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The third topic is a verbal introduction to the major areas of actuarial practice — life insurance, superannuation, and general insurance. This is very helpful in giving an idea of the context of actuarial science.

The final chapter provides an introduction to life contingencies. More than an introduction, in fact, it provides the formulae for premiums for traditional annual premium life insurance. All this the student will meet again, in a more rigorous setting, in a higher level life contingencies course, but introducing the ideas at an early stage may help students understand early the nature of actuarial calculations. It should also make the whole subject easier the second time they meet it.

The book is very easily readable; the pedagogical expertise of the authors comes through in simple, clear and intuitive explanations of actuarial concepts that most students find difficult.

There are some omissions: it would have been nice to see more coverage of background ideas which are important to insurance and pensions practice, but not necessarily to be covered in a lot more detail later. Examples would include utility and insurance, moral hazard and risk management, and some discussion of the asset side of insurance and pensions management. There is not much that connects the chapters to each other.

Overall though, this book provides a well written introduction to actuarial science. For a student just embarking on the actuarial examinations, it provides a swift overview of the actuarial material that they will be poring over for the next few years.

MARY HARDY

Lundberg Approximations for Compound Distributions with Insurance Applications. By G. E. WILLMOT AND X. S. LIN (Springer, 2000)

Willmot and Lin are well-known researchers in actuarial science, and this book might more aptly be titled *Collected Works*, 1994-2000. It reflects the many contributions they have made to the literature in actuarial science and applied probability over that period. By contrast, their chosen title hides the fact that, at the heart of this work, is the authors' fondness for reliability theory, in particular reliability classifications of distributions. I offer this comment as a warning that, if this is not a subject close to your heart, you will find many parts of the text hard going. This review may be jaundiced by the fact that I include myself in this category!

So, briefly, what is the book about? The main topics are compound distributions, bounds for tail probabilities of these distributions, defective renewal equations and their application to selected problems in ruin theory.

Rather strangely, compound distributions are not discussed in detail until Chapter 4, some fifty pages into the text. Following a brief introductory chapter, Chapter 2 is devoted to reliability theory. Here we are introduced to concepts such as failure rate (force of mortality in actuarial-speak), equilibrium distributions and residual lifetime distribution. The authors then bombard us with the language of reliability theory and classifications of distributions: NWU (new worse than used); IMRL (increasing mean residual lifetime); and UBAE (used better than aged in expectation), to name but a few bits of jargon. I have the advantage of having read many of the authors' research papers, so I had a good idea where this was all leading, but, for a first-time reader, this chapter may appear both dry and baffling.

Chapter 3 contains a short discussion on mixed Poisson distributions. It contains a number of bounds for ratios of tail probabilities, including bounds based on reliability classifications of the mixing distribution. From counting distributions in Chapter 3, we move to compound distributions in Chapter 4. These are the standard tool for modelling aggregate claims distributions. After a discussion of basic ideas and some illustrations of cases where tail probabilities can be found exactly, the authors derive general upper and lower bounds for tail probabilities of compound distributions. These are based on inductive arguments. A feature of the text is that it is written in a style that: "involves the use of elementary analytic

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techniques which require only undergraduate mathematics as a prerequisite". This is both a strength and a weakness of the book. Other (shorter) proofs of results are available using different techniques. The authors illustrate this, in Chapter 4, by also deriving the main results using a martingale argument, but this is not a general feature of the text.

Chapter 5 discusses bounds based on reliability classifications, whilst Chapter 6 discusses parametric bounds — both exponential and Pareto bounds. These chapters are very technical, and a disappointing aspect is that there is no attempt to place the results in context. We are not told, for example, which distributions are commonly used in insurance applications and what their reliability classification is. Further, for a book on bounds and approximations, there is not a single table of numbers or any graph to illustrate the application of bounds. Whilst parametric bounds are neat and convenient for calculation purposes, simple numerical bounds can be calculated for many compound distributions using Panjer's (1981) recursion formula. I was disappointed that the authors had not attempted to compare and contrast such approaches.

Chapter 7 discusses both compound geometric and compound negative binomial distributions. Although the former is a special case of the latter, it is worth separate study, as it is crucial in the study of ruin theory. However, the authors make very little reference to ruin theory in this chapter, although we do meet the famous Lundberg inequality. Chapter 8 discusses a simple approximation, known as a Tijms approximation, to the tail of a compound geometric distribution, which is a mixture of two exponentials. This approximation is applied in later chapters, particularly when integrals to the tail probability of a compound geometric distribution are involved.

Chapter 9 is, in my view, the most interesting chapter of the book. It contains very recent research on the relationship between defective renewal equations and compound geometric distributions. The authors use this relationship to discuss the time of ruin and related quantities. These ideas are also used in Chapter 10 to study the distribution of the deficit at ruin, given that ruin occurs. In this chapter I came across a new reliability classification. In Theorem 10.5.1, the authors provide a bound for the conditional distribution of the deficit at ruin, given that ruin occurs. The proof hinges on the NWU property of the compound geometric distribution. This result was basically proved by Dickson (1989) using a simple probabilistic argument. In my humble, and unbiased, opinion, this is a clear example of a NWU proof! In the final chapter, there is a discussion of the Sparre Andersen risk model, with generalisations of some results from previous chapters.

This book has much to offer both teachers and researchers in risk theory. It would be a simple exercise to put together a one or two semester course based on the text (one semester if your courses are reliability-free zones). Also, the above mentioned lack of numerical examples provides a wealth of potential projects for students. From a research point of view, the text provides an excellent summary of many important results. The references in the text are excellent and the bibliography is extensive. This book is a must for any serious student of risk theory.

References

DICKSON, D.C.M. (1989). Recursive calculation of the probability and severity of ruin. *Insurance: Mathematics & Economics*, **8**, 145-148.

PANJER, H.H. (1981). Recursive calculation of a family of compound distributions. *ASTIN* Bulletin, **12**, 22-26.

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