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IN DEFENCE OF THE SOVEREIGNTY OF PHILOSOPHY: AL-BAGHDĀDĪ'S CRITIQUE OF IBN AL-HAYTHAM'S GEOMETRISATION OF PLACE*

NADER EL-BIZRI

I

This paper examines the critical objections that were raised by the philosopher ʻAbd al-Laṭīf al-Baghdādī (d. ca. 1231 CE) against al-Ḥasan ibn al-Haytham's (Alhazen; d. *after* 1041 CE) geometrisation of place.[1] The philosophical propositions that were advanced by al-Baghdādī in his tract: *Fī al-Radd ʻalā Ibn al-Haytham fī al-makān* (*A refutation of Ibn al-Haytham's place*), are contrasted herein with the geometrical demonstrations that Ibn al-Haytham presented in his groundbreaking treatise on place, entitled: *Qawl fī al-Makān* (*Discourse on place*). In this line of enquiry, we shall attempt to establish careful distinctions between physical and mathematical methods of investigating the problems that respectively result from the definition of τόπος in terms of *enveloping / containing surfaces* or the contrasting conception of *al-makān* as a *postulated / imagined void*. We shall moreover situate al-Baghdādī's critique in the context of his defence of the Aristotelian definition of place as delineated in Book IV (*Delta*) of the *Physics*, which was refuted on mathematical grounds by Ibn al-Haytham.[2]

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[1] I take this opportunity to thank Professor Roshdi Rashed for his thoughtful remarks on an earlier version of this study, which assisted me in refining its propositions.

[2] The historical significance of this dispute points to classical responses to Aristotle's conception of place within corollaries attributed to Philoponus and

II

It was not uncommon in the mediaeval intellectual history in Islam that selected problems in theoretical philosophy were solved, or attempted to be resolved, with the assistance of mathematics. This was for instance the case with Abū Sahl Wayjan ibn Rustam al-Qūhī's geometrical demonstration of the possibility of achieving 'an infinite motion in a finite time' (*fī al-zamān al-mutanāhī ḥaraka ghayr mutanāhiya*),[3] which aimed at contesting Aristotle's views on this matter as they were delineated in Book VI of the *Physics*. Using a geometrical model, al-Qūhī showed that if the arc of a given semicircle can be traversed in a uniform motion in a finite time, then its simultaneously projected variable motion on an infinite branch of a hyperbola would likewise be covered in a finite time. Moreover, we attest similar endeavours to deploy mathematics in solving selected problems of speculative onto-theology; like it was the case with Naṣīr al-Dīn al-Ṭūsī's use of mathematical *combinatorial analysis* in explicating the onto-theological phenomenon of emanation from the One.[4] Such instances, which accentuated the epistemic and cognitive priority of the mathematical disciplines over the other classical traditions in science and philosophy, under certain conditions in research, were elaborately investigated by Roshdi Rashed in his monumental volumes of *Les mathématiques infinitésimales* (particularly in vol. 4),[5] as well as in his recently published

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Simplicius. Moreover, prolongations of the epistemic consequences of the geometrisation of place are encountered in the early-modern scientific and mathematical research of Descartes and Leibniz in reference to '*extensio*' and '*analysis situs*'.

[3] See: Roshdi Rashed, *Geometry and Dioptrics in Classical Islam* (London, 2005), p. 986. Refer also to: Roshdi Rashed, 'Al-Qūhī *vs.* Aristotle: On motion', *Arabic Sciences and Philosophy*, 9 (1999): 7–24. It is worth noting in this regard that al-Qūhī praised the epistemic and foundational value of mathematics in comparison with the merits of the other sciences in the preamble of his treatise *On the Trisection of a Known Angle* (*Istikhrāj qismat al-zāwiya al-ma'lūma bi-thalāthat aqsām mutasāwiya*); Rashed, *Geometry and Dioptrics*, pp. 494–5.

[4] See: Rashed, *Geometry and Dioptrics*, p. 975, and: Roshdi Rashed, 'Metaphysics and mathematics in classical Islamic culture: Avicenna and his successors', in Ted Peters and Muzaffar Haq (eds.), *God, Life, and the Cosmos: Christian and Islamic Perspectives* (Aldershot, 2002), pp. 151–71; Roshdi Rashed, 'Combinatoire et métaphysique: Ibn Sīnā, al-Ṭūsī et al-Ḥalabī', in Roshdi Rashed and Joël Biard (eds.), *Les doctrines de la science de l'antiquité à l'âge classique* (Leuven, 1999), pp. 61–86.

[5] Roshdi Rashed, *Les mathématiques infinitésimales du IX^{e} au XI^{e} siècle*, vols. 1–4 (London, 1993–2002).

groundbreaking treatise: *Geometry and Dioptrics in Classical Islam*.[6] It is perhaps in this classical spirit that Ibn al-Haytham endeavoured to present his geometrical definition of *al-makān* (place) as a solution to a longstanding problem that remained philosophically unresolved, which, to our knowledge, did also constitute in its own right the first viable attempt to geometrise 'place' in the history of mathematics and science.

Ibn al-Haytham's primary objectives aimed at promoting a geometrical conception of place that is akin to *extension* in an attempt to address selected mathematical problems and needs that emerged in reference to unprecedented developments in geometrical transformations (*al-naql*), the introduction of motion (*al-ḥaraka*) in geometry, the anaclastic research in conics, and the founding of dioptrics in the 9th / 10th century prolongations of the school of the Apollonian-Archimedean Arabic legacy of mathematicians like the Banū Mūsā ibn Shākir, Thābit ibn Qurra, Ibrāhīm ibn Sinān, Abū Sa'd al-'Alā' ibn Sahl, Abū Sahl Wayjan ibn Rustam al-Qūhī and Aḥmad ibn Muḥammad ibn 'Abd al-Jalīl al-Sijzī.

For instance, based on catoptrics and the study of burning mirrors and lenses, Ibn Sahl devised the foundations of dioptrics, and furthermore established a principle akin to the so-called '*Snell's Law*' of refraction (a formula used in the calculation of the refraction of light between two media of differing refractive indexes).[7] Moreover, Ibn Sahl's projective method, his study of the squaring of curvilinear figures and of the continuous drawing of conic sections, all paralleled the research of al-Qūhī, who attempted to offer solutions to geometrical problems that could not be resolved using an unmarked ruler and a compass; like the duplication of a cube, the trisection of an angle, and the construction of the regular heptagon. Combining the geometry of conics with *neusis* (namely, a *verging* geometric construction that adjusts a

[6] See *supra* n. 3.

[7] This law is named after the Dutch mathematician Snellius Willebrord (d. 1626), which was also investigated by the English mathematician Thomas Harriot (d. 1621) and by René Descartes (d. 1650). However, the investigation of Ibn Sahl's dioptrics necessitates the reformulation of the problem of the successive rediscoveries of this 'law of refraction'; as Rashed eloquently states: 'In other words, to the names of Snell, Harriot and Descartes we must henceforth add that of Ibn Sahl' (Rashed, *Geometry and Dioptrics*, p. 63). See also: Roshdi Rashed, 'A pioneer in anaclastics: Ibn Sahl on burning mirrors and lenses', *Isis*, 81 (1990): 464–91.

measured length on a marked straightedge to fit a diagram) bestowed legitimacy to the construction and use of scientific instruments (astrolabes, sundials, compasses), and evolved into procedures deploying the intersections of conics and their applications; in addition to introducing novel directions in mathematics involving loci and quadratic surfaces (resonating with Pierre Fermat's *Loci in surfaces*; 17th century early-modern science).

