.

We begin exploring these issues by allowing the factors of production to *respond* to biased technological changes, first in a comparative statics experiment in Section 2.2, and then in a fully specified general equilibrium model in Section 3. This is an important feature to consider, given the common observation that developing economies often have too many factors allocated to low-productivity work.<sup>10</sup>

## 2.2. Unbalanced Growth: A Comparative Statics Experiment

Here we consider changes in total output, Y, that can occur when we have the factors of production respond to exogenous unbalanced technological growth.



**FIGURE 1.** Relative technologies versus relative factors and output ( $\sigma = 0.5$ ).

First, let us totally differentiate the production function given by (1):

$$dY = \left(\frac{\partial Y}{\partial A_l}\right) dA_l + \left(\frac{\partial Y}{\partial A_h}\right) dA_h + \left(\frac{\partial Y}{\partial L}\right) dL + \left(\frac{\partial Y}{\partial H}\right) dH. \tag{3}$$

Both types of technologies and both types of factors have the potential to change. Let us assume that when technologies are biased toward factor L, it induces L to rise and H to fall (higher unskilled-intensive productivity makes some people become unskilled laborers instead of skilled ones). On the other

hand, technological growth that is biased toward H induces L to fall and H to rise (higher skilled-intensive productivity makes some erstwhile unskilled laborers become skilled ones). That is,  $dA_l > 0 \Rightarrow -dH = dL > 0$  and  $dA_h > 0 \Rightarrow -dL = dH > 0$ .

Let us consider two possibilities. The first is where  $dA_l = 1$  and  $dA_h = 0$ . This is the case of unskilled-biased technological change (where the total change in output can be written as  $dY_{\rm unsk}$ ). The second case is where  $dA_h = 1$  and  $dA_l = 0$ . This is the case of skilled-biased technological change (where the total change in output can be written as  $dY_{\rm sk}$ ).

When there is unskilled-biased technological change, the total change in income can be written as

$$dY_{\text{unsk}} = [(A_l L)^{\sigma} + (A_h H)^{\sigma}]^{\frac{1-\sigma}{\sigma}} \cdot ((A_l L)^{\sigma-1} (L \cdot dA_l + A_l \cdot dL) + (A_h H)^{\sigma-1} (A_h \cdot (-dH))),$$
(4)

where  $dY_{\rm unsk}$  is the total change in income with unskilled intensive growth. Note that here  $dA_h = 0$  and the change to H is negative. On the other hand, when there is skilled-biased technological change, the total change in income can be written as

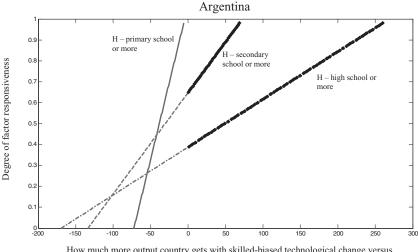
$$dY_{sk} = [(A_l L)^{\sigma} + (A_h H)^{\sigma}]^{\frac{1-\sigma}{\sigma}} \cdot ((A_l L)^{\sigma-1} (A_l \cdot (-dL)) + (A_h H)^{\sigma-1} (A_h \cdot dA_h + (A_h dH))),$$
 (5)

where  $dY_{sk}$  is the total change in income with skilled intensive growth. Note that here  $dA_l = 0$  and the change to L is negative.

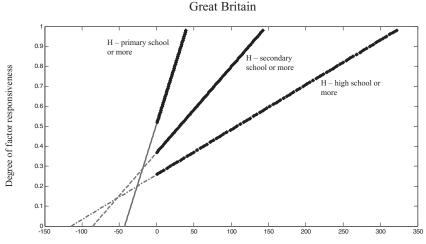
Does skilled labor–biased technological growth produce more output than unskilled labor–biased technological growth? If labor is strictly fixed, the answer is no. With Caselli and Coleman (2006)'s categorization and calculations of L and H, L > H even for wealthy nations. Because factors are grossly substitutable, technologies used by the more abundant factor will generate the greater aggregate gain.

However, the more responsive factors are to biased technological changes, the greater are the relative output gains from skilled-biased technological change. This follows simply from the fact that H is inherently the more productive factor. This comes both from its relative scarcity (so that its marginal productivity tends to be higher even if technologies are symmetrical) and from the higher productivity coefficient on H compared to that for L. So if labor tends to switch readily from one type to the other with unbalanced technical progress, skilled-intensive growth tends to produce more output.

Combining the two observations, we see that each country has a *threshold level* of factoral responsiveness, whereby  $dY_{\rm sk} = dY_{\rm unsk}$ . Figure 2 plots  $dY_{\rm sk} - dY_{\rm unsk}$  against the degree of factoral response for two illustrative countries, Argentina and Great Britain. If we consider H to be those with at least some secondary schooling, we can see that a one-unit change in  $A_h$  would require a 0.64 unit shift



How much more output country gets with skilled-biased technological change versus unskilled-biased technological change  $(dY_{\rm sk}-dY_{\rm unsk})$ 

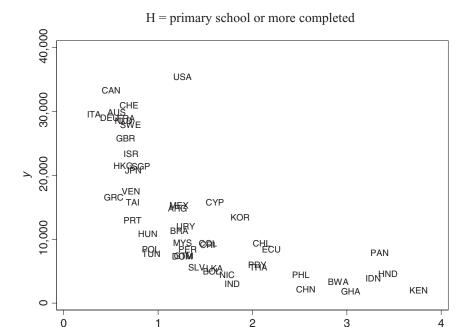


How much more output country gets with skilled-biased technological change versus unskilled-biased technological change  $(dY_{\rm sk} - dY_{\rm unsk})$ 

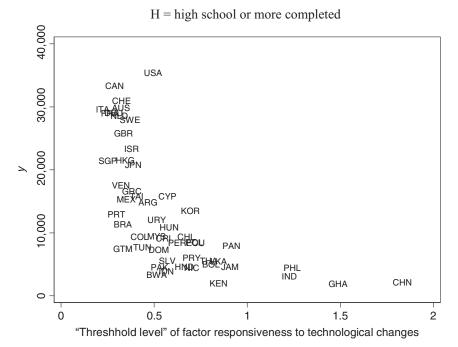
**FIGURE 2.** Skilled versus unskilled technological growth when factors respond: comparing two countries.

from L to H to produce more output than a similar change in  $A_l$  in Argentina, whereas it would require only a 0.35 unit shift from L to H in Britain.

Thus we see that because countries have their own unique pairs of factor supplies and productivities, they will have different factoral response threshold levels. Figure 3 plots each country's threshold level of factor responsiveness (where  $dY_{\rm sk} = dY_{\rm unsk}$ ) against its GDP per capita. We can see that the poorer



"Threshhold level" of factor responsiveness to technological changes



**FIGURE 3.** How much must factors respond to make skilled-biased technological growth "better?": cross country comparison.

the nation is on average, the more will factors need to respond to technological changes for skill-intensive growth to be the superior path to development. Because poorer nations tend to have greater relative quantities of unskilled labor, and also tend to have relatively less productive skilled labor, factors need to respond with greater magnitude for skill-intensive technical growth to produce relatively more output.

However, as we compare the top and bottom scatterplots, we can see that the more narrow is our definition of H, the smaller is the threshold factor responsiveness. This is simply because increases from the relatively more scarce factor produces greater benefits, for the marginal productivities of the more scarce factor tends to be larger. This in effect flips Acemoglu's discussion of so-called "market-size effects" on its head: if factors are allowed to respond to technological change, such change that augments the less abundant factor may produce more output in the longer run.

Yet to suggest from this partial equilibrium analysis that skilled-biased technological growth in the context of responsive factors generates faster growth would be premature. If we believe there exists a quality-quantity trade-off in child-rearing [Becker and Lewis (1973); Becker and Barro (1988)],  $^{12}$  educational changes will also generate fertility changes. And education and fertility changes may themselves lead to subsequent changes in biased technologies. So we must move beyond this comparative static analysis to a model that endogenizes both factors and technologies in a general equilibrium framework. That is, by actually endogenizing the microeconomic incentives for researchers and families, we can simulate values for  $dA_l$ ,  $dA_h$ , dL, and dH for a hypothetical economy over time.

