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ANATOMIZING INCOMPLETE-MARKETS SMALL OPEN ECONOMIES: POLICY TRADE-OFFS AND EQUILIBRIUM DETERMINACY

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We propose a simple incomplete-markets small-open-economy model that is amenable to analytical dissection of its policy-relevant mechanisms. In contrast to its complete-markets limit, the equilibrium real exchange rate is irreducible from the incomplete-markets equilibrium. Market incompleteness exacerbates the domestic-inflation and output-gap monetary-policy trade-off in two ways: its steepness and its resulting endogenous cost-push to the trade-off. The latter depends on an equilibrium combination of structural shocks and on agents' beliefs of future events. Thus, in comparison to its complete-markets and closed-economy limits, standard Taylor-type rules are less capable of inducing determinate rational expectations equilibrium in our environment. Despite the larger policy trade-off under incomplete markets, simple policies that also respond to exchange-rate growth are able to manage expectations that drive the endogenous cost-push term. However, policies that respond directly to expectations may turn out to exacerbate the cost-push trade-off further, and thus, to be more likely to fuel self-fulfilling multiple or unstable equilibria.

Keywords: Incomplete Markets, Monetary Policy Isomorphism, Exchange Rate, Equilibrium Determinacy

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1. INTRODUCTION

Why should small-open-economy monetary authorities care about international exchange rates? Is there a justification for managing exchange rates, and if possible, expectations thereof? What is its connection to incomplete international risk sharing of country-specific shocks? In practice, in many small open economies with floating-exchange-rate regimes, the dynamics of the exchange rate matter in structural modeling, and for monetary policy design. Also, it remains unclear in the literature which monetary policy is better equipped for inducing equilibrium stability, when the dynamics of the exchange rate cannot be decoupled from inflation and output gap in an equilibrium characterization.

In standard monetary-policy small-open-economy models, the exchange rate is a reducible variable in equilibrium. In other words, its explicit dynamics can be decoupled from necessary equilibrium conditions. Specifically, under certain restrictions on inter- and intratemporal elasticities of substitution, the open-economy dimension merely alters the equilibrium conditions that are familiar from a closed-economy model in terms of the slopes of an IS curve and a Phillips curve [see Benigno and Benigno (2003); Galí and Monacelli (2005); Clarida et al. (2001)]. More generally, if these parametric restrictions are relaxed, Benigno and Benigno (2003) have shown that the monetary policy implication for the open economy is no longer isomorphic to its closed-economy limit. That is, the design of monetary policy for the small open economy must also take into account the trade-offs arising from the open-economy channels. However, the explicit dynamics of the exchange rate is still redundant in these systems as long as the open economy has access to a complete international state-contingent asset market.

Our considerations in this paper are different from the well-known question regarding the "isomorphism" between closed- and open-economy monetary policies in the context of New Keynesian models. Ours are predicated on the role of international asset market incompleteness in explaining the irreducibility of the exchange rate from an equilibrium description of a small open economy. More importantly, we ask how this single feature of market incompleteness alters well-known monetary policy trade-offs arising in complete-markets small-open-economy and closed-economy counterparts.[1] This then leads us to ask how the feature matters for simple and operational monetary policy design, when one is concerned about equilibrium determinacy.

We propose a tractable small-open-economy model with incomplete international asset markets in order to address these two questions. Our model nests the canonical complete-markets small-open-economy model of Clarida et al. (2001), which is similar to that of Galí and Monacelli (2005), and the standard New Keynesian closed-economy model [see, e.g., Woodford (2003)] as special cases. Our contribution is twofold.

Our first contribution is the following observation. Incomplete markets result in an irreducible and explicit exchange-rate channel in the model's equilibrium characterization. This result manifests itself in terms of two aspects relevant to

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monetary policy. We show that the complete-markets open economy has a less onerous domestic-inflation-to-output-gap trade-off than its closed-economy counterpart. [This repeats the insights from Clarida et al. (2001) and Llosa and Tuesta (2008).] However, for all empirically plausible values of risk aversion, we also show that the incomplete-markets open economy has a steeper (conditional) trade-off relative to the same closed-economy counterpart. These new insights are obtained analytically. Second, the irreducible-exchange-rate channel also shows up as an endogenous cost-push term that perturbs the conditional domestic-inflation–output-gap trade-off. This cost-push term is composed of conditional expectations of future output gap and exchange rate, along with an equilibrium combination of primitive exogenous shocks.[2] As a corollary, we also obtain a break in the "monetary-policy isomorphism" between the small open economy and its closed-economy limit.

As our second contribution, we show that established lessons on local stability of *rational expectations equilibrium* (REE) under alternative monetary policies are reversed as a result of the fact that the economy cannot completely insure against country-specific risks. This poses additional restrictions on the admissibility of policy rules in inducing determinate REE. We show that although the inability of a small open economy to insure against its country-specific technology risk reduces such admissible sets of monetary policies, it can be improved by a family of simple policies that take exchange rate growth into account as well.

The intuition for these numerical findings is given by our first observation—that the additional constraints on policy in the incomplete-markets setting arise through (i) an exacerbated conditional trade-off between domestic inflation and output gap and (ii) the endogenous cost-push channel. In the incomplete-markets setting, the latter yields another means for monetary policy to prevent self-fulfilling multiple equilibria, or worse, equilibrium instability. This other means is effected through monetary policies that can "correctly" manage expectations entering the endogenous cost-push term. By smoothing out output gap and real exchange rates, and therefore instilling non-self-fulfilling or nonexplosive conditional expectations, policies responding to growth in the exchange rate are better at inducing a determinate REE.

We thus provide a simple theoretical rationale for standard monetary policy modeling and practice in small open economies with floating exchange rates. In practice, modelers and policy makers in these economies take explicit exchange rate dynamics into account in model equilibrium conditions and also in policy objectives. For example, clause 4(b) of New Zealand's 2002 *Policy Targets Agreement* states that[3]

> "[I]n pursuing its price stability objective, the Bank shall seek to avoid unnecessary instability in output, interest rates and the exchange rate."

Our analysis in this paper also complements existing studies of business cycles and/or welfare consequences of alternative monetary policies assuming incomplete-market large or small open economies [e.g., McCallum and

Nelson (1999); Chari et al. (2002); Benigno and Thoenissen (2008); Leitemo and Söderström (2008); de Paoli (2009b)]. Although these papers focus on business-cycle accounting and/or quantifying welfare under alternative policies, there has not been a clear dissection of how a notion of market incompleteness impacts equilibrium monetary-policy trade-offs. Moreover, a clear exposition of the role of international asset-market incompleteness in affecting REE determinacy or indeterminacy under alternative monetary-policy rules has not been studied in either two-country or small-open-economy environments.[4]

Therefore, our contribution is to fill a gap in the literature by providing a tractable version of a small-open-economy model, whose equilibrium characterization allows for a careful dissection of the role of incomplete markets in altering an existing monetary policy trade-off and delivering an endogenous cost-push to that trade-off. That is, we can provide analytical and comparative policy insights with respect to well-known closed- and complete-markets open-economy models. This then allows us to revisit and contrast with well-known results [e.g., Bullard and Mitra (2002); Llosa and Tuesta (2008)] in terms of indeterminacy of REE under standard simple monetary policy rules.

The rest of the paper is organized as follows. In Section 2, we describe the details of our alternative model. We characterize competitive equilibrium in Section 3. In Section 4, we provide an analytical dissection of how asset-market incompleteness in our model can result in an exacerbated and endogenous monetary-policy trade-off. In Section 5, we analyze the implications of market incompleteness—and therefore the additional restrictions on stability-inducing monetary-policy rules-for equilibrium determinacy. Finally, in Section 6, we conclude.

2. MODEL

Consider a small-open-economy model consisting of monopolistically competitive domestic goods markets with nominal pricing rigidity, and, households that have access only to a restricted set of internationally traded non-state-contingent assets—viz., the incomplete-international-asset-markets assumption. The domestic economy is small in the sense that local equilibrium outcomes do not have any impact on the rest of the world, but the converse is not true. The foreign economy (or the rest of the world) is treated as a large closed economy. We will use variables with an asterisked superscript (e.g., X^*) to refer to the foreign country and variables without an asterisk to denote the small domestic economy. Subscripts "H" (for Home) and "F" (for Foreign) on certain variables will denote the country of origin for quantities and their supporting prices.

2.1. Representative Household

As in McCallum and Nelson (1999) or Benigno and Thoenissen (2008), individuals in our small open economy have access only to a pair of domestic and foreign nominal uncontingent bonds denominated in their own currencies, respectively B_t

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and B_t^*. More precisely, let $h^t := (z_0, \ldots, z_t)$ denote the t-history of aggregate shocks, where $z_t = (A_t, Y_t^*)$ is a vector of domestic productivity and foreign output levels, respectively. $B_{t+1}(h^t)$ or $B_{t+1}^*(h^t)$ denotes a claim on one unit of currency following h^t and is independent of any continuation state z_{t+1} that may occur at $t+1$. Let $S_t(h^t)$ denote the nominal exchange rate, defined as the domestic currency price of a unit of foreign currency. In domestic currency terms, the prices of one unit of the nominal bonds $B_{t+1}(h^t)$ and $B_{t+1}^*(h^t)$ are, respectively, $1/[1+r_t(h^t)]$ and $S_t(h^t)/[1+r_t^*(h^t)]$, where r_t and r_t^* are the respective domestic and foreign nominal interest rates.

The representative consumer in the domestic country faces the following sequential budget constraint, for each $t \in \mathbf{N}$ and each (measurable) history h^t,

$$P_t(h^t)C_t(h^t) + \frac{B_{t+1}(h^t)}{1+r_t(h^t)} + \frac{S_t(h^t)B_{t+1}^*(h^t)}{1+r_t^*(h^t)} \leq W_t(h^t)N_t(h^t) + B_t(h^{t-1}) + S_t(h^t)B_t^*(h^{t-1}) + \Pi_t(h^t), \quad \textbf{(1)}$$

where P_t is the domestic consumer price indexes, C_t is a composite consumption index, W_t is the nominal wage rate, N_t denotes the hours of labor supplied, and Π_t is the total nominal dividends received by the consumer from holding equal shares of the domestic firms.

