A.A.S. **1**, I, 103-128 (2006)

ON A SIMPLE GRAPHICAL APPROACH TO MODELLING ECONOMIC FLUCTUATIONS WITH AN APPLICATION TO UNITED KINGDOM PRICE INFLATION, 1265 TO 2005

By W. S. Chan, M. W. Ng and H. Tong

ABSTRACT

Structural instability in economic time series is widely reported in the literature. It is most prevalent in such series as price indices and inflation related data. Many methods have been developed for analysing and modelling structural changes in a univariate time series model. However, most of them assume that the data are generated by one fixed type (linear or non-linear) of the time series processes. This paper proposes a strategy for modelling different segments of an economic time series by different linear or non-linear models. A graphical procedure is suggested for detecting the model change points. The proposed procedure is illustrated by modelling annual United Kingdom price inflation series over the period 1265 to 2005. Stochastic modelling of inflation rates is an important topic to actuaries for dealing with long-term index linked insurance business. The proposed method suggests dividing the U.K. inflation series into four segments for modelling. Inflation projections based on the latest segment of the data are obtained through simulations. To get a better understanding of the impact of structural changes on inflation projections we also perform a forecasting study.

KEYWORDS

Actuarial Simulations; ARIMA Model; Outliers; SETAR Model; Tests for Non-Linearity; Time Series Analysis; U.K. Price Inflation; Wilkie Model

CONTACT ADDRESSES

W. S. Chan, Ph.D., F.S.A., C.Stat., Department of Finance, The Chinese University of Hong Kong, Shatin, N.T., Hong Kong. Tel: +852-2609-7715; Fax: +852-2603-6586; E-mail: chanws@cuhk.edu.hk
M. W. Ng, M.Phil., Department of Statistics and Actuarial Science, The University of Hong Kong, Pokfulam Road, Hong Kong. Tel: +852-2859-2466; Fax: +852-2858-9041; E-mail: h9500301@hkusua.hku.hk
H. Tong, Ph.D., Hon. F.I.A., Department of Statistics, Columbia House, London School of Economics, Houghton Street, London WC2A 2AE, U.K. Tel: +44 (0)2079-556879; Fax: +44 (0)2079-557416; E-mail: h.tong@lse.ac.uk

1. Introduction

1.1 As mentioned by Rosenberg & Young (1999) discrete time series models are useful in analysing actuarial assumptions (such as non-issue rates, lapse rates, investment rates, incidence rates, and severity rates) for pricing and reserving insurance products.

1.2 Time series modelling is also important to actuaries for generating economic scenarios in a dynamic financial analysis (DFA) model or in a cash flow testing (CFT) model (Dempster *et al.*, 2003).

1.3 In addition to analysing time-dependent variables which are specific to the pricing, reserving or dynamical analysing of insurance products, advanced time series models have been used for insurance and financial risk management (see, e.g., Embrechts *et al.*, 1999; Longin, 2000; Lucas, 2000; and Hardy, 2003).

1.4 Non-linear stochastic asset modelling has attracted considerable interest from actuaries around the world in recent years. Substantial empirical evidence for non-linearities in economic time series fluctuations has been reported in the actuarial literature. (See, e.g., Clarkson, 1991; Whitten & Thomas, 1999; Hardy, 2001; and De Gooijer & Vidiella-i-Anguera, 2003; among many others.)

1.5 Non-linear time series models have the advantage of being able to capture asymmetries, jumps and time irreversibility, which are 'stylised' facts observed in many financial and economic time series (Franses & Van Dijk, 2000).

1.6 On the other hand, linear time series models (particularly the class of ARMA models) have been reasonably successful as a practical tool for actuarial modelling and economic forecasting. The computation time required for obtaining a parsimonious ARMA model for most economic data is well within the reach of practitioners. Ready-made computer packages are widely available. Over the years, much experience has been accumulated in the actuarial application of linear ARMA models (see, e.g., Foster, 1994; Wilkie, 1995; and Lai & Frees, 1995).

1.7 Furthermore, multivariate generalisation of linear ARMA models is fairly straightforward (Sims, 1980; Tiao & Box, 1981; and Chan, 2002), while research in multivariate non-linear time series modelling is still at its infancy.

1.8 Non-linear time series models provide a much wider range of possible dynamics for the economic data, at the cost of additional complexity as compared to linear models. There are certainly tradeoffs between linear and non-linear models in analysing economic time series.

1.9 In modelling economic fluctuations, we often assume that all of the time series data are generated by a single type (linear or non-linear) of the models. However, in practice, different segments of the observed series may behave quite differently. For example, McCulloch & Tsay (1994) and Chen *et al.* (1997) discuss a general Bayesian approach, allowing each observation to 'choose' one of two pre-specified models such as the well-known ARMA model, the GARCH model of Bollerslev (1986), the bilinear model of Granger & Andersen (1978), and the threshold autoregressive model of Tong (1978).

1.10 In this paper we propose a procedure for modelling different segments of an economic time series by linear or non-linear models. Unlike the Bayesian method of Chen *et al.* (1997), the orders of the competing models need not be pre-specified. The proposed procedure also allows more than two competing models for each segment of the data.

1.11 The stochastic model of retail price inflation is a core part of the Wilkie composite model (Wilkie, 1986 and 1995). It provides, directly or indirectly, inputs to other component variables. Kitts (1990) gives an empirical review on the Wilkie price inflation model. The long-term validity of the model was questioned, because it does not accommodate possible structural changes. Similar concerns have been expressed by some other actuaries (see, e.g., Daykin & Hey, 1990; Geoghegan *et al.*, 1992; and Huber, 1997). In this paper, we apply the proposed procedure to modelling annual United Kingdom price inflation series over the period 1265 to 2005. Three structural change points are identified. Our method suggests dividing the inflation series into four segments for modelling. Inflation projections based on the latest segment of the data are obtained through simulations.

2. Linear and Non-Linear Time Series Modelling

2.1 *Linear ARMA Modelling*

2.1.1 The orthodox linear ARMA model (Box & Jenkins, 1976) has the form:

$$\phi(L)Y_t = \mu + \theta(L)a_t$$

where $\phi(L) = 1 - \phi_1 L - \ldots - \phi_p L^p$ and $\theta(L) = 1 - \theta_1 L - \ldots - \theta_q L^q$ are polynomials in L of degrees p and q, respectively, μ is a constant, L is the lag operator such that $L^s Y_t = Y_{t-s}$ and $\{a_t\}$ is a sequence of independent random variables drawn from a distribution with mean zero and constant variance $\sigma_a^2 < \infty$. All the zeros of $\phi(L)$ and $\theta(L)$ are required to lie outside the unit circle to meet the stationarity and invertibility requirements. However, time series arising from economic and business areas are often non-stationary. The class of homogeneous non-stationary ARIMA models can be used to analyse the data. It assumes that the differenced series follows a stationary ARMA process.

2.1.2 Although there are numerous successful examples in economic applications of ARIMA models, this class of linear models has a number of serious shortcomings for studying economic fluctuations (Potter, 1995). One of the defects is that they are not capable of accommodating large shocks (outliers), shifting trends and structural changes. (See, e.g., Balke & Fomby, 1991 and 1994; Clements & Hendry, 1996; Atkinson *et al.*, 1997; De Jong & Penzer, 1998).

2.1.3 Tsay (1988) and Chen & Liu (1993) extend the class of ARMA models by adding an intervention (outlier) component, i.e.:

$$Y_t^* = Y_t + \eta_t(T, \omega)$$

where Y_t follows an ordinary ARMA process in (1), and $\eta_t(T, \omega)$ is used to describe the type, location (T) and magnitude (ω) of the outlier (shock). Tsay (1988) considers four commonly encountered types of outlier. They are additive outlier (AO), innovational outlier (IO), level shift (LS) and temporary change (TC). The form of $\eta_t(T, \omega)$ for each type of outlier is given as:

$$\begin{aligned} AO&: \ \eta_t(T, \omega) = \omega D_t^{(T)} \\ IO&: \ \eta_t(T, \omega) = \omega \frac{\theta(L)}{\phi(L)} D_t^{(T)} \\ LS&: \ \eta_t(T, \omega) = \frac{\omega}{1 - L} D_t^{(T)} \\ TC&: \ \eta_t(T, \omega) = \frac{\omega}{1 - \delta L} D_t^{(T)} \end{aligned}$$

where:

$$D_t^{(T)} = \begin{cases} 1 & \text{if } t = T \\ 0 & \text{if } t \neq T \end{cases}$$

is the indicator variable representing the presence or absence of an outlier at time T.

2.1.4 Recall that an additive outlier affects only the level of the given observation; an innovational outlier affects all observations beyond the given time through the memory of the underlying ARMA dynamics; a level shift is an event which affects a time series at a particular time point whose effect becomes permanent; a temporary change is an event having an initial impact and whose effect decreases exponentially according to a fixed dampening parameter, say δ. In practice, the value of δ is often set at $\delta = 0.7$ (Chen & Liu, 1993). More generally, a time series may contain m outliers of different types, and we have the following general time series outlier model:

$$Y_t^* = Y_t + \sum_{j=1}^{m} \eta_t(T_j, \omega_j).$$

2.1.5 Chen & Liu (1993) propose a three-stage (detection, estimation and adjustment) iterative procedure for modelling ARMA processes with outlier components. It substantially widens the use of linear ARMA models. Balke & Fomby (1994), Bizovi *et al.* (1998), and Junttila (2001) report many

https://doi.org/10.1017/S1748499500000075 Published online by Cambridge University Press

successful applications of Chen & Liu's methodology to various disciplines, ranging from economics to medicine.