#### 3. THE MODEL

## 3.1. Production

Consider a discrete-time economy. We use the production function given by (1), but now we explicitly specify factor-specific technologies. Specifically, production of aggregate output is specified as the following:

$$Y = [(A_l L)^{\sigma} + (A_h H)^{\sigma}]^{1/\sigma}, \tag{6}$$

$$A_{l} \equiv \int_{0}^{M_{l}} \left(\frac{x_{l}(j)}{L}\right)^{\alpha} dj, \qquad A_{h} \equiv \int_{0}^{M_{h}} \left(\frac{x_{h}(k)}{H}\right)^{\alpha} dk. \tag{7}$$

Here both types of labor (unskilled, L, and skilled, H) work with intermediate "machines" to produce a homogenous final output. A machine is designed for use either by skilled labor or unskilled labor, but not both. Machines (of type j) that complement unskilled labor are denoted by  $x_l(j)$ , whereas machines (of type k) that complement skilled labor are denoted by  $x_h(k)$ .

The parameter  $\sigma$  indicates the degree of substitutability between the skilledand unskilled-intensive "sectors" in aggregate production. As mentioned in Section 2.1, estimates of this elasticity clearly place  $\sigma$  above zero; thus we will assume that these sectors are grossly substitutable.

Echoing Kiley (1999) and Acemoglu (2002), technological advance is assumed to come in two varieties. In the "unskilled-labor sector," technical advance comes about from an expansion in the number of intermediate machines specialized for unskilled labor (that is, an increase in  $M_l$ ). Similarly, in the "skilled-labor sector," technical advance means an expansion in the number of intermediate machines specialized for skilled labor (an increase in  $M_h$ ).

Final goods output produced by different firms is identical, and can be used for consumption, for the production of different intermediate machines, and for research and development to expand the varieties of skill-augmenting and unskilled-augmenting machines. For each time period (suppressing time subscripts), these firms endeavor to maximize

$$\max_{\{L,H,x_l(j),x_h(k)\}} Y - w_l L - w_h H - \int_0^{M_l} p(j) x_l(j) dj - \int_0^{M_h} p(k) x_h(k) dk, \quad (8)$$

where p(j) is the price of machine  $x_l(j)$  and p(k) is the price of machine  $x_h(k)$ . Endogenous growth theory suggests that research is generally profit-motivated. It typically assumes that the gains from innovation flow indefinitely, and that this flow depends on the price of the product being produced and the factors required for production at each moment in time. However, not only is this structure unrealistic in models where agents are finitely lived, it is analytically intractable in models where factors evolve endogenously.<sup>13</sup>

To simplify the analysis and also achieve greater realism, we assume that the gains from innovation last one time period only. More specifically, we assume that intermediate machines are produced either in monopolistic or in competitive environments. An inventor of a new machine at time t enjoys monopoly profits for machine production only at t. After this, patent rights expire, and subsequent production of this brand of machine is performed by many competitive manufacturers. Whether a machine is produced monopolistically or competitively will be conveyed in its rental price, denoted either as p(j) for an unskilled labor–using machine j or p(k) for a skilled labor–using machine k, and explained in the next section. Also, for simplicity, we assume that all machines depreciate completely after use, and that the marginal cost of production is simply unity in terms of the final good.

Given technology levels  $M_l$  and  $M_h$  and labor types L and H, an equilibrium can be characterized as machine demands for  $x_l(j)$ 's and  $x_h(k)$ 's that maximize final-good producers' profits (from equation (8)), machine prices p(j) and p(k) that maximize machine producers' profits, and factor prices  $w_l$  and  $w_h$  that clear the labor market.

The first-order conditions for final-good producers yield intermediate-machine demands:

$$x_l(j) = \left[ (A_l L)^{\sigma} + (A_h H)^{\sigma} \right]^{\frac{1-\sigma}{(1-\alpha)\sigma}} A_l^{\frac{1-\sigma}{\alpha-1}} \left( \frac{p(j)}{\alpha} \right)^{\frac{1}{\alpha-1}} L^{\frac{\sigma-\alpha}{1-\alpha}}, \tag{9}$$

$$x_h(k) = \left[ (A_l L)^{\sigma} + (A_h H)^{\sigma} \right]^{\frac{1-\sigma}{(1-\alpha)\sigma}} A_h^{\frac{1-\sigma}{\alpha-1}} \left( \frac{p(k)}{\alpha} \right)^{\frac{1}{\alpha-1}} H^{\frac{\sigma-\alpha}{1-\alpha}}$$

Note that greater levels of employment of a factor raise the demand for intermediate goods augmenting that factor so long as  $\sigma > \alpha$ , an idea consistent with Acemoglu's so-called "market-size" effect. We will assume throughout the analysis that this condition is met.

The other first-order conditions for final-good producers illustrate that workers receive their marginal products:

$$w_l = \left[ (A_l L)^{\sigma} + (A_h H)^{\sigma} \right]^{\frac{1-\sigma}{\sigma}} A_l^{\sigma} L^{\sigma-1}, \tag{10}$$

$$w_h = \left[ (A_l L)^{\sigma} + (A_h H)^{\sigma} \right]^{\frac{1-\sigma}{\sigma}} A_h^{\sigma} H^{\sigma-1}. \tag{11}$$

### 3.2. Research

In this section we describe the growth paths of  $M_l$  and  $M_h$ . Researchers expend resources (rather than time) to develop new types of machines, and these resource costs can change over time.<sup>14</sup> We make this modeling choice to stress that unbalanced growth can occur when research costs differ between different sectors. We will assume that these costs depend both on the number of machine types already extant (indexed by  $M_l$  and  $M_h$ ), and on some factor-specific technology variable (denoted by  $z_l$  and  $z_h$ , and discussed later). Specifically, the up front cost of developing the blueprint of a new machine, c, is given simply by

$$c\left(\frac{M_l}{z_l}\right) = \frac{M_l}{z_l}$$

for an unskilled labor-augmenting machine, and

$$c\left(\frac{M_h}{z_h}\right) = \frac{M_h}{z_h}$$

for a skilled labor–augmenting machine. These functional forms illustrate that the costs of invention are negligible when there is little machine variety. As factor-specific technologies grow, however, costs can become increasingly prohibitive.<sup>15</sup>

Given these costs of technological advance, innovating firms must receive some profits from the development of a new technology in order to make research worth the expense. As mentioned earlier, we assume that developers of new machines

receive monopoly rights to the production and sale of their machines for only one period. As a result, we must make a distinction between *old* machines (those invented before *t*) and *new* machines (those invented at *t*).

Assuming unitary marginal costs of machine production, the excess revenue generated from new machines of both types is given by the "value" functions

$$V_l = (p(j) - 1) x_l(j),$$
  
$$V_h = (p(k) - 1) x_h(k).$$

Because demand is isoelastic, the price which maximizes monopolists' profits equals  $1/\alpha$  for both skilled- and unskilled-augmenting machines, so that demand for *new* intermediate machines (those invented at t) is

$$x_{l,\text{new}}(j) = x_{l,\text{new}} = \alpha^{\frac{2}{1-\alpha}} [(A_l L)^{\sigma} + (A_h H)^{\sigma}]^{\frac{1-\sigma}{(1-\alpha)\sigma}} A_l^{\frac{1-\sigma}{\alpha-1}} L^{\frac{\sigma-\alpha}{1-\alpha}},$$

$$x_{h,\text{new}}(j) = x_{h,\text{new}} = \alpha^{\frac{2}{1-\alpha}} [(A_l L)^{\sigma} + (A_h H)^{\sigma}]^{\frac{1-\sigma}{(1-\alpha)\sigma}} A_h^{\frac{1-\sigma}{\alpha-1}} H^{\frac{\sigma-\alpha}{1-\alpha}}.$$
(12)

On the other hand, because older machines are competitively produced, their prices equal unitary marginal costs, so that demand for old intermediate machines (those invented before t) is simply

$$x_{l,\text{old}}(j) = x_{l,\text{old}} = \alpha^{\frac{1}{1-\alpha}} [(A_l L)^{\sigma} + (A_h H)^{\sigma}]^{\frac{1-\sigma}{(1-\alpha)\sigma}} A_l^{\frac{1-\sigma}{\alpha-1}} L^{\frac{\sigma-\alpha}{1-\alpha}},$$

$$x_{h,\text{old}}(j) = x_{h,\text{old}} = \alpha^{\frac{1}{1-\alpha}} [(A_l L)^{\sigma} + (A_h H)^{\sigma}]^{\frac{1-\sigma}{(1-\alpha)\sigma}} A_h^{\frac{1-\sigma}{\alpha-1}} H^{\frac{\sigma-\alpha}{1-\alpha}}.$$
(13)