A minor difference of our model from Galí and Monacelli (2005) is that consumers exhibit an endogenous discount factor that we denote by ρ_t. This assumption is introduced to ensure a unique nonstochastic steady-state consumption level, following Schmitt-Grohé and Uribe (2003).[5] However, this is not a fundamental assumption for our conclusions with respect to the endogenous monetary-policy trade-off arising from the real-exchange-rate channel.[6] The consumers' preferences are given by the following present-value total expected utility function:

$$\mathrm{E}_0\left\{\sum_{t=0}^{\infty} \rho_t\{U[C_t(h^t)] - V[N_t(h^t)]\}\right\}, \quad \rho_t = \begin{cases} \beta(C_{t-1}^a(h^{t-1}))\rho_{t-1} & \text{for } t > 0 \\ 1 & \text{for } t = 0, \end{cases} \quad \textbf{(2)}$$

where E_0 denotes the expectations operator conditional on time-0 information and C_t^a denotes the cross-economy average level of consumption.

For concreteness, we will consider the following parametric form for the function $\beta : \mathbf{R}_+ \to (0, 1)$, following Ferrero et al. (2010):

$$\beta(C_t^a) = \frac{\bar{\beta}}{1 + \phi(\ln C_t^a - \vartheta)}; \quad \bar{\beta} \in (0, 1). \quad \textbf{(3)}$$

We do not impose a priori any condition on the sign of the dependence of the discount factor on average consumption; i.e., we assume only that $\beta'(C_t^a) \neq 0$. We also assume that per-period utility of consumption and labor have the respective forms $U[C_t(h^t)] = C_t(h^t)^{1-\sigma}/(1-\sigma)$ and $V[N_t(h^t)] = \psi N_t(h^t)^{1+\varphi}/(1+\varphi)$, where $\sigma > 0$, $\varphi > 0$, and $\psi > 0$.

The household chooses an optimal plan $\{C_t(h^t), N_t(h^t), B_{t+1}(h^t), B^*_{t+1}(h^t)\}_{t\in\mathbf{N}}$ to maximize (2) subject to (1). Unilaterally, the household will take the aggregate outcome $C^a_t(h^t)$, nominal prices $\{W_t(h^t), P_t(h^t), S_t(h^t)\}_{t\in\mathbf{N}}$, and policy $\{r_t(h^t)\}_{t\in\mathbf{N}}$ as fixed for each measurable h^t, and also the household will take $B_0(h^0)$ and $B^*_0(h^0)$ as given. To simplify notation hereinafter, we denote a measurable selection $X_t(h^t) =: X_t$ implicitly. Define the real exchange rate as $Q_t := S_t P^*_t / P_t$. Given the functional forms, the respective first-order conditions of the household's problem, for each h^t and $t \in \mathbf{N}$, are

$$\psi N_t^{\varphi} C_t^{\sigma} = \frac{W_t}{P_t}, \tag{4}$$

$$C_t^{-\sigma} = (1 + r_t)\,\mathrm{E}_t \left\{ \beta\left(C_t^a\right) \left(\frac{P_t}{P_{t+1}}\right) C_{t+1}^{-\sigma} \right\}, \tag{5}$$

$$C_t^{-\sigma} = \left(1 + r_t^*\right)\mathrm{E}_t \left\{ \beta\left(C_t^a\right) \left(\frac{P_t^* Q_{t+1}}{P_{t+1}^* Q_t}\right) C_{t+1}^{-\sigma} \right\}. \tag{6}$$

Each optimally chosen C_t will be consistent with the household's intraperiod choice of a home-produced final consumption good, $C_{\mathrm{H},t}$ and an imported final good $C_{F,t}$, where C_t is defined by a CES aggregator

$$C_t = \left[(1-\gamma)^{\frac{1}{\eta}} (C_{\mathrm{H},t})^{\frac{\eta-1}{\eta}} + \gamma^{\frac{1}{\eta}} (C_{F,t})^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}; \quad \gamma \in (0,1),\ \eta > 1. \tag{7}$$

Furthermore, each type of final good, $C_{\mathrm{H},t}$ and $C_{\mathrm{F},t}$, are aggregates of a variety of differentiated goods indexed by $i, j \in [0, 1]$. Respectively, these aggregates are $C_{\mathrm{H},t} = [\int_0^1 C_{\mathrm{H},t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di]^{\frac{\varepsilon}{\varepsilon-1}}$ and $C_{\mathrm{F},t} = [\int_0^1 C_{\mathrm{F},t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj]^{\frac{\varepsilon}{\varepsilon-1}}$, where $\varepsilon > 1$. As is well known from Galí and Monacelli (2005), optimal allocation of the household expenditure across each good type gives rise to static demand functions for $(C_{\mathrm{H}}(i), C_{\mathrm{F}}(i), C_{\mathrm{H}}, C_{\mathrm{F}})$ and price indices. Details of these demand functions and prices are given in our online Supplementary Appendix (see Section A).

2.2. Differentiated Goods Technology and Pricing

We assume a production sector similar to that in Galí and Monacelli (2005). This is purely to keep our expositions later transparent and comparable to the mainstream models in the literature [i.e., Clarida et al. (2001); Galí and Monacelli (2005); Llosa and Tuesta (2008)].[7] Each domestic firm $i \in [0, 1]$ produces a differentiated good. Production is represented by a linear technology

$$Y_t\left(i, h^t\right) = A_t N_t^d\left(i, h^t\right), \tag{8}$$

where $N_t^d(i, h^t)$ is labor hired by the firm and the random variable $A_t := \exp\{a_t\}$ is an exogenous embodied labor productivity. With a homogeneous production function of degree one, the first-order conditions (for cost minimization with

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respect to labor) can be written in the aggregate as

$$\frac{W_t(h^t)}{P_t(h^t)} = \frac{\mathrm{MC}_t^n(h^t)}{P_t(h^t)} A_t, \tag{9}$$

where MC_t^n is nominal marginal cost.

Because each firm $i \in [0, 1]$ is assumed to be imperfectly competitive, it gets to set an optimal price $P_{\mathrm{H},t}(i, h^t)$ given a Calvo-style random time-independent signal to do so. With a per-period probability $(1 - \theta)$ the firm gets to reset price. For every date t and history h^t, the firm's optimal pricing decision is characterized by a first-order condition,

$$\mathrm{E}_t \left\{ \sum_{k=0}^{\infty} \theta^k \left(\prod_{\tau=t}^{t+k-1} \beta(C_\tau^a) \right) \frac{\xi_{t+k}}{\xi_t} Y_{t+k}(i) \left[\tilde{P}_{\mathrm{H},t}(i) - \left(\frac{\varepsilon}{\varepsilon - 1} \right) \mathrm{MC}_{t+k}^n \right] \right\} = 0, \tag{10}$$

where $\xi_t := U_C(C_t)$, and the demand faced by the firm at some time $t + k$ (and following history h^{t+k}), conditional on the firm maintaining a sale price of $\tilde{P}_{\mathrm{H},t}(i)$, is

$$Y_{t+k}(i) = \left[\frac{\tilde{P}_{\mathrm{H},t}(i)}{P_{\mathrm{H},t+k}} \right]^{-\varepsilon} \left(C_{\mathrm{H},t+k} + C_{\mathrm{H},t+k}^* \right). \tag{11}$$

In a symmetric pricing equilibrium, where $\tilde{P}_{\mathrm{H},t} := \tilde{P}_{\mathrm{H},t}(h^t) = \tilde{P}_{\mathrm{H},t}(i, h^t)$, the law of motion for the aggregate price is $P_{\mathrm{H},t} = [\theta P_{\mathrm{H},t-1}^{1-\varepsilon} + (1 - \theta) \tilde{P}_{\mathrm{H},t}^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}}$. As this part of the model is quite standard in the literature, we derive the details separately (see Supplementary Appendix B).

2.3. Market Clearing

In a competitive equilibrium we require that given monetary policy and exogenous processes, the decisions of households and firms are optimal, as characterized earlier, and that markets clear. First, the labor market must clear, so that (4) equals (9) for all states and dates: $N_t(i, h^t) = N_t^d(i, h^t)$. Second, the final Home-produced goods market for each variety $i \in [0, 1]$ clears, so that

$$Y_t(i, h^t) = C_{\mathrm{H},t}(i, h^t) + C_{\mathrm{H},t}^*(i, h^t). \tag{12}$$

Third, the no-arbitrage condition for international bonds will be given by the equality of (5) and (6). In the rest of the world, assumed to be the limiting case of a closed economy, we have market clearing as $Y_t^* = C_t^*$.

3. LOCAL EQUILIBRIUM DYNAMICS

In this section we characterize the log-linearized REE dynamics of our small open economy. To this end, consider the gap between each aggregate variable and its potential level defined in an equilibrium with fully flexible domestic prices—i.e., when the percentage deviation (from steady state) of real marginal cost, denoted by mc_t, is zero at any time t and in any state. Let lower case variables denote the percentage deviation of its level X from its nonstochastic steady state point X_{ss}; e.g., $x := \ln(X/X_{\mathrm{ss}})$. Define the potential output and the real exchange rate, respectively $\overline{y}_t$ and $\overline{q}_t$, as the levels of output and real exchange rate, respectively, at the flexible-price equilibrium. It can be shown that the levels of both $\overline{y}_t$ and $\overline{q}_t$ depend only on exogenous variables. Let $\widetilde{x}_t$ and $\widetilde{q}_t$ denote the domestic output gap and the real exchange rate gap (in percentage deviation), respectively, where $\widetilde{x}_t = y_t - \overline{y}_t$ and $\widetilde{q}_t = q_t - \overline{q}_t$. The REE characterization can be approximated to first-order accuracy as a system of forward-looking stochastic dynamic equations for $\widetilde{x}_t$, $\pi_{\mathrm{H},t}$ and $\widetilde{q}_t$. (Derivations are provided in Supplementary Appendix C.)