2.2 *Non-Linear SETAR Modelling*

2.2.1 The class of self-exciting threshold autoregressive (SETAR) models (Tong, 1978 and 1983) has been widely employed in the literature to explain various empirical phenomena observed in economic time series. See, e.g., Krağer & Kugler (1993), Peel & Speight (1994) and Chappell *et al.* (1996) for foreign exchange rate variables; Yadav *et al.* (1994) for futures market; Tiao & Tsay (1994) and Potter (1995) for United States GNP; Montgomery *et al.* (1998) for U.S. unemployment; and De Gooijer & Vidiella-i-Anguera (2003) for monthly inflation rates. Statistical properties and forecasting performance of SETAR models have been extensively examined. See, e.g., Tong (1990); Clements & Smith (1997 and 1999); Kepetanios (2000); and De Gooijer (2001).

2.2.2 A k-regime SETAR(d; $p_1, p_2, \dots, p_k$) model has the form:

$$Y_t = \sum_{l=0}^{p_j} \phi_l^{(j)} Y_{t-l} + a_t \qquad \text{if} \quad Y_{t-d} \in (r_{j-1}, r_j]$$

where $j = 1, 2, \dots, k$, $-\infty = r_0 < r_1 < \cdots < r_k = \infty$ are the threshold values, d, k, and $(p_1, p_2, \dots, p_k)$ are positive integers, and a_t is a sequence of i.i.d. random variables with zero mean and constant variance $\sigma_a^2 < \infty$.

2.2.3 Tsay (1989) has proposed a test for threshold nonlinearity, which we shall generalise in the next section. Now, for an observed time series $\{Y_t, t = 1, 2, \dots, n\}$, we define $p = \max(p_1, p_2, \dots, p_k)$ as the maximum AR order in the SETAR model. Given a fixed delay parameter d, let $h = \max(1, p + 1 - d)$. The observations $\{Y_h, Y_{h+1}, \dots, Y_{n-d}\}$ can be arranged in ascending order as $\{Y_{\pi_1}, Y_{\pi_2}, \dots, Y_{\pi_{n-d-h+1}}\}$, where π_i denotes the index of the ith smallest values of the unsorted series. An arranged autoregression can be written as:

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\Phi} + \mathbf{a}$$

where $\mathbf{Y} = (Y_{\pi_1+d}, \dots, Y_{\pi_{n-d-h+1}+d})'$, $\mathbf{X}$ is an $(n-d-h+1) \times (p+1)$ matrix with the first column being an unit vector and the remaining columns containing the corresponding lagged Y_{π_i+d} values, $\boldsymbol{\Phi} = (\phi_0, \phi_1, \dots, \phi_p)'$ and $\mathbf{a}$ is a vector of noise.

2.2.4 Let the number of startup observations be $m > p + 1$. Stepwise regressions can be performed by regressing the first r rows of $\mathbf{Y}$ on the first r rows of $\mathbf{X}$. Then the corresponding one-step-ahead predictive residuals $\hat{e}_{d+\pi_{r+1}}$ can be computed successively for $r = m, m+1, \dots, n-d-h$.

2.2.5 Tsay's test (*op. cit.*) utilises the orthogonality property between these predictive residuals and the regressors $\{Y_{\pi_i+d-v} | v = 1, \ldots, p, \quad i = 1, \ldots, n - d - h - m + 1\}$ under the null hypothesis of linearity. If the true model is a non-linear SETAR process, the orthogonality will be destroyed. Hence, the usual F-statistic for the regression between these predictive residuals and the regressors:

$$\hat{e}_{\pi_i+d} = \alpha_o + \sum_{v=1}^{p} \alpha_v Y_{\pi_i+d-v} + \varepsilon_{\pi_i+d}$$

for $i = m + 1, \ldots, n - d - h + 1$, can be used to test the orthogonality, and thus test for threshold-type non-linearity. In most situations, p and d are not known. As a quick-and-dirty method, Tsay (1989) selects p by the sample partial autocorrelation function (PACF) of Y_t. Once p is selected, d is chosen such that it gives the most significant F-statistic. For the number of regime k and the threshold parameter(s), he proposes using various scatterplots (e.g. standardised predictive residuals versus Y_{t-d}) to locate them. Finally, the AIC (Akaike, 1974) is used to specify the AR order of each regime.

3. A New Modelling Strategy

3.1 Structural instability in economic time series relations is not uncommon. (See, e.g., Stock & Watson, 1996.) The underlying data generating process (DGP) of an economic time series may be changing with time. Hence, we may wish to adopt a modelling procedure which allows different models for different segments of the observed series.

3.2 In this paper, we assume that segments of an economic time series are generated from either the classical linear ARMA models or non-linear SETAR models (Tong, 1978 and 1983). We argue that it will substantially enrich the possible dynamics for an economic time series if we allow piecewise switching between these two classes of models.

3.3 Given a time series, it remains for us to search the 'change point(s)' in order to divide it into segments. Instead of developing a rigorous statistical test for testing for change points, we propose a simple graphical procedure for locating the possible switches.

3.4 Inspired by Tsay (1991) and Chan & Cheung (1994), we suggest a rolling local non-linearity testing procedure. The F-statistic discussed in ¶2.2.5 is first computed using a rectangular window with a fixed number of data points inside the window. Let $\hat{F}_t$ be the corresponding F-statistic for the window of observations $\{Y_{t-w}, \ldots, Y_t, \ldots, Y_{t+w}\}$ (i.e., the window width is $2w + 1$). The graph $\hat{F}_t$ versus t, for $t = w + 1, \ldots, n - w$, generally contains useful information on the change points.

3.5 We illustrate the proposed graphical procedure by a simulated

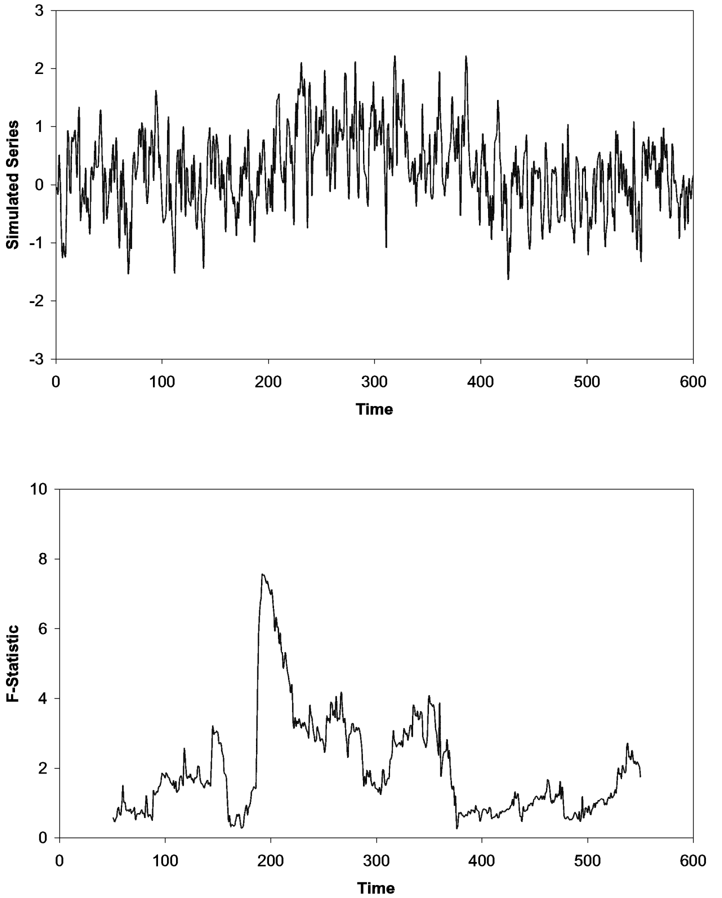

Figure 3.1. Time plot and rolling F-statistic plot of the simulated series (Model A)

example. Figure 3.1 (upper panel) shows a time plot of 600 observations generated from the process:

$$\text{Model A:} \quad Y_t = \begin{cases} 0.5Y_{t-1} + a_t & \text{if } t = 1, \ldots, 200 \\ \begin{cases} 0.8Y_{t-1} + a_t & \text{if } Y_{t-1} \geq 0 \\ -0.8Y_{t-1} + a_t & \text{if } Y_{t-1} < 0 \end{cases} & \text{if } t = 201, \ldots, 400 \\ a_t + 0.3a_{t-1} + 0.4a_{t-2} & \text{if } t = 401, \ldots, 600 \end{cases}$$

https://doi.org/10.1017/S1748499500000075 Published online by Cambridge University Press

where $Y_0 = 0$ and a_t are i.i.d. Gaussian random variates with zero mean and $\sigma_a = 0.5$. The sample PACF of the simulated series has a 'cut-off' pattern after lag 2. Therefore, we employ $p = 2$ for the F-test. The delay parameter ($d \leq p$) is chosen such that it gives the largest F-statistic for the full-length simulated series. We obtain $d = 1$. For computing the rolling F-statistics, we set the window size = 101 (i.e., $w = 50$) in this example. The graph $\hat{F}_t$ versus t for $t = 51, \ldots, 550$ is given in Figure 3.1 (lower panel). The graph clearly indicates that 'something happened' around $t = 190$ and again around $t = 375$. These two points are fairly close to the structural change points in Model A, $t = 201$ and $t = 401$, respectively. Once the time series is divided into three pieces, ARMA and SETAR modelling procedures, described in Section 2, can be applied to the corresponding segments.