Thus factor-specific TFPs given by equation (6) can be rewritten as an aggregation of two kinds of machines, illustrating the cumulation of all past and current innovation. If  $M_{z,\text{old}}$ ,  $M_{z,\text{new}}$ , and  $M_z$  are, respectively, the number of existing old, new, and total machine types used by factor z, we can write factor productivity as

$$A_{l} \equiv \int_{0}^{M_{l}} \left(\frac{x_{l}(j)}{L}\right)^{\alpha} dj = \left[\int_{0}^{M_{l,\text{old}}} x_{l,\text{old}}(j)^{\alpha} dj + \int_{M_{l,\text{old}}}^{M_{l}} x_{l,\text{new}}(j)^{\alpha} dj\right] (1/L)^{\alpha}$$

$$= \frac{M_{l,\text{old}} x_{l,\text{old}}^{\alpha} + M_{l,\text{new}} x_{l,\text{new}}^{\alpha}}{L^{\alpha}},$$
(14)

$$A_{h} \equiv \int_{0}^{M_{h}} \left(\frac{x_{h}(k)}{H}\right)^{\alpha} dk$$

$$= \left[\int_{0}^{M_{h,\text{old}}} x_{h,\text{old}}(k)^{\alpha} dk + \int_{M_{h,\text{old}}}^{M_{h}} x_{h,\text{new}}(k)^{\alpha} dk\right] (1/H)^{\alpha}$$

$$= \frac{M_{h,\text{old}} x_{h,\text{old}}^{\alpha} + M_{h,\text{new}} x_{h,\text{new}}^{\alpha}}{H^{\alpha}}.$$
(15)

Substituting the monopoly price into our value functions yields

$$V_l = \left[\frac{1-\alpha}{\alpha}\right] x_{l,\text{new}},$$

$$V_h = \left\lceil \frac{1-\alpha}{\alpha} \right\rceil x_{h,\text{new}},$$

where  $x_{l,\text{new}}$  and  $x_{h,\text{new}}$  are given by (13). Finally, an individual is free to research, guaranteeing that

$$V_l(L, H, A_l, A_h) \le c \left(\frac{M_{l,\text{old}} + M_{l,\text{new}}}{z_l}\right), \tag{16}$$

$$V_h(L, H, A_l, A_h) \le c \left(\frac{M_{h, \text{old}} + M_{h, \text{new}}}{z_h}\right).$$
(17)

If resource costs of research were actually *less* than discounted profits, entry into research would occur, driving technology levels, and hence costs, up. We assume this happens quickly enough so that valuations never exceed costs in any time period. Further, because applied research is irreversible (a society cannot forget how to make something once it has learned), the variety of machines remains unchanged when the inequalities in (16) or (17) do not bind with equality.

The levels of our technology variables  $z_l$  and  $z_h$  in the economy are key determinants of the costs of developing new "production processes"; higher levels of  $z_u$  lower the costs of developing intermediate machines that complement factor u. Conceivably the evolution of technological variables can be shaped by many things, such as factor endowments, government policies, trade patterns, institutional features, and technological diffusion from other countries. With this in mind, we will consider three cases,

Case 1: Exogenous growth:  $\Delta A_l/A_l = g$  or  $\Delta A_h/A_h = g$ ,

Case 2: Semiendogenous growth:  $\Delta z_l/z_l = g$  or  $\Delta z_h/z_h = g$ ,

Case 3: Endogenous growth:  $\Delta z_l/z_l = \mu H^{\lambda}$  or  $\Delta z_h/z_h = \mu H^{\lambda}$ ,

where 0 < g < 1,  $\mu > 0$ , and  $\lambda > 0$ . That is, we wish to compare the growth prospects of either path, looking at three alternative growth regimes.

A "steady state" can be characterized as one where the share of labor devoted to each sector (skilled and unskilled) remains fixed, whereas output, the technology variables  $z_l$  and  $z_h$ , the varieties of skilled and unskilled machines, and wages all grow at the same rate, g (for Case 2). This will occur so long as equations (16) and (17) hold with strict equality. But as these inequalities imply, there may be a considerable period of time when growth is *unbalanced*; this would occur if only one of the equations held with equality. What kind of unbalanced growth is likely to unfold will depend on a number of things, including the available supply of different factors (a relatively large L, for example, raises  $V_l$  and thus increases

the chance that growth will be unskilled-biased) and the relative "skewness" of the technology variables (a relatively large  $z_l$ , for example, lowers  $c_l$  and likewise increases the chance for unskilled-biased growth).

No doubt unbalanced growth will be slower than balanced steady-state growth ceteris paribus, but it also seems logical that growth in the *bigger* sector (in this case, the unskilled sector) will produce faster growth than growth in the smaller sector (the skilled sector). At the same time, there is wide recognition among development economists of the importance of skill accumulation in economic growth. The centrality of human capital in economic development is so established that most economists now treat education and modernity as going hand in hand. From this perspective, a country's relative abundance in unskilled labor scarcely matters; the skilled-intensive path is the *only* viable path to sustainable progress.

This paper suggests that forces that change the factors of production themselves are an important part of our answer to the question of which is the more appropriate growth path. Specifically, changes in the relative rewards to factors due to technological developments surely will alter the incentives to become educated or to remain an unskilled laborer. From the model we can write the "skill premium," the skilled wage relative to the unskilled wage, as

$$\frac{w_h}{w_l} = \left(\frac{\alpha^{\frac{\alpha}{1-\alpha}} M_{h,\text{old}} + \alpha^{\frac{2\alpha}{1-\alpha}} M_{h,\text{new}}}{\alpha^{\frac{\alpha}{1-\alpha}} M_{l,\text{old}} + \alpha^{\frac{2\alpha}{1-\alpha}} M_{l,\text{new}}}\right)^{\frac{\sigma-\alpha\alpha}{1-\sigma\alpha}} \cdot \left(\frac{H}{L}\right)^{\frac{\sigma-1}{1-\sigma\alpha}}.$$
 (18)

In the absence of any demographic response, skilled-biased technological growth will raise the skill premium (by raising  $M_{h,\text{new}}$ ), whereas unskilled-biased technological growth will lower it (by raising  $M_{l,\text{new}}$ ). But surely if unskilled-intensive growth lowered the relative returns to skill, this would induce some people to remain unskilled. Conversely, increases in the returns to skills should induce individuals to increase human capital, and thus lower fertility rates through quality–quantity trade-offs. Indeed, from the previous section, we suggest that the more responsive these factors are to changes in their relative returns, the more likely will skilled-biased technological growth yield greater income per capita growth. But we have yet to analyze how such demographic responses can influence *subsequent* technological developments. These considerations compel us to merge this growth model with a simple theory of demography. The next sections do precisely that.

# 3.3. Endogenous Demography

To capture the symbiotic relationship between technologies and factors, we introduce households into the model in an overlapping-generations framework, where individuals have two stages of life: young and old. Only old people are allowed to make any decisions regarding demography. Specifically, the representative household is run by an adult who maximizes her utility by deciding two things: how many children to have (denoted by n) and the fraction of these children that will receive an education (denoted by e).

An individual born at time t works either as an unskilled laborer (earning  $w_l$ ) or as a skilled laborer (earning  $w_h$ ). The individual becomes old at t + 1. At this point she decides how many children to have herself, and the fraction of these children that will get an education and work as skilled workers.

Specifically, individuals wish to maximize both their own income and the income of their children. <sup>18</sup> Let utility for the household planner be described by the function

$$U = w_i (1 - c(n, e)) + ln [w_l (1 - e)n + w_h en],$$

where  $c(\cdot)$  is the function denoting child-rearing costs, and  $w_j$  is the wage of the parent (who could be either a skilled worker or an unskilled worker, depending on what *her* parent chose for her last time period, so j = l, h). A fraction (1 - e) of young work as unskilled workers, whereas a fraction e of young work as skilled workers. This quasi-linear utility form simply conveys that adults face diminishing returns to enjoyment in their children's income, but not in their own.<sup>19</sup>

The first-order condition for the number of children is

$$\frac{1}{n} = w_j c_n, \tag{19}$$

where  $c_n$  is the derivative of the cost function with respect to fertility. The left-hand side illustrates the marginal benefit of an additional child (which falls with the total number of children), whereas the right-hand side denotes the marginal cost (the income foregone to raise an additional child).