DEFINITION 1 (Incomplete Markets (IM)). *Given a monetary policy process $\{r_t\}_{t\in\mathbf{N}}$ and exogenous processes $\{\epsilon_t, u_t\}_{t\in\mathbf{N}}$, a (locally approximate) rational expectations competitive equilibrium (REE) in the IM model is a bounded stochastic process $\{\pi_{\mathrm{H},t}, \widetilde{x}_t, \widetilde{q}_t\}_{t\in\mathbf{N}}$ satisfying*

$$\pi_{\mathrm{H},t} = \bar{\beta}\mathrm{E}_t\{\pi_{\mathrm{H},t+1}\} + \lambda(\kappa_1\widetilde{x}_t + \kappa_2\widetilde{q}_t), \tag{13}$$

$$\widetilde{x}_t = \varpi\mathrm{E}_t\{\widetilde{x}_{t+1}\} - \mu[r_t - \mathrm{E}_t\{\pi_{\mathrm{H},t+1}\}] + \chi\mathrm{E}_t\{\widetilde{q}_{t+1}\} + \epsilon_t, \tag{14}$$

$$\widetilde{q}_t = \mathrm{E}_t\{\widetilde{q}_{t+1}\} - (1-\gamma)[r_t - \mathrm{E}_t\{\pi_{\mathrm{H},t+1}\}] + u_t, \tag{15}$$

where

$$\overline{\beta} = \beta(C_{\mathrm{ss}}),$$

$$\lambda = \frac{(1-\theta)(1-\theta\bar{\beta})}{\theta},$$

$$\kappa_1 = \varphi + \frac{\sigma}{1-\gamma},$$

$$\kappa_2 = -\frac{\sigma\eta\gamma(2-\gamma)}{(1-\gamma)^2} + \frac{\gamma}{(1-\gamma)},$$

$$\varpi = \frac{\sigma}{\sigma-\phi}, \quad \mu = \left[\frac{1-\gamma}{\sigma-\phi}\right]\left[1-\gamma + \frac{\eta\gamma\,(2-\gamma)\,(\sigma-\phi)}{1-\gamma}\right],$$

and

$$\chi = \frac{\eta\gamma\phi(2-\gamma)}{(1-\gamma)(\sigma-\phi)}.$$

Consider the equilibrium IS functional equation (14). In our small open economy the real exchange rate indirectly affects the output gap via the ex ante real interest rate (through μ). This indirect channel is similar to the standard models of Clarida et al. (2001) and Galí and Monacelli (2005) and depends on the degree of openness γ. Note, however, that movements in the conditional expectation of the real exchange rate in our model also affect the output gap (via χ) directly: (i) by modifying the marginal rate of substitution of consumption between different

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periods and across states (i.e., ϕ); and (ii) through the interaction of these effects with the substitution between home- and foreign-produced goods (via η). This direct channel is just an artefact of the endogenous discount rate model and is negligible when we assume the limiting case for the elasticity of the discount rate with respect to aggregate consumption, $\phi \searrow 0$. In this case, $\chi \approx 0$. This assumption follows Ferrero et al. (2010). Furthermore, ϕ affects the elasticities of output gap with respect to the ex-ante real interest rate μ. Again, with $\phi \searrow 0$, this indirect channel introduced by endogenous discounting will be negligible.

Equation (13) is an augmented New Keynesian Phillips curve representing the dynamics of the short-run aggregate supply. Consider first the term $\lambda\kappa_1$, representing the direct equilibrium link between output gap and domestic inflation. This term has the textbook interpretation of a conditional slope of the Phillips curve in output-gap–domestic-inflation space. It indexes the domestic-inflation–output-gap (or monetary policy) trade-off. This trade-off connects the domestic labor market equilibrium relation (hence the dependency of κ_1 on φ and σ) and goods market clearing (hence γ) to the firms' wage bills (or real marginal cost) and their optimal pricing plans. For example, when output demand gap $\tilde{x}_t$ goes up, all else unchanged, there is a rise in the domestic firms' demand for labor input to meet the rise in demand for their final goods. This raises the firms' real marginal cost and therefore domestic inflation, as some firms can and optimally would like to readjust prices upward to maintain their optimal markup plan. Variation in $\tilde{x}_t$ also has effects on the real exchange rate. Hence the degree of openness γ further steepens this domestic-inflation–output-gap trade-off. This feature is also common to standard complete markets models [e.g., Clarida et al. (1999, 2001)].

Next, consider the term involving κ_2, which is only present in the IM economy. This direct link between real exchange rate movements and the real marginal cost encapsulates two effects arising from demand channels corresponding to the two terms in the composite parameter κ_2 in equation (13). Consider an increase in the (current) real exchange rate—i.e., an exchange rate depreciation. This increases the relative prices of the imported consumption goods faced by domestic consumers. This effect has a substitution and a wealth effect on real marginal cost, and thus on domestic inflation in the equilibrium Philips curve (13). On one hand (i.e., the first term in κ_2), this leads consumers to reduce the demand for imported goods, and therefore to reduce aggregate consumption and to substitute more leisure for it. This translates into an increase in marginal product of labor that drives the marginal cost up. On the other hand (i.e., the second term in κ_2), this relative increase in the price of imported consumption goods reduces the real wage income faced by consumers, who react by increasing labor supply in response to the lower purchasing power of their given income. This leads to a reduction in the marginal product of labor, which pushes the marginal cost down.

Observe that the substitution effect dominates if agents are sufficiently risk-averse—i.e., $\sigma > (1-\gamma)/[\eta(2-\gamma)]$), so that $\kappa_2 < 0$. This implies that the effect of an increase in the relative price of the imported consumption goods on marginal cost is always negative. Therefore, the overall impact of the real exchange rate on domestic inflation will also be negative.[8] Moreover, the larger the measure of agents' risk aversion σ, the more sensitive is the previously discussed domestic-inflation–output-gap (or monetary-policy) trade-off, which is indexed by κ_1, to the real exchange rate. That is, we can imagine the monetary-policy trade-off shifting around more, the more sensitive it is to real exchange rate movements—i.e., the larger κ_2. In Section 4, we will relate to these observations again when we study the role of market incompleteness in affecting the policy trade-off.

Therefore, in contrast to standard models in the literature, we do not need to assume exogenous "cost-push shocks" to create a nontrivial monetary-policy trade-off.[9] Moreover, in contrast to standard open-economy models [e.g., Benigno and Benigno (2003); Galí and Monacelli (2005); de Paoli (2009a)], the relevant monetary-policy trade-off embedded in the Phillips curve—between $\widetilde{x}_t$ and $\pi_{H,t}$—is now perturbed by an endogenous "cost-push" channel (via $\lambda\kappa_2$).[10]

4. DISSECTING THE IM MECHANISM

We will now study the role of international asset-market incompleteness in this model in two parts. In Section 4.1, we demonstrate how IM implies an irreducible (i.e., explicit) real-exchange-rate channel. This is done by contrast to its two limit-economy observations—a complete-markets (CM) model and a closed (CD)-economy model. In Section 4.2, we complete the study by looking at what these limit economies mean for comparative monetary policy trade-offs across the three models. The following exposition on IM's exchange-rate irreducibility and IM's limit CM and CD economies will allow us to form sharper insights into how market incompleteness alters monetary-policy trade-offs relative to the well-known CM and CD assumptions. These insights will be useful for understanding the results of our experiments on alternative monetary policies and equilibrium determinacy later.

4.1. Two Limit Economies of IM

The IM model nests familiar CM [e.g., Clarida et al. (2001)] and CD [e.g., Woodford (2003)] counterparts. Let κ_1 and κ_2 be as stated in Definition 1.

In the CM version of our model, complete international risk sharing results in a tight link between the real exchange rate and the marginal rate of substitution between cross-country consumption, $q_t = \sigma(c_t - c_t^*)$, in every date and state of nature.[11]

Using this relationship and from market clearing, we obtain that

$$y_t = \frac{(1-\gamma)^2 + \sigma\eta\gamma(2-\gamma)}{\sigma(1-\gamma)} q_t + y_t^*. \quad \textbf{(16)}$$

Equation (16) also holds when output and the real exchange rate are at their respective potentials, $\overline{y}_t$ and $\overline{q}_t$. Because y_t^* is exogenous and assuming it is at its

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potential level, this implies that output gap is proportional to the real exchange rate gap, or

$$\tilde{q}_t = \frac{\sigma(1-\gamma)}{(1-\gamma)^2 + \sigma\eta\gamma(2-\gamma)}\tilde{x}_t \equiv \tau\tilde{x}_t. \qquad \textbf{(17)}$$

Using this fact, we arrive at the following characterization of a REE for the CM economy:

PROPOSITION 1 (Complete Markets). *If the small open economy has access to complete international Arrow securities, then the real exchange rate is reducible from—i.e., it has no direct role in—the dynamic characterization of equilibrium. The competitive equilibrium is then described by*

$$\pi_{\mathrm{H},t} = \overline{\beta}\mathrm{E}_t\{\pi_{\mathrm{H},t+1}\} + \lambda\kappa^{\mathrm{CM}}\widetilde{x}_t, \qquad \textbf{(18)}$$

$$\widetilde{x}_t = \varpi\mathrm{E}_t\{\widetilde{x}_{t+1}\} - \mu^{\mathrm{CM}}[r_t - \mathrm{E}_t\{\pi_{\mathrm{H},t+1}\}] + \epsilon_t, \qquad \textbf{(19)}$$

where

$$\begin{aligned} \kappa^{\mathrm{CM}} &= \frac{\sigma}{(1-\gamma)^2 + \sigma\eta\gamma(2-\gamma)} + \varphi \equiv \kappa_1 + \tau\kappa_2, \\ \tau &:= \frac{\sigma(1-\gamma)}{(1-\gamma)^2 + \sigma\eta\gamma(2-\gamma)}, \\ \mu^{\mathrm{CM}} &= \left[\frac{1-\gamma}{\sigma - \phi(1-\gamma)}\right]\left[1-\gamma + \frac{\eta\gamma\sigma(2-\gamma)}{1-\gamma}\right]. \end{aligned} \qquad \textbf{(20)}$$

The first term on the right of $\kappa^{\mathrm{CM}} \equiv \kappa_1 + \tau\kappa_2$ in (20) captures the direct link between output gap and domestic inflation. This channel is in common with its counterpart in the IM model, which was explained earlier. In contrast to IM, the second term in κ^{CM} captures a compound effect. Recall that in the IM economy, because real-exchange-rate variation $\tilde{q}_t$ is explicitly decoupled from output gap $\tilde{x}_t$ (because of incomplete international risk sharing), exogenous shocks causing movements in $\tilde{q}_t$ would directly impact domestic inflation via the equilibrium trade-off term κ_2. However, as we showed in (17), under CM, complete international risk sharing means that movements in $\tilde{q}_t$ are directly absorbed in $\tilde{x}_t$, reflecting equilibrium shifts of state-contingent allocations that satisfy the state-by-state and date-by-date no-arbitrage asset-pricing restriction. Therefore any impact of movements in $\tilde{q}_t$ on domestic inflation—i.e., κ_2 in the equivalence in (20)—will only arise indirectly via domestic output gap adjustments in the CM economy—i.e., the compound term $\tau\kappa_2$.