3.6 In order to further explore the effectiveness of the proposed graphical procedure, in addition to Model A described above, we consider three more data generating processes:

$$\text{Model B:}\ \ Y_t = \begin{cases} \begin{cases} 0.8Y_{t-1} + a_t & \text{if } Y_t - 1 \geq 0 \\ -0.8Y_{t-1} + a_t & \text{if } Y_t - 1 < 0 \end{cases} & \text{if } t = 1, \ldots, 200; 401, \ldots, 600 \\ Y_{t-1} + a_t & \text{if } t = 201, \ldots, 400 \end{cases}$$

$$\text{Model C:}\ \ Y_t = 0.6Y_{t-1} + 0.3Y_{t-2} + a_t \qquad t = 1, \ldots, 600$$

$$\text{Model D:}\ \ Y_t = \begin{cases} 0.8Y_{t-1} + a_t & \text{if } Y_t - 1 \geq 0 \\ -0.8Y_{t-1} + a_t & \text{if } Y_t - 1 < 0 \end{cases} \quad t = 1, \ldots, 600.$$

3.7 There are two structural change points in Model B ($t = 201$ and $t = 401$). Model C is a pure AR(2) process, while Model D is a pure non-linear SETAR model without any structural breaks. Time series observations are generated from each model. We apply the proposed graphical procedure to each simulated series, and the results are plotted in Figure 3.2.

3.8 The rolling F-statistic plot of the Model B simulated series shows big jumps at around $t = 180$ and $t = 400$, which are fairly close to the true structural break points. For Model C, the rolling F-statistic values are all insignificant (the critical value for the local rolling F-test is around 2.7 at the 5% level). On the other hand, almost all (except one) of the $\hat{F}_t$ values for Model D are above the critical value. No dipped period is found. Therefore, the rolling F-statistic plots do not suggest any structural breaks for the simulated series from Models C and D, as expected.

3.9 In summary, our experience suggests that the proposed graphical procedure is reasonably effective in locating possible switches between a linear ARMA model and a non-linear SETAR process. It has the particular advantage of being simple and visual. We will further demonstrate its usefulness through a real application in the next section.

https://doi.org/10.1017/S1748499500000075 Published online by Cambridge University Press

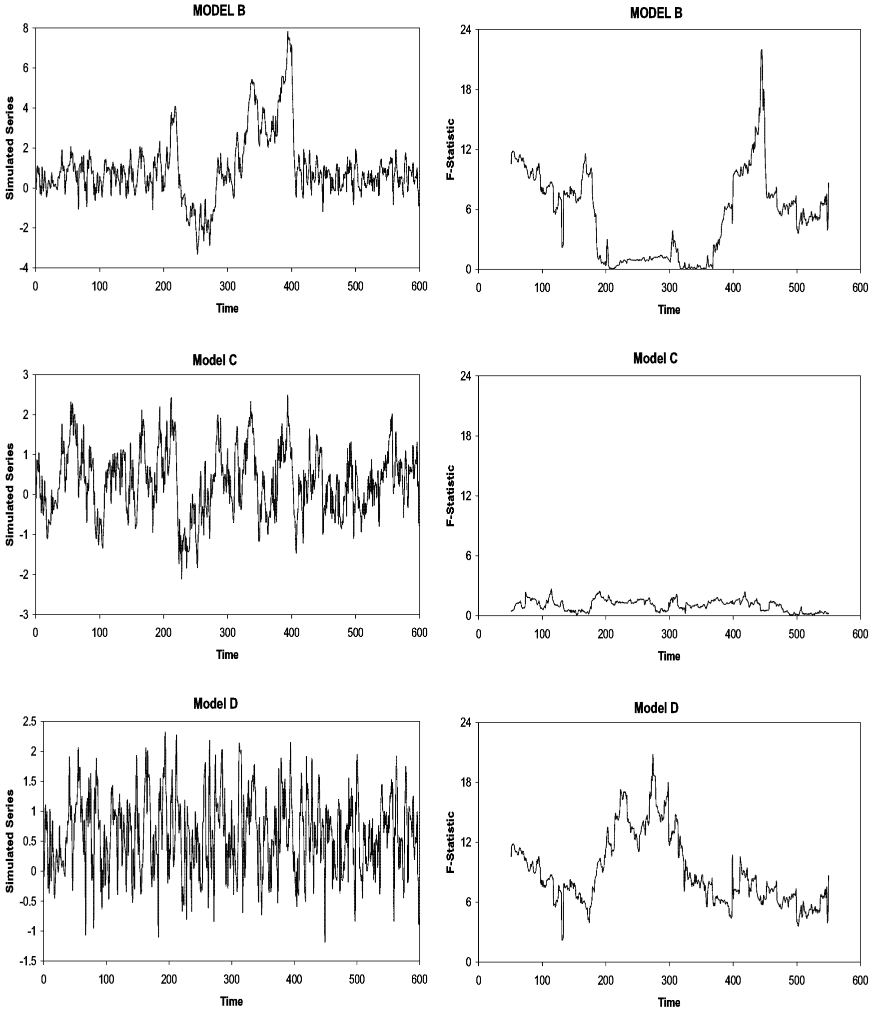

Figure 3.2. Time plots and rolling *F*-statistic plots of the simulated series (Models B, C and D)

4. Modelling U.K. Price Inflation, 1265 to 2005

4.1 *The Data*

4.1.1 Inflation is an important topic for many disciplines. For example, numerous articles and books have been written on the topic by economists. (See, e.g., Dicks-Mireaux & Dow, 1959; Sargan, 1964; Friedman, 1977;

https://doi.org/10.1017/S1748499500000075 Published online by Cambridge University Press

Rowlatt, 1988; De Brouwer & Ericsson, 1998; and Hendry, 2001). Actuaries have to examine inflation for indexing long-term insurance contracts (Wilkie, 1981). Financial analysts need to study inflation for pricing inflation-indexed bonds (Roll, 1996).

4.1.2 In this section we consider the time series modelling of U.K. inflation. Following Wilkie (1995), an annual Retail Prices Index (RPI) series P_t can be constructed starting as early as 1264. A detailed description of the data sources is given in the Appendix. The annual price inflation is defined as:

$$Y_t = \ln P_t - \ln P_{t-1}.$$

It is often called the 'force of inflation' in the actuarial literature. Figure 4.1 plots the inflation series from 1265 to 2005. For the past seven and a quarter centuries or so, U.K. inflation has fluctuated greatly in response to many political, economical and technological changes. We will illustrate the proposed modelling strategy, discussed in Section 3, using this long and important economic time series.

4.2 *Detecting the Model Change Points*

4.2.1 The sample PACF for the inflation series is first computed:

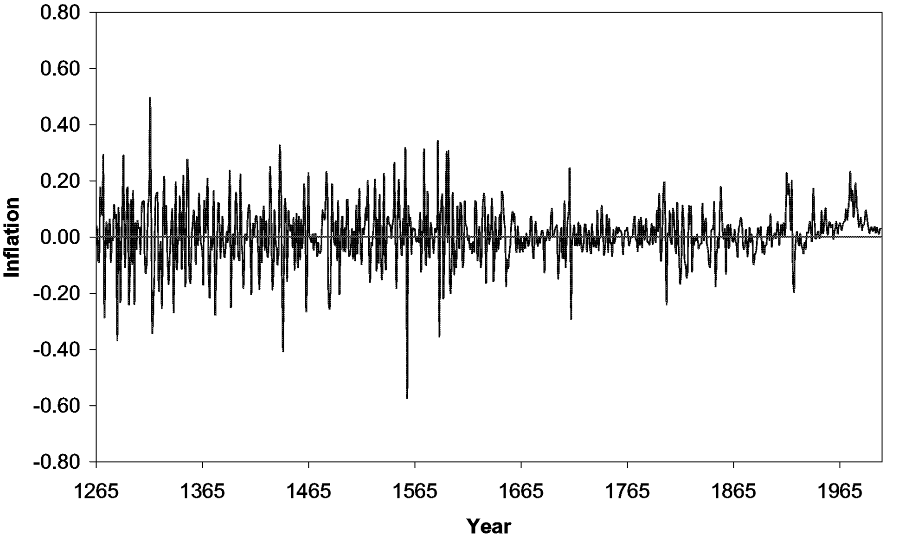

Figure 4.1. Time plot of U.K. price inflation, 1265 to 2005

https://doi.org/10.1017/S1748499500000075 Published online by Cambridge University Press

Lag	1	2	3	4	5	6	7	8
Sample PACF	0.06	−0.23	−0.11	−0.02	−0.02	0.01	0.04	0.07
S.E.	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04

It suggests that $p = 3$ for the F-tests for non-linearity. Next, we carry out the F-tests with $p = 3$ and $d \leq p$. The results are obtained:

Delay (d)	1	2	3
F-statistic	2.25	2.06	0.35

The delay parameter $d = 1$ is chosen, since it gives the largest F-statistic among all $d \leq p$.