The first-order condition for education is

$$\frac{w_h}{w_l(1-e) + w_h e} = \frac{w_l}{w_l(1-e) + w_h e} + w_j c_e,$$
 (20)

where  $c_e$  is the derivative of the cost function with respect to education. Again, the left-hand side is the marginal benefit and the right-hand side the marginal cost. At the optimum, the gains received from the added skilled income offsets the foregone unskilled and adult income requisite for giving more children an education.

Note that the results of this simple optimization problem are consistent with the negative correlations between income and fertility and between education and fertility that are observed in developing countries [Kremer and Chen (2002)]. For example, rising skilled wages induce households to increase education; the rise in child-rearing costs this produces, however, will also incentivize households to decrease fertility.

Completing the model requires us to relate fertility and education rates to aggregate levels of unskilled labor and skilled labor. At time t, labor types are given by

$$L = N_t(1 - e_{t-1}) (1 - c(n_t, e_t)) + N_t n_t (1 - e_t),$$
(21)

$$H = N_t e_{t-1} (1 - c(n_t, e_t)) + N_t n_t e_t,$$
(22)

where *N* is simply the adult population. Note that each type of labor is composed of both young and old workers, and that old workers spend only a fraction of their

time in the workforce (spending the rest of their time raising children). Finally, population growth is given by

$$N_t = n_{t-1} N_{t-1}. (23)$$

Combining this model of demography with our model of biased technologies is straightforward. Through the simultaneous solving of (10), (11), (14), (15), (16), (17), (19), (20), (21), and (22), a unique set of variables  $w_l$ ,  $w_h$ ,  $A_l$ ,  $A_h$ ,  $M_l$ ,  $M_h$ , n, e, L, and H can be determined for every time period. <sup>20</sup> We can perhaps synopsize our findings by initially focusing only on the economy's choice of e and  $M_h$ . If an adult expects researchers to develop new skilled-biased technologies (and so to increase  $w_h$ ), he or she will want to endow his or her children with more human capital. Similarly, if researchers anticipate a larger pool of human capital, they may wish to invent and build new skill-intensive machines, raising  $M_{h,\text{new}}$  and thus  $M_h$ overall. Consequently we can plot the two "reaction functions" of each group as two upward-sloping curves; the development of new skill-using machines and the accumulation of skills are strategic complements. From the intersection of these reaction curves, we find the unique simultaneous solution of the level of education and the new skilled-biased technical coefficient. This is done in Figure 4. We can similarly plot two upward-sloping curves to determine an economy's choice of nand  $M_1$ .

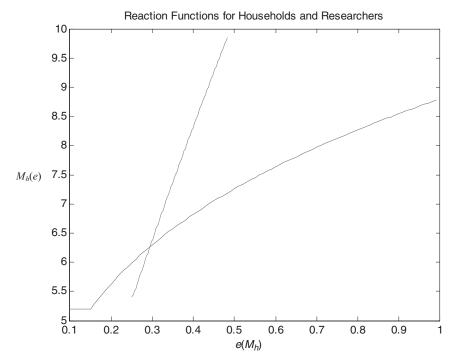
To summarize, potential researchers look to the skill composition of the workforce (influenced by households) to determine the direction and scope of technical change. Households look to wages (influenced by researchers) to determine the levels of skilled and unskilled workers. Together they jointly determine the overall composition of the economy.

# 4. "APPROPRIATE" GROWTH PATHS FOR A DEVELOPING COUNTRY—SOME SIMULATIONS<sup>21</sup>

With a model that endogenizes both technologies and factors, we may better assess the appropriateness of alternative development paths. Let us consider a hypothetical developing country endowed with a fairly sizable amount of unskilled labor and a modest amount of skilled labor. We will set initial conditions such that n > 1 (to represent a growing population) and  $e_{t-1} = e_t$  (to represent a stable education rate). We can then test the effects of unbalanced growth on incomes per capita ( $y_t = Y_t/N_t$ ) by allowing either only unskilled-labor technology or only skilled-labor technology to rise, run the "horse-race," and compare the two paths.<sup>22</sup> Each simulation is run for 30 time periods.

# 4.1. Case 1: Simulation with Exogenous Unbalanced Growth<sup>23</sup>

Our first horse-race simply has either  $A_l$  or  $A_h$  grow exogenously, and compares the two paths. In other words, we ignore our discussion about endogenous technical growth in Section 3.2 for the moment, and assume that unbalanced growth happens



**FIGURE 4.** "Reaction curves." The steeper line represents the fraction of educated young a parent would choose for a given technological parameter  $M_h$ . The flatter curve represents the skilled-biased technical coefficient that would result from a given fraction of educated young.

simply as an exogenous process, such as through technological diffusion from other countries (see the Appendix for the system of equations being solved each time period). Specifically, each technological parameter grows 5% each period.

Figure 5 illustrates the results of these simulations. The lighter lines are where only  $A_l$  grows; darker lines are where only  $A_h$  grows. Growth in both cases lowers fertility, because it raises the opportunity costs to raise children. However, it is clear that skilled-biased growth lowers fertility more dramatically, because it induces families to provide more of their offspring with education; this raises the costs of children even further. Unskilled-biased growth, on the other hand, puts downward pressure on skill premia, exerting upward pressure on fertility and downward pressure on education.<sup>24</sup>

We can see how these demographic shifts affect the factors of production. Initial fertility rates above one induce increases in both labor types; once fertility falls below one, both labor types begin to fall. It is also clear that unbalanced growth creates changes in *relative* factors. We can see that H/L falls with unskilled-biased growth, and rises with skilled-biased growth. Recall from our discussion in Section 2.2 that the latter means there is relative growth in the more productive

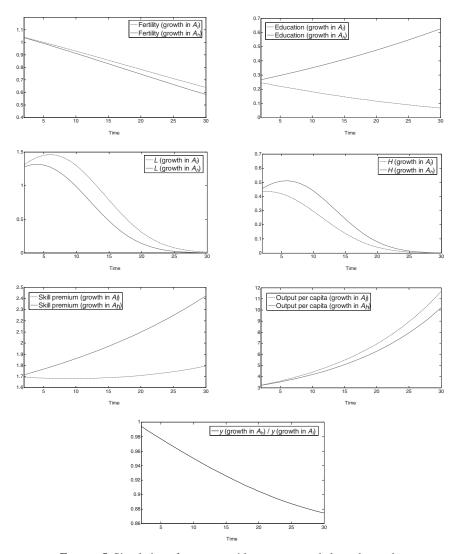


FIGURE 5. Simulation of economy with exogenous unbalanced growth.

factor (H is more productive even though we start with  $A_l = A_h$  because it is more scarce than L), and this should be a boost to overall income. On top of this, the overall population grows faster with unskilled-biased growth than with skilled-biased growth.

And yet despite these apparent benefits of skilled-biased growth, over time it actually generates *less* income per person than unskilled-biased growth. The final two graphs compare simulated per capita GDPs (y = Y/N) for the two paths. Why does skilled-biased growth underperform? Consistent with our earlier

discussions, skilled-biased technologies are not "appropriate," in the sense that they augment a relatively smaller workforce, so unskilled-biased technical growth generates relatively more overall growth. The roughly 3:1 ratio of unskilled to skilled workers assumed as the initial condition here is a conservative one; larger ratios would generate even greater divergence between the two growth paths.<sup>25</sup> Thus for reasonable skilled and unskilled labor endowments, unskilled-biased growth appears the more appropriate, despite the ostensibly negative effects of higher fertility and lower education. This result is also robust to different starting values of fertility and education (see the Appendix, where initial fertility is higher).

Note that with our quasi-linear utility specification, economic growth serves as a form of birth control—increases in income, whether from unskilled- or skilled-biased technologies, induce parents to have fewer children. This raises a question—what kinds of feedback effects will falling fertility have on *endogenous* technological growth? Do our conclusions change?

# 4.2. Case 2: Simulation with Semiendogenous Unbalanced Growth<sup>26</sup>

In this case we use the full system of equations that jointly solve for technological levels and for demographic variables. Technologies in this case are "semiendogenous," in that we have research costs exogenously fall in order to observe the endogenous technological and demographic responses (see the Appendix for the full system of equations). Specifically, for unskilled-intensive growth, we set  $z_l$  such that  $V_l = c_l$  at the start of the simulation. Then we simply have  $z_l$  grow 5% each period, inducing research that produces new varieties of unskilled labor-using machines. For skilled-intensive growth we do the same to  $z_h$ ,  $V_h$  and  $c_h$ .