Observe that these indirect effects of the real exchange rate on the dynamics of domestic inflation (13) through marginal cost disappear when $\gamma = 0$. Furthermore, if $\phi = 0$, then there is no direct real exchange rate channel in the IS relation (14). Moreover, Clarida et al. (2001) have shown that such an economy is qualitatively similar to the CM economy. That is, PROPOSITION 2 (Closed Economy). *If (i) the economy does not rely on imported final consumption goods, $\gamma = 0$, and (ii) thus endogenous discounting is an irrelevant assumption (i.e., $\phi = 0$), then the model is equivalent to the canonical New Keynesian closed-economy model:*

$$\pi_{H,t} = \overline{\beta} E_t\{\pi_{H,t+1}\} + \lambda \kappa^{CD} \widetilde{x}_t \tag{21}$$

$$\widetilde{x}_t = \varpi E_t\{\widetilde{x}_{t+1}\} - \mu^{CD}[r_t - E_t\{\pi_{H,t+1}\}] + \epsilon_t, \tag{22}$$

where

$$\kappa^{CD} = \varphi + \sigma, \quad \mu^{CD} = \frac{1}{\sigma}, \quad \omega = 1. \tag{23}$$

This limit economy is isomorphic to the complete-markets small open economy characterized by (18) and (19).

Finally, note that our model admits another source through which the exchange rate may explicitly matter: the endogenous-discount-factor channel. However, as discussed earlier, this remains inconsequential to this result (i.e., when $\phi \searrow 0$). That is, if endogenous discounting were not present, an irreducible exchange rate dynamic would still remain; and the latter is purely a result of the existence of incomplete international asset markets.

4.2. Limit Economies and Comparative Policy Trade-Offs

We are now ready to discuss comparative monetary-policy trade-offs between IM and its limit economies: CM and CD. These comparisons can be conveniently cast in terms of the constant-relative-risk-aversion (CRRA) parameter σ. That is, under specific values of σ, we would have, respectively, an equivalence between IM and CM and an equivalence between IM and CD in terms of REE and monetary-policy trade-offs. For values of σ away from these limiting equivalence points, we can compare monetary-policy trade-offs implied by these three economies' different REE.[12] We will also discuss which of the cases of REE policy trade-offs considered are relevant for quantitatively plausible values of σ.

In the following observations, we maintain the assumption that $\phi \searrow 0$, which was justified earlier. First, consider the case when IM has the same REE characterization as CM. From Definition 1 and Proposition 1, we can see that this occurs when $\kappa_2 = 0$. A sufficient condition, written in terms of the risk aversion parameter σ, is $\sigma = \sigma^* := \frac{1-\gamma}{\eta(2-\gamma)}$. Denote this REE equivalence as $CM(\sigma^*) \equiv CD(\sigma^*)$. Perturbing the $IM(\sigma)$ economy away from this special case, we have that $\partial\kappa_2/\partial\sigma = \eta\gamma(2-\gamma)/(1-\gamma)^2 > 0$—i.e., in the IM economies with high risk aversion at some $\sigma \neq \sigma^*$ (i.e., with consumers who are more sensitive to inter- and intratemporal realloaction of risky consumption), the trade-off between output gap and domestic inflation (as indexed by κ_1) will face larger "shifts" due to movements in the real exchange rate. (Also, recall the earlier observation on this point in Section 3.)

Second, consider the case when CM is equivalent to CD, or $\kappa^{\mathrm{CM}} = \kappa^{\mathrm{CD}}$. Comparing Proposition 1 and Proposition 2, a sufficient condition for this economy to arise is that $\sigma = \hat{\sigma} := 1/\eta$. Denote this REE equivalence as $\mathrm{CM}(\hat{\sigma}) \equiv CD(\hat{\sigma})$. Away from this REE equivalence point, the CM economies are such that $\partial\kappa^{\mathrm{CM}}/\partial\sigma = (1-\gamma)^2/[(1-\gamma)^2 + \sigma\eta\gamma(2-\gamma)]^2 > 0$. We summarize these intermediate observations in Lemma 1. In short, what this means is that for $\sigma > \hat{\sigma}$ away from $\mathrm{CM}(\hat{\sigma}) \equiv CD(\hat{\sigma})$, a CM economy with higher risk aversion will face a steeper REE monetary-policy trade-off between domestic inflation and output gap.

LEMMA 1. *Assume* $\phi \searrow 0$.

- *IM and CM have equivalent REE characterizations when* $\sigma = \sigma^* := \frac{1-\gamma}{\eta(2-\gamma)} \Rightarrow \kappa_2 = 0$. *Furthermore, in the IM economy, we have* $\partial\kappa_2/\partial\sigma < 0$.
- *CM and CD have equivalent REE characterizations when* $\sigma = \hat{\sigma} := \frac{1}{\eta}$. *Furthermore, in the CM economy,* $\partial\kappa^{\mathrm{CM}}/\partial\sigma > 0$.

IM versus CM. We are now ready to show that the equilibrium policy trade-off between domestic inflation and output gap (conditional on given agents' expectations) can be steeper in IM than in CM, when agents are sufficiently risk-averse. First, consider the IM economy. We can equivalently derive the equilibrium relation between output gap and the real exchange rate, using the IS (14) and UIP (15) relations, as

$$\begin{aligned}\tilde{q}_t = &-\mu^{-1}(1-\gamma)\tilde{x}_t + \mu\omega(1-\gamma)\mathrm{E}_t\tilde{x}_{t+1}\\ &+[1+\mu^{-1}(1-\gamma)\chi]\mathrm{E}_t\tilde{q}_{t+1} + \mu^{-1}(1-\gamma)\epsilon_t + u_t. \end{aligned} \qquad \mathbf{(24)}$$

Using (24) in the Phillips relation (13), we can equivalently write the incomplete-markets equilibrium conditional trade-off between domestic inflation and output gap as

$$\begin{aligned}\pi_{\mathrm{H},t} &= \overline{\beta}\mathrm{E}_t\pi_{\mathrm{H},t+1} + \lambda\kappa^{\mathrm{IM}}\tilde{x}_t + \lambda\kappa_2\varpi_t,\\ \varpi_t &:= \mu\omega(1-\gamma)\mathrm{E}_t\tilde{x}_{t+1} + 1 - \mu^{-1}(1-\gamma)\chi\mathrm{E}_t\tilde{q}_{t+1} + \mu^{-1}(1-\gamma)\epsilon_t + u_t,\end{aligned} \qquad \mathbf{(25)}$$

where $\kappa^{\mathrm{IM}} = \kappa_1 - \kappa_2\mu^{-1}(1-\gamma) \equiv \kappa^{\mathrm{CD}} + \frac{\sigma\gamma}{1-\gamma} - \kappa_2\mu^{-1}(1-\gamma)$, and ϖ_t is another representation of the *endogenous cost-push term* that arises under the incomplete-markets equilibrium. In this representation we can also see that the cost-push term ω_t depends not only on underlying shocks, but also on random variables that are conditional expectations of future output and real exchange rate gaps. The first term composing κ^{IM} captures the direct effect of output gap on domestic inflation; the term κ_2 captures the direct link between the real exchange rate and domestic inflation; and the term $\mu^{-1}(1-\gamma)$ is the indirect effect of output gap, via adjustments in the ex ante real interest rate in the IS relation, on the real exchange rate in the UIP.

Now compare IM with CM. There are three cases to consider. From Lemma 1, it is clear that when $\sigma = \sigma^*$, we have equivalent trade-offs in the two types of economies. When $\sigma > \sigma^*$, $\kappa_2 < 0$ and κ_2 is increasingly negative with increasing σ. This implies that $\kappa^{\mathrm{IM}} > \kappa^{\mathrm{CM}}$. Last, when we have $\sigma < \sigma^*$, the term κ_2 becomes positive. However, it is ambiguous how these trade-offs are ordered, for arbitrary parameters. Nevertheless, we can still deduce that for $\sigma \searrow 0$ (i.e., small enough), the term $\mu^{-1} \searrow 0$, so that the trade-offs across all economies converge to the same limit of φ. This delivers us the following result, which summarizes all three cases.

PROPOSITION 3. *Consider identically parameterized economies IM(σ) and CM(σ).*

1. *If $\sigma = \sigma^*$, then the two economies have identical REE trade-offs.*
2. *If $\sigma > \sigma^*$, then IM(σ) has a steeper REE inflation–output-gap trade-off than CM(σ), where*

$$\kappa^{\mathrm{IM}} = \frac{\sigma}{1-\gamma} - \kappa_2 \mu^{-1}(1-\gamma)$$
$$> \varphi + \frac{\sigma}{(1-\gamma)^2 + \sigma\eta\gamma(2-\gamma)} = \kappa^{\mathrm{CM}}.$$

3. *If $\sigma^* > \sigma \searrow 0$, then $\kappa^{\mathrm{IM}} \to \kappa^{\mathrm{CD}} \searrow \kappa^{\mathrm{CM}} \searrow \varphi$.*

CM versus CD. Next, compare CM with CD. Recall that a sufficient condition for CM to exhibit REE equivalent to that for CD is that $\sigma = \hat{\sigma} := 1/\eta$, which implies that $\kappa^{\mathrm{CM}} = \sigma + \varphi = \kappa^{\mathrm{CD}}$.