4.2.2 The rolling $\hat{F}_t$ values are computed using $(p, d) = (3, 1)$ with window size = 101. The results are plotted in Figure 4.2. It should be noted that the critical value for the local F-test is 2.5 at the 5% level. There are three obvious sustained 'cross-over' upward jumps at around 1480, 1605 and 1914. Interestingly, the period between the first two detected breaks matches closely to the famous House of Tudor in the history of England. The Tudors were a Welsh-English family which ruled England from 1485 to 1603. It was a great time of change in England, and Tudor inflation is an important research topic for many economic historians (see, e.g., Phelps Brown & Hopkins, 1956; Brenner, 1961 and 1962; and Fisher, 1965). The third detected break is at around 1914, the beginning of the First World War.

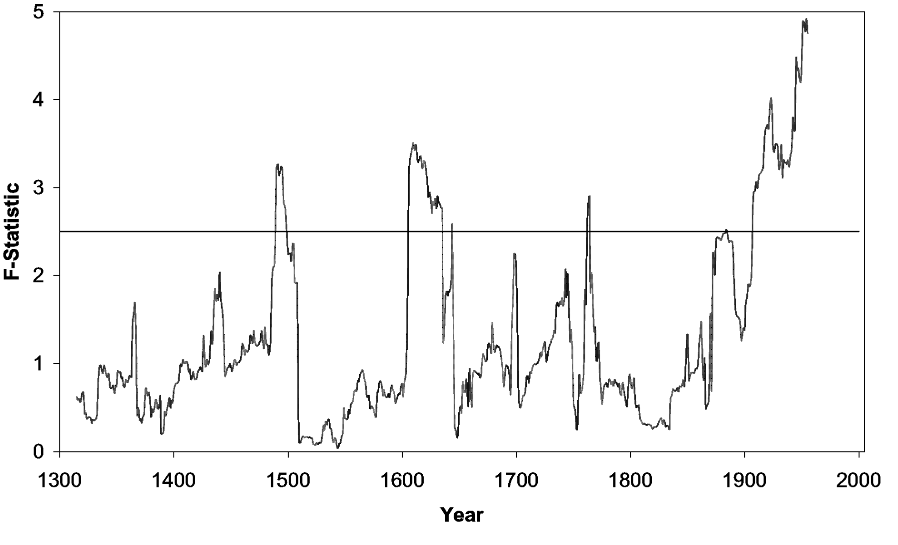

Figure 4.2. Rolling F-statistic plot (U.K. price inflation series)

https://doi.org/10.1017/S1748499500000075 Published online by Cambridge University Press

Table 4.1. Results of Tsay's F-test on U.K. inflation data

	Period I 1265-1484	Period II 1485-1603	Period III 1604-1913	Period IV 1914-2005
Length of period	220	119	310	92
Chosen AR order (p)	4	3	6	6
Delay ($d \leq p$)		p-value of the F-test		
$d = 1$	0.7802	0.6390	0.2677	0.0004*
$d = 2$	0.6889	0.2244	0.0003*	0.2210
$d = 3$	0.6023	0.5926	0.1763	0.0392*
$d = 4$	0.7705		0.2257	0.7030
$d = 5$			0.0015*	0.3767
$d = 6$			0.5155	0.0820

*Note: Asterisk indicates rejection of linearity at the 5% level

4.2.3 We roughly subdivide the whole time series into four periods. They are: Period I (1265-1484), Period II (1485-1603), Period III (1604-1913) and Period IV (1914-2005). Figure 4.3 shows the annual U.K. Retail Prices Index series (in vertical logarithmic scale) separated by the corresponding sub-periods.

4.2.4 Table 4.1 displays the ordinary (i.e., non-rolling) F-test results for these four periods. It is quite easy to conclude that Period I and Period II are linear. For Period III, non-linearity is detected at $d = 2$ and $d = 5$. For the final period, non-linearity is found at $d = 1$ and $d = 3$. By looking at Figure 4.3, it can be seen that there are some spurious observations around 1919 to 1922, which suggest that outliers may exist in Period IV. As discussed in Chan & Ng (2004), existence of outliers may affect the accuracy of threshold-type non-linearity tests. Therefore, a linear model with outliers and a non-linear SETAR model are both possible choices for this period. Both classes of models will be fitted and compared later.

4.3 *Empirical Results*

(a) *Period I: 1265-1484*

4.3.1 To fit a linear ARMA model, the first step is to specify the AR and MA orders. Using the SCA-EXPERT system (Liu, 1996), an MA(3) model is tentatively suggested for this period. The fitted model is:

$$Y_t = a_t - \underset{(0.0663)}{0.0485}a_{t-1} - \underset{(0.0599)}{0.4468}a_{t-2} - \underset{(0.0671)}{0.1889}a_{t-3}$$

where Y_t denotes inflation series and $\hat{\sigma}_a = 0.1177$.

4.3.2 Diagnostic checking of the residuals was performed. The Ljung & Box (1978) portmanteau statistic (with ten lags) for testing independence of the residuals is 4.9, which is highly insignificant (the critical value of the test

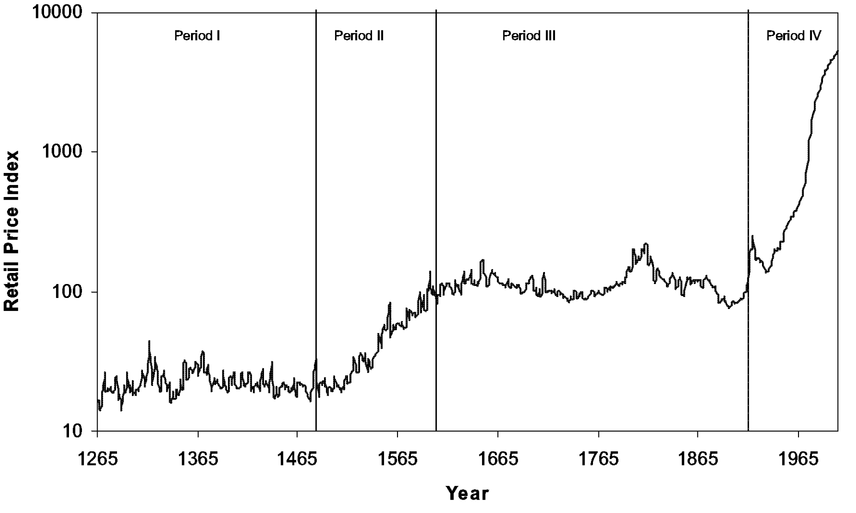

Figure 4.3. United Kingdom Retail Prices Index, 1264 to 2005

is $\chi^2_{7,0.95} = 14.067$). The Jarque & Bera's (1980) statistic is 32.42, which shows that non-normality is significant at the 5% level (the critical value of the test is $\chi^2_{2,0.95} = 5.99$).

4.3.3 Figure 4.4 (upper panel) shows the standardised residual plot of the model. There is an obvious outlying residual at around $t = 50$. The corresponding histogram has a heavy tail on the right. It is therefore suspected that non-normality is caused by the outlying residual. Outlier detection procedure, proposed by Chen & Liu (1993), is performed, and an IO at $t = 52$ (corresponding to the year 1316) is detected with a t-ratio of 4.75. Checking the history of England in the medieval ages, the harvest failed due to appalling weather during 1315 and 1316. Besides, there were widespread cattle and sheep murrains. These natural disasters caused prices to increase. These findings lend support to an outlier adjusted model:

$$
\begin{aligned}
Y_t = a_t &- 0.0795a_{t-1} - 0.4597a_{t-2} - 0.1720a_{t-3} \\
&+ 0.537(1 - 0.0795L - 0.4597L^2 - 0.1720L^3)D_t^{(1316)}
\end{aligned}
$$

with $\hat{\sigma}_a = 0.1130$.