Moreover, Ibn Sahl, al-Qūhī and al-Sijzī endeavoured to ground the perfection of instruments (compasses, astrolabes) on mathematics, and they applied geometry to solving problems in astronomy. This also resulted in the definition of conic curves by the motions generating them (like for instance that of the perfect compass), and it led al-Sijzī to establish a distinction between geometrical curves and mechanical ones (reminiscent of what is encountered seven centuries later in René Descartes' separation of geometrical-algebraic curves from mechanical-transcendental ones).[8]

Besides the penchant to offer mathematical solutions to problems in theoretical philosophy that were challenged by longstanding historical obstacles and epistemic impasses, Ibn al-Haytham's remarkable and successful endeavour in geometrising place was undertaken in view of sustaining and grounding his research in mathematical analysis and synthesis (*al-taḥlīl wa-al-tarkīb*),[9] and in response to the needs associated with the unfurling of his studies on knowable mathematical entities (*al-ma'lūmāt*), in order to reorganise most of the notions of geometry and rethinking them in terms of motion.[10]

[8] This classical line in mathematical research was explored in detail, with accompanying sets of Arabic critical editions and annotated English (and French) translations of selected foundational treatises composed by Ibn Sahl, al-Qūhī and al-Sijzī, as advanced in: Rashed, *Geometry and Dioptrics*, Roshdi Rashed, *Géométrie et dioptrique au X^e siècle* (Paris, 1993); translated also into Arabic as: *'Ilm al-handasa wa-al-manāẓir fī al-qarn al-rābi' al-hijrī* (Beirut, 1996; 2nd ed. 2001). See also: Philippe Abgrall, *Le développement de la géométrie aux IX^e–XI^e siècles: Abū Sahl al-Qūhī* (Paris, 2004).

[9] The Arabic critical edition (based on 4 MSS) and the annotated French translation of this treatise (*Fī al-Taḥlīl wa-al-tarkīb*; *L'analyse et la synthèse*) are established in: Rashed, *Les mathématiques infinitésimales*, IV, 230–391.

[10] The Arabic critical edition (based on 2 MSS) and annotated French translation of this treatise (*Fī al-Ma'lūmāt*; *Les connus*) are established in: Rashed, *Les mathématiques infinitésimales*, IV, 444–583. See also: Roshdi Rashed, 'La philosophie mathématique d'Ibn al-Haytham, II: *Les Connus*', *MIDEO*, 21 (1993): 87–275.

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Consequently, he had to critically reassess the dominant philosophical conceptions of place in his age, which were encumbered by inconclusive theoretical disputes that were principally developed in response and reaction to Aristotle's *Physics*.

One ought to add herein that, while most philosophers (including Ibn Sīnā in the *Shifā'* and *Kitāb al-Ḥudūd*) adopted the Aristotelian conception of place, the dialectical theologians (mainly the exponents of *Mu'tazilī kalām*) affirmed the existence of the void, and reflected on place as being akin to spatiality (*ḥayyiz* or *taḥayyuz*) in physical deliberations that were partly founded on geometric adaptations of the physical theories of Greek atomism.

It is in this context that Ibn al-Haytham's *Qawl fī al-Makān* (*Discourse on Place*)[11] can be grasped as being a critique of the thrust of the Aristotelian conception of *topos* as an enveloping surface (*saṭḥ muḥīṭ*) following the thesis of Book IV of the *Physics*.[12]

In reflection of the purposes of our enquiry, and prior to assessing the merits of 'Abd al-Laṭīf al-Baghdādī's attempted refutation of Ibn al-Haytham's geometrisation of place, which can be seen as a defence of the sovereignty of philosophy against mathematical critiques, we ought firstly to present highlights regarding Aristotle's notion of *topos*, to be followed by selected critical geometrical demonstrations deployed by Ibn al-Haytham to refute it as well as reinforce his own novel mathematical definition of *al-makān*.

III

Even though Aristotle affirmed that *topos* has the three dimensions of length, width and depth (διαστήματα μὲν οὖν ἔχει τρία, μῆκος καὶ πλάτος καὶ βάθος; *Physics*, IV, 209a 5), he rejected the theories that posited place as being the form (εἶδος), the matter (ὕλη) or the interval (διάστημα) between the extremities of the body that it contains (*Physics*, IV, 212a 3–5). He ultimately defined *topos* as: 'the innermost primary surface-boundary of the containing body that is at rest, and is in contact with the

[11] The Arabic critical edition (based on 5 MSS) and annotated French translation of this treatise (*Fī al-Makān*; *Traité sur le lieu*) are established in: Rashed, *Les mathématiques infinitésimales*, IV, 666–85.

[12] Aristotle, *Physics*, ed. W. David Ross (Oxford, 1936).

outermost surface of the mobile contained body' (ὥστε τό τοῦ περιέχοντος πέρας ἀκίνητον πρῶτον, τοῦτ' ἔστιν ὁ τόπος; *Physics*, IV, 212a 20–21). Based on this thesis, one would add that a place can be grasped as a *vessel* (ἀγγεῖον) that is immovable (ἀμετακίνητον). Moreover, when something moves inside another that is also in motion, like a boat in a river, it uses the containing body as a *vessel* (ἀγγεῖον), while the river-basin acts as the motionless (ἀκίνητος) place (*Physics*, IV, 212a 15–20).[13] 'A *place* is together with the [contained] *thing*, since the *limit* [of that which contains] *coincides* with that which is *limited*' (ἔτι ἅμα τῷ πράγματι ὁ τοπος. ἅμα γὰρ τῷ πεπερασμένῳ τὰ πέρατα; *Physics*, IV, 212a 29–30); and this is the case given that the inner boundary of a containing body coincides with the shape of the contained body.

For the purposes of our enquiry (and with the permission of ardent Aristotelians who correctly construe the *Physics* as being principally an investigation of the principles of motion, κίνησις), let us bracket references made to motion and rest, as well as suspend our judging of their philosophical (Aristotelian) implications. So, by way of elucidating the theme of our investigation, we shall say that *topos* is: 'the inner surface of the containing body that is in contact with the outer surface of what it contains'. Hence, it is an enveloping surface of containment; which resulted in the grasp of *al-makān* as a *saṭḥ muḥīṭ* or *saṭḥ ḥāwī* (surrounding or containing surface).

Nonetheless, we ought to highlight herein that this definition of *topos* refers principally to what I shall call a '*local* place', which is the *specific containing body* that a given thing occupies, in contrast with the '*cosmic* qua *natural place*', namely the one towards which things tend to go back to due to their own

[13] To further illustrate the significance of Aristotle's definition of *topos* in reference to motion, let us take the example of a bottle filled with water that is within a vessel (boat) floating down a river. The specific and local place of the water would be the inner surface-boundary of the bottle that contains it, which is at *rest* in reference to the water it contains, and is in contact with it; and this is the case even though the bottle is *within* a vessel that is sailing on the river. Hence, the *place* of the vessel would not be the water of the river as such, but the *riverbed* (the river basin), which is at *rest*, and is in contact with the *mobile* water. This enveloping and containment physical principle continues outwards in macro scales in the shape of concentric spheres (envelopers / containers), whereby the ultimate limit (that is perhaps the boundary of 'all there is') would be the outermost, furthest and immobile extremity of heavens (*Physics*, IV, chapters 4–5).