Figure 6 illustrates the results of these two simulations. In both cases unbalanced technological growth generates a robust demographic transition, lowering fertility and raising living standards. But soon thereafter, growth in y slows dramatically; further, skilled-biased growth actually lowers income per capita, whereas unskilled-biased growth continues to raise it, albeit slightly. Thus it seems that in this case the skilled-biased path is very inappropriate. This may be surprising, because as in Case 1 unskilled-intensive growth lowers H/L, and H is the more productive factor because of its relative scarcity. And these results once again buck conventional wisdom in development economics, which suggests that fertility declines should bolster per capita income, and that declines in education can be destructive for long-term prosperity.

In this case, however, the fall in population due to the demographic transition contributes to subsequent technological stagnation in two ways. First, it induces final-goods producers to demand and use fewer *existing* machines, making workers less productive (equation (9)). Second, it shrinks the scale of the market for *new* innovation (equations (16) and (17)). These perverse effects on factor productivities offset the exogenous decreases in research costs, creating economic stagnation. Because unskilled-biased growth puts some upward pressure on fertility (after it

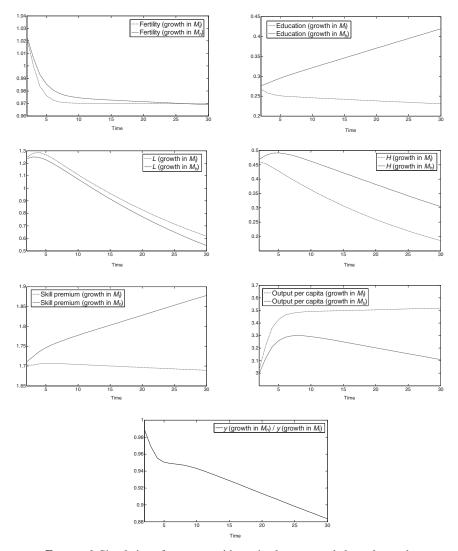


FIGURE 6. Simulation of economy with semiendogenous unbalanced growth.

lowers it significantly because of large productivity gains early on), the damage to technological progress is less severe.

A lesson here is that fertility declines, although inevitable in the process of economic development, can hurt subsequent growth when technologies are endogenous. Scale matters in this case, as indeed it does in nearly all semiendogenous or endogenous growth models, because researchers require a large group of workers to purchase and use their new machines to recoup their fixed costs.<sup>27</sup> Because

unskilled labor-biased growth limits the decline in fertility, it limits such negative effects in market size.

## 4.3. Case 3: Simulation with Endogenous Unbalanced Growth<sup>28</sup>

Although the previous case implies that unskilled-intensive growth produces more per capita income in the long run than the alternative, we should acknowledge that this case is based on the assumption that growth in  $z_l$  or  $z_h$  is exogenous. If in fact such variables can rise only through a skilled workforce (for example, through a higher capacity to innovate [Nelson and Phelps (1966)], or through human capital externalities [Lucas (1988)]), the deskilling effects of unskilled-intensive growth may hinder overall growth. That is, perhaps our previous cases underemphasize the importance of the skilled-intensive path?

Our final case explores this by assuming that growth in  $z_l$  or  $z_h$  is given by the endogenous process suggested in Section 3.2, where human capital generates beneficial spillovers to knowledge creation. Again, we allow for growth only in  $z_l$  or  $z_h$ , and compare trends created by each in Figure 7. Perhaps surprisingly, after the initial burst of economic growth from technological progress and demographic transition, per capita GDPs fall in both cases! Now the demographic transition adversely affects growth through two channels. The first is the same as before—factor productivities stagnate or outright shrink due to falling population. The second is a slowdown in growth in  $z_l$  and  $z_h$  due to declines in H. For a certain interval of time, skilled-biased growth generates faster overall growth than unskilled-biased growth; increases in  $z_h$  spur increases in education, and this creates a virtuous cycle of more human capital and technological progress. But through the quality-quantity trade-off, it also generates a dramatic drop in fertility. Ultimately the declines in fertility outweigh the increases in education, such that overall human capital falls. Because this happens more dramatically with skilled-biased growth, income prospects deteriorate faster in this case.

At this point one could perhaps suggest that human capital externalities simply need to be sufficiently strong in for the skilled-biased path to generate more growth (that is, have a larger  $\mu$  and/or  $\lambda$ ). But the dynamics remains the same (results not shown). A larger  $\lambda$ , for example, generates more income growth over the skilled-intensive path early on, but also produces more dramatic decreases in fertility and thus *worse* growth prospects later on.

Cases 2 and 3 suggest an educational version of the "paradox of thrift." As skilled-biased technological growth incentivizes individual households to raise education levels, aggregate levels of human capital can outright shrink. The classic paradox suggests that individuals who raise their rates of savings can end up depressing aggregate demand and therefore lower savings and output [Keynes (1935)]. Similarly, individuals here can raise their rates of education only by lowering their rates of fertility; this ends up depressing technological growth and therefore lowers aggregate skilled labor and output.

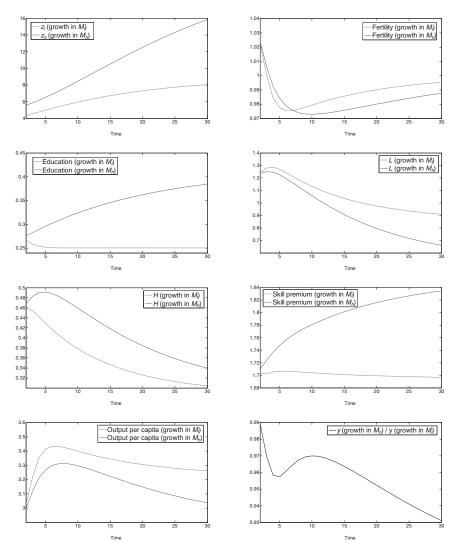


FIGURE 7. Simulation of economy with endogenous unbalanced growth.

Of course there may be other longer-term consequences of unskilled-intensive growth that we are not considering here. For example, it may become harder to switch to the skilled-intensive path the longer the economy remains on the unskilled-intensive path. Indeed, where multiple equilibria exist, the economy may fall into a poverty-trap where children work full time and never get an education [Bell and Gersbach (2009)]. This paper suggests that there are mediumrun benefits for developing economies in taking the unskilled-intensive route, without ruling out such potential longer-run consequences.

## 5. CONCLUSION

This paper models the simultaneity of factors and technologies to evaluate different growth paths. Unlike approaches that credit either technological progress [Christensen and Cummings, (1981)] or factor accumulation [Young (1995)] alone for economic success, the interaction of both can lend us new insights into which development path will breed the greatest rewards.

We see that the answer depends on the structure of the macro economy. Generally, a skilled-intensive path will generate more benefits the more plentiful skilled labor already is. It also produces more benefits the more *responsive* are factors to technological changes, *provided* there is no or limited feedback on technologies from these changes. This is because the falling population growth caused by skilled-intensive growth, normally a boom to income per capita, would hurt economic growth if technologies are endogenous. Thus skilled-biased technological diffusion, of the kind generated by the worldwide pervasiveness of skilled-intensive technologies [Berman et al. (1998); Berman and Machin (2000)] can generate robust growth *because* of its exogenous nature, somewhat offsetting its apparent inappropriateness because of a low endowment of H.

Yet in general, unskilled-intensive growth appears to be the best path for developing counties. Not only does it militate against factor income inequality (a focus of previous directed technical change papers), but also it actually produces greater *aggregate* wealth for a variety of growth scenarios. Even developed countries may wish for developing regions to be more productive in unskilled-intensive production because of greater trade potential that can benefit both regions [Fernandes and Kumar (2007)].

The bottom line is that the proper path to macro prosperity depends on lots of things—here we provide only the broadest brushstrokes delineating some major concerns. This subject, however, is relevant to all developing nations. Should India focus more on labor-heavy manufacturing or skill-heavy services? Should China's fiscal stimulus channel resources to build infrastructure using skill-intensive or labor-intensive techniques? Questions such as these dominate discussions over macroeconomic strategy in these countries; the answers will depend on some of the issues raised here.