4.3.4 The standardised residuals are computed for diagnostic checking. The Ljung & Box statistic is reduced to 3.4 and the Jarque & Bera statistic is 0.92. Both values are statistically insignificant. The standardised residuals

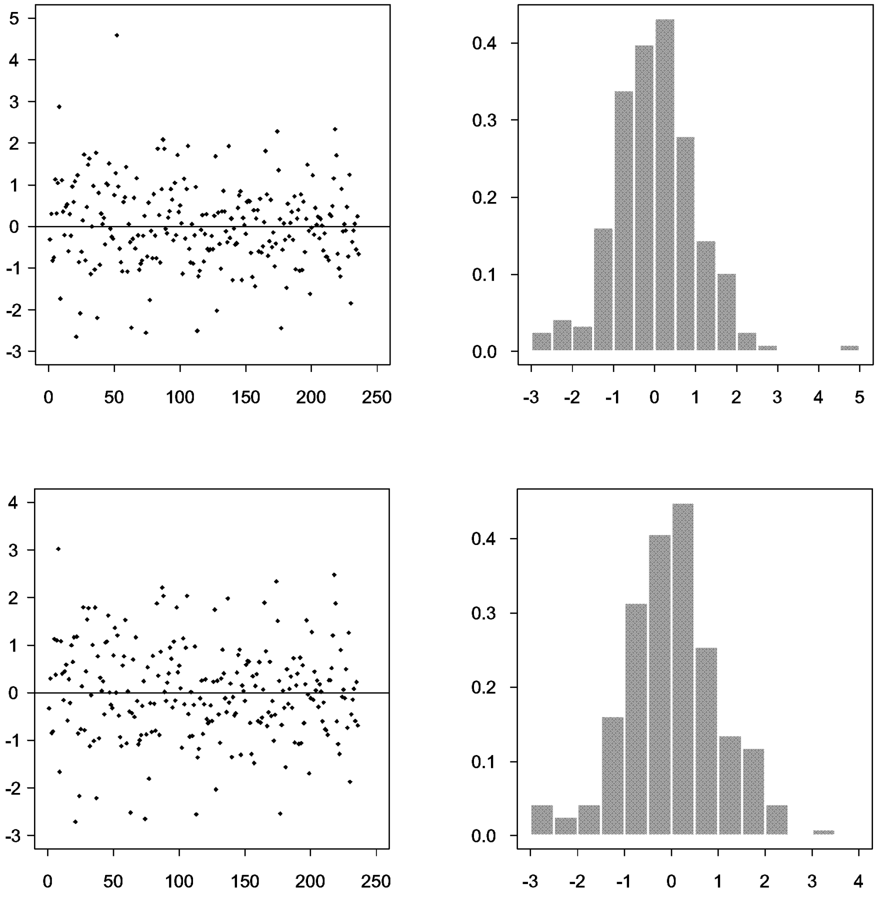

Figure 4.4. Scatterplot and histogram of standardised residuals of models in ¶4.3.1 and ¶4.3.3

may also be assumed to be randomly and normally scattered (see Figure 4.4, lower panel). Therefore, the model in ¶4.3.3 gives a good fit for Period I.

(*b*) *Period II: 1485-1603*

4.3.5 Our proposed graphical procedure detects the first two structural breaks for the U.K. price inflation series in approximately 1480 and 1605 (see Figure 4.2). The period within these two breaks matches closely to the era of Tudor England (1485 to 1603). The inflation of prices in Tudor England was part of a European-wide phenomenon, long considered to have been

https://doi.org/10.1017/S1748499500000075 Published online by Cambridge University Press

Table 4.2. Outlier detection results for Period II

Year	Sizes	*t*-ratio	Type	Event and plausible explanations
1546	0.300	3.27	IO	Henry VIII's debasement of the currency in England started (Hill & Long, 2001, p494)
1556	0.343	3.91	AO	Catastrophic harvests in the years 1555-6 (Outhwaite, 1969, p14)
1558	−0.556	−5.78	IO	Severe influenza epidemic led to a pause in inflation (Fisher, 1965, p125)
1564	0.312	3.40	IO	Missing data in 1563-4 (Phelps Brown & Hopkins, 1956, p312)
1587	0.344	3.75	IO	Poor harvests, increasing speculation and the accursed activities of middlemen (Outhwaite, 1969, p19)
1595	0.298	3.24	IO	The mid-1590s witnessed four terrible harvests in a row (Outhwaite, 1969, p43)
1597	0.398	4.33	IO	Poor harvests, years of war and heavy taxation pushed prices (Outhwaite, 1969, p43)

caused by monetary factors; specifically, gold and silver from America began to flow into Europe at almost the same time as the House of Tudor began (Outhwaite, 1969). Therefore, we choose the period of Tudor England as Period II.

4.3.6 Results of Tsay's *F*-test in Table 4.1 indicate that Period II is linear. Using the SCA-EXPERT system, an MA(2) model is tentatively fitted. We perform the outlier detection procedure suggested by Chen & Liu (1993). Table 4.2 gives the locations, sizes, types and plausible explanations of the detected outliers in this period.

4.3.7 The final fitted model for the outlier-adjusted series, $Y_t = Y_t^* - \sum_{j=1}^{m} \eta_t(T_j, \omega_j)$ in ¶2.1.4, is:

$$Y_t = a_t - 0.2276a_{t-1} - 0.2712a_{t-2}$$

with $\hat{\sigma}_a = 0.0919$. The standardised residuals are computed for diagnostic checking. The Ljung & Box (1978) portmanteau statistic (with ten lags) for testing the independence of the residuals is 7.4, which is insignificant (the critical value of the test is $\chi^2_{7,0.95} = 14.067$). The Jarque & Bera's (1980) statistic is 1.41, which does not indicate any non-normality problem in the residuals (the critical value of the test is $\chi^2_{2,0.95} = 5.99$). Therefore, the above model gives a good fit for Period II.

(*c*) *Period III: 1604-1913*

4.3.8 In this section, SETAR model fitting for Period III is performed. As shown in Table 4.1, the most significant non-linearity test result is

https://doi.org/10.1017/S1748499500000075 Published online by Cambridge University Press

obtained for the case $(p, d) = (6, 2)$. Therefore, $p = 6$ and $d = 2$ are selected for the preliminary analysis.

4.3.9 Scatterplots are useful tools for suggesting possible threshold values (Tsay, 1989). The scatterplot of recursive t-ratios of the lag-1 AR coefficient against ordered Y_{t-2} is shown in Figure 4.5. Plots for other lags carry similar information, and hence are not shown here. The figure shows two obvious breaks: one near $Y_{t-2} = -0.0184$; and another near $Y_{t-2} = 0.1011$, which give two possible threshold values.

4.3.10 Finally, the AIC (Akaike, 1974) is used to refine the AR order in each regime. The AR orders are 0, 6, 0; the number of observations are 112, 169 and 23; and the residual variances are 0.0035, 0.0042 and 0.0058 for the three regimes, respectively. The fitted SETAR model using the STAR programme (Tong, 1990) is:

$$Y_t = \begin{cases} \underset{(0.0056)}{0.0097} + a_t^{(1)} & \text{if } Y_{t-2} \leq -0.0184 \\[2ex] \underset{(0.0719)}{0.1882}Y_{t-2} - \underset{(0.0723)}{0.1365}Y_{t-3} - \underset{(0.0713)}{0.2610}Y_{t-6} + a_t^{(2)} & \text{if } -0.0184 < Y_{t-2} \leq 0.1011 \\[2ex] \underset{(0.0159)}{-0.0407} + a_t^{(3)} & \text{if } Y_{t-2} > 0.1011. \end{cases}$$

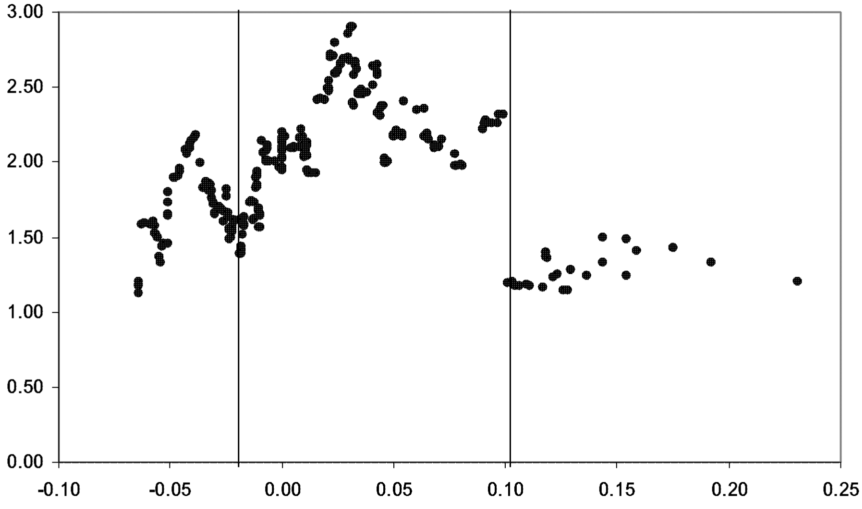

Figure 4.5. Scatterplot of recursive t-ratios of the lag-1 AR coefficient versus ordered Y_{t-2} for U.K. inflation of Period III

https://doi.org/10.1017/S1748499500000075 Published online by Cambridge University Press

4.3.11 Diagnostic checking of the fitted model does not show any model inadequacy. The above model has quite a simple economic interpretation. The data are divided into three regimes at thresholds −0.0184 and 0.1011. Regime 1 corresponds to a period of deflation. Regimes 2 and 3 correspond to periods of normal and high inflation respectively. Most of the observations lie in the normal period, while only 23 observations lie in the high inflation period. It is well-known that both high inflation and severe deflation would cause serious damages to the economy. Therefore, in times of these extreme situations, the government might carry out interventions to re-direct the inflation rate back to the 'normal regime'. This is confirmed by the fitted model: for times of high inflation, that is $Y_{t-2} > 0.1011$, the inflation rate follows a negative-intercept process; on the other hand, for times of severe deflation, it follows a positive-intercept process.