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nature (φύσει; κατὰ φύσιν) if not prevented from doing so;[14] like heavy bodies *travel by their nature downwards* towards the Earth in a fall in the direction of the centre of the Universe (κόσμος; τὸ πᾶν), or that light bodies *travel by their nature upwards* towards the heavens (*Physics*, IV, 4, 212a 24). This view does furthermore accentuate the Aristotelian presupposition of the so-called '*power* of place' (τόπου δύναμις) in assertion of the existential anteriority of *topos* (πρῶτον εἶναι) with respect to all beings (*Physics*, IV, 208b 33–34, 209a 1–2).

IV

In contesting the longstanding Aristotelian *physical* conception of *topos* (specifically in its 'local' containment sense; *vide supra*, 'Section III') Ibn al-Haytham posited *al-makān* as an imagined void (*khalā' mutakhayyal*; postulated void) whose existence is secured in the imagination, like is the case with invariable geometrical entities.[15] He moreover held that the 'imagined void' *qua* 'geometrised place' consisted of imagined immaterial distances that are between the opposite points of the surfaces surrounding it (*al-khalā' al-mutakhayyal huwa al-ab'ād al-mutakhayyala allatī lā mādda fīhā, allatī hiya bayna al-nuqaṭ al-mutaqābila min al-saṭḥ al-muḥīṭ bi-al-khalā'*).[16] He furthermore noted that the imagined distances of a given body, and those of its containing place, get superposed and united in such a way that they become the same distances (*qua* dimensions) as mathematical lines having lengths without widths / breadths. Consequently, it is worth noting in this regard that Ibn al-Haytham's geometrical conception of place as a relational extension is 'ontologically' neutral.[17] This is the case

[14] For instance, when a cup rests on the surface of a table, its fall is arrested, given that the surface of this table on which it rests prevents it from travelling further towards the Earth in the direction of the centre of the Universe.

[15] Following the Platonist tradition, geometry transcends the transitory nature of visual experiences and points towards the clarity of ideas. It is rather a mode of knowing '*what always is*'; namely, *being*: the realm of eternal existents. See: Plato, *Republic*, the Greek text with English translation by Paul Shorey (Cambridge, Massachusetts, 1930–1935); trans. rep. Harvard University Press, 1980; 526e, 527b.

[16] Rashed, *Les mathématiques infinitésimales*, IV, 669.

[17] In order that I do not reproduce complementary research efforts that I have engaged elsewhere on this topic (The latest in the context of an international colloquium held in Perugia in September 2005, where I presented a study titled: '*Le problème de l'espace*' in a panel that I had the honour to join with Professor

given that his mathematical notion of *al-makān* was not simply obtained through a 'theory of abstraction' as such, nor was it derived by way of a 'doctrine of forms', nor was it grasped as being the (phenomenal) 'object' of 'immediate experience' or 'common sense'. It is rather the case that his geometrised place resulted from a mathematical isometric 'bijection' function between two sets of relations or distances.[18] Nothing is retained of the properties of a body other than *extension*, which consists of mathematical distances that underlie the geometrical and formal ('*formelle*') conception of place. Accordingly, the *makān* of a given object is a '*region of extension* that is defined by the distances between its points, on which the distances of that object can be applied *bijectively*'.[19] One would thus note that it is precisely this essential aspect of Ibn al-Haytham's mathematical thesis that eluded al-Baghdādī's critique.[20]

Roshdi Rashed) I refer the reader to the following titles: Nader El-Bizri, 'Le problème de l'espace', in Graziella Federici Vescovini (ed.), *Oggetto e spazio*, Micrologus (Firenze, 2006), forthcoming; Nader El-Bizri, 'A philosophical perspective on Alhazen's *Optics*', *Arabic Sciences and Philosophy*, 15, 2 (2005): 189–218; Nader El-Bizri, 'La perception de la profondeur: Alhazen, Berkeley et Merleau-Ponty', *Oriens-Occidens: sciences, mathématiques et philosophie de l'antiquité à l'âge classique* (Cahiers du Centre d'Histoire des Sciences et des Philosophies Arabes et Médiévales, CNRS), 5 (2004): 171–84; Nader El-Bizri, 'ON KAI KHÔRA: Situating Heidegger between the *Sophist* and the *Timaeus*', *Studia Phaenomenologica*, IV, 1–2 (2004): 73–98; Nader El-Bizri, 'Ontopoièsis and the interpretation of Plato's Khôra', *Analecta Husserliana: The Yearbook of Phenomenological Research*, LXXXIII (2004): 25–45; Nader El-Bizri, 'A phenomenological account of the "ontological problem of space"', *Existentia Meletai-Sophias*, XII, 3–4 (2002): 345–64; Nader El-Bizri, '*Qui êtes-vous Khôra?* Receiving Plato's *Timaeus*', *Existentia Meletai-Sophias*, XI, 3–4 (2001): 473–90; Nader El-Bizri, 'The body and space', *CAST*, 3 (2000): 92–5.

[18] 'Bijection' (or 'bijectivity') refers herein to an equivalence relation or function of mathematical transformation that is both an 'injection' (*i.e.* a 'one-to-one' correspondence) and 'surjection' (designated in mathematical terms as 'onto') between two sets.

[19] Rashed describes this mathematical conception as '*ensembliste et relationelle*' (*Les mathématiques infinitésimales*, IV, 658, 901).

[20] It is perhaps worth noting that a tacit mathematical inclination to '*de-ontologise*' place may have been encountered prior to Ibn al-Haytham's *Qawl fī al-Makān* in Thābit ibn Qurra's negation of the Aristotelian doctrine of 'natural *place*'. It is relegated to us on the authority of Fakhr al-Dīn al-Rāzī (d. ca. 1209 CE) that Thābit stated: 'whomever thinks that Earth seeks the place where it is (*ṭāliba li-al-makān alladhī hiya fīh*; *i.e.* tends towards its own place) holds a false opinion, since one ought not represent (*yatawahham*) a given place as having a certain state (*ḥāl*) that is exceptionally its own'. See: Rashed, *Les mathématiques infinitésimales*, IV, 658, note 14; also refer to: Fakhr al-Dīn al-Rāzī, *Kitāb al-Mabāḥith al-mashriqiyya* (Tehran, 1966), vol. II, p. 63; as also cited by Rashed.

In interrogating al-Baghdādī's objections to Ibn al-Haytham's geometrisation of place, it is worth highlighting in this context that Aristotle's definition of place received bold classical critiques in the commentaries on his work, including the reflections of Theophrastus on this matter and the poignant objections advanced by Philoponus in defence of a conception of *topos* as extension or interval (διάστασις; διάστημά). Additional doubts concerning Aristotle's conception of *topos* were also delineated in Simplicius' corollary on place.[21] However, what primarily distinguishes Ibn al-Haytham from his predecessors is that his critique of Aristotle was mathematical, and, that it was partly auxiliary to his response to the epistemic and mathematical need to geometrise place, while what preceded his efforts (including Philoponus' corollaries) mainly restricted their critical objections to the Aristotelian notion of *topos* to philosophical deliberations in physics. This state of affairs may have led sceptics amongst the philosophers / physicists to assume positions that affirmed (intentionally or not) the physical possibility of the existence of the void (κενόν; *Physics*, IV, 208b 25–27, 213a 15–19, 214a 2–17); hence displaying a clear opposition to Aristotle's physical theory as a whole rather than merely raising doubts regarding his definition of *topos* as such.