## **NOTES**

- 1. Ray (2010) suggests that the typical abstraction of balanced growth is both unrealistic and not particularly helpful for many important questions in development.
  - 2. This example comes from Caselli and Coleman (2006)
- 3. Papers that do consider interactions between technology and human capital include Stokey (1988), Chari and Hopenhayn (1991), Grossman and Helpmann (1991), Young (1993), Redding (1996), Galor and Moav (2000), and Galor and Weil (2000). None, however, assess the appropriate path to development for an economy in the context of such simultaneity.
- 4. Galor and Mountford (2006) stress changes in fertility in explaining divergent growth paths in history.

- 5. According to Temple (2003), "A balanced growth path is a special case which is likely to demand restrictive assumptions..it would be a mistake to assume that a good model of growth necessarily gives rise to a balanced growth path."
- 6. Acemoglu (1998) relegates the possibility of endogenously determined human capital in the Appendix to his paper, whereas he does not discuss the possibility either in Acemoglu (2002) or in the chapter on directed technical change in his growth textbook [Acemoglu (2008)].
- 7. This functional form resembles the production function used in Section 3, where we endogenize technological growth; efficiency coefficients will proxy for the breadth and depth of factor-complementary machines.
- 8. The data are also from Caselli and Coleman (2006). Y is average GDP per capita for 1985–1990, taken from the Penn World Tables. Labor levels are constructed using the implied Mincerian coefficients from Bils and Klenow (2000). Wages for skilled and unskilled are constructed using Mincerian coefficients and the duration in years of the various schooling levels. See their paper for more details.
- 9. Ciconne and Peri (2005) themselves estimate  $\sigma$  to be 0.5 when considering U.S. high school dropouts as unskilled labor and high school graduates as skilled labor (although their preferred measure is 0.33).
- 10. See, for example, Graham and Temple (2003), Rustuccia (2004), Chanda and Dalgaard (2008), and Vollrath (2009).
- 11. Note that only when  $\sigma > 0$  can we consider  $A_l$  unskilled-biased and  $A_h$  skilled-biased. This is a reasonable assumption, given previous estimates of  $\sigma$ . See Acemoglu (2002) for a fuller discussion.
- 12. This is also stressed by Galor and Mountford (2006, 2008). In these papers population changes come from trade specialization patterns. Technological changes, however, can affect demographic patterns in very similar ways.
- 13. For example, the seminal Romer (1990) model describes the discounted present value of a new invention as a positive function of  $L L_R$ , where L is the total workforce and  $L_R$  are the number of researchers. Calculating this value function is fairly straightforward if labor supplies of production workers and researchers are constant. If they are not, however, calculating the true benefits to the inventor may be difficult.
  - 14. This echoes Rivera-Batiz and Romer (1991)'s "lab-equipment" assumption for research.
- 15. This approach of varying the cost of research echoes the leader–follower model illustrated in Barro and Xala-i-Martin 2003), where costs depend on the distance from the frontier of general knowledge. We can thus alternatively consider  $c(\cdot)$  the cost of *adopting* technologies from a pool of global knowledge. This interpretation is most relevant for undeveloped regions, where "research" often amounts to amending foreign-developed technologies to make them more appropriate for local conditions.
- 16. If  $\Delta a/a = g$  and  $\Delta b/b = 0$ ,  $\Delta (a+b)/a + b = \Delta a/a + b$ , which is smaller than, but converges to, g. The smaller is b relative to a, the closer will this growth be to g.
- 17. By one recent study's account, "Anything that harms the accumulation of human capital harms our economic well-being" [Remler and Pema (2009)]. For a brief history of the study of human capital, see Ehrlich and Murphy (2007).
- 18. This echoes Moav (2005), who models parents that decide both the number of children and the level of human capital of each child in order to simply maximize their potential income.
- 19. Such quasi-linear utility functions to model demography have been used by, among others, Kremer and Chen (2002) and Weisdorf (2007). Further, incorporating child-rearing costs strictly as opportunity costs faced by the parent is standard in this literature [see Galor and Weil (2000)].
  - 20. This 10-by-10 system is reiterated with more detail in the Appendix.
- 21. Note that for the lessons of the simulations to hold, we require only that  $0 < \alpha < \sigma < 1$ . This simply means that factors of production must be substitutable "enough." Specifically, we assume that  $\alpha = 0.33$  and  $\sigma = 0.5$ . We also specify the simple cost function  $c(n, e) = \gamma \left(n^2 + (ne)^2\right)$ .  $\gamma$  is set to ensure that costs rise in both n and e, and that c(n, e) remains bounded between 0 and 1. Results are not sensitive to the precise form of c, so long as costs are convex in n and e.

- 22. This is strictly a comparison of growth paths. For interesting works on *shifts* from one path to another see Yuki (2008) and Valente (2011).
- 23. We wish to establish plausible initial conditions for our hypothetical economy. Figure 1 strongly suggests that countries with large relative endowments of unskilled labor tend also to have low relative levels of skilled-biased technologies. The high skill premia we observe in developing countries are therefore mostly attributable to low relative endowments of skilled labor and not to high levels of skilled-biased technologies. With this in mind, we set initial values of  $A_l$  and  $A_h$  to 1 and normalize N to 1. For this and the next two cases, we set  $\gamma = 0.5$ ; this gives an initial equilibrium where  $n \approx 1.05$  and  $e \approx 0.25$ , producing initial factor endowments of L = 1.3 and H = 0.45, and initial wages of  $w_l = 1.59$  and  $w_h = 2.7$ .
- 24. Note that income growth lowers fertility because of quasi-linear preferences. What is important here is the *comparison* between fertility changes generated by the two growth paths. The analysis would thus be consistent for alternative functions. For example, Cobb-Douglas preferences over consumption and aggregative income of children [used, for example, in Galor and Weil (2000)] would generate *rising* fertility with unskilled-biased technological growth, and *falling* fertility with skilled-biased technological growth.
- 25. For example, if we use the Caselli and Coleman (2006) country estimates for skilled workers (defined as those with secondary schooling or more) and for unskilled workers (those with anything less than secondary schooling), the world average over all countries is roughly 4 to 1, with obviously much higher ratios for developing countries.
- 26. For this case initial levels of machines are set to  $M_l = 0.87$  and  $M_h = 0.75$ . This gives us initial values of  $A_l$  and  $A_h$  of 1, thus giving us the same initial fertility, education, wages, and factors as in Case 1.
- 27. More specifically, in such seminal endogenous growth models as Romer (1986, 1990), Segerstrom et al. (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992), a smaller labor force implies lower growth of technology. In "semiendogenous" growth models such as those of Jones (1995), Young (1998), and Howitt (1999), a smaller labor force implies a lower level of technology.
- 28. Initial values are the same as in Case 2.  $\lambda = 0.1$ , and  $\mu$  is set so that growth in  $z_l$  or  $z_h$  starts at 5% per time period.

#### REFERENCES

Acemoglu, Daron (1998) Why do new technologies complement skills? Directed technical change and wage inequality. *Quarterly Journal of Economics* 113, 1055–1089.

Acemoglu, Daron (2002) Directed technical change. Review of Economic Studies 69, 781-809.

Acemoglu, Daron (2008) Introduction to Modern Economic Growth. Princeton, NJ: Princeton University Press.

Acemoglu, Daron (2010) Theory, general equilibrium, and the political economy in development economics. *Journal of Economic Perspectives* 24, 17–32.

Acemoglu, Daron and Fabrizio Zilibotti (2001) Productivity differences. Quarterly Journal of Economics 116, 563–606.

Aghion, Philippe and Peter Howitt (1992) A model of growth through creative destruction. *Econometrica* 60, 323–351.

Autor, David, Lawrence Katz, and Alan Krueger (1998) Computing inequality: Have computers changed the labor market? *Quarterly Journal of Economics* 113, 1169–1213.

Basu, Susanto and David N. Weil (1998) Appropriate technology and growth. Quarterly Journal of Economics 113, 1025–1054.

Barro, Robert J. (2001) Human capital and growth. American Economic Review Papers and Proceedings 91, 12–17.

Barro, Robert J. and Xavier Sala-i-Martin (2003) *Economic Growth*, 2nd ed. Cambridge, MA: MIT Press.