(*d*) *Period IV: 1914-2005*

4.3.12 As discussed in ¶4.2.4, the type (linear or non-linear) of models for this period is not clearly identified. There might be some confounding effects of the aberrant observations affecting the rolling F tests. We first proceed to fit a linear ARMA model with outlier components using Chen & Liu's (1993) methodology. An AR(1) model is tentatively specified and four outliers are detected. Table 4.3 shows the details of the detected outliers and the corresponding events. Chan & Wang (1998) and Hendry (2001) identify similar turbulent points for the U.K. inflation series.

4.3.13 The fitted linear model is:

$$\begin{aligned}(Y_t - 0.0413) = 0.5278(Y_{t-1} - 0.0413) \\ + \frac{0.166}{1 - 0.5278L} D_t^{(1920)} - \frac{0.268}{1 - 0.7L} D_t^{(1921)} \\ + \frac{0.156}{1 - 0.5278L} D_t^{(1940)} + \frac{0.135}{1 - 0.7L} D_t^{(1975)} + a_t\end{aligned}$$

with $\hat{\sigma}_a = 0.0421$. Diagnostic checking of the model shows that it is adequate for the data.

Table 4.3. Outlier detection results for Period IV

Year	t-ratio	Type	Event
1920	3.94	IO	Post WWI
1921	−6.54	TC	Post WWI
1940	3.70	IO	World War II
1975	3.31	TC	Oil crisis shock

https://doi.org/10.1017/S1748499500000075 Published online by Cambridge University Press

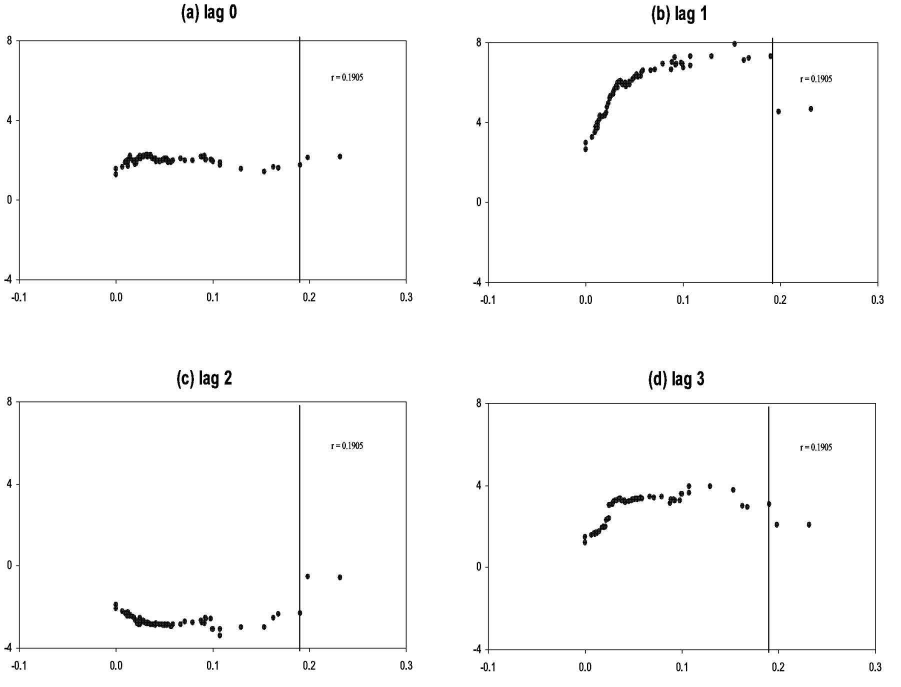

Figure 4.6. Scatterplot of recursive t-ratios of the lag-k AR coefficient versus ordered Y_{t-1} for U.K. inflation of Period IV

4.3.14 A non-linear SETAR model is also fitted for $p = 6$ and $d = 1$, based on the test results in Table 4.1. Figure 4.6 shows the scatterplot of recursive t-ratios for the AR parameters $(k = 0, 1, 2, 3)$. The threshold parameter $\hat{r} = 0.1905$ is selected, as there is a sudden drop of the t-ratios for the lag-1 AR parameter. However, there are only two observations in the second regime, which is inadequate to produce efficient estimates for the AR parameters. On the other hand, these two observations can be regarded as outliers with respect to a linear model. Furthermore, we apply the F-test for non-linearity to the outlier adjusted (see Table 4.3) series, and all the test results are insignificant. Therefore, we conclude that a linear AR(1) model with outlier components, as fitted in ¶4.3.13, is more appropriate for the data in this period.

4.3.15 One of the main objectives of actuarial modelling is to provide a realistic simulation of the variables (Wilkie, 1995, p804). In Figure 4.7 we show a set of ten projected paths of the U.K. inflation rate variable (Y_t) from 2006 to 2015, along with the record since 1914, using the fitted model in ¶4.3.13 for the latest segment of the series.

https://doi.org/10.1017/S1748499500000075 Published online by Cambridge University Press

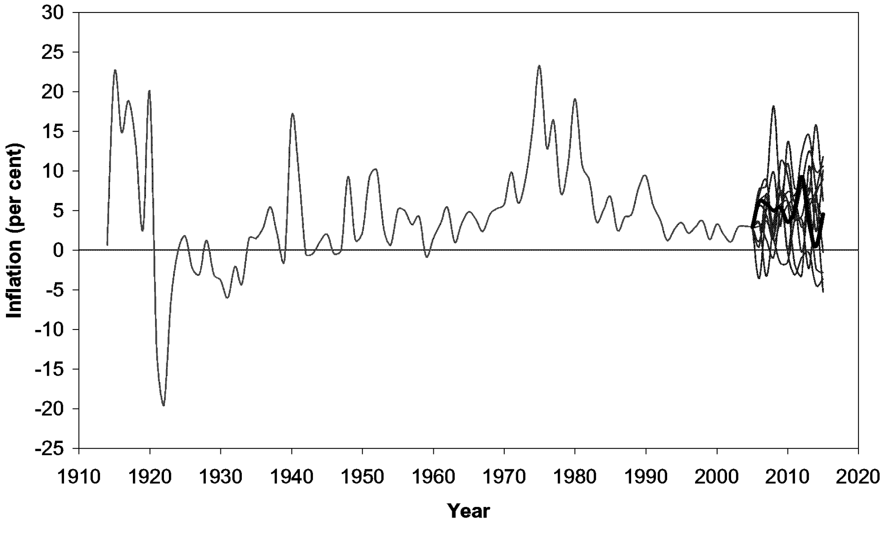

Figure 4.7. U.K. price inflation, 1914 to 2005, and projections, 2006 to 2015 (with mean projected path drawn with a thicker line)

4.3.16 Whether or not it is appropriate to adjust the data for the outliers depends on the purpose to which the model so derived will be used. If the model is to be used in an application for which extreme stochastic fluctuations are less important (e.g. to ensure that premiums are adequate in most, but not extreme, scenarios), then it may be preferable to use a model based on outlier-adjusted data. If, however, the model is to be used in an application for which extreme stochastic fluctuations are important (such as pricing catastrophe risks or ensuring that investment guarantee reserves are sufficient to keep an insurance company solvent in all but the most extreme scenarios), then a model which is sympathetic to outliers in the data ought to be used.

4.3.17 Various alternative approaches have been proposed for dealing with outliers (Chan, 2002). For example, one would use a model allowing for outliers to estimate the parameters, and then to re-calculate the residual standard error applying these estimated parameters to the data including outlier years, but with no outlier effects in the model. One gets a larger value of residual variance, of course, but this fitted model is more sympathetic to outliers in the data.

4.4 *Forecasting Study*

4.4.1 To study the potential impact of the proposed procedure on the

U.K. price inflation projection, we compare empirically both one-step and multi-step forecasts generated from various models. We examine the post-sample predication for the period 1981 to 2005 (i.e., holdout sample size is 25). Six forecasting methods are considered.

4.4.2 The Naive (NAIVE) method is based on the historical mean of the U.K. inflation series. The simple exponential smoothing (SES) technique is a very popular method in the business community for forecasting economic time series with no trend. An SES model with smoothing constant $\alpha = 0.1$ (Newbold, 1995, p217) is included. The third model in our study is the first-order autoregressive (AR(1)) process which we recommended for the latest segment of the U.K. inflation series. To explore the value of identifying structural breaks, the 'old' models (i.e.: MA(3) in Period I; MA(2) in Period II; and SETAR(2; 0,6,0) in Period III) are also considered. To achieve comparability, we re-fit the 'old' models over the extended periods.

4.4.3 The forecast methods are compared by reference to the mean squared forecast error (MSFE). We also consider the relative performance of the methods to the Naive model. The results are given in Table 4.4. It is clear that the linear AR model based on the last-known stable fundamental structure of the series outperforms all other methods considered in this forecasting exercise.