V

To offer some highlights of Ibn al-Haytham's critique of Aristotle's definition of *topos* as an enveloping surface, we could, for instance, evoke the mathematical model that he deployed in demonstrating the incoherence of a conception of place that rests on a 'containment by envelopment' thesis. In a mathematical interpretation of Ibn al-Haytham's arguments let us consider the case of a parallelepiped (*mutawāzī al-suṭūḥ*; *i.e.*

[21] I refer the reader to: *Simplicii in Aristotelis Physicorum Libros Quattuor Priores Commentaria*, ed. Hermann Diels in *Commentaria in Aristotelem Graeca*, vol. IX (Berlin, 1882); Simplicius, *Corollaries on Place and Time*, trans. James Opie Urmson (London, 1992); 601,25–611,10; 604,5–11. See also: Simplicius, *On Aristotle, Physics 4.1–5, 10–14*, trans. James Opie Urmson (London, 1992); Philoponus, *Corollaries on Place and Void*, and: Simplicius, *Against Philoponus on the Eternity of the World*, trans. David Furley and Christian Wildberg (London, 1991) – Fragments from the *Physics* commentaries of Theophrastus of Eresos were gathered in a volume edited by William W. Fortenbaugh, Pamela M. Huby, Robert W. Sharples and Dimitri Gutas (Leiden, 1991).

a geometric solid bound by six parallelograms) that occupies a given place defined by the surfaces enclosing it. If that parallelepiped were to be divided into two parts by a rectilinear plane that is parallel to one of its surfaces, and is then recomposed, the cumulative size of the parts resulting from its partition would be equal to the magnitude of that parallelepiped prior to being divided, while the total sum of the surface areas of the parts would be greater than that of the parallelepiped prior to its division. Following the Aristotelian definition of *topos* as enveloping surface, and in reference to the case of this partitioned parallelepiped, one would conclude that: an object divided into two parts occupies a place that is larger than the one it occupied prior to its division. Hence, the place of a given body increases while that body does not (*makān al-jism yaʿẓim wa-al-jism lam yaʿẓim*); consequently, an object of a given magnitude is contained in unequal places, which seems to be an untenable proposition.[22] Likewise, if we consider the case of a parallelepiped that we carve with a variety of carefully selected geometrical shapes, we would diminish its bodily magnitude while the total sum of its surface areas would increase. Following the Aristotelian definition of *topos* as enveloping surface, and in reference to the case of this carved parallelepiped, one would conclude that: an object that diminishes in scale occupies a larger place prior to its diminution in magnitude, which seems to be an untenable thesis. Moreover, using mathematical demonstrations, in reference to geometrical figures of equal surface-areas (*isépiphaniques*), which are based on studies conducted on figures that are of equal perimeters (*isopérimétriques*), Ibn al-Haytham showed that the sphere is the largest in size with respect to all other solids that have equal areas of their enveloping outer surfaces (*al-kura aʿẓam al-ashkāl allatī iḥāṭatuhā mutasāwiya*).[23]

[22] Refer to Ibn al-Haytham's consideration of this parallelepiped in: Rashed, *Les mathématiques infinitésimales*, IV, 670–3. Let us suppose that the parallelepiped is P1 with a corresponding *makān* M1. If P1 is divided into P1a and P1b, then M1 will be correlatively divided into two places M1a and M1b. Following the Aristotelian definition of *topos*, the magnitude of the place M1 will be equal to the total sum of the surface areas of P1, and so is the case with M1a and M1b in reference to the total sums of the surface areas of P1a and P1b respectively. Consequently, we obtain the following results: M1a + M1b > M1, and P1a + P1b = P1; given that P1a and P1b will have together two additional surface areas generated along the cut of the plane that divided P1.

[23] This also referred to the title of one of Ibn al-Haytham's studies that was mentioned in his *Qawl fī al-Makān*. One ought to highlight that a similar line of

Ultimately, Ibn al-Haytham's critique of Aristotle's definition of *topos*, and his own geometrical positing of *al-makān* as an 'imagined void' (*khalā' mutakhayyal*), both substituted the grasping of the body as being a totality bound by physical surfaces to construing it as being a set of mathematical points that are joined by geometrical line segments. Hence, the qualities of a body are posited as an *extension* that consists of mathematical lines, which are invariable in magnitude and position, and that connect points within a region of the *three-dimensional space* independently of any physical body.

The geometrical place of a given object is posited as a 'metric' of a region of the so-called 'Euclidean' *qua* 'geometrical *space*', which is occupied by a given body that is in its turn also conceived extensionally, and corresponds with its geometrical place by way of 'isometric bijection'. Consequently, Ibn al-Haytham's geometrisation of place points to what later was embodied in the conception of the 'anteriority of spatiality' over the demarcation of a metric of its regions by means of mathematical lines and points, as explicitly implied by the notion of a 'Cartesian space'.[24]

Based on Ibn al-Haytham's geometrical demonstrations that contrasted volumetric magnitudes with surface areas, Aristotle's definition of *topos* seems to be groundless. However, a closer look at the consequences of these demonstrations may still show that what appears as being an evident proof does potentially hold tacit implications that do not readily entail the radical incoherence of Aristotle's conception of *topos* as an enveloping surface. It is in this regard that we ought to rethink

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enquiry is encountered in the research of Abū Ja'far Muḥammad ibn al-Ḥusayn al-Khāzin. See: Rashed, *Les mathématiques infinitésimales*, I, 776, 828; II, 381–2, 451–7.

[24] See: Rashed, *Les mathématiques infinitésimales*, IV, 661–2, and associated notes 25–26 on p. 662. It is also worth noting what Descartes stated in this regard, that: 'L'objet des géomètres, que je concevais comme un corps continu, ou un espace indéfiniment étendu en longueur, largeur et hauteur ou profondeur, divisible en diverses parties, qui pouvaient avoir diverses figures et grandeurs, et être mues ou transposées en toutes sortes'; René Descartes, *Discours de la méthode*, in *Œuvres de Descartes*, ed. Charles Adam and Paul Tannery (Paris, 1965), vol. VI, p. 36 – also cited by Rashed, p. 662. Moreover, Leibniz noted that a place (*situs*) is a fragment of the geometrical *space* that describes an invariable relation between the points of a given configuration of an object, like '$A \cdot B$' which designates an *extensum* that ties A with B mathematically with invariance. See: Gottfried Wilhelm Leibniz, *La caractéristique géométrique*, ed. Javier Echeverria, trans. Marc Parmentier, *Mathesis* series (Paris, 1995), p. 235 – also quoted by Rashed, p. 662.

the merits of al-Baghdādī's philosophical interrogation of Ibn al-Haytham's geometrical critique of Aristotle's physical definition of *topos*. Nonetheless, in doing so, I do not question the fact that Ibn al-Haytham's geometrisation of place offered efficient and working solutions to emerging problems in mathematics. I do not also raise doubts regarding its epistemic pertinence or its celebrated historical significance, which were confirmed in the maturation of mathematics and science as embodied by the 17th century conceptions of place as extension *qua* space; particularly in reference to the works of Descartes and Leibniz. Nevertheless, while Ibn al-Haytham's geometrisation of place is a coherent historical and intellectual undertaking, which was epistemologically grounded by subsequent research in mathematics, and is akin to our modern sensibility in grasping *place* as a three dimensional, uniform, isotropic and homogeneous space, al-Baghdādī's objections to his critique of *topos* as an enveloping surface are not to be refuted outright.