PROPOSITION 4. *Consider identically parameterized economies CM(σ) and CD(σ).*

1. *If $\sigma = \hat{\sigma}$, then CM(σ) and CD(σ) have equivalent REE.*
2. *If $\sigma > \hat{\sigma}$, then*

$$\kappa^{\mathrm{CM}} = \varphi + \frac{\sigma}{(1-\gamma)^2 + \sigma\eta\gamma(2-\gamma)}$$
$$< \varphi + \sigma = \kappa^{\mathrm{CD}}.$$

3. *If $\sigma < \hat{\sigma}$, then $\kappa^{\mathrm{CM}} > \kappa^{\mathrm{CD}}$.*

Note that for empirically plausible $\eta \in (1, 2)$ and $\gamma \in (0, 1)$, $\sigma^* < \hat{\sigma}$. This implies that the relevant range of σ that one ought to be concerned with is given by $\sigma > \hat{\sigma} > \sigma^* > 0$. Therefore, Propositions 3 (part 2) and 4 (part 2) are the only quantitatively relevant propositions that we will need to focus on later. These observations lead us to the following statement.[13]

PROPOSITION 5. *Assume the quantitatively plausible case where* $\sigma > \hat{\sigma} > \sigma^* > 0$. *Then we have the following ordering of (conditional) policy trade-offs:*

$$\kappa^{\text{IM}} > \kappa^{\text{CD}} > \kappa^{\text{CM}}.$$

Explaining the policy trade-off comparisons. To summarize, if we assume a quantitatively plausible and sufficiently large risk-aversion parameter σ for agents, then in the CM economy, the conditional trade-off between domestic inflation and output gap is relatively flatter than its IM and CD counterparts. In contrast, in our IM economy, the trade-off becomes steeper relative to the same CD counterpart.

To explain these comparative trade-offs summarized in Propositions 3 and 4, we just need to reconsider the channels that make up κ^{IM} in the IM economy, and those that make up κ^{CM} and κ^{CD}, respectively.

For a plausible parameterization of $\sigma > \hat{\sigma} > \sigma^*$, openness of the CM economy to trade, $\gamma \in (0, 1)$, reduces κ^{CM} relative to κ^{CD}. This is because openness under international market completeness allows the small open economy to have access to perfect cross-country insurance of its domestic fluctuations, as shown in the condition (16). This renders the real exchange rate a complete shock absorber for the economy, so that consumption is smoothed across countries, state by state and date by date. Thus innovations to domestic output gap in CM have a weaker impact on domestic inflation than in CD, because domestic agents now can borrow or lend (i.e., switch consumption expenditures) internationally in complete contingent-claims markets. This was originally pointed out by Clarida et al. (2001).

What then happens with the IM economy is that although domestic agents can borrow or lend internationally to attempt to smooth out domestic fluctuations in consumption, they do not have the perfect international risk sharing present in the case of CM. Risk sharing is only in conditional expectations terms. Hence the UIP-type condition (15). This shows up in relation to domestic inflation, in reduced form, as

$$\kappa^{\text{IM}} \equiv \underbrace{\kappa^{\text{CD}} + \frac{\sigma\gamma}{1-\gamma}}_{\kappa_1\text{: domestic marginal cost channel}} + \underbrace{\left[-\kappa_2\mu^{-1}(1-\gamma)\right]}_{\text{incomplete risk sharing channel}} > 0,$$

in (25) where $\mu \approx [\frac{1-\gamma}{\sigma}][1-\gamma+\frac{\eta\gamma(2-\gamma)(\sigma)}{1-\gamma}]$. Observe that the first two terms, $\kappa^{\text{CD}} + \frac{\sigma\gamma}{1-\gamma} \equiv \kappa_1$, are what would have been the trade-off component due purely to output gap via the domestic real-marginal-cost channel. In other words, these terms would capture qualitatively the same explanations for the trade-off as in a purely CD economy, but one that is weakened by trade openness, $\gamma \in (0, 1)$. Therefore, the last term, $-\kappa_2\mu^{-1}(1-\gamma)$, captures the additional channel arising under international asset-market incompleteness. Recall from Section 3 that the term κ_2 encodes additional substitution and wealth effects on labor supply as a result of direct variations in the international real exchange rate, which in turn determine variations in domestic inflation. This term, under a plausible parameterization of $\sigma > \hat{\sigma} > \sigma^*$, is negative, and increasingly negative with σ. The interaction of κ_2 with $\mu^{-1}(1-\gamma)$ summarizes the indirect effect of innovations through incomplete international risk sharing (via the domestic ex ante real interest rate) on domestic inflation. Specifically, note that μ^{-1} is increasing in magnitude with risk aversion σ. In words, the additional impact of incomplete markets on domestic inflation is more severe the more agents dislike large reallocations of consumption across states and dates, because the real exchange rate cannot be a complete-insurance shock absorber, unlike the case of CM.

Also, note that market incompleteness affects the equilibrium relation between output gap and the real interest rate, given by μ in (14). In the CM version this parameter would be μ^{CM}, as defined in (20).

These explanations will help shed light on the implications of market incompleteness for equilibrium determinacy under alternative policy rules later.

Managing expectations and endogenous cost push. Another observation, which we will come back to later when discussing alternative policies, is that in (25), the endogenous cost-push term ϖ_t, can play a vital linkage between stabilizing policies and expectations management. The intuition works as follows. Under incomplete markets, we have an exacerbation of the contemporaneous policy trade-off as stated in Proposition 5. However, if an interest-rate policy can also "correctly" manipulate the conditional expectation terms in ϖ_t, then it can alleviate this trade-off somewhat. We say "correctly" because it is not clear that a policy that directly responds to these expectational variables may be stabilizing. In fact, by doing so, it may create more inflationary expectation spirals. On the contrary, as we will illustrate later, managing these expectations indirectly by conditioning policy of past growth in the variables will turn out to be more desirable, from an equilibrium determinacy perspective.

In contrast, in the CD and CM economies, this endogenous cost-push term is nonexistent. Thus, one would expect that a policy that directly manipulates conditional expectations will not do better in yielding stable REE in these environments.

However, if $\phi \searrow 0$, then $\mu \approx \mu^{\mathrm{CM}}$. Therefore, the effect of market incompleteness in the equilibrium relation between output gap and the real interest rate will be negligible.

5. IMPLICATIONS FOR POLICY RULES AND REE DETERMINACY

In this section, we show how the additional incomplete-asset-markets friction alters the space of alternative policy rules that can feasibly deliver a unique REE. The main conclusion here is that incomplete asset markets have implications for REE stability under various policy rules that are drastically different from the well-known wisdom for the closed economy [e.g., Bullard and Mitra (2002)] and complete-markets small open economy [e.g., Llosa and Tuesta (2008)] literature.

Unfortunately, the various REE determinacy characterizations for the IM economy cannot be derived analytically, unlike its special cases of CM [see Llosa and Tuesta (2008)] and CD [see Bullard and Mitra (2002)]. Nevertheless, we can illustrate our insights from Section 4 numerically.

TABLE 1. Parameterization for IM model

Parameter	Value	Source
Risk aversion, σ	5	LT
Disutility of labor, ψ	1	GM
Inverse Frisch elasticity, φ	0.47	LT
Discount factor elasticity, ϕ	10^{-6}	
Steady state discount factor, $\bar{\beta}$	0.99	GM
Home–Foreign goods elasticity of substitution, η	1.5	LT
Share of Home goods in C, γ	0.4	GM
Elasticity of substitution between good varieties, ε	6	GM
Price stickiness probability, θ	0.75	GM

Notes: GM: Galí and Monacelli (2005); LT: Llosa and Tuesta (2008). In the generalized model with imported inputs there are two additional parameters, which we set according to McCallum and Nelson (1999). These are the labor-imported-input elasticity of substitution ($\upsilon = -2$) and the steady-state-imported-input share of output ($\delta = 0.144$). See our Supplementary Appendix for this generalized model.

Our baseline economy (IM) is parameterized in line with Llosa and Tuesta (2008).[14] Llosa and Tuesta (2008) use the same parameterization as Galí and Monacelli (2005) with the exception of the constant relative-risk-aversion coefficient (σ), the inverse of Frisch labor supply elasticity (φ), and the elasticity of substitution between domestic and foreign goods (η). For a majority of parameters, we follow Llosa and Tuesta (2008) for two reasons: (i) ease of comparison of their findings with ours in terms of the REE stability analyses; and (ii) the setting in Llosa and Tuesta (2008) is a more general parameterization. Furthermore, these parameters do not affect the results qualitatively, although they may have important quantitative effects. This is especially true in the case of σ.[15] We summarize the model parameters in Table 1.

Note that this set of parameters is also used for the limit economies CM and CD. That is, by using the relevant composite parameters, we have (i) the small open economy with complete markets (CM): $\kappa_2 = 0$, $\kappa_1 = \kappa^{CM}$, and $\mu = \mu^{CM}$; and (ii) the closed economy (CD): $\gamma = 0$.

5.1. Numerical Illustration of Trade-Offs

As a preliminary exercise we demonstrate, for the baseline parameterization, the REE policy trade-off comparisons explained earlier in Section 4. From Table 2 we conclude the following. First, the positive trade-off between domestic inflation and output gap, given by $\lambda\kappa_1$, is much larger (around six times larger) with incomplete markets. The intuition for this was shown and discussed in Proposition 5. In short, in the absence of complete international risk sharing, a given external shock to the small open economy cannot be fully insured against by a single incomplete market claim. Hence the effect of the shock gets amplified or transmitted more to domestic allocations via the inflation process. Second, the equivalent version

TABLE 2. Comparing REE characterizations

	IM	M	CD
$\lambda\kappa_1$	0.756	0.124	0.470
$\lambda\kappa_2$	−1.087	0	0
μ	1.032	1.032	0.200

Note: This is for the baseline parameterization, where $\sigma = 5$.

TABLE 3. Alternative monetary policy rules

DITR	$r_t = \phi_\pi \pi_{H,t} + \phi_x \widetilde{x}_t;$	Domestic inflation targeting
FB-DITR	$r_t = \phi_\pi E_t \left\{\pi_{H,t+1}\right\} + \phi_x E_t \{\widetilde{x}_{t+1}\}$	Forecast-based version of DITR
CPITR	$r_t = \phi_\pi \pi_t + \phi_x \widetilde{x}_t$	CPI inflation targeting rule
FB-CPITR	$r_t = \phi_\pi E_t \{\pi_{t+1}\} + \phi_x E_t \{\widetilde{x}_{t+1}\}$	Forecast-based CPITR
MERTR	$r_t = \phi_\pi \pi_{H,t} + \phi_x \widetilde{x}_t + \phi_s \triangle s_t$	Managed exchange rate rule
FB-MERTR	$r_t = \phi_\pi E_t \pi_{H,t+1} + \phi_x E_t \widetilde{x}_{t+1} + \phi_s E_t \triangle s_{t+1}$	Forecast-based MERTR

Notes: The elasticities ϕ_π, ϕ_x, and, ϕ_s are nonnegative policy reaction parameters. π_t is the inflation rate of the CPI index. $\Delta s_t := s_t - s_{t-1}$ denotes the growth rate in the nominal exchange rate S_t. In the paper we focus discussions on the DITR, MERTR, and FB-MERTR families.

of $\lambda\kappa_1$ in CD is between the value in the incomplete-market version and in the complete-market version. Given that ϕ is very close to zero, the response of the output gap to the interest rate, given by μ, is the same in the two versions of open economies. Last, the relation between the output gap and the interest rate, given by μ, is much smaller in the closed economy. The reason for this is as in Galí and Monacelli (2005)—viz., trade openness presents an *indirect* terms-of-trade (or real exchange rate) variation on aggregate demand.