4.4.4 Clement & Hendry (1995) observe that predictive failure is not uncommon in macroeconomic forecasting. They argue that economic forecasting performance might be improved by allowing for interventions or mean shifts. The underlying data generating process is, however, still assumed to have a fixed fundamental structure (say linear or non-linear). The forecasting comparisons in this study suggest that it is also important to identify portions of an observed economic time series which might not be useful for the purpose of forecasting or actuarial simulation, due to possible fundamental changes of economies.

Table 4.4. Comparison of post-sample forecasts of U.K. price inflation, 1981 to 2005

Forecasting method	Fitting period	One-step MSFE	Forecast ratio*	20-step MSFE	Forecast ratio*
NAIVE	1265-1980	0.00186	1.00	0.00191	1.00
SES	1265-1980	0.00106	0.57	0.01535	8.04
AR(1)	1914-1980	0.00044	0.24	0.00109	0.57
SETAR(2; 0,6,0)	1604-1980	0.00298	1.60	0.00204	1.07
MA(2)	1485-1980	0.00162	0.87	0.00868	4.55
MA(3)	1265-1980	0.00258	1.39	0.00994	5.21

*Ratio of the MSFE of the forecasting method to that of the Naive model

5. Concluding Remarks

5.1 In time series modelling, the conventional approach is to examine how well each individual model fits *all* of the data. However, in reality it is often difficult to find a single model which captures all the features all the time (such as unconditional leptokurtosis, volatility clustering, jumps, structural breaks and regime shifts) in an economic time series entirely to our satisfaction. In this paper, we propose a procedure for modelling different segments of an economic time series by different models, which may be either linear ARMA models or non-linear SETAR models. A rolling version of Tsay's (1989) *F*-statistic is proposed to detect model change points graphically. The time series is then segmented into sub-periods for modelling. The proposed modelling strategy deals effectively with the situation in which different portions of the data favour different types of models. This flexibility is of particular value in time series analysis where the underlying data generating process (i.e. the economic force) may be changing over time.

5.2 We should emphasise that, in this paper, the class of SETAR models is chosen for its convenience and popularity as a class of non-linear models. As far as our proposed procedure is concerned, it can be easily be replaced by other classes of non-linear models. Using similar arguments as above, we may develop a rolling version of the Lagrange multiplier test (Tong, 1990, p320) to cater for the possibility of switchings between linear ARMA models and bilinear models. For switchings between linear ARMA models and GARCH models, we may employ a rolling version of McLeod & Li's (1983) portmanteau test. A rolling Likelihood Ratio (LR)-statistic (Franses & van Dijk, 2000, p104) can be used to locate model changes between linear AR models and Markov-Switching models (Tong, 1983, p62; and Hamilton, 1989).

5.3 In our study of the annual U.K. price inflation series, we have found that the data split roughly into four segments. In the first segment (1265 to 1500), a linear MA(3) model with an innovational outlier at the year 1316 is preferred. The second segment (1485 to 1603) is the era of Tudor England. A linear MA(2) model is fitted with seven outliers identified. Our analysis in this period quantifies the aberrant inflation periods in Tudor England, which might be useful to economic historians. In the third segment (1604 to 1913), a three-regime SETAR model is fitted, which has an interesting economic interpretation. For the final segment (1914 to 2005), a linear AR(1) process is fitted with several outliers; we suggest that the outliers may be accounted for by the turbulences due to the world wars and the oil crises. U.K. price inflation forecasts obtained using only the final segment of the data are found to be more accurate than those forecasts computed using the full observed series. The folklore of 'using all the data for forecasting' has been challenged. The proposed procedure in this paper might be useful in identifying the relevant portion of an observed time series for the purpose of economic forecasting and actuarial projections.

Acknowledgment

The work described in this paper was partially supported by a grant from the Research Grants Council of the Hong Kong Special Administrative Region, China (Project No. CUHK7413/05H). The authors acknowledge contributions from the Editor, Professor Mary Hardy, and an anonymous referee.

References

Akaike, H. (1974). A new look at the statistical model identification. *IEEE Transaction on Automatic Control*, **AC-9**, 716-723.

Atkinson, A., Koopman, S. & Shephard, N. (1997). Detecting shocks, outliers and breaks in time series. *Journal of Econometrics*, **80**, 387-422.

Balke, N. & Fomby, T. (1991). Shifting trends, segmented trends, and infrequent permanent shocks. *Journal of Monetary Economics*, **28**, 61-85.

Balke, N. & Fomby, T. (1994). Large shocks, small shocks, and economic fluctuations, outliers in macroeconomic time series. *Journal of Applied Econometrics*, **9**, 181-200.

Bizovi, K.E., Leikin, J.B., Hryhorczuk, D.O. & Frateschi, L.J. (1998). Night of the sirens: analysis of carbon monoxide-detector experience in suburban Chicago. *Annals of Emergency Medicine*, **31**, 737-740.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroscedasticity. *Journal of Econometrics*, **31**, 307-327.

Box, G.E.P. & Jenkins, G.M. (1976). *Time series forecasting and control*. Second Edition. Holden-Day: San Francisco, CA.

Brenner, Y.S. (1961). The inflation of prices in early Sixteenth Century England. *Economic History Review*, 2nd series, **XIV**, 266-284.

Brenner, Y.S. (1962). The inflation of prices in England, 1551-1650. *Economic History Review*, 2nd series, **XV**, 266-284.

Central Statistical Office (1991). *Retail prices, 1914-1990*. H.M.S.O.: London.

Central Statistical Office (1994). *Annual abstract of statistics*. H.M.S.O.: London.

Chan, W.S. (2002). Stochastic investment modelling: a multiple time series approach. *British Actuarial Journal*, **8**, 545-591.

Chan, W.S. & Cheung, S.H. (1994). On robust estimation of threshold autoregressions. *Journal of Forecasting*, **13**, 37-49.

Chan, W.S. & Ng, M.W. (2004). Robustness of alternative non-linearity tests for SETAR models. *Journal of Forecasting*, **23**, 215-231.

Chan, W.S. & Wang, S. (1998). Wilkie stochastic model for retail price inflation revisited. *British Actuarial Journal*, **4**, 637-652.

Chappell, D., Padmore, J., Mistry, P. & Ellis, C. (1996). A threshold model for the French Franc/Deutschmark exchange rate. *Journal of Forecasting*, **15**, 155-164.

Chen, C. & Liu, L.M. (1993). Joint estimation of model parameters and outlier effects in time series. *Journal of the American Statistical Association*, **88**, 284-297.

Chen, C.W.S., McCulloch, R. & Tsay, R.S. (1997). A unified approach to estimating and modelling linear and nonlinear time series. *Statistica Sinica*, **7**, 451-472.

Clarkson, R.S. (1991). A non-linear stochastic model for inflation. *Transactions of the 2nd AFIR International Colloquium*, Brighton, **3**, 233-253.

Clements, M. & Hendry, D. (1995). Macro-economic forecasting and modelling. *Economic Journal*, **105**, 1001-1013.

Clements, M. & Hendry, D. (1996). Intercept corrections and structural change. *Journal of Applied Econometrics*, **11**, 475-494.

Clements, M. & Smith, J. (1997). The performance of alternative forecasting methods for SETAR models. *International Journal of Forecasting*, **13**, 463-475.

Clements, M. & Smith, J. (1999). A Monte Carlo study of the forecasting performance of empirical SETAR models. *Journal of Applied Econometrics*, **14**, 123-141.

Daykin, C.D. & Hey, G.B. (1990). Managing uncertainty in a general insurance company. *Journal of the Institute of Actuaries*, **117**, 173-277.

De Brouwer, G. & Ericsson, N.R. (1998). Modelling inflation in Australia. *Journal of Business and Economic Statistics*, **16**, 433-449.

De Jong, P. & Penzer, J. (1998). Diagnosing shocks in time series. *Journal of the American Statistical Association*, **93**, 796-806.

De Gooijer, J.G. (2001). Cross-validation criteria for SETAR model selection. *Journal of Time Series Analysis*, **22**, 267-281.

De Gooijer, J.G. & Vidiella-i-Anguera, A. (2003). Nonlinear stochastic inflation modelling using SEASETARs. *Insurance: Mathematics and Economics*, **32**, 3-18.

Dempster, M.A.H., Germano, M., Medova, E.A. & Villaverde, M. (2003). Global asset liability management. *British Actuarial Journal*, **9**, 137-216.

Dicks-Mireaux, L.A. & Dow, J.C.R. (1959). The determinants of wage inflation: United Kingdom, 1946-1956. *Journal of the Royal Statistical Society*, **A 122**, 145-184.

Embrechts, P., Resnick, S.I. & Samorodnitsky, G. (1999). Extreme value theory as a risk management tool. *North American Actuarial Journal*, **3**, 30-41.

Fisher, F.J. (1965). Influenza and inflation in Tudor England. *Economic History Review*, 2nd Series, **XVIII**, 120-129.

Foster, R.S. (1994). A stochastic evaluation of the short-range economic assumptions in the 1994 OASDI trustees report. *Actuarial Study No. 109*, Baltimore, MD: Social Security Administration.