There is no doubt that the maturation of Euclidean geometry and its prolongations benefited immensely from the geometrisation of place, which among other developments resulted in the emergence of what came to be known in periods following Ibn al-Haytham's age as being a 'Euclidean space'; namely, an appellation that is coined in relatively modern times, and describes a notion that is historically posterior to the geometry of figures as embodied in Euclid's *Stoikheia* (*The Elements*; *Kitāb Uqlīdis fī al-Uṣūl*).[25] After all, the expression deployed by Euclid that is closest to a notion of '*space*' as expressed in the Greek term: χώρα, is the appellation: χωρίον, which designates 'an area enclosed within the perimeter of a specific geometric abstract figure' – As for instance noted in Euclid's *Data* (*Dedomena*; *al-Mu'ṭayāt*) Prop. 55 (related to: *Elements*, VI, Prop. 25): 'if an *area* [χωρίον] be given in form and in magnitude, its sides will also be given in magnitude'.[26]

[25] Euclid, *The Thirteen Books of Euclid's Elements*, vols. 1–3, translated with introduction and commentary by Thomas L. Heath, 2nd edition (New York, 1956). The Greek edition of Euclid's *Elements* is preserved in the Teubner Classical Library in 8 volumes with a supplement, entitled: *Euclides opera omnia*, ed. Johann Ludvig Heiberg and Heinrich Menge (Leipzig, 1883–1916).

[26] Some contemporary architectural historians and theorists argue that: 'the notion of a homogeneous Euclidean space is a modern invention that coincides with the development of perspective, leading to the formation of the Cartesian space'. See: Dalibor Vesely, *Architecture in the Age of Divided Representation: The Question of Creativity in the Shadow of Production* (Cambridge, Massachusetts,

In spite of the evidence associated with Ibn al-Haytham's sound and compelling geometrical demonstrations, al-Baghdādī's attempted defence of the sovereignty of philosophy in reaction to the mathematical doubts raised about its explications still merits rethinking.

VI

In his attempted refutation of Ibn al-Haytham's conception of place (*Fī al-Radd 'alā Ibn al-Haytham fī al-makān*),[27] al-Baghdādī displayed great rigour and attentiveness in presenting the cases raised by Ibn al-Haytham with accuracy, wherein his critique offers historians some valuable highlights regarding the textual transmission of the latter's treatise (*Qawl fī al-Makān*) up to the beginnings of the 13th century CE.

Closely following each of Ibn al-Haytham's arguments, and not failing to admire the mathematical acumen of the author subjected to his critique, al-Baghdādī claimed that Ibn al-Haytham did not logically account for a correspondence / concomitance between a given object and its 'place' *qua* 'enveloping surfaces' as both being subject to change.[28]

If a given object changes by way of division and / or diminution, its place changes as well, due to the transformation of its shape and its associated surface areas. To explore this proposition, let us reconsider the case of the parallelepiped which was divided and / or carved; in both instances it has been transformed in its shape and associated surfaces, hence its place changed as well. If a divided object becomes two distinct entities, then its shape is likewise transformed into two separate shapes, and its original place is transmuted into two different places with distinct surface areas. The fact that the parallelepiped is divided or carved entails that it is no longer

2004), pp. 113, 140–1. Refer also to: David Rapport Lachterman, *The Ethics of Geometry: A Genealogy of Modernity* (London, 1989), p. 80; Morris Kline, *Mathematics: The Loss of Certainty* (Cambridge, 1980), p. 87.

[27] The Arabic edition (based on 1 manuscript: Bursa, Hüseyin Çelebi, MS. 823; copied prior to the 16th cent.) and annotated French translation of this treatise (*Fī al-Radd 'alā Ibn al-Haytham fī al-makān*; *La réfutation du lieu d'Ibn al-Haytham*) are established in: Rashed, *Les mathématiques infinitésimales*, IV, 908–53.

[28] Rashed, *Les mathématiques infinitésimales*, IV, 914–15.

the same entity that it was prior to its division or carving; and so is the case with its place, shape and the total sum of its surface areas, which get transformed into something else. According to al-Baghdādī, Ibn al-Haytham's geometrical proofs neglected the fact that a change in a given object leads to a transformation in its shape, the total sum of its surface areas, and the place it occupies. Failing to recognise that the parallelepiped becomes something other than itself, when partitioned or carved, results in neglecting the fact that its shape, place, and the total sum of its surface areas are also transformed. It is hence valid to say that an object occupies a different place when it is divided and / or carved, given that it is no longer the same object *per se*, but is rather transformed into another sort of entity with a distinct shape, and a different total sum of its surface areas; even if its recomposed (volumetric) size remained the same. Similarly, a sphere that has a surface area equal to that of a cube would be (volumetrically) larger than the cube; and yet, this does not readily entail that the Aristotelian definition of *topos* is unsound, since that sphere is distinct from the cube, and it has a different shape from it, even though both have equal surface areas. The same can also be said about a sphere that is equal in (volumetric) magnitude to a cube, which entails that its surface area is smaller than that of the cube; and, hence, as a distinct entity, with a differing shape and surface areas, it would occupy a smaller place than that of the cube.

In all of this, al-Baghdādī presupposed philosophical accounts of the individuation of bodies as a modality by virtue of which he attempted to offer counterexamples to Ibn al-Haytham's geometrical demonstrations, while also unguardedly and erroneously assuming that the latter's propositions were reducible to one and the same type of arguments. After all, al-Baghdādī kept on emphasizing that his responses to Ibn al-Haytham's claims assumed that the ambiguities (*al-shubah*) that were pointed out by the latter were all the same with variations in examples (*aqūl innamā hiya shubha wāḥida lahā amthila kathīra*);[29] while, the mathematical contents of Ibn al-Haytham's objections to the Aristotelian conception of *topos* are in fact not simply reducible to the same type of geometrical demonstrations.

[29] Rashed, *Les mathématiques infinitésimales*, IV, 916–17, 924–5.

Attempting to refute Ibn al-Haytham's objections regarding Aristotle's definition of place, al-Baghdādī argued that the judgement of a given body in-itself differs from judging its surrounding surfaces; since the surfaces of a body change in the magnitude of their areas with the transformation of the shape of that body, while the body is unchanged in-itself (*ḥukm al-jism fī dhātih ghayr ḥukm suṭūḥih al-muḥīṭa; fa-inna suṭūḥ al-jism takhtalif misāḥatuhā bi-ikhtilāf ashkāl al-jism, wa-al-jism fī nafsih lā yataghayyar*).

To illustrate al-Baghdādī's counterargument in response to Ibn al-Haytham, let us for instance consider the case of reshaping the same quantity of wax from being a sphere to becoming a cube. Consequently, the wax maintains its (volumetric) magnitude while the surface areas of its transformed shape become larger. Accordingly, al-Baghdādī held that, even though the surface area of the sphere is in this case smaller than that of the cube, the substantial body of the wax remained the same, without increase or decrease in magnitude (*al-jism al-jawharī wāḥid lam yazid wa-lam yanquṣ*); given that this body stays the same in its substance, like is the case with this wax (*al-jism yakūn wāḥidan fī jawharih, ka-al-sham'a mathalan*).[30]

In all of this, al-Baghdādī believed that Ibn al-Haytham's mathematical doubts were not only raised with respect to place as an enveloping surface, but were moreover applicable to the essence of the body that occupies it (*al-shubah al-mu'tarada laysat fī al-makān fa-ḥasb bal fī dhāt al-jism*); given that a body is in a place by way of its actual surfaces not its internal potential distances (*al-jism fī makān bi-suṭūḥih lā bi-ab'ādih fī nafsih*).