- Becker, Gary S. and Robert J. Barro (1988) A reformulation of the economic theory of fertility. Quarterly Journal of Economics 103, 1–25.
- Becker, Gary S. and H. Gregg Lewis (1973) On the interaction between the quantity and quality of children. *Journal of Political Economy* 81, S279–S288.
- Bell, Clive and Hans Gersbach (2009) Child labor and the education of a society. *Macroeconomic Dynamics* 13, 220–249.
- Berman, Eli, John Bound, and Stephen Machin (1998) Implications of skill-biased technological change: International evidence. *Quarterly Journal of Economics* 113, 1245–1279.
- Berman, Eli and Stephen Machin (2000) Skill-Biased Technology Transfer: Evidence of Factor-Biased Technological Change in Developing and Developed Countries. Unpublished manuscript, University of California, San Diego.
- Bils, Mark and Peter J. Klenow (2000) Does schooling cause growth? *American Economic Review* 90, 1160–1183.
- Caselli, Francesco and Wilbur J. Coleman II (2006) The world technology frontier. American Economic Review 96, 499–522.
- Chanda, Areendam and Carl Johan Dalgaard (2008) Dual economies and international total factor productivity differences. *Economica* 75, 629–661.
- Chari, V. V. and Hugo Hopenhayn (1991) Vintage human capital, growth, and the diffusion of new technology. *Journal of Political Economy* 99, 1142–1165.
- Christensen, Laurits R. and Diane Cummings (1981) Real product, real factor input, and productivity in the Republic of Korea, 1960–1973. *Journal of Development Economics* 8, 285–302.
- Ciccone, Antonio and Giovanni Peri (2005) Long-run substitutability between more and less educated workers: Evidence from U.S. states 1950–1990. Review of Economics and Statistics 87, 652– 663
- Ehrlich, Issac and Kevin M. Murphy (2007) Why does human capital need a journal? *Journal of Human Capital* 1, 1–5.
- Fernandes, Ana and Krishna B. Kumar (2007) Inappropriate technology. *Macroeconomic Dynamics* 11, 487–518.
- Galor, Oded and Omer Moav (2000) Ability-biased technological transition, wage inequality and growth. *Quarterly Journal of Economics* 115, 469–498.
- Galor, Oded and Andrew Mountford (2006) Trade and the great divergence: The family connection. American Economic Review Papers and Proceedings 96, 299–303.
- Galor, Oded and Andrew Mountford (2008) Trade and the great divergence: Theory and evidence *Review of Economic Studies* 75, 1143–1179.
- Galor, Oded and David N. Weil (2000) Population, technology, and growth: From Malthusian stagnation to the demographic transition and beyond. *American Economic Review* 90, 806–828.
- Graham, Bryan S. and Jonathan Temple (2003) Rich nations, poor nations: How much can multiple equilibrium explain? *Journal of Economic Growth* 11, 5–41.
- Grossman, Gene M. and Elhanan Helpmann (1991) *Innovation and Growth in the Global Economy*. Cambridge, MA: MIT Press.
- Howitt, Peter (1999) Steady endogenous growth with population and R&D inputs growing. *Journal of Political Economy* 107, 715–730.
- Jones, Charles I. (1995) R&D-based models of economic growth. *Journal of Political Economics* 103, 759–784.
- Katz, Lawrence F. and Kevin M. Murphy (1992) Changes in relative wages, 1963–1987: Supply and demand factors. Quarterly Journal of Economics 107, 35–78.
- Keynes, John M. (1935) *The General Theory of Employment, Interest, and Money*. New York: Harcourt, Brace and Co.
- Kiley, Michael T. (1999) The supply of skilled labor and skill-biased technological progress. Economic Journal 109, 708–724.
- Kremer, Michael and Daniel L. Chen (2002) Income distribution dynamics with endogenous fertility. *Journal of Economic Growth* 7, 227–258.

- Lucas, Robert (1988) On the mechanics of economic development. Journal of Monetary Economics 22, 3–42.
- Moav, Omer (2005) Cheap children and the persistence of poverty. Economic Journal 115, 88-110.
- Nelson, Richard, and Edmund Phelps (1966) Investment in humans, technological diffusion, and economic growth. *American Economic Review* 61, 69–75.
- Owen, Ann L., Julio Videras, and Lewis Davis (2009) Do all countries follow the same growth process? Journal of Economic Growth 14, 265–286.
- Ray, Debraj (2010) Uneven growth: A framework for research in development economics. *Journal of Economic Perspectives* 24, 45–60.
- Redding, Stephen (1996) The low-skill, low-quality trap: Strategic complementarities between human capital and research and development. *Economic Journal* 106, 458–470.
- Remler, Dahlia K. and Elda Pema (2009) Why Do Institutions of Higher Education Reward Research While Selling Education? NBER working paper 14974.
- Restuccia, Diego (2004) Barriers to capital accumulation and aggregate total factor productivity. *International Economic Review* 45, 225–238.
- Rivera-Batiz, Luis A. and Paul M. Romer (1991) Economic integration and endogenous growth. *Quarterly Journal of Economics* 106, 531–555.
- Romer, Paul M. (1986) Increasing returns and long-run growth. *Journal of Political Economy* 94, 1002–1037.
- Romer, Paul M. (1990) Endogenous technological change. *Journal of Political Economy* 98, S71–S102.
  Segerstrom, Paul S., T. C. A. Anant, and Elias Dinopoulos (1990) A Schumpeterian model of the product life cycle. *American Economic Review* 80, 1077–1092.
- Stokey, Nancy L. (1988) Learning by doing and the introduction of new goods. *Journal of Political Economy* 96, 701–717.
- Temple, Jonathan (2003) The long-run implications of growth theories. *Journal of Economic Surveys* 17, 497–510.
- Valente, Simone (2011) Endogenous growth, backstop technology adoption, and optimal jumps. Macroeconomic Dynamics 15, 293–325.
- Vollrath, Dietrich (2009) How important are dual economy effects for aggregate productivity? *Journal of Development Economics* 88, 325–334.
- Weisdorf, Jacob (2007) Malthus revisited: Fertility decision making based on quasi-linear preferences. *Economic Letters* 99, 127–130.
- Xu, Bin (2001) Endogenous Technology Bias, International Trade, and Relative Wages. University of Florida working paper.
- Young, Alwyn (1993) Substitution and complementarity in endogenous innovation. *Quarterly Journal of Economics* 108, 775–807.
- Young, Alwyn (1995) The tyranny of numbers: Confronting the statistical realities of the East Asian growth experience. *Quarterly Journal of Economics* 110, 641–680.
- Young, Alwyn (1998) Growth without scale effects. *Journal of Political Economy* 106, 41–63.
- Yuki, Kazuhiro (2008) Sectoral shift, wealth distribution, and development. *Macroeconomic Dynamics* 12, 527–559.
- Zeira, Joseph (1998) Workers, machines, and economic growth. Quarterly Journal of Economics 113, 1091–1117.

# APPENDIX A: SIMULATIONS

First, for all simulations, we assume that the parent's wage,  $w_j$ , is simply an average of unskilled and skilled wages (specifically, we assume it is  $w_l^{0.5}w_h^{0.5}$ ). This simplifying assumption allows us to treat all parents as the same. Alternatively, we could have skilled

parents earn  $w_h$  and unskilled parents earn  $w_l$ . This would complicate the analysis without altering any of the qualitative results.