5.2. Policy Rules and REE (In)determinacy

Next, we study the implications of IM for REE stability under alternative monetary-policy rules. Overall, we consider six classes of simple contemporaneous and forecast-based Taylor-type monetary policy rules used in the literature [see, e.g., Bullard and Mitra (2002); Llosa and Tuesta (2008)]. These are summarized in Table 3. For the main discussion hereinafter, we will focus on the simple DITR rule and then also discuss two other examples with the MERTR and FB-MERTR families of policy rules. We consider these examples here because they are sufficient to illustrate the additional implications of our IM economy for stabilization policy. In particular, the endogenous cost-push term in the IM economy's monetary-policy trade-off was shown to depend on expectations of future variables, and we conjectured that policy rules that can manage these expectations may turn out to be more robust in a REE determinacy sense. We relegate discussions of the other alternative policy rules to our Supplementary Appendix.

Where relevant, we will compare within each policy class the REE stability and indeterminacy implications across the three economies: (a) The CM limit; (b) the CD limit; and (c) the IM model.

Given each policy rule, and the competitive equilibrium conditions (13), (14), and (15), the equilibrium system can be reduced to

$$\mathrm{E}_t \mathbf{x}_{t+1} = \mathbf{A}\mathbf{x}_t + \mathbf{C}\mathbf{w}_t, \tag{26}$$

where $\mathbf{x} := (\pi_{\mathrm{H}}, \tilde{x}, \tilde{q})$ and $\mathbf{w} := (\varepsilon, u)$, and where $\mathbf{A} := \mathbf{A}(\vec{\theta}, \vec{\phi})$ and $\mathbf{C} := \mathbf{C}(\vec{\theta}, \vec{\phi})$ depend on the parameters in (13), (14), and (15), $\vec{\theta} := (\overline{\beta}, \gamma, \lambda, \kappa_1, \kappa_2, \omega, \mu, \chi)$, and also the policy parameters $\vec{\phi} := (\phi_\pi, \phi_x, \phi_s)$.

Local stability of a REE depends on the eigenvalues of matrix $\mathbf{A}$. Following the terminology of Blanchard and Kahn (1980), we can see that there are three nonpredetermined variables. Therefore, the equilibrium under DITR will be determinate if the three eigenvalues of $\mathbf{A}$ are outside the unit circle, whereas it will be indeterminate when at least one of the three eigenvalues of $\mathbf{A}$ is inside the unit circle. Unfortunately, we are not able to obtain analytical characterizations of the stability conditions for each class of policy rules. We numerically check for determinate REE (and similarly check for multiplicity of REE) as functions of the policy parameters. In particular, we consider $\phi_\pi \in [0, 4]$ and $\phi_x \in [0, 4]$, as in Llosa and Tuesta (2008), and vary ϕ_s where relevant.

We will state the overall conclusions for our baseline model parameterization. First, market incompleteness results in a conclusion opposite to the finding in Llosa and Tuesta (2008). Llosa and Tuesta (2008) showed that the set of admissible DITR (that respond to contemporaneous variables) inducing unique REE, in a small open economy with complete markets, is larger than that in its closed-economy limit. In general, we find that market incompleteness makes the admissible policy sets smaller than when we have the CM limit. In the specific case of the DITR, international asset market incompleteness also reduces the admissible policy space relative to when we have the CD limit. Second, if the policy rules are of the forecast-based families (FB-DITR, FB-CPITR, and FB-MERTR), then market incompleteness in our model also shrinks the sets of these policies that can induce unique REE, relative to their counterparts in the special case of the complete-markets small open economy model. Third, if monetary policy can be described by simple policy rules, then a contemporaneous rule (MERTR) that responds not only to domestic inflation and output gap, but also to the real exchange rate growth, can greatly expand the feasible set of such policies in inducing determinate rational expectations equilibrium. This result is also well known in the context of small open economies with complete markets [see, e.g., Llosa and Tuesta (2008)].

DITR. Figure 1 reports the simulation results for DITR across the three economies, under the baseline parameterization. Each shaded region refers to the set of DITR policy rules, indexed by (ϕ_π, ϕ_x), that would have induced a determinate (i.e., stable) REE in each of the economies CM, CD, and IM. The

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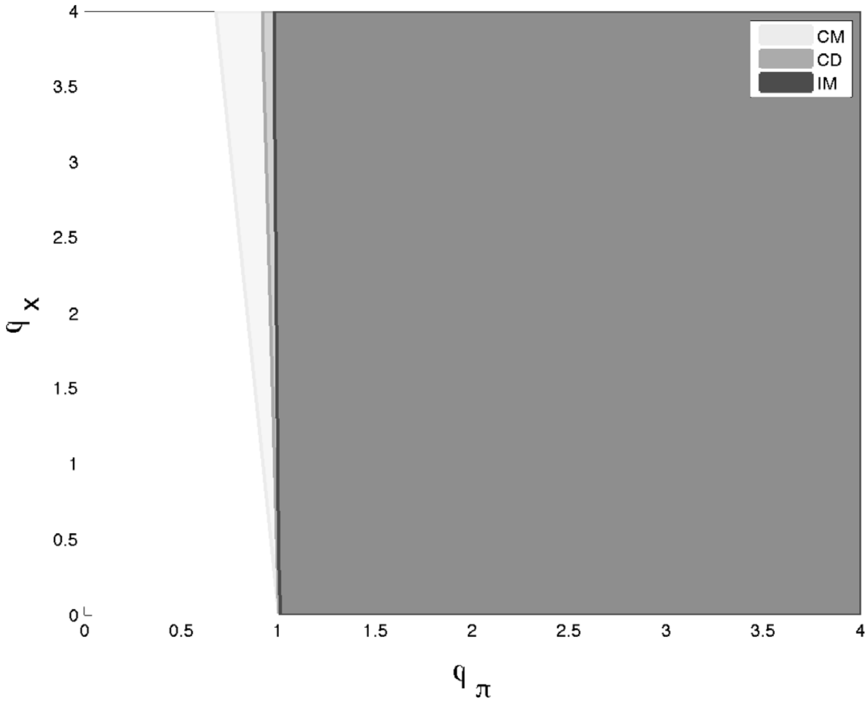

FIGURE 1. Domestic inflation targeting rule (DITR) and indeterminacy regions (shaded) for three economies. Each complementary region refers to stable REE cases.

complement set of each shaded region represents the region with multiple or indeterminate REE.

Consider our baseline IM economy under the DITR family of policy rules. We observe that the highest value of ϕ_π for which REE indeterminacy arises is 1, which corresponds to $\phi_x = 0$. The highest value of ϕ_x for which we find indeterminacy is 4, which corresponds to $\phi_\pi = 0.97$. In fact, the points $(\phi_\pi, \phi_x) = (0.97, 4)$ and $(\phi_\pi, \phi_x) = (1, 0)$ determine the length of the locus in Figure 1 that separates the region of DITR policies that induce REE indeterminacy (i.e., to its left) and the region of DITR policies that induce REE stability (i.e., to its right).

From this figure, we can see that the monetary authority is not constrained if the policy reaction to inflation ϕ_π is greater than unity (i.e., the "Taylor principle"). However, provided that $\phi_\pi < 1$, the smaller this policy parameter is, the greater the authority's response to the output gap.

Further, from Figure 1, we can see a qualification of existing results [e.g., Llosa and Tuesta (2008)] that openness to trade reduces the indeterminacy of REE under the DITR family of policy rules. Now, openness to trade under complete markets (CM) reduces the set of DITR policies that induces REE indeterminacy, compared to the CD economy. However, incomplete asset markets (IM) expand

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the set of indeterminate REE from that of the CD economy. This observation is new to the literature. In other words, whereas trade openness reduces the constraints for DITR policy makers if markets are complete, this openness increases the constraints if markets are incomplete. However, note that the result requires that the parameterization of the CRRA parameter σ be "sufficiently large.'

The intuition for this is not surprising once we recall our observations in Section 4.2, and in particular, Proposition 5. As discussed in Section 4.2, and also numerically verified in Table 2, market incompleteness does two things: (i) it exacerbates the slope of the inflation–output-gap trade-off and (ii) it amplifies the shifts to this trade-off due to the endogenous cost-push–real-exchange-rate channel. The additional sensitivity of inflation to output gap and the real exchange rate in the Phillips curve means that a DITR policy maker in the IM economy will have to counter movements in inflation much more than its counterparts in the CM or CD economies, in order to deliver a determinate REE. Finally, under CM, the trade-off is flatter than under CD. Thus the same observation as in Llosa and Tuesta (2008) applies here: That for a given response to domestic inflation, a CM–DITR policy intending to deliver a determinate or stable REE needs to respond less heavily to output gap than its counterpart in the CD setting, provided that the expenditure-switching channel is sufficiently strong, i.e., $\sigma\eta > 1$.

For completeness, we also consider numerical examples where σ is "small,' and in particular, when $\sigma = \sigma^*$ (equivalence between IM and CM) or $\sigma = \hat{\sigma}$ (equivalence between CM and CD). The numerical results are reported in Table 4. From Propositions 3 and 4, we can already expect the results for values of the CRRA parameter σ that are "too small." The first row of Table 4 is the baseline case across all three economies—i.e., the tabulation of Figure 1. Next, consider the second row as an example where $\sigma = \hat{\sigma}$ (equivalence between CM and CD). This example shows that the regions of DITR policies that induce REE stability (or indeterminacy) are identical for the CM and CD economies. Moreover, the area of indeterminacy for the IM economy is larger. The intuition for this comes from observing that the policy trade-off in this case for IM, $\kappa_{\hat{\sigma}}^{\mathrm{IM}} = \kappa_{\hat{\sigma}}^{\mathrm{CD}} + \frac{\hat{\sigma}\gamma}{1-\gamma} - \kappa_2\mu^{-1}(1-\gamma) > \kappa_{\hat{\sigma}}^{\mathrm{CD}} \equiv \kappa_{\hat{\sigma}}^{\mathrm{CM}} > 0$, is steeper than the equivalent CM and CD economies. We summarize the main finding in the preceding and its alternative numerical results from Table 4 in the following three statements.