Moreover, and in response to Ibn al-Haytham's positing of place as an 'imagined void' (*khalā' mutakhayyal*), al-Baghdādī wondered how the actual distances (*bi-al-fi'l*) of a given body can be superposed on the imagined potential distances (*bi-al-quwwa*) of its place. He was unsure whether Ibn al-Haytham considered the distances of a body and those of its place as being potentialities and not actualities; hence positing them as non-existents. He furthermore rejected the claim that the presumably 'superposed distances' (*al-ab'ād al-mutaṭābiqa*) can be actual existents, since this implies a co-penetration of

30 Rashed, *Les mathématiques infinitésimales*, IV, 924–5.

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material entities, which seems to be an implausible state of affairs;[31] hence failing to recognise the epistemic entailments of Ibn al-Haytham's mathematisation of place as geometric *extension*.

After all, al-Baghdādī asserted that the mathematician judges distances insofar that they are imagined in the mind as being abstracted from matter (*wa-hādhā sha'n al-rajul al-ta'līmī alladhī yaḥkum 'alā al-ab'ād bi-mā hiya muta-khayyala fī al-dhihn*), while the physicist grasps them as existing externally (*mawjūda fī al-khārij*). And yet, the difference between the research of the *physicist* and that of the *mathematician* does not only reflect the contrast between an Aristotelian metaphysics *cum* physics and a Platonist theory of forms, it rather points primarily to a 'third' classical tradition that we could refer to as being 'Archimedean', which was not satisfied with the mere philosophical cognition of 'natural phenomena', but essentially aimed at investigating them mathematically; and it is precisely this pathway that Ibn al-Haytham followed.[32]

Moreover, al-Baghdādī noted that a single body can be in many differing places that have diverse magnitudes (*maqādīr*), but not simultaneously, given that it occupies places of differing magnitudes due to changes in its shapes. This position rested in principle on ontological determinations that equated the existence of bodies and their places with physical manifestations and observations; hence to exist is to be an externally present entity, subject to visual perception, and to concrete modes of experiential verification and inspection.[33] One could perhaps note that al-Baghdādī endeavoured to advance his objections to Ibn al-Haytham's mathematical demonstrations in a manner that accommodated Aristotelian methods in philosophical analysis, which presupposed the individuation of bodies, and the presupposition that they ought to be analysed in terms of the structuring notions of matter / form and

[31] Rashed, *Les mathématiques infinitésimales*, IV, 916–17.

[32] Rashed, *Les mathématiques infinitésimales*, IV, 928–9.

[33] The ontological entailments of thinking about given objects in *eidetic* terms (in reference to εἶδος) encompass a distinction between mathematical objects (μαθήματα) and the entities of sense perception (αἴσθησις), which evoke a separation of the *noêtic* from the *aisthêtic* (respectively in reference to νόησις and αἴσθησις), and the manner the former is closer to what is seen as being knowable or *epistêmonic* (in reference to ἐπιστήμων).

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actuality / potentiality (relying in this on concepts that were doubted by Ibn al-Haytham).

It is perhaps pertinent in this context to indicate that, based on theories of visual perception, which place some emphasis on the physicality of existents, it is hardly plausible that a polymath like Ibn al-Haytham, who articulates clear and distinct ideas, would have accepted a duality (or dualism) in his conception of place, whereby his *Qawl fī al-Makān* geometrised place and grasped it as extension (namely, as a mathematical and abstracted demarcation of place in metric space), while his *Kitāb al-Manāẓir* (*Optics*; *De aspectibus*; *Perspectivae*) would have retained an Aristotelian notion of *topos* as a two-dimensional surface. It is therefore unclear in this context why a scholar like A. Mark Smith argued that the perception of distance (*qua* depth) in Ibn al-Haytham's *Optics* was rooted in the Aristotelian conception of *topos*, which is relative to the surrounding objects that define it. As if by this, Mark Smith was not aware of the remarkable accomplishment brought forth by Ibn al-Haytham's *Qawl fī al-Makān*;[34] or he might have possibly confused our polymath al-Ḥasan ibn al-Haytham (Alhazen) with his mediaeval contemporary, the philosopher / physicist of the Aristotelian tradition, Muḥammad ibn al-Haytham, who commented on place and time within the Peripatetic legacy in reflection of his adoption of the definition of *topos* in Book IV of the *Physics*,[35] which clearly ought not to be mistaken for Alhazen's

[34] See: A. Mark Smith, 'The Alhacenian account of spatial perception and its epistemological implications', *Arabic Sciences and Philosophy*, 15 (2005): 219–40, p. 225, n. 14. Also refer to Book II, Chapter 3, Paragraphs 67–126 of: Ibn al-Haytham, *Kitāb al-Manāẓir*, ed. Abdelhamid I. Sabra (Kuwait, 1983); or in: Alhazen, *The Optics of Ibn al-Haytham, Books I–III, On Direct Vision*, trans. Abdelhamid I. Sabra (London, 1989) – A Latin version of the text is also preserved under the cover: *Opticae thesaurus Alhazeni*, ed. Friedrich Risner (Basel, 1572). I have discussed elsewhere the question concerning the perception of depth in Ibn al-Haytham's *Optics* and its affirmation of the grasp of place as *extension*; see: El-Bizri, 'La perception de la profondeur'; El-Bizri, 'A philosophical perspective on Alhazen's *Optics*'; Nader El-Bizri, 'La phénoménologie et l'optique géométrique', in André Allard (ed.), *Actes du congrès de la Société Internationale d'Histoire des Sciences et des Philosophies Arabes et Islamiques* (Namur, 2006), forthcoming; also, *vide supra*, note 17.

[35] Muḥammad ibn al-Haytham, *Kitāb al-Makān wa-al-zamān 'alā mā wajadah yalzam ra'y Arisṭūṭālīs fīhimā* (*A treatise on place and time as he* [*Muḥammad ibn al-Haytham*] *found corresponding to Aristotle's opinion*) – Mentioned in: Ibn Abī Uṣaybi'a, *'Uyūn al-anbā' fī ṭabaqāt al-aṭibbā'*, ed. Nizar Rida (Beirut, 1965), p. 558.

(al-Ḥasan ibn al-Haytham) treatise on place and its geometric imports.[36]

Notwithstanding, we ought to highlight in this context that Alhazen (al-Ḥasan ibn al-Haytham) maintained a rigorous geometrical account of place in his *Qawl fī al-Makān*, which did not allude to aspects or enquiries that were not determined mathematically; and he did this despite the substantial optical research he conducted in his *Kitāb al-Manāẓir* on the perception of place.