If technologies are exogenously determined, the simulation predetermines  $A_l$  and  $A_h$ , and solves the following system of equations for  $w_l$ ,  $w_h$ ,  $n_t$ ,  $e_t$ , L, and H for each time period t, given  $n_{t-1}$ ,  $e_{t-1}$ , and  $N_t$ :

$$w_l = \left[ (A_l L)^{\sigma} + (A_h H)^{\sigma} \right]^{\frac{1-\sigma}{\sigma}} A_l^{\sigma} L^{\sigma-1}, \tag{A.1}$$

$$w_h = \left[ (A_l L)^{\sigma} + (A_h H)^{\sigma} \right]^{\frac{1-\sigma}{\sigma}} A_h^{\sigma} H^{\sigma-1}, \tag{A.2}$$

$$\frac{1}{n_t} = 2\gamma w_j \left( n_t + n_t e_t^2 \right), \tag{A.3}$$

$$\frac{w_h}{w_l(1-e_t) + w_h e_t} = \frac{w_l}{w_l(1-e_t) + w_h e_t} + 2\gamma w_j n_t^2 e_t,$$
 (A.4)

$$L = N_t (1 - e_{t-1} (1 - \{ \gamma [n_t^2 + (n_t e_t)^2] \}) + N_t n_t (1 - e_t),$$
(A.5)

$$H = N_t e_{t-1} \left( 1 - \left\{ \gamma \left[ n_t^2 + (n_t e_t)^2 \right] \right\} \right) + N_t n_t e_t.$$
 (A.6)

If, on the other hand, technologies are semiendogenously or endogenously determined by the process discussed in Section 3.2, the following equations are solved for  $M_{l,\text{new}}$ ,  $M_{h,\text{new}}$ ,  $A_l$ ,  $A_h$ ,  $w_l$ ,  $w_h$ ,  $n_t$ ,  $e_t$ , L, and H for each time period t, given  $n_{t-1}$ ,  $e_{t-1}$ ,  $N_t$ ,  $z_l$  and  $z_h$ :

$$\left(\frac{1-\alpha}{\alpha}\right)\alpha^{\frac{2}{1-\alpha}}\left[\left(A_{l}L\right)^{\sigma}+\left(A_{h}H\right)^{\sigma}\right]^{\frac{1-\sigma}{(1-\alpha)\sigma}}A_{l}^{\frac{1-\sigma}{\alpha-1}}L^{\frac{\sigma-\alpha}{1-\alpha}}\leq \left(\frac{M_{l,\text{old}}+M_{l,\text{new}}}{z_{l}}\right),\tag{A.7}$$

$$\left(\frac{1-\alpha}{\alpha}\right)\alpha^{\frac{2}{1-\alpha}}\left[(A_lL)^{\sigma}+(A_hH)^{\sigma}\right]^{\frac{1-\sigma}{(1-\alpha)\sigma}}A_h^{\frac{1-\sigma}{\alpha-1}}H^{\frac{\sigma-\alpha}{1-\alpha}} \leq \left(\frac{M_{h,\text{old}}+M_{h,\text{new}}}{z_h}\right) \tag{A.8}$$

$$A_{l} = \left[\alpha^{\frac{\alpha}{1-\alpha}} M_{l,\text{old}} + \alpha^{\frac{2\alpha}{1-\alpha}} M_{l,\text{new}}\right] \left( (A_{l}L)^{\sigma} + (A_{h}H)^{\sigma} \right)^{\frac{(1-\sigma)\alpha}{(1-\alpha)\sigma}} A_{l}^{\frac{(\sigma-1)\alpha}{1-\alpha}} L^{\frac{\alpha(\sigma-1)}{1-\alpha}}, \tag{A.9}$$

$$A_h = \left[\alpha^{\frac{\alpha}{1-\alpha}} M_{h,\text{old}} + \alpha^{\frac{2\alpha}{1-\alpha}} M_{h,\text{new}}\right] \left( (A_l L)^{\sigma} + (A_h H)^{\sigma} \right)^{\frac{(1-\sigma)\alpha}{(1-\alpha)\sigma}} A_h^{\frac{(\sigma-1)\alpha}{1-\alpha}} H^{\frac{\alpha(\sigma-1)}{1-\alpha}}, \quad (\mathbf{A.10})$$

$$w_l = [(A_l L)^{\sigma} + (A_h H)^{\sigma}]^{\frac{1-\sigma}{\sigma}} A_l^{\sigma} L^{\sigma-1}, \tag{A.11}$$

$$w_h = \left[ (A_l L)^{\sigma} + (A_h H)^{\sigma} \right]^{\frac{1-\sigma}{\sigma}} A_h^{\sigma} H^{\sigma-1}, \tag{A.12}$$

$$\frac{1}{n_t} = 2\gamma w_j \left( n_t + n_t e_t^2 \right), \tag{A.13}$$

$$\frac{w_h}{w_l(1-e_l)+w_he_l} = \frac{w_l}{w_l(1-e_l)+w_he_l} + 2\gamma w_j n_i^2 e_t,$$
 (A.14)

$$L = N_t(1 - e_{t-1}) \left( 1 - \left\{ \gamma \left[ n_t^2 + (n_t e_t)^2 \right] \right\} \right) + N_t n_t (1 - e_t), \tag{A.15}$$

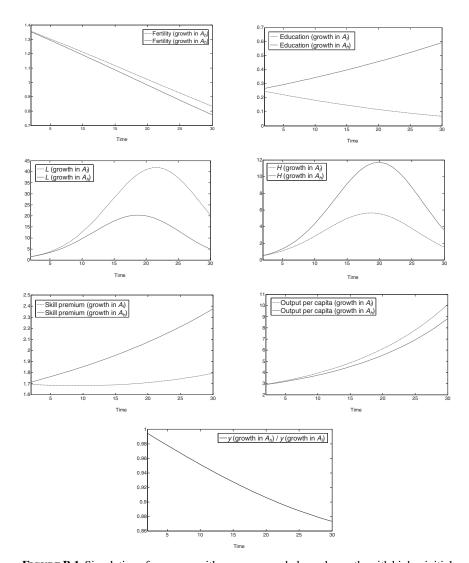
$$H = N_t e_{t-1} \left( 1 - \left\{ \gamma \left[ n_t^2 + (n_t e_t)^2 \right] \right\} \right) + N_t n_t e_t.$$
 (A.16)

Equations (A.7) and (A.8) illustrate the benefits and costs of innovation; (A.9) and (A.10) are factor-specific TFP levels as functions of the demand for old and new machines and factors of production; (A.11) and (A.12) are wages; (A.13) and (A.14) are the benefits and costs of having children and educating them; (A.15) and (A.16) describe how fertility and education choices translate into aggregate factors of production. Note that if either of the first two equations holds with strict inequality, the algorithm sets the value of  $M_{\text{new}}$  to zero and simply solves the the rest of the system.

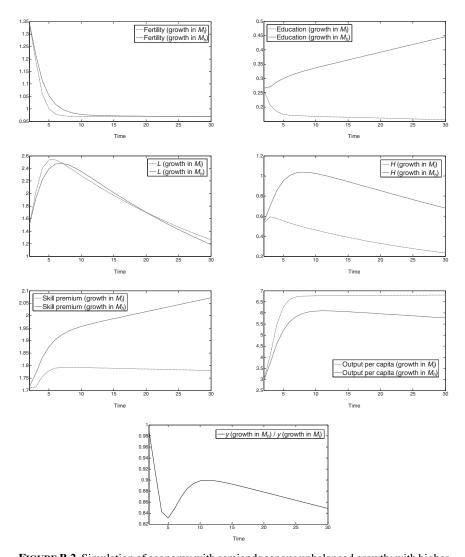
# APPENDIX B: SIMULATIONS WITH HIGHER INITIAL FERTILITY RATES

Finally we demonstrate simulation results when fertility begins at a higher level. By setting  $\gamma = 0.3$ , we obtain  $n_1 \approx 1.35$ . Rerunning all simulations with this condition does not alter our general story. Indeed, we can raise fertility by as much as possible—similar dynamics emerge. Thus, high-fertility and low-fertility countries face similar divergent growth path prospects.

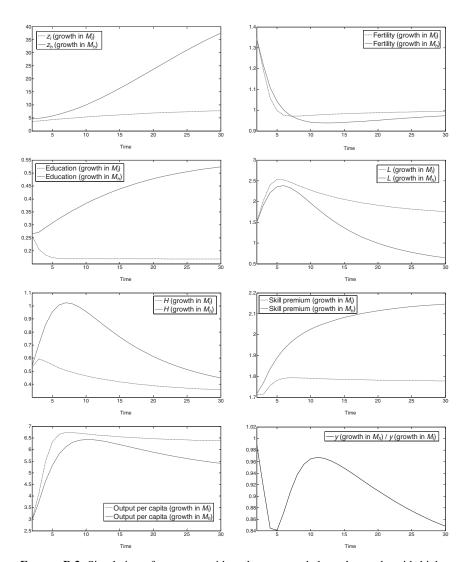
See Figures B.1-B.3



**FIGURE B.1.** Simulation of economy with exogenous unbalanced growth: with higher initial fertility.



 $\textbf{FIGURE B.2.} \ Simulation \ of \ economy \ with \ semiendogenous \ unbalanced \ growth: \ with \ higher initial \ fertility.$ 



**FIGURE B.3.** Simulation of economy with endogenous unbalanced growth: with higher initial fertility.