RESULT 1. *Assume the DITR family of monetary policy rules. Comparing IM(σ) and the CM(σ) open economies, we observe that*

- *If $\sigma > \sigma^*$ (i.e., $\kappa_2 < 0$) then the set of DITR policy rules indexed by (ϕ_π, ϕ_x) that would have induced indeterminate (stable) REE is larger (smaller) in the IM(σ) economy.*
- *If $\sigma \leq \sigma^*$ (i.e., $\kappa_2 \leq 0$) then the set of DITR policy rules indexed by (ϕ_π, ϕ_x) that would have induced indeterminate (or stable) REE is the same in IM(σ) and in CM(σ).*

TABLE 4. Regions of DITR policies that induce REE indeterminacy: Alternative cases of σ

	IM		CM		CD	
σ	Corners[†] (ϕ_π, ϕ_x)	Area[‡]	Corners[†] (ϕ_π, ϕ_x)	Area[‡]	Corners[†] (ϕ_π, ϕ_x)	Area[‡]
5 (baseline)	(0.99, 0), (0.97, 4)	3.942	(1, 0), (0.67, 4)	3.336	(1, 0), (0.91, 4)	3.825
$\hat{\sigma} = 0.67$	(0.99, 0), (0.79, 4)	3.580	(1, 0), (0.59, 4)	3.160	(1, 0), (0.59, 4)	3.160
$\sigma^* = 0.25$	(0.99, 0), (0.47, 4)	2.923	(0.99, 0), (0.47, 4)	2.923	(1, 0), (0.35, 4)	2.673
0.15	(0.99, 0), (0.39, 4)	2.754	(0.99, 0), (0.39, 4)	2.754	(1, 0), (0.24, 4)	2.459
0.1	(0.99, 0), (0.32, 4)	2.614	(0.99, 0), (0.32, 4)	2.614	(1, 0), (0.18, 4)	2.322

Notes: "Corners" refer to the two interior vertices of region of policies that yield indeterminate REE. See Figure 1 for an example of the baseline parameterization with $\sigma = 5$. "Area" refers to the area of the polygonal region of policies that yield indeterminate REE. The sample policies in the relevant region are given by the interior "corners" and the origin (0, 0) and (0, 4) in (ϕ_π, ϕ_x)-space.

RESULT 2. *Comparing the CD(σ) and the CM(σ) economies, we observe that*

- *If $\sigma > \hat{\sigma}$, then the set of DITR policy rules indexed by (ϕ_π, ϕ_x) that would have induced indeterminate (or stable) REE is smaller (larger) in the CM(σ) economy.*
- *If $\sigma = \hat{\sigma}$, then the set of DITR policy rules indexed by (ϕ_π, ϕ_x) that would have induced indeterminate (or stable) REE is the same in the CM(σ) economy and in the CD(σ) economy.*
- *If $\sigma < \hat{\sigma}$, then the set of DITR policy rules indexed by (ϕ_π, ϕ_x) that would have induced stable (indeterminate) REE is larger (smaller) in the CD(σ) economy.*

RESULT 3. *Comparing the IM(σ) and the CD(σ) economies, we observe that the set of DITR policy rules that would have induced an indeterminate (stable) REE is always larger (smaller) in the open IM economy.*

These numerical results corroborate our theoretical insights from Propositions 3, 4, and 5. However, recall that some of these possibilities would be moot, from a quantitative point of view, because one typically parameterizes $\sigma \geq 1$, and for plausible calibrations, we will have $\sigma^* < \hat{\sigma} < 1 < \sigma$.

MERTR. Consider the managed exchange rate Taylor rule (MERTR). Figure 2 reports the simulation results for MERTR across the three economies, under the baseline parameterization. Using the definition of the nominal exchange rate, $\Delta s_t = \Delta q_t + \pi_t - \pi_t^*$, where without loss of generality we set $\pi_t^* = 0$, we then obtain the relation that $\Delta s_t = \frac{1}{1-\gamma}\Delta q_t + \pi_{\mathrm{H},t}$. Using this, we have an equivalent representation of the MERTR as

$$r_t = (\phi_\pi + \phi_s)\pi_{\mathrm{H},t} + \phi_x \widetilde{x}_t + \left(\frac{\gamma\phi_\pi + \phi_s}{1-\gamma}\right)(\widetilde{q}_t - \widetilde{q}_{t-1}). \quad \textbf{(27)}$$

By combining this rule with the equilibrium conditions for the IM economy, we can again characterize REE stability numerically.

Similarly, we can derive a representation of the MERTR for the case of the CM economy as

$$r_t = (\phi_\pi + \phi_s)\pi_{\mathrm{H},t} + \left[\phi_x + \frac{\tau(\gamma\phi_\pi + \phi_s)}{1-\gamma}\right]\widetilde{x}_t - \frac{\tau(\gamma\phi_\pi + \phi_s)}{1-\gamma}\widetilde{x}_{t-1}, \quad \textbf{(28)}$$

where $\tau := \frac{\sigma(1-\gamma)}{(1-\gamma)^2 + \sigma\eta\gamma(2-\gamma)}$.

We fix $\phi_s = 0.6$ as in Llosa and Tuesta (2008).[16] Relative to the DITR, the admissible set of the MERTR inducing determinate REE equilibrium is larger. However, relative to the CM economy, asset market incompleteness in the IM economy reduces this set.

An interesting observation about this policy is its equivalence to one that also responds to a quasi-difference in output gap growth in the CM economy. One can interpret this as a policy that places a limit on the speed in the

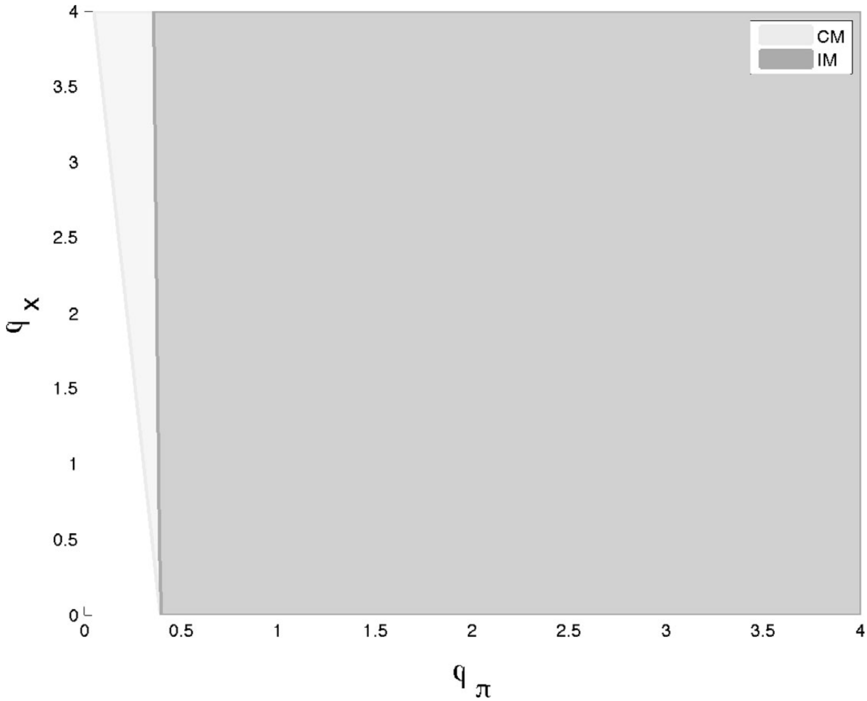

FIGURE 2. Managed-exchange-rate targeting rule (MERTR) and indeterminacy for three economies. Each complementary region refers to stable REE cases.

domestic-inflation–output-gap trade-off. It does so by responding to a measure of output gap growth and growth in the domestic-goods price level, and it thus is able to prevent a self-fulfilling prophecy spiral in unstable inflation. This is achieved to a lesser degree by (27) in the IM economy, because it still faces a larger policy trade-off. However, in contrast to the DITR policy, the MERTR family of policies manage real exchange rate growth directly. By doing so, the policies can better regulate expectations of output gap and real exchange rates. The latter expectational variables feature in the composition of the endogenous cost-push shock term ϖ_t in the policy trade-off (25). Thus, preventing a self-prophesying spiral in these variables through the endogenous cost-push term is crucial. This point is made stronger if we contrast with the next class of policy rules that attempt to manipulate expectations directly.

FB-MERTR. Figure 3 reports the simulation results for FB-MERTR across the three economies, under the baseline parameterization. It can be shown that the FB-MERTR rule has the equivalent form

$$r_t = (\phi_\pi + \phi_s)\mathrm{E}_t\pi_{\mathrm{H},t+1} + \phi_x \mathrm{E}_t\widetilde{x}_{t+1} + \left(\frac{\gamma\phi_\pi + \phi_s}{1-\gamma}\right)(\mathrm{E}_t\widetilde{q}_{t+1} - \widetilde{q}_t) \qquad \mathbf{(29)}$$

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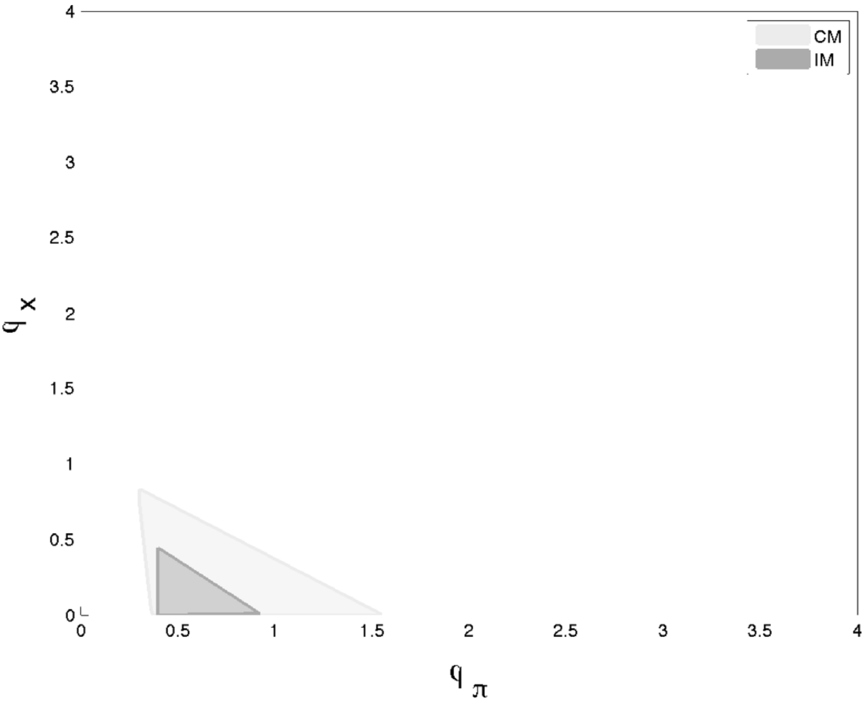

FIGURE 3. Forecast-based managed-exchange-rate targeting rule (FB-MERTR) and indeterminacy for three economies. Each complementary region refers to stable REE cases.

in the IM economy and

$$r_t = (\phi_\pi + \phi_s)\mathrm{E}_t\pi_{\mathrm{H},t+1} + \left[\phi_x + \tau\left(\frac{\gamma\phi_\pi + \phi_s}{1-\gamma}\right)\right]\mathrm{E}_t\widetilde{x}_{t+1} - \tau\left(\frac{\gamma\phi_\pi + \phi_s}{1-\gamma}\right)\widetilde{x}_t \tag{30}$$

in the CM economy, where $\tau := \frac{\sigma(1-\gamma)}{(1-\gamma)^2+\sigma\eta\gamma(2-\gamma)}$.

Relative to the CM economy, asset-market incompleteness in the IM economy results in a smaller admissible set of the FB-MERTR inducing determinate REE equilibrium.

This example policy illustrates our intuition in Section 4.2 most starkly. Now, instead of responding to real exchange rate growth, the policy responds to expectations of real exchange rate growth, inter alia. From this, we can see that this family of policies is less capable of delivering stable REE. The intuition is that responding directly to expections of future variables exacerbates further the already larger domestic-inflation–output-gap trade-off in the IM economy (relative to CM) by causing more self-fulfilling spirals in exchange rate and inflation expectations, amplified through the endogenous cost-push term ϖ_t in (25).

Discussion. We have seen in the preceding illustrations, for a quantitatively plausible risk aversion σ, that the existence of incomplete international risk sharing results in a reduction of the sets of admissible rules inducing determinate REE, relative to when the environment is the standard complete-markets small open economy; and where relevant, relative to when the environment is the standard closed economy. However, given international asset-market incompleteness, the admissible set of stabilizing policy rules can be greatly expanded by a family of simple policies that take into account contemporaneous real exchange rate growth as well. In the Supplementary Appendix, we also discuss a similar result (with similar intuitions) for the CPITR rule.

These results make sense, because the additional constraints on policy in the incomplete-markets setting arose through (i) an exacerbated conditional trade-off between domestic inflation and output gap and (ii) the endogenous cost-push channel. As hinted in Section 4.2, the latter gave us another means of preventing self-fulfilling multiple equilibria, or worse, equilibrium instability. This other means is effected through monetary policies that can "correctly" manage expectations that affect the endogenous cost-push term. By smoothing out output gaps and real exchange rates, it turns out that the CPITR and, better yet, the MERTR policies are better at inducing a determinate REE. This is perhaps one reason that practicing small-open-economy inflation targeters do worry about exchange-rate management in monetary-policy designs.

6. CONCLUDING REMARKS

In this paper, we have developed a small open economy whose monetary policy implications are no longer similar to those of its closed-economy counterpart. We showed, in a transparent manner, that asset market incompleteness essentially exposes the supply side of the model's equilibrium characterization to a notion of an endogenous cost-push shock. Our notion of an endogenous cost-push trade-off here is different from existing models with complete markets [cf. Monacelli (2005)]. In our model, this is a consequence of an irreducible and explicit exchange-rate equilibrium dynamic channel. Moreover, this term involves endogenous random variables that comprise conditional expectations of future output gap and real exchange rate gap. We then showed how this alters the relevant monetary-policy trade-off between stabilizing domestic inflation and stabilizing output gap in an analytical and comparative way. Finally, we revisit the lessons on equilibrium determinacy under alternative rules in a small open economy. We show that asset market incompleteness now results in conclusions opposite to those in the existing literature utilizing the workhorse CM model.

Although our model is a simple and transparent illustration of the relation between international asset-market incompleteness, equilibrium exchange rate irreducibility, and its implications for monetary-policy trade-off and REE determinacy, it is probably too simple for normative business-cycle and welfare analysis. These questions have been addressed by larger and more quantitative models [see, e.g., de Paoli (2009b); Monacelli (2005)].

NOTES

1. As a corollary, we will also find that with incomplete markets, as in the more general settings with complete markets [see, e.g., Benigno and Benigno (2003); Monacelli (2005); de Paoli (2009b)], there is a break in the monetary policy isomorphism between the small open economy and its closed-economy limit.

2. We also consider a more general version of the model presented here. The general model admits another source through which the exchange rate may explicitly matter: The possibility of an imported input in the small economy's production structure. The model in this paper is a limit of the general model, and thus in the absence of this additional channel, an irreducible-exchange-rate dynamic still remains. In short, this result is purely due to the existence of incomplete international asset markets.

3. The Reserve Bank of New Zealand pioneered inflation targeting, implementing this policy in 1990.

4. An exception is Linnemann and Schabert (2004), who considered an incomplete-markets small open economy with an additional predetermined state variable in the form of a net foreign asset level (i.e., current account). They showed how a simple monetary policy rule that reacts to the backward-looking state variable can help to instill a determinate REE. However, it is not precisely clear how market incompleteness in their model works with respect to monetary-policy trade-offs. In contrast, we present an alternative incomplete-markets model that can be analytically dissected in terms of its mechanism and its implications for monetary-policy trade-offs. Moreover, our approach allows us to also contrast with well-known complete-markets and closed-economy structures in the literature in an analytical and comparable way.

5. See also Lubik (2007), who expand on the results of Schmitt-Grohé and Uribe (2003) in terms of a real-business-cycle model with debt-dependent interest rate on net foreign asset positions. In contrast, Galí and Monacelli (2005) assume the existence of an international market for complete state-contingent claims. In doing so, they avoid the problem of steady-state allocations being dependent on initial conditions. McCallum and Nelson (1999) assume incomplete markets, which would mean the opposite for steady-state consumption; but this issue is not discussed by the authors. In a continuous-time setting, Linnemann and Schabert (2004) also offer an alternative "closure" for this problem, similar to Lubik (2007), but in a sticky-price model. However, such an alternative introduces an additional predetermined state variable, and if applied to our setting, would hinder a clean dissection and comparison of the role of incomplete markets vis-à-vis well-known complete-markets and closed-economy characterizations.

6. Other ways of closing open-economy models are also discussed in Schmitt-Grohé and Uribe (2003). In our framework the most natural alternative could be to assume endogenous transaction costs of taking positions in foreign bonds [see, e.g., Benigno and Thoenissen (2008)]. The model with this alternative assumption would be analytically less tractable, and the equilibrium dynamics requires a specific law of motion for bonds. Our assumption will make clear that what is crucial for the policy trade-off is just the incompleteness of financial markets, and not the random walk property of the asset/consumption dynamics implied by this incompleteness (in the absence of the endogenous discounting assumption).

7. In the online Supplementary Appendix to this paper, we consider a more general production model that admits domestic labor and imported intermediate factors of production, as in McCallum and Nelson (1999). Qualitatively, this will not matter for the implications of incomplete asset markets for our monetary policy trade-off. In fact, the extension generalizes our main points and conclusions in this paper.

8. When we generalize the production side to include imported intermediate inputs, the sign of κ_2 is then ambiguous, and it depends on the degree of openness γ and the share of imported intermediate

inputs $(1 - \alpha)$. For an empirically plausible parameterization, we show that in such a more general model, the overall sign of this slope is still negative.

9. See Clarida et al. (1999, 2001) for a detailed discussion of this ad hoc cost-push term.

10. In our model the real marginal cost is not fully tied to the output gap but also depends on the real exchange rate, as is shown in the Supplementary Appendix. Moreover, as (15) shows, the dynamics of the real exchange rate depends on the exogenous variable u_t, given some endogenous nominal interest rate, r_t, policy outcome. This feature of our model does not rely on price stickiness in an additional imported goods sector as in Monacelli (2005).

11. With complete markets, the Euler condition (within the conditional expectations operator) in (5) will in fact hold for every state of nature, following every history, such that by equating the Home Euler condition to that of the rest of the world, one can derive the condition that $Q_t = (C_t^*/C_t)^{-\sigma}$, and a log-linear transform of this expression is $q_t = \sigma(c_t - c_t^*)$.

12. It is important to keep in mind that we are always comparing like with like—i.e., identical model parameters (for each instance of a common value for σ) across economies.

13. In general, if we introduce the possibility of imported intermediate goods on the production side, $\alpha \in (0, 1)$, then an arbitrary setting of the elasticity of substitution between domestic labor and imported inputs, ν, may switch the ordering $\kappa^{\text{CM}} < \kappa^{\text{closed}} < \kappa^{\text{IM}}$. However, given the plausible parameterization, this order is still preserved. This general setting is dealt with in our Supplementary Appendix.

14. For the generalized version of this model, we parameterize its additional imported production input components according to McCallum and Nelson (1999).

15. The goal in this paper is to understand the qualitative implications of incomplete markets for equilibrium stability using a simple but salient model, and not to quantify or match business cycle regularities. However, we do perform some sensitivity analysis on this parameter when it is required. Results of these alternative experiments are available from the authors.

16. Additional sensitivity results with respect to ϕ_s are available from the authors. We show that the qualitative ordering of the sets of determinate or indeterminate equilibria is not affected by the feasible choice of this parameter.

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