VII

By reflecting on the matter at hand from a situational and contextual perspective, which speculates about the circumstances surrounding textual authorship or its exegesis, it is not that clear why al-Baghdādī attempted to refute Ibn al-Haytham's definition of *al-makān* in the course of his enquiries about the essence of place. After all, the urge to respond to Ibn al-Haytham's thesis might not have been necessarily restricted to didactic *cum* pedagogic exercises aiming at demonstrating to apprentices the arguable 'incoherence' of Ibn al-Haytham's endeavour, or the 'fragility' of the mathematicians' attempts to address or resolve selected problems in theoretical philosophy. And yet, one cannot assert with decisive historical documentation that Ibn al-Haytham's *Qawl fī al-Makān* and its geometrisation of place sustained common or widespread epistemic practices within the scholarly circles of al-Baghdādī's time, which would have thus favoured the mathematical conception of place over the physical Aristotelian notion of *topos*; this being the case even though Ibn al-Haytham was known in that period as being one of the principal classic polymaths. And yet, manuscripts of Ibn al-Haytham's *Qawl fī al-Makān* must have been in circulation

[36] Regarding the debate over the distinction between our polymath, al-Ḥasan ibn al-Haytham (Alhazen) and the philosopher / physicist of the Aristotelian tradition, Muḥammad ibn al-Haytham, I refer the reader to the following: Abdelhamid I. Sabra, 'One Ibn al-Haytham or two? An exercise in reading the bio-bibliographical sources', *Zeitschrift für Geschichte der arabisch-islamischen Wissenschaften*, Band 12, ed. Fuat Sezgin (Frankfurt am Main, 1998), pp. 1–50; and furthermore refer to the clarification of this matter as advanced by Roshdi Rashed in his chapter: 'Al-Ḥasan ibn al-Haytham et Muḥammad ibn al-Haytham: Le mathématicien et le philosophe, *Sur le Lieu*', *Les mathématiques infinitésimales*, IV, 957–9.

amongst theologians and philosophers, since we know that at least a scholar like Fakhr al-Dīn al-Rāzī (d. ca. 1209 CE) did indeed read it.[37]

We cannot conclusively ascertain that Ibn al-Haytham's geometrisation of place constituted an explicit and directly pressing challenge to al-Baghdādī's philosophical teachings, nor to his standing amongst his contemporaries. However, given the poignant conceptual menace confronting the hegemony of Aristotle's physical account of *topos* as a result of Ibn al-Haytham's geometrical conception of place, an Aristotelian like al-Baghdādī had to endeavour to refute Ibn al-Haytham's *Qawl fī al-Makān* despite the intellectual risk of calling further attention to its content, and thus dialectically contributing to its promotion even by way of opposition.

After all, in order to refute Ibn al-Haytham's thesis, al-Baghdādī reconstructed it to the best of his expository skills; hence, he also made it implicitly available to the learned in the form of a critical commentary in such a manner that they had the opportunity to assess whether its content was epistemologically superior to his own position or not. He thus indirectly exposed his own thesis to the peril of being potentially discredited; as the history of mathematics and science ultimately proved to us to be the case.

Albeit, al-Baghdādī gave hints regarding what incited him to attempt to refute Ibn al-Haytham's thesis. After all, al-Baghdādī noted that what motivated him to compose his critical treatise (*alladhī ḥarrakanī ʿalā waḍʿ hādhih al-maqāla*) is the observation that Ibn al-Haytham's discourse on place expressed a thesis that cannot be readily attributed to someone of his eminent station as a celebrated polymath, given that it does not befit his erudition, even though it is akin to his writing style or utterances (*namaṭṭ kalāmih*). Ultimately, al-Baghdādī was concerned about sheltering 'truth' wherein an illustrious scholar like Ibn al-Haytham may have errantly concealed it or buried it (*yukhāf ʿalā al-ḥaqq idhā taʿarraḍa rajul nabīh li-ṭamsih*).[38] His response to Ibn al-Haytham's

[37] A synopsis of Ibn al-Haytham's *Qawl fī al-Makān* was reported by Fakhr al-Dīn al-Rāzī in a fragment from his *Kitāb al-Mulakhkhaṣ*. The Arabic critical edition (based on 1 manuscript: Tehran, Majlis Shūrā, MS. 827, fols. 92–93), and annotated French translation of al-Rāzī's fragment are established in: Rashed, *Les mathématiques infinitésimales*, IV, 955–6.

[38] Rashed, *Les mathématiques infinitésimales*, IV, 910–11.

treatise was not only a reactionary attempt to defend the Peripatetic tradition in philosophy, but may have also resulted from his endeavour to secure his own personal investigation of the notion of place in the commentaries he composed on the versions of Aristotle's *Categories* (*al-Manṭiq*; *Qāṭīghūriyās*) and *Physics* (*al-Samā' al-ṭabī'ī*) as they were available in his time.[39] Refuting the philosophical doubts that were raised against Aristotle's conception of *topos*, and further explicating their entailments, al-Baghdādī came across Ibn al-Haytham's treatise and was compelled to respond to it in view of completing his own research on the essence of *al-makān*.

It is worth adding that al-Baghdādī's critique of Ibn al-Haytham, in defence of philosophy and the physical conceptions of place, rested on his own grasp of Aristotle's *Physics*, and was not squarely reproducing the Aristotelian arguments, but rather grounding his own interpretations and speculations on them. The objections and counterexamples that were delineated by al-Baghdādī against Ibn al-Haytham's geometrisation of place offered adaptive extensions and interpretive prolongations of Aristotelian concepts. After all, al-Baghdādī's critical arguments can be assessed in their own right as being expressions of his philosophical acumen, and consequently offer a testimony to the epistemic applications of the Aristotelian tradition in thinking. Furthermore, al-Baghdādī's treatise was composed in specific intellectual circumstances as a direct and careful response to the propositions that he faithfully reconstructed based on Ibn al-Haytham's *Discourse on Place*.

Ultimately, al-Baghdādī believed, with unfairness and inaccuracy, that mathematicians like Ibn al-Haytham neglected logic (*al-manṭiq*) and engaged a little exercising of this art (*li-qilat riyāḍatih bi-ṣinā'at al-manṭiq*), while being errantly reliant on dialectical methods (*ṭuruq jadaliyya*; διαλεκτικός). Based on al-Baghdādī's judgement, the lack of logical grounding led mathematicians like Ibn al-Haytham to commit aberrations that did not befit polymaths of the latter's eminent station. It may have well been the case that al-Baghdādī assumed that Ibn al-Haytham was purely a mathematician (*rajul ta'līmī*) whose art did not carry philosophical bearings;[40] consequently failing to recognise the

[39] Rashed, *Les mathématiques infinitésimales*, IV, 908–9.
[40] Rashed, *Les mathématiques infinitésimales*, IV, 910–15, 916–17, 920–7.

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significance of mathematics in solving selected problems of theoretical philosophy.

In our mediation on the topic at hand, a circumspectly nuanced distinction ought to be drawn between the fragility (or failure) of some classical philosophers *cum* physicists in responding to the doubts raised by mathematicians concerning their theories and the actual epistemic merits of their art. If the arguments of Aristotle were successfully rejected by Ibn al-Haytham, this does not readily undermine or radically compromise the epistemological significance of Aristotelian physics altogether.

VIII

The multidisciplinary character of the dispute over the definition of place, between physics and geometry, between Aristotle and al-Baghdādī from one side, and (if I may say) Ibn al-Haytham and subsequently Descartes / Leibniz from the other side, may be interpreted as being the result of 'an epistemic shift' in 'paradigmatic' perspective.