Franses, P.H. & Van Dijk, D. (2000). *Nonlinear time series models in empirical finance*. Cambridge: Cambridge University Press.

Friedman, M. (1977). Nobel lecture: Inflation and unemployment. *Journal of Political Economy*, **85**, 451-473.

Geoghegan, T.J., Clarkson, R.S., Feldman, K.S., Green, S.J., Kitts, A., Lavecky, J.P., Ross, F.J.M., Smith, W.J. & Toutounchi, A. (1992). Report on the Wilkie stochastic investment model. *Journal of the Institute of Actuaries*, **119**, 173-228.

Granger, C. & Andersen, A. (1978). *An introduction to bilinear time series models*. Vandehoek and Ruprecht: Gottingen.

Hamilton, J. (1989). A new approach to the economic analysis of nonstationary time-series and the business cycle. *Econometrica*, **57**, 357-384.

Hardy, M.R. (2001). A regime-switching model of long-term stock returns. *North American Actuarial Journal*, **5**, 41-53.

Hardy, M.R. (2003). *Investment guarantees: modelling and risk management for equity-linked life insurance.* John Wiley & Sons, New York.

Hendry, D. (2001). Modelling U.K. inflation, 1875-1991. *Journal of Applied Econometrics*, **16**, 255-275.

Hill, E.D. & Long, W.B. (2001). *Tudor England: an encyclopedia*. London: Garland Publishing, Inc.

Huber, P.P. (1997). A review of the Wilkie's stochastic investment model. *British Actuarial Journal*, **3**, 181-210.

JARQUE, C.M. & BERA, A.K. (1980). Efficient tests for normality, homoscedasticity and serial independence of regression residuals. *Economics Letters*, **6**, 255-259.

JUNTTILA, J. (2001). Structural breaks, ARIMA model and Finnish inflation forecasts. *International Journal of Forecasting*, **17**, 203-230.

KEPETANIOS, G. (2000). Small sample properties of the conditional last squares estimator in SETAR models. *Economics Letters*, **69**, 267-276.

KITTS, A. (1990). Comment on a model of retail price inflation. *Journal of the Institute of Actuaries*, **117**, 104-412.

KRAĞER, H. & KUGLER, P. (1993). Nonlinearities in foreign exchange markets: a different perspective. *Journal of International Money and Finance*, **12**, 195-208.

LAI, S.W. & FREES, E.W. (1995). Examining changes in reserves using stochastic interest models. *Journal of Risk and Insurance*, **62**, 535-574.

LIU, L.M. (1996). *Modelling and forecasting time series using SCA-EXPERT capabilities*. Scientific Computing Associates: Chicago.

LJUNG, G.M. & BOX, G.E.P. (1978). On a measure of lack of fit in time series models. *Biometrika*, **65**, 297-303.

LONGIN, F.M. (2000). From value at risk to stress testing: the extreme value approach. *Journal of Banking and Finance*, **24**, 1097-1130.

LUCAS, A. (2000). A note on optimal estimation from a risk-management perspective under possible misspecified tail behaviour. *Journal of Business and Economic Statistics*, **18** 31-39.

MCCULLOCH, R. & TSAY, R.S. (1994). Bayesian inference of trend- and difference-stationarity. *Econometric Theory*, **10**, 596-608.

MCLEOD, A.I. & LI, W.K. (1983). Diagnostic checking ARMA time series models using squared-residual autocorrelations. *Journal of Time Series Analysis*, **4**, 269-273.

MITCHELL, BR.R. & DEANE, P. (1962). *Abstract of British historical statistics.* Cambridge: Cambridge University Press.

MONTGOMERY, A., ZARNOWITZ, V., TSAY, R.S. & TIAO, G. (1998). Forecasting the U.S. unemployment rate. *Journal of the American Statistical Association*, **93**, 478-493.

NEWBOLD, P. (1995). *Statistics for business and economics*. New Jersey: Prentice-Hall.

OFFICE OF NATIONAL STATISTICS (1996-2005). *Annual abstract of statistics*. H.M.S.O.: London. (www.statistics.gov.uk/downloads/theme_economy/Rp02.pdf)

OUTHWAITE, R.B. (1969). *Inflation in Tudor and early Stuart England*. London: MacMillan.

PEEL, D.A & SPEIGHT, A. (1994). Testing for non-linear dependence in inter-war exchange rates. *Weltwirtschaftsliches Archiv*, **130**, 391-417.

PHELPS BROWN, E.H. & HOPKINS, S.V. (1956). Seven centuries of the prices of consumables, compared with builder's wages. *Economica*, **23**, 269-312.

POTTER, S. (1995). A nonlinear approach to U.S. GNP. *Journal of Applied Econometrics*, **10**, 109-125.

ROLL, R. (1996). Treasury inflation-indexed bonds: the design of a new security. *Journal of Fixed Income*, **6**, 9-27.

ROSENBERG, M.A. & YOUNG, V.R. (1999). A Bayesian approach to understanding time series data. *North American Actuarial Journal*, **3**, 130-143.

ROWLATT, P.A. (1988). Analysis if the recent path of U.K. inflation. *Oxford Bulletin of Economics and Statistics*, **50**, 335-360.

SARGAN, J.D. (1964). Wages and prices in the United Kingdom: a study in econometric methodology (with discussion). In *Econometric analysis for national economic planning*, HART, P.E., MILLS, G. & WHITAKER, J.K. (eds). Vol. 16 of *Colston Papers*. Butterworth Co: London.

SIMS, C. (1980). Macroeconomics and reality. *Econometrica*, **48**, 1-48.

STOCK, J. & WATSON, M. (1996). Evidence on structural instability in macroeconomic time series relations. *Journal of Business and Economic Statistics*, **14**, 11-30.

TIAO, G. & BOX, G.E.P. (1981). Modelling multiple time series with applications. *Journal of the American Statistical Association*, **76**, 802-816.

https://doi.org/10.1017/S1748499500000075 Published online by Cambridge University Press

TIAO, G. & TSAY, R.S. (1994). Some advances in non-linear and adaptive modelling in time-series. *Journal of Forecasting*, **13**, 109-131.

TONG, H. (1978). On a threshold model. In *Pattern recognition and signal processing*, CHEN, C.H. (ed.). Sijhoff and Noordoff: Amsterdam.

TONG, H. (1983). *Threshold models in non-linear time series analysis*. Springer-Verlag: New York.

TONG, H. (1990). *Non-linear time series: a dynamical system approach*. Clarendon Press: Oxford.

TSAY, R.S. (1988). Outliers, level shifts and variance changes in time series. *Journal of Forecasting*, **7**, 1-20.

TSAY, R.S. (1989). Testing and modelling threshold autoregressive processes. *Journal of the American Statistical Association*, **84**, 231-240.

TSAY, R.S. (1991). Detecting and modelling nonlinearity in univariate time series analysis. *Statistica Sinica*, **1**, 431-451.

WHITTEN, S. & THOMAS, G. (1999). A non-linear stochastic asset model for actuarial use. *British Actuarial Journal*, **5**, 919-953.

WILKIE, A.D. (1981). Indexing long-term financial contracts. *Journal of the Institute of Actuaries*, **108**, 299-360.

WILKIE, A.D. (1986). More on a stochastic asset model for actuarial use. *Transactions of the Faculty of Actuaries*, **39**, 341-403.

WILKIE, A.D. (1995). A stochastic investment model for actuarial use. *British Actuarial Journal*, **1**, 777-964.

YADAV, P.K., POPE, P.F. & PAUDYAL, K. (1994). Threshold autoregressive modelling in finance: the price difference of equivalent assets. *Mathematical Finance*, **4**, 205-221.

APPENDIX

Data

As suggested by Wilkie (1995) the annual U.K. inflation data have been constructed by taking several indices and splicing them together. Since 1914, June values of the monthly series have been used. The whole series is then rebased to the year 1914. The detailed sources are summarised in the following table.

Period	Source
1264-1661	Appendix B of Phelps Brown & Hopkins (1956). The five missing values in the series are replaced by the average of the two adjacent observations.
1661-1696	Schumpeter-Gilboy Price Indices, Part A, consumers' goods. On page 468 of Mitchell & Deane (1962).
1696-1790	Schumpeter-Gilboy Price Indices, Part B, consumers' goods. On pages 468 to 469 of Mitchell & Deane (1962).
1790-1850	Indices of British Commodity Prices on page 470 of Mitchell & Deane (1962).
1850-1871	The Rousseaux Price Indices (overall index) on pages 471 to 472 of Mitchell & Deane (1962).
1871-1914	Board of Trade Wholesale Price Indices (total index) on page 476 of Mitchell & Deane (1962).
1914-1947	'All Items' Cost of Living Index, Table 84 of Central Statistical Office (1991).
1947-1990	'All Items' Retail Prices Index, Table 1 of Central Statistical Office (1991).
1990-1993	'All Items' General Index of Retail Prices, Table 18.7 of Central Statistical Office (1994).
1993-2005	'All Items' Retail Prices Index, Table 18.7 of Office of National Statistics (1996 to 2005).