To see something is to interpret it; whereby there is some sort of merging between the seeing of an aspect and the 'dawning' of another. What I describe as being a place is also a factor of my viewpoint on it. It is ultimately my perception that is described as well as what I interpretively see. This also reflects a logical interchange of designators, or a 'play of signifiers', where the total sum of surface areas is designated as *topos*, in contrast with assigning the name 'place' (or 'space') to a volumetric interval (imagined void) between the innermost surfaces of a containing body. This reflects the ambiguity that surrounds a 'homonym' (*al-ism al-mushtarak*), as evoked by al-Baghdādī in reference to Ibn al-Haytham's arguable 'reductive' conflation of a given body unto its mathematical continuous quantifications.[41] In the exchange of the cognitive implications of 'naming', the designation of *place* as 'surface' seems plausible from a logical viewpoint, though of little use for geometry and science when compared with the mathematical and scientific accomplishments that the (volumetric) geometrisation of place led to.

[41] Rashed, *Les mathématiques infinitésimales*, IV, 922–3.

This state of affairs solicits further reflections on ways of breaking away from the dominance of a *disjunctive* logic (of the: '*either* this, *or* that'), in view of admitting novel possibilities of thinking about the topic at hand in reference to a *conjunctive* logic of 'the *complementarity* of opposites / contraries' (of the: 'this *and* that'). In this case, one ought not tend to place excessive emphasis on 'incommensurability' or 'anomalies', and rather rethink how duality (or dualism) points to 'indeterminacy' in assessing the truth-value of theories. Ultimately, preferences become justified in reference to the epistemic and cognitive entailments or consequences of theories. We thus face what can be referred to as: 'three-value' logic; namely that of: truth / falsity / indeterminacy. The *indeterminate* middle-ground admits both theories, while potentially giving primacy of one over the other (volume *vs*. surface) in terms of its bearings and applications in particular epistemological and methodological conditions, without entailing the epistemic or veridical equivalence of both theories.

One ought to add in this context that, from a mathematical and epistemic standpoint, there is no equivalence between the 'Aristotelian *topos*' and the 'Alhazenian-Cartesian *extensio* / space', even from the viewpoint of the historical maturation of non-Euclidean geometries.[42] After all, the Aristotelian theory regarding *topos* investigated 'entities' from the perspectives of 'common sense' doctrines and 'descriptions' of the phenomenal 'objects' of immediate experience, which are foreign (if not even 'inconsequential') to mathematics, while the Alhazenian-Cartesian '*extensio* / space' can be posited (even from the standpoint of non-Euclidean systems) as being a case of Euclidean geometry that is not simply refutable.

[42] We point herein to the geometries that questioned Euclid's tradition, mainly in reference to the refutability of the universal applicability of the 'Fifth Postulate' of parallelism, as principally embodied in the non-Euclidean geometrical systems / models that were advanced by the Italian scholar Girolamo Saccheri, the Russian mathematician Nicolai Lobachevsky, the Hungarian mathematician Janos Bolyai, and the German mathematician Bernhard Riemann. It is worth reminding the reader that *Postulate 5* of Euclid's *Elements* may be interpreted as follows: 'If two lines are drawn which intersect a third in such a way that the sum of the inner angles on one side is less than two right angles, then the two lines inevitably must intersect each other on that side if extended far enough'. Moreover, Def. 23 of Book I of the *Elements* also addressed the question of 'parallelism' by stating that: 'Parallel lines are straight lines which, being in the same plane and being produced indefinitely in both directions, do not meet one another in either direction'.

While we unreservedly celebrate the historical and conceptual achievement made by Ibn al-Haytham in his geometrical conception of place, it is still the case that the fragile line in thinking adopted by al-Baghdādī is not necessarily to be negated, although its applications and intellectual prolongations are restrictive and limited. The difference between both scholars is that they did not enquire about the same 'object' (nor the same '*makān*'); while al-Baghdādī's 'object' was that of common sense and as the given of immediate experience, Ibn al-Haytham's '*object*' was 'constructed' (*i.e.* postulated / posited) as an 'entity' (schema) of scientific / mathematical research. This being the case, given that Ibn al-Haytham's *makān* has a reality that is independent of the corporeality of physical objects, which is that of 'imagined distances' construed within the '*more geometrico*' research in Euclidean geometry.[43]

In a digression from geometrical and physical / philosophical epistemic accounts of place and / or space (extension), the diverse applications of geometry in architecture do not neglect the tactile and experiential approaches to the uniqueness of the heterogeneous *physical place*, as opposed to solely focusing on the homogeneity of the isotropic *mathematical space*. While I do not question the epistemological, scientific and mathematical value of the geometrisation of place, the 'containment by envelopment' physical definition seems to be applicable under restrictive conditions within certain practices in architecture and its associated plastic arts. After all, architecture and perspective share a sense of coherent spatiality as embodied in the notion of the '*room*'. This idealised representation acquired in its longstanding history the characteristics of the isotropic space of geometry, which was already 'anticipated' in the inherent perspectivity of architecture with the parallelism of its structuring components like columns, pillars and walls, in addition to the axial regularity of its spatial articulations and arrangements.[44] And yet, while architecture centres on the design of space in terms of geometric determinants, the preoccupation with the production of space and its technological imports retains the physical concreteness of place in sight. Hence, the isotropic and *homogeneous mathematical space* of

[43] See: Rashed, *Les mathématiques infinitésimales*, IV, 661.
[44] See: Vesely, *Architecture*, pp. 113, 140–1.

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abstraction is co-entangled with the unique and *heterogeneous physical place* of experiential concreteness;[45] albeit, without entailing that architectural thinking (historical, theoretical or critical) advocates a definition of *place* as an 'enveloping surface' that compromises the deeply entrenched conception of *place* as 'space'.

What emerges in reference to the conception of place and / or space as a consequence of the classical traditions in physics and mathematics, as exemplified by al-Baghdādī's philosophical reaction to Ibn al-Haytham's mathematical break from the Peripatetic model, is the guiding observation that: '*a lower-dimensional entity bounds a higher-dimensional one*'.[46]

[45] My enquiry on place and space in reference to Aristotle, Ibn al-Haytham, and al-Baghdādī, as well as my appeal to the corollaries of Philoponus and Simplicius, or to early-modern reflections on this matter to be thought in the works of Descartes and Leibniz, and through them in the tradition of Kant and in phenomenology, all coincide with my training, and with my pedagogic and didactic concerns, as well as research orientation in the field of architectural history, theory and criticism. It is in this sense that my preoccupation with theories of place and space in architectural humanities corresponds with parallel accounts in the history of mathematics, science and philosophy, like what I have attempted to address in this paper and other complementary studies that I have conducted elsewhere; for some references, *vide supra* note 17.

[46] For instance, a three-dimensional cube is bound by two-dimensional squared-surfaces; or, a 'hypercube', within the 'fourth hidden dimension', is hypothetically delimited by three-dimensional cubes; again: '*the lower-dimensional entity bounds a higher-dimensional one*'. This principle is central to contemporary discussions in Particle Physics, mainly in the domain of exploring theories that underlie the Standard Model (including String Theory and Model Building). This is particularly the case with research on the 'multidimensionality' of the universe as embodied in theories articulated around the notion of '*brane*' (namely: 'a mem*brane*-like entity in higher-dimensional space that can carry energy and confine particles and forces; wherein, a '*brane*-world', like our visible universe, is a physical setup in which matter and forces are confined to '*branes*' and bound by them). See: Lisa Randall, *Warped Passages: Unravelling the Universe's Hidden Dimensions* (London, 